A spare parts stocking policy based on equipment criticality

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Abstract

Demand for spare parts can sometimes be classified into critical and non–critical demand, depending on the criticality of the equipment in which it is installed. To effectively handle this situation in spare parts inventory control, we propose a stocking policy where some of the stock is reserved for critical demand. For Poisson demand and a lot–for–lot stocking policy with positive replenishment leadtime, approximations of the service levels for both classes of demand are derived, and verified by simulation. Finally, a numerical example is presented to illustrate the advantages of using the proposed policy.

Keywords: Spare parts, inventory control, equipment criticality, demand classes.

1 Introduction

Customer segregation, i.e. distinguishing classes of customers and giving them different service, is an ill–researched topic in inventory control. The topic does not appear in several reviews on the area (e.g. Veinott [13], Chikán [2], Lee and Nahmias [5] and Porteus [7]). All papers listed consider one type of customers only. Recently we did a case–study on the inventory control of slow moving spare parts in a large petrochemical plant. Management explicitly asked us to study the possibility of having *equipment criticality determine stock levels.* Some equipment in a plant may be quite critical, while other may be almost redundant. It is surprising therefore that equipment criticality plays only a minor role, in theory and in practice, in setting stock levels. Equipment criticality is defined here as the importance of equipment for sustaining production in a safe and efficient way. In the petrochemical plant one could distinguish between vital, essential and auxiliary equipment. If spare parts would always belong to equipment in one criticality class, then one could give each criticality class its own service level or stockout penalty. This case can be studied with standard models. It is not uncommon, however, that similar parts are installed in equipment of different criticality.

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Equipment criticality is a function of the use of equipment, rather than of equipment itself and a certain pump may in one case be critical and in another auxiliary. If this occurs, one faces the problem of maintaining different service levels for the same type of pump.

For simplicity we assume that it is possible to distinguish between two types of demand, i.e. critical and non–critical demand, where critical demand originates from failures of parts installed in vital equipment, and non–critical demand from parts installed in essential and auxiliary equipment. We analyze a continuous review, lot–for–lot stocking policy with Poisson demand, where some of the stock is reserved for critical demand only. A lot–for–lot policy is optimal for low demand items with high holding and shortage costs relative to the ordering costs (e.g. Lee and Nahmias [5]). Reserving stock for critical demand generally increases the service level for critical demand and decreases the service level for non–critical demand.

The policy has originally been proposed by Veinott [12], and analyzed by Topkis [11], who considered a periodic review inventory system with zero leadtime and n demand classes. He divided every period into a number of subintervals, and at the end of each subinterval, after all demand has been observed, an allocation decision is made in the following way (for the case of only two demand classes): all demand is satisfied until the stock level reached a critical value, after which only high priority demand is satisfied. As a motivation for their model, Veinott [12] remarked that "the item is used in several types of equipment of varying degrees of essentiality" and Topkis mentioned as an example "a spare part which is more critical in some locations than in others". Other contributions incorporating different demand classes are made by Kaplan [4] and Evans [3]. They both assume that at some known date in the future the stock will be replenished. Kaplan [4] suggested to let the critical level depend on the time until the next replenishment. More recently, Atkins and Katircioğlu [1] considered a periodic review system where different service levels were assigned to different demand classes. Although they developed an interesting heuristic allocation and order replenishment policy, they also did not solve the problem where demand has to be satisfied directly. Nahmias and Demmy [6] analyzed an approximate continuous review model with Poisson demand, and they assumed that stock is replenished according to a fixed lot size, reorder point policy. They also incorporated a fixed positive leadtime, but they assumed that there is never more than a single order outstanding. In this case one knows that whenever a replenishment order is triggered, the net stock equals the inventory position.

In this paper we analyze the problem where it is possible that more than one order is outstanding. This provides additional complexities that to our knowledge have not been discussed in literature so far. It will be shown that the way incoming replenishment orders are handled has a significant impact on the performance of the system. The main contribution of this paper is an approximation of the service level for critical demand, which takes into account the policy for handling incoming orders. In Section 2 the model is explained in detail and expressions for the service levels of both critical and non–critical demand are derived. In Section 3 a numerical example is presented which illustrates the benefits of the stocking policy we consider, and finally in the last section the main results are summarized.

2 The model

Consider a spare parts inventory system, where demand can be classified into two classes, i.e. critical and non–critical demand. It is assumed that the penalty costs for not satisfying critical demand from stock on hand are much higher than the corresponding costs for non–critical demand. We also assume that inventory is controlled by a lot–for–lot policy, i.e. every time demand occurs a replenishment order is placed, which arrives after some nonnegative, deterministic leadtime L. Finally, it is assumed that demand follows a Poisson process, where λ_c and λ_n respectively denote the average critical and non–critical demand. Since demand for spare parts often originates from many installed components with low failure rates, the choice for the Poisson distribution seems a reasonable one (e.g. Tijms [10]).
In order to effectively deal with critical demand, the following policy is proposed. Let S_c and S be nonnegative integers. Every time demand occurs, an order is placed to raise the inventory position to the level $S_c + S$. However, when the stock on hand drops to the level S_c, the so–called *critical level*, all remaining stock is reserved for critical demand and thus all non–critical demand will be backordered until the stock level again exceeds the level S_c.
The main operating characteristics of any inventory system are the inventory costs and the service level. In this paper we define the service level as the percentage of demand which is satisfied directly from stock on hand, which is the so–called fill rate. For standard $(S-1, S)$ models with Poisson demand it is easy to determine the fill rate, since it in this case is equal to the probability of a stockout (e.g. Silver and Peterson [9]). A stockout occurs when the total leadtime demand exceeds the order–up–to–level and thus the fill rate equals the probability that the leadtime demand does not exceed the order–up–to–level. However, in our model the determination of the service level is not straightforward. In the next subsection approximations of the service levels for critical and non–critical demand will be derived.

2.1 Service level for non–critical demand

To determine the service level for non–critical demand, the following reasoning is used. At an arbitrary time point t, the inventory position is equal to $S + S_c$. Hence, since all outstanding orders at time t will have arrived at time $t + L$, a stockout for non–critical demand at time $t + L$ will occur if and only if the total demand (i.e. critical and non–critical) during $(t, t + L)$ is greater than or equal to S. Observing that the total leadtime demand is Poisson distributed with parameter $(\lambda_c + \lambda_n)L$, it follows that the service level β_n for non–critical demand is given by

$$\beta_n = \sum_{i=0}^{S-1} \frac{e^{(\lambda_c+\lambda_n)L}[(\lambda_c+\lambda_n)L]^i}{i!} \tag{1}$$

2.2 Service level for critical demand

To determine the service level for critical demand, the following reasoning is used. Let H be a nonnegative random variable representing the time until the first S demands have arrived (Nahmias and Demmy [6] refer to H as the *hitting time* of the critical level). Then, using the same arguments as above, we observe that a stockout for critical demand at time $t+L$ will occur if $H < L$ and the critical demand during $(t+H, t+L)$ is greater than or equal to S_c. Since the inter–occurrence times of the demands are exponentially distributed with parameter $\lambda_c + \lambda_n$, it follows that H is Erlang–S distributed with parameter $\lambda_c + \lambda_n$ (Tijms [10]). Conditioning on H, we obtain an expression for the service level β_c for critical demand.

$$\begin{aligned}\beta_c &= 1 - \int_0^L (\lambda_c+\lambda_n)^S \frac{y^{S-1}}{(S-1)!} e^{-(\lambda_c+\lambda_n)y} \left(1 - \sum_{i=0}^{S_c-1} \frac{e^{\lambda_c(L-y)}[\lambda_c(L-y)]^i}{i!}\right) dy \\ &= \beta_n + \sum_{i=0}^{S_c-1} \int_0^L (\lambda_c+\lambda_n)^S \frac{y^{S-1}}{(S-1)!} e^{-(\lambda_c+\lambda_n)y} \frac{e^{\lambda_c(L-y)}[\lambda_c(L-y)]^i}{i!} dy \end{aligned} \tag{2}$$

where β_n is given by (1).

However, this service level is an approximation of the realized service level, since the latter will depend on the way incoming replenishment orders are handled. Clearly, when upon arrival of a replenishment order there is a backorder for critical demand, it is optimal to use the incoming order for this backorder. Also, when the stock level is at least S_c, it is optimal to first use the incoming order to satisfy any backorders for non–critical demand if necessary, and then to replenish stock. However, when upon arrival of a replenishment order no backorder for critical demand is present, but there is a backorder for non–critical demand *and* the stock on hand is below the level S_c, one has to choose between either increasing the stock on hand for critical demand, or satisfying the backorder for non–critical demand. We will consider three ways of handling incoming replenishment orders.

1) priority is given to *backorders for non–critical demand*

2) priority is given to *stock reserved for critical demand*

3) if the order was triggered by critical demand, then priority is given to *stock reserved for critical demand*, otherwise priority is given to *backorders for non–critical demand*

In the first method an incoming order is first used to satisfy backorders for critical resp. non–critical demand. If no such backorders exist, the stock level is increased. The second method is to first satisfy backorders for critical demand and increase the stock until the level S_c is reached, before satisfying backorders for non–critical demand. These two methods do not require keeping track of the type of demand which triggered the replenishment order. However, in the last method this distinction is made. If an incoming order was triggered by critical demand, then increasing the stock level to S_c has priority over satisfying backorders for non–critical demand. If the order was triggered by non–critical demand the priority is reversed. Although method 3 will give lower service levels for critical demand than method 2, the main advantage of the last method is the fact that it limits the length of a stockout for non–critical demand.

To illustrate the implications of the different methods to handle incoming orders, we consider the following examples. In Figure 1 two realizations of demand during $[t-L, t+L]$ are plotted, where C and N denote the arrival of critical demand and non–critical demand respectively. The two arrows in the interval $[t, t+L]$ denote the arrivals of the replenishment orders placed during $[t-L, t]$.

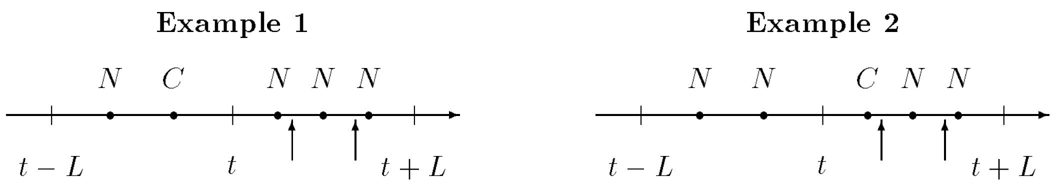

Figure 1: Examples to illustrate implications of method 1, 2 and 3

Assume that both S_c and S are equal to 1, and at time $t-L$ no orders are outstanding. Then a critical order at time $t+L$ can be satisfied if incoming orders are handled according to method 2 or 3, but not with method 1. At time t there is no stock on hand, since N used the normal stock and C used the reserved stock. When method 1 is applied, the two replenishment orders that arrive during $[t, t+L]$ are used to satisfy the outstanding backorders for non–critical demand, and thus at $t+L$ no stock is available. When method 2 or 3 is applied, then one of the arriving replenishment order after t is used to increase the stock level for critical demand, and thus a critical order at time $t+L$ can be satisfied from stock on hand. Similarly as in Example 1, it can be seen in Example 2 that a critical order at time $t+L$ can only be satisfied if method 2 is applied. Because the replenishment orders that arrive during $[t, t+L]$ were caused by non–critical demand, with method 3 they will be used to satisfy the backorders of non–critical demand, and thus at time $t+L$ no stock is available. However, with method 2 the first incoming order after t will be used to increase the reserved stock, and thus a critical demand at $t+L$ can be delivered from stock on hand.

We used simulation to analyze the quality of the approximation for the service levels. The simulated service levels for non–critical demand were equal to β_n. However, the service levels for critical demand obtained by simulation differed from β_c, depending on which method was used to handle incoming replenishment orders. We have simulated 200 different cases, where the lead time was always set equal to 6 months. Moreover, only situations where the service level for critical demand was at least 90% were considered, since in our opinion lower service levels for critical demand are not realistic. The results were obtained based on 10^6 simulated demands. The maximum and average difference between the approximated and simulated service levels for critical demand are presented in Table 1.

	maximum difference	average difference
method 1	+6.6%	+1.5%
method 2	-2.6%	-0.9%
method 3	+2.7%	+0.7%

Table 1: Difference between approximation and simulation of service levels for critical demand

One can see that method 2 gives the best service levels for critical demand, and method 1 provides the worst. The service levels for non–critical demand are not affected by the way incoming orders are handled. However, the length of a stockout for non–critical demand does depend on which method is applied. Method 1 gives the shortest average duration of a stockout, while method 2 gives the longest.

The signs of the differences for method 1 and 2 are intuitively clear, since in method 1 some orders triggered by critical demand will be used for backorders for non–critical demand, and in method 2 some orders triggered by non–critical demand will be used to replenish stock for critical demand. The reason for the positive sign for method 3 is that in most of the cases considered, the arrival rate of non–critical demand was much higher than the arrival rate of critical demand, and thus on average more priority is given to backorders for non–critical demand. In Appendix 1 a table is presented where the results of 20 cases are given.

Since the way the incoming orders are handled proved to have a significant influence on the service level, another approximation is presented in which this effect is taken into account. The idea is to let the possibility of a stockout at time $t+L$ depend on the demand during

$[t-L,t]$, and on the demand during $[t,t+L]$. Assume that the stock on hand at time $t-L$ equals $S+S_c$. Let $N_1 \geq 0$, resp. $N_2 \geq 0$, be random variables representing the total demand during $[t-L,t]$ and $[t,t+L]$ respectively. Let $\mathcal{V}(n_1,n_2)$ be the set of all possible orderings of critical and non–critical demand if $N_1 = n_1$ and $N_2 = n_2$. For example, if C resp. N denotes a critical resp. non–critical demand, $n_1 = 1$ and $n_2 = 2$, the set $\mathcal{V}(n_1,n_2)$ is given by $\{CCC, CCN, CNC, CNN, NCC, NCN, NNC, NNN\}$. Assuming that the demand is equally spread over time, we know for each possible ordering of the demand during $[t-L,t]$ and $[t,t+L]$ if a critical demand at time $t+L$ can be satisfied from stock on hand. For an ordering $j \in \mathcal{V}(n_1,n_2)$ let $p_j(n_1,n_2)$ denote the probability of this ordering, and let $\varphi(j,n_1,n_2)$ be defined as

$$\varphi(j,n_1,n_2) = \begin{cases} 0 & \text{if a critical demand at time } t+L \text{ can be satisfied from stock on hand,} \\ & \text{given } N_1 = n_1,\ N_2 = n_2 \text{ and ordering } j \in \mathcal{V}(n_1,n_2) \\ 1 & \text{otherwise} \end{cases}$$

Thus, a second approximation for the service level for critical demand is given by

$$\boxed{\beta_c^2 = 1 - \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{j\in\mathcal{V}(n_1,n_2)} P[N_1=n_1]P[N_2=n_2]p_j(n_1,n_2)\varphi(j,n_1,n_2) \qquad (3)}$$

Since both N_1 and N_2 are Poisson distributed with parameter $(\lambda_c+\lambda_n)L$ it is easy to calculate $P[N_1 = n_1]$ resp. $P[N_2 = n_2]$. Observing that the probability of an ordering, where the critical demand equals r and the total demand equals n $(r \leq n)$, is given by $\left(\frac{\lambda_c}{\lambda_c+\lambda_n}\right)^r \left(\frac{\lambda_n}{\lambda_c+\lambda_n}\right)^{n-r}$, it is not difficult to determine $p_j(n_1,n_2)$. Finally, we remark that by assuming that demand is equally spread over time, it follows when $n_1 = n_2$ that during $[t,t+L]$ the replenishment orders arrive at the same time as the demand. However, one can verify that the value of $\phi(j,n_1,n_2)$ is not affected by assuming that replenishment orders arrive just before demand occurs.

Consider again Example 1 and 2, presented in Figure 1. In both cases $N_1 = 2$ and $N_2 = 3$. The arrival times of the orders are assumed to be equally spread over $[t-L,t]$ and $[t,t+L]$. For the orderings in Example 1 and 2 it follows that $p_j(2,3) = \left(\frac{\lambda_c}{\lambda_c+\lambda_n}\right)\left(\frac{\lambda_n}{\lambda_c+\lambda_n}\right)^4$. In Example 1 we obtain that $\phi(j,2,3) = 1$ for method 1 only, and for Example 2 it follows that $\phi(j,2,3) = 1$ for method 1 and 3.

We again used simulation to analyze the quality of this second approximation of the service level for critical demand, and it turned out to be a great improvement over the first approximation. In Table 2 the average and maximum difference between the second approximation and the simulation results are presented. Note that we only considered method 2 and method

3 for handling incoming orders, since the previous results showed that method 1 gave the worst performance with respect to the service levels. In Appendix 1 the results for 20 selected cases are presented completely.

	maximum difference	average difference
method 2	-0.6%	-0.3%
method 3	+0.6%	+0.3%

Table 2: Difference between second approximation and simulation of service levels for critical demand

One can see that the second approximation yields better results than the first one. However, a main disadvantage concerns the running time, which increases exponentially with the average demand. Nevertheless, in cases where the critical demand is only a small fraction of the total demand, the running time is not too large.

In the next section an example is presented which shows that using the model we proposed can lead to a decrease in the average costs.

3 Example

Before introducing the numerical example, we explain how the total expected costs were determined. The total costs are composed of holding costs, shortage costs and ordering costs, which are calculated by

$$
\begin{aligned}
\text{holding costs} &= h(S - \lambda_n L) + h(S_c - \lambda_c L) \\
\text{penalty costs} &= p_n \lambda_n (1 - \beta_n) + p_c \lambda_c (1 - \beta_c) \\
\text{ordering costs} &= K(\lambda_c + \lambda_n)
\end{aligned}
$$

where

h = holding costs per unit per year

p_n = penalty cost per stockout for non–critical demand

p_c = penalty cost per stockout for critical demand

K = fixed cost for placing a replenishment order

Note that the expression for the average holding costs is an approximation, which is only justified if the service levels are relatively high, say $\geq 90\%$ (Tijms [10], Silver and Peterson [9]). Observe that when calculating the penalty costs, we implicitely assume that these are independent of the length of a stockout.

The example we present is based on a real life situation (see De Rooij [8]) at the previously mentioned petrochemical plant. The most important characteristics of the part we consider, a bearing, are given in Table 3.

Price	563
Demand	5 units every 3 year
Leadtime	6.5 months
# installed in critical equipment	4
# installed in non–critical equipment	26

Table 3: Characteristics of the bearing

It is assumed that the probability of a failure of the part is independent of the equipment it is installed in. Hence, the average demand per year for critical (λ_c), resp. non–critical (λ_n), demand equals 0.222, resp. 1.444. The leadtime L is equal to 0.541 years. The company calculates the annual unit holding cost by taking 25% of the price, and thus $h = 140.75$ in this case. The ordering costs are equal to $K = 42$. The Economic Order Quantity is equal to $\sqrt{2K(\lambda_c + \lambda_n)/h} = 0.99$, so the lot–for–lot policy is a good policy in this case. The penalty cost for non–critical demand was equal to $p_n = 650$. For p_c different values were analyzed. To determine the service level for critical demand the second approximation was used, assuming that incoming orders are handled according to method 3 (see Section 2.2). The optimal values S and S_c were obtained by using enumeration. In the Tables 4 and 5 results are presented for different values of p_c, both for the standard $(S-1, S)$ model and the model we proposed. Observe that the standard $(S-1, S)$ model is equivalent to the new model with the restriction that $S_c = 0$.

p_c	model	(S_c, S)	av. penalty costs (cr.)	av. penalty costs (non–cr.)	total
7500	$(S-1, S)$	(0,3)	54	30	84
	new	(0,3)	54	30	84
15000	$(S-1, S)$	(0,3)	108	30	138
	new	(1,2)	0	138	138
30000	$(S-1, S)$	(0,3)	216	30	246
	new	(1,2)	0	138	138

Table 4: Average penalty costs for different values of p_c

In all cases the annual expected holding and ordering costs were equal to 295 and 70 respec-

(S_c, S)	service level critical demand	service level non–critical demand
$(0,3)$	96.8%	96.8%
$(1,2)$	100%	85.3 %

Table 5: Service levels for different policies

tively. From Table 4 one can see that for p_c high enough (i.e. $p_c \geq 15000$) the new model yields lower costs. This is mainly due to the almost 100% service level for critical demand obtained by taking $S_c = 1$, which more than compensates the extra expected penalty costs for non–critical demand, caused by a decrease in the service level from 96.8% to 85.3% (see Table 5).

4 Conclusions and implementation issue

In spare parts inventory systems the situation may occur that identical equipment can be installed in different machines, with different importance for the production process. Hence, some of the demand may have very high penalty costs in case of a stockout. In this paper we discussed a model that enabled us to handle equipment criticality. We analyzed a stocking policy where some of the stock is reserved for spare parts demand originating from failures of critical equipment. The inventory was controlled by a lot–for–lot policy and the leadtime was assumed to deterministic and nonnegative. It was shown in Section 2.2 that the way incoming replenishment orders are handled has a great impact on the service level for critical demand. The service level for non–critical was not affected by the policy for handling incoming orders, although the average length of a stockout for non–critical demand was influenced.

For this model we presented approximations of the service level for critical and non–critical demand, which were tested by using simulation, with satisfactory results. The second approximation for the service level of critical demand enables to take into account the way incoming orders are handled, and is therefore to be preferred over the first approximation. A negative aspect, though, is the computing time, which grows exponentially with the average demand. In Section 3 a numerical example was presented that illustrated the advantages of using the proposed model.

Although in the company, considered in the case study, the spare parts warehouse was fully automated, the parameters controlling the stock of about 25000 items, representing an inventory value of tens of millions, had been entered manually and no automatical optimization procedure was used. This may suggest that our policy is far too complex for practical purposes. However, management was quite enthusiastic about our results, since it proved that reserving stock for critical demand was cost–effective. Because spare parts stock in practice is often an insurance, it is important that spare parts management can guarantee high service levels for critical equipment, because otherwise, as was the case in the company, so–called squirrel stocks may be created, i.e. secret stocks maintained outside the inventory system by the owners of critical equipment. Moreover, the spare parts inventory managament system did allow to reserve part of the stock and hence allowed a critical reserve which could only be issued manually by a stock analyst.

Although there is still a gap between inventory theory and actual spare parts management, we believe that the proposed policy is of practical value. The company did start to introduce the critical stock level manually. However, developing a decision support system was unfortunately deferred because of possible changes in the spare parts inventory management system, which blocked all other new developments.

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Appendix 1: Simulation results

In order to analyze the quality of the approximations (2) and (3) for the service level for critical demand, we simulated the service levels for 200 different cases. We considered situations where the average critical and non–critical demand were equal, and situations where the average non–critical demand was ten times the average critical demand. The leadtime was set equal to 6 months in all cases. In Table 6 20 of the results are listed. In the first columns the average critical (λ_c) and non–critical demand (λ_n) per year are presented. In column 3 and 4 the optimal values of S_c and S are given, which were determined by enumeration. In column 5, 6 and 7 the simulated service levels are listed, for each methods of handling incoming orders. Column 8 presents the value of the approximation given by (2), and the last columns reports the values of the approximation given by (3), for method 2 and 3 of handling incoming orders.

λ_c	λ_n	S_c	S	Sim.1	Sim.2	Sim.3	App.1	App.2, meth.2	App.2, meth.3
0.03	0.3	1	1	99.9%	99.9%	99.9%	99.9%	99.9%	99.9%
0.08	0.8	1	1	99.0%	99.2%	99.0%	99.1%	99.1%	99.1%
0.17	1.7	1	1	95.6%	97.5%	96.2%	96.7%	97.1%	96.4%
0.25	2.5	1	1	90.6%	96.0%	92.5%	93.7%	95.4%	92.9%
0.25	2.5	1	2	96.6%	98.2%	96.8%	97.5%	97.9%	97.2%
0.5	5.0	1	3	90.7%	96.9%	92.2%	94.3%	96.4%	92.7%
0.5	5.0	1	4	95.8%	98.4%	96.2%	97.3%	98.1%	96.7%
0.5	5.0	2	1	91.7%	99.5%	97.6%	98.3%	99.7%	97.6%
1.0	10.0	1	7	91.7%	97.5%	92.6%	95.3%	97.8%	93.0%
0.03	0.03	1	1	99.2%	99.4%	99.4%	99.4%	99.3%	99.3%
0.08	0.08	1	1	95.1%	95.8%	95.2%	95.4%	95.5%	95.5%
0.08	0.08	1	2	99.3%	99.3%	99.3%	99.3%	99.3%	99.3%
0.17	0.17	1	2	94.8%	95.9%	95.0%	95.4%	95.4%	95.3%
0.17	0.17	2	1	96.5%	97.9%	97.0%	97.3%	97.9%	97.4%
0.25	0.25	2	1	90.3%	95.2%	92.6%	93.2%	95.4%	93.2%
0.25	0.25	1	3	95.8%	96.7%	95.9%	96.2%	96.6%	96.5%
0.25	0.25	3	1	97.8%	99.2%	98.5%	98.7%	99.3%	98.8%
0.50	0.50	1	5	94.0%	95.8%	94.2%	94.8%	96.3%	94.8%
1.00	1.00	2	8	94.6%	97.9%	95.1%	96.3%	98.4%	95.4%
1.00	1.00	2	9	97.4%	99.0%	97.5%	98.2%	99.2%	97.9%

Table 6: Examples to illustrate differences between simulation and approximation