Conditional Downside Risk and the CAPM

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THIERRY POST* AND PIM VAN VLIET

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ABSTRACT

The mean-semivariance CAPM strongly outperforms the traditional mean-variance CAPM in terms of its ability to explain the cross-section of US stock returns. If regular beta is replaced by downside beta, the traditional risk-return relationship is restored. The downside betas of low-beta stocks are substantially higher than the regular betas, while high-beta stocks involve less systematic downside risk than suggested by their regular betas. This pattern is especially pronounced during bad states-of-the-world, when the market risk premium is high. In sum, our results provide evidence in favor of market portfolio efficiency, provided we account for conditional downside risk.

A WELL-KNOWN LIMITATION OF THE MEAN-VARIANCE (MV) CAPM is that variance is a questionable measure of investment risk. While investors generally dislike deviations below the mean and like deviations above the mean, this measure treats downside volatility and upside volatility in the same manner. This is a powerful argument for replacing variance with measures of downside risk. Hogan and Warren (1974) and Bawa and Lindenberg (1977) develop the mean-semivariance (MS) CAPM, which replaces variance with semivariance and replaces the regular beta with a downside beta that measures the co-movements with the market portfolio in a falling market.

The MS CAPM preserves all key characteristics of the MV CAPM, including the two-fund separation principle, efficiency of the market portfolio and the linear risk-return relationship. The only difference is the use of the relevant risk measures – variance and regular beta vs. semivariance and downside beta. The importance of this difference depends on the shape of the return distribution. For a normal return distribution, regular beta and downside beta are identical. However, for skewed distributions such as the lognormal, the two models diverge.

Price, Price and Nantell (1982) show that the historical downside betas of US stocks systematically differ from the regular betas. Specifically, the regular beta systematically underestimates the downside beta for low-beta stocks and overestimates the downside beta for high-beta stocks. This (little known) finding may help to explain why low-beta stocks appear systematically underpriced and high-beta stocks appear systematically overpriced in empirical tests of the MV CAPM (see for example Reinganum (1981) and Fama and French (1992)).

Surprisingly, despite the theoretical limitations of variance, the empirical problems of the MV CAPM and the differences between regular betas and downside betas, the MS CAPM thus far has not been subjected to rigorous empirical testing. The few studies devoted to testing the model suffer from problems related to the data and the methodology. Jahankani (1976) focuses on the relatively small sample period 1951-1969 and find no support for the MS model. By contrast, Harlow and Rao (1989) report strong evidence in favor of the general mean-lower partial moment (MLPM) CAPM, which replaces the regular beta with a general LPM beta.[1] However, in the empirical methodology they do not actually estimate the LPM beta and the risk measure that is estimated instead is not economically meaningful.[2] Thus, we may conclude that the MS CAPM thus far has not been subjected to unambiguous testing.

The purpose of this study is to fill the gap by providing an empirical comparison of the MV and MS models. The study has three distinctive features. First, we pay special attention to obtaining economically meaningful results. Specifically, we require the pricing kernels to be economically well-behaved in the sense that they obey the basic regularity conditions of non-satiation and risk aversion. One approach to achieve this is by using nonparametric stochastic dominance tests (see Post (2003)); these tests start from the regularity conditions rather than a parameterized model. Interestingly, using these tests, Post and Vliet (2004) show that downside risk helps to explain the high returns earned by small value winner stocks. This study takes an alternative approach; we fix the model parameters independently of the errors, in the spirit of the time-series methodology of Gibbons, Ross and Shanken (1989). In this respect, our study differs from Ang, Chen and Xing (2004), where the model parameters are fitted rather than fixed using a cross-sectional regression methodology.

Second, we employ data sets that are specially tailored to the analysis of downside risk. When analyzing risk, it is particularly important to include periods during which investment risks are high and investors are sensitive to risk. For this reason, we use an extended sample (1926-2002) that includes the prolonged bear markets of the 1930s, 1970s and early 2000s. Further, we will use benchmark portfolios that are formed on downside beta. After all, if downside beta drives asset prices, then sorting on other stock characteristics may lead to a lack of variation in means and betas and erroneous rejections of the MS CAPM.

Third, we employ unconditional tests as well as conditional tests that account for the economic state-of-the-world. The conditional models are particularly relevant given the mounting evidence in favor of time-varying risk and time-varying risk aversion. Guaranteeing a well-behaved kernel is especially important for conditional tests. Such tests frequently calibrate the model parameters to optimize the statistical fit of the model (e.g. Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). Unfortunately, this approach may yield economically questionable results. Specifically, the results of unrestricted conditional tests frequently violate the basic regularity conditions of nonsatiation (no arbitrage) and risk aversion. For example, Dittmar (2002), Section IIID) shows that the apparent explanatory power of skewness and kurtosis in addition to variance can be attributed almost entirely to violations of risk aversion.

In this paper, we find a strong indication that conditional downside risk drives asset prices. If regular beta is replaced by downside beta, the traditional risk-return relationship is restored. The conditional and unconditional MS CAPM outperform the traditional MV CAPM for the beta decile portfolios. The low (high) beta stocks involve more (less) systematic downside risk than expected based on their regular betas. This pattern is especially pronounced during bad states-of-the-world, when the market risk premium is high.

Figure 1 illustrates our main findings with the empirical risk-return relationship of ten beta decile portfolios. Later on in this paper we will discuss the empirical and methodological details. Panel A shows the weak relationship between regular betas and mean returns; a low annualized mean spread (3.5%) is combined with a high beta spread (1.1). Consistent with earlier empirical studies on the MV CAPM ((e.g. Black, Jensen and Scholes (1972), Fama and MacBeth (1973), Reinganum (1981) and Fama and French (1992)). If the MV CAPM holds, then low (high) beta stocks are seriously over (under) priced. As shown in Panel B, the results greatly improve if the regular beta is replaced by the downside beta of the MS CAPM. The annualized mean spread increases from 3.5 percent to 5.5 percent and the beta spread decreases from 1.1 to 0.9. Finally, Panel C shows a near perfect fit between means and downside betas during bad states-of-the world. The mean spread is consistent with the beta spread and the equity premium during bad states. The relation between conditional downside and average returns will be formally tested in this paper.

[Insert Figure 1 about here]

The remainder of this study is structured as follows. Section I first formulates the competing capital market models in terms of pricing kernels and explains how we will select the unknown parameters of the pricing kernels and determine the empirical support for the competing models. Section II discusses the data used to test the competing models. Next, Section III gives our results. Subsequently, Section IV provides a discussion of the results and finally Section V concludes.

I. Competing Asset Pricing Models

A. Kernels

MV CAPM and MS CAPM are relatively simple single-period, portfolio-oriented, representative-investor models of a perfect capital market. Both models predict that the value-weighted market portfolio of risky assets (*M*) is efficient and that the expected return of individual assets is determined solely by their contribution to the risk of the market portfolio. In our analysis, it is useful to formulate both capital market models in terms of a pricing kernel.

The investment universe consists of N risky assets with excess returns $\boldsymbol{r} \in \Re^N$ and a riskless asset with a zero excess return.[3] The return on the market portfolio is given by $r_M \equiv \boldsymbol{r}^{\mathrm{T}} t$, where $t \in \Re^N$ denotes the weights of the market portfolio or the relative market capitalization of the risky assets. Capital market equilibrium can be characterized using a pricing kernel $m(r_M)$ that assigns weights to the return of the market portfolio. Specifically, in equilibrium, the following equality must hold:

$$E[m(r_M)\boldsymbol{r}] = \boldsymbol{0}_N \qquad \textbf{(1)}$$

In words, the average risk-adjusted excess return of all assets must equal zero. For different specifications of the pricing kernel, this equality is our null hypothesis throughout this study.

The pricing kernel can be seen as the marginal utility function of the representative investor and equality (1) as the first-order condition for portfolio optimization for the representative investor. In this study, we use this preference-based perspective. The shape of the pricing kernel and the restrictions placed on its parameters are governed by the properties of a well-behaved utility function.

It is useful to reformulate (1) as the following a risk-return trade-off:

$$m = m_M b \qquad \textbf{(1')}$$

where the mean returns $m \equiv E[\boldsymbol{r}]$ equal the market risk premium $m_M = m^{\mathrm{T}} t$ times the market betas $b \equiv (E[m(r_M)\boldsymbol{r}] - E[m(r_M)]E[\boldsymbol{r}])(E[m(r_M)r_M] - E[m(r_M)]m_M)^{-1}$. The betas are generalizations of the traditional market betas. Specifically, the betas measure the covariance of the assets with the pricing kernel, standardized with the covariance of the market portfolio with the pricing kernel. For the UMV model, the generalized beta reduces to the traditional beta.

Different capital market models impose different assumptions about the pricing kernel. In our analysis, we will analyze the following four models:

Model	**Kernel** ($m(r_M)$)
Unconditional mean-variance (UMV)	$b_0 + b_1 r_M$
Unconditional mean-semivariance (UMS)	$b_0 + b_1 \min(r_M, 0)$
Conditional mean-variance (CMV)	$(b_0 + b_1 z) + (b_2 + b_3 z) r_M$
Conditional mean-semivariance (CMS)	$(b_0 + b_1 z) + (b_2 + b_3 z)\min(r_M, 0)$

In the unconditional mean-variance (UMV) CAPM, the kernel is a linear function of market return. The unconditional mean-semivariance model (UMS) deviates from the MV models by using a kernel that is a linear function of market return in case of losses ($r_M < 0$) only; for gains, the kernel is not affected by the market return. In the conditional versions of these models (CMV and CMS), the two parameters are a linear function of a single conditioning variable z. Then, the shape of the kernel also depends on the state of-the-world. The CMS model assumes that investors fear negative stock returns during bad states-of-the-world most. In the empirical analysis, we will condition on the dividend yield (D/P) and show that similar results are obtained for other conditioning variables, such as the credit spread and the earnings yield (E/P).

In practice, we cannot directly check the equilibrium condition (1), because the return distribution of the assets is unknown. However, we can estimate the return distribution using time-series return observations and employ statistical tests to determine if the equilibrium condition is violated to a significant degree. Throughout the text, we will represent the observations by the matrix $\mathbf{R} \equiv (\boldsymbol{r}_1 \cdots \boldsymbol{r}_T)$, with $\boldsymbol{r}_t \equiv (r_{1t} \cdots r_{Nt})^{\mathrm{T}}$. The values of the kernel will be denoted by the vector $\boldsymbol{m} \equiv (m(r_{M1}) \cdots m(r_{MT}))^{\mathrm{T}}$. Finally, we will use $\boldsymbol{r}_M \equiv \mathbf{R}^{\mathrm{T}} t$,

The empirical deviations from the equilibrium equation (1), also known as pricing errors or alphas, are defined as

$$\hat{a} \equiv T^{-1}\mathbf{R}^{\mathrm{T}}\boldsymbol{m} \qquad \textbf{(2)}$$

The alphas can equivalently be formulated as

$$\hat{a} = \hat{m} - \hat{m}_M \hat{b} \qquad \textbf{(2')}$$

with $\hat{m} \equiv T^{-1}\mathbf{R}^{\mathrm{T}}\boldsymbol{e}$, $\hat{m}_M = \hat{m}^{\mathrm{T}} t$ and $\hat{b} \equiv (\mathbf{R}^{\mathrm{T}}\boldsymbol{m} - \hat{m}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{m}))(\boldsymbol{r}_M^{\mathrm{T}}\boldsymbol{m} - \hat{m}_M(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{m}))^{-1}$ for the sample means and sample betas respectively.

In practice, the empirical researcher faces two issues: the selection of the parameters of the kernel and the statistical inference about the equilibrium condition based on the alphas relative to the kernel.

B. Selecting the model parameters

Some empirical asset pricing studies select the parameters of the pricing kernel so as to minimize the alphas. Unfortunately, this approach can yield economically questionable parameter values. Most notably, the parameter values may imply arbitrage possibilities (a negative kernel) and/or risk seeking (a kernel that increases with market return).

Arbitrage opportunities are inconsistent with the basic economic concept of increasing utility of wealth, or nonsatiation. Risk seeking entails two economic problems. First, risk seeking is inconsistent with the basic economic concept of diminishing marginal utility of wealth. Second, the interpretation of the test results in terms of utility maximizing investors breaks down if we allow for risk seeking. Recall that the equilibrium condition (1) can be seen as the first-order condition for portfolio optimization. The first-order condition in general is not a necessary condition for a global maximum, because minima and local maxima may arise in case of risk seeking. For these reasons, a good statistical fit may come at the cost of a poor economic fit. Section IIIA will give some striking examples of this problem; when selected to minimize the alphas, the CMV and CMS kernels take negative values and are increasing for favorable states-of-the-world.

The UMV efficiency test of Gibbons, Ross and Shanken (GRS, 1989) circumvents this problem by fixing the kernel independently of the alphas. Specifically, this test requires a zero alpha for the market portfolio:

$$T^{-1}\boldsymbol{r}_M^{\mathrm{T}}\boldsymbol{m} = 0 \tag{3}$$

Also, the test standardizes the kernel by setting its sample average equal to unity:

$$T^{-1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{m} = 1 \tag{4}$$

Combined, the two restrictions (3) and (4) completely fix the two parameters of the UMV kernel. The resulting kernel typically is well-behaved, that is, it obeys nonsatiation and risk aversion provided the historical market risk premium takes a moderate and positive value. We will use these two restrictions for all models evaluated in this study. This means that our UMV alphas are identical to the GRS alphas. As for the UMV kernel, imposing (3) and (4) completely fixes the UMS kernel. Provided the historical market risk premium is positive, the resulting kernel will be well behaved. For the CMV and CMS models, the restrictions (3) and (4) do not suffice to guarantee a well-behaved pricing kernel for every value of the conditioning variable *z*, and further restrictions are required. For this purpose, we introduce a "utopia state", characterized by an extremely favorable value for the conditioning variable, say z^*. For example, our analysis below will condition on the dividend yield (D/P) and will use a zero dividend yield for the utopia state. We assume that the representative investor is satiated (the kernel equals zero) and risk neutral (the kernel is flat) in the "utopia state". This boils down to imposing the following two restrictions:

$$(b_0 + b_1 z^*) = 0 \tag{5}$$

$$(b_2 + b_3 z^*) = 0 \tag{6}$$

The four parameters of the conditional models are completely fixed by the four equalities (3)-(6). By imposing satiation and risk neutrality for the utopia state, we effectively avoid the possibility of a negative and/or increasing kernel for favorable states-of-the-world.

We stress that our approach of fixing the kernel necessarily leads to a worse statistical fit than optimizing the kernel. However, our approach ensures that the kernel is economically well behaved, in the sense that arbitrage possibilities and risk seeking are excluded. An additional advantage of this approach is that a single kernel can be used for different benchmark sets. Thus, we do not explain different benchmark sets with different kernels.

C. Statistical inference

We now turn to the issue of statistical inference about the equilibrium condition (1) based on the estimated alphas. Under the null, the alphas have means $E[\hat{a}] = \mathbf{0}_N$. The covariance matrix $\text{W} \equiv E[\hat{a}\hat{a}^{\text{T}}]$ of the alphas can be estimated in a consistent manner by

$$\hat{\text{W}} \equiv T^{-1}(\boldsymbol{m} \otimes \mathbf{R})^{\text{T}}(\boldsymbol{m} \otimes \mathbf{R}) \qquad \textbf{(7)}$$

In the spirit of the Generalized Method of Moments, we can use the following test statistic to aggregate the individual alphas:

$$JT \equiv T\hat{a}^{\text{T}}\hat{\text{W}}^{-1}\hat{a} \qquad \textbf{(8)}$$

Assuming that the observations are serially independently and identically distributed (IID) random draws, the test statistic obeys an asymptotic chi-squared distribution with N-1 degrees of freedom. The "loss" of one degree of freedom occurs due to the restriction that the alpha of the market portfolio should equal zero (3). Thus, in case of a single risky asset (N=1), the market portfolio is fully efficient and $JT = 0$ by construction. More generally, for N assets, the test statistic behaves as the sum of squares of N-1 contemporaneously IID random variables.

II. Data

A. Data sources and stock data requirements

In the empirical analysis we use individual stock returns, index returns, hedge portfolio returns and conditional variables. The monthly stock returns (including dividends and capital gains) are from the Center for Research in Security Prices (CRSP) at the University of Chicago. The one-month US Treasury bill is obtained from Ibbotson. The monthly hedge portfolio returns (SMB and HML) are taken from the data library of Kenneth French. The dividend and earnings yield are obtained from Robert Schiller's homepage. The credit spread is the difference between the Aaa and Baa corporate bond yields and are from the St. Louis Fed. The CRPS total return index is a value-weighted average of all US stocks included in this study. We subtract the risk-free rate from nominal returns to obtain excess returns.

We select ordinary common US stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ markets. We exclude ADRs, REITs, closed-end-funds, units of beneficial interest, and foreign stocks. Hence, we only include stocks that have a CRSP share type code of 10 or 11. At formation date a stock needs to have (1) at least 60 months of data available and (2) market capitalization information (defined as price times the number of outstanding shares). Portfolio formation takes place at December of each year (except for momentum). For ex ample, to be included at December 1930 a stock must have trading information since January 1926 and a positive market capitalization for December 1930. A stock is excluded from the analysis if there is no more price information available. In case of exclusion, the delisting return or partial monthly return provided by CRSP is used for the last observation.

B. Sample period

When analyzing risk, it is particularly important to include periods during which investment risks are high and investors are sensitive to risk. In this respect, the failure of the SV model to improve upon the MV model in the analysis of Jahankhani (1976) is presumably caused by the focus on a sample period (1951-1969) that excludes the important bear markets of the 1930s, 1970s and 2000s. Nowadays, empirical researchers often confine themselves to the post-1963 period to avoid biases associated with the Compustat database. Still, the CRSP database is free of delisting bias and survivorship bias for the total 1926-2002 period, since 1999. Therefore, when CRSP data is used only (without Compustat requirements) there is no reason to exclude the pre-1963 period. Contrary, because this early period includes the Great Depression and the (recovery of) Second World War excluding this period would lead to a loss of useful information. To address these issues, our study will use data from a bng sample period (1926-2002). Furthermore, we analyze the role of downside risk in different subsamples.

C. Benchmark portfolios

At the end of December of each year, starting in 1930, all stocks that fulfill our data requirements are sorted into decile portfolios. [4] We sort stocks into portfolios based on historical 60-month (1) regular beta and (2) downside beta. For each portfolio we calculate the value-weighted returns for the following next 12 months, thereby closely resembling a buy-and-hold strategy.[5] When a stock is delisted or removed from the database after formation date, the portfolio return is calculated as the average for the remaining stocks in the portfolio during the holding period.

We will start with an analysis of benchmark portfolios that are formed on regular market beta, because the MV CAPM predicts that the regular beta is the relevant measure. In fact, if regular beta drives asset prices, then sorting on other stock characteristic may lead to a lack of variation in means and erroneous rejections of the MV CAPM. Similarly, if the MS CAPM holds, then sorting stocks on downside beta maximizes the mean spread and minimizes the probability of erroneous rejections of the MS CAPM. To disentangle the effect between regular and downside beta on stock returns we also apply a double-sorting routine. Sorting takes place on regular beta first (quintiles) and subsequently on downside beta (quintiles), and visa versa. Later on, in the discussion, we will also control for size. In this case, we first place stocks in NYSE size decile portfolios and then sort on the two risk measures. Thus, we obtain 100 size/regular-beta and 100 size/downside-beta portfolios. Finally, we will also test the MS CAPM relative to ten momentum decile portfolios. Momentum is defined as the past 12-2 month return performance and rebalancing takes place at the end of each month. The sample period, data requirements, and portfolio return calculation, for all sorts are identical to the beta portfolios.

III. Results

A. Pricing Kernels

Figure 2 shows the conditional and unconditional pricing kernels for the mean-variance and mean-semivariance models.

The shape of the unconditional kernels is determined by the historical market risk premium. Since the UMS model explains the market risk premium only with the distribution of losses, the degree of risk aversion (for losses) in this model exceeds the degree of risk aversion in the UMV model. Specifically, the slope of the UMV kernel is -/-0.022, while the slope of the UMS kernel in the loss segment is -/-0.049.

The shape of the conditional kernels is determined by the unconditional historical market risk premium and the requirement of satiation and risk neutrality in the utopia state. Since the kernels are flat in the utopia state, the slope during the worst states is much higher during the conditional models. Specifically, for a dividend yield of 10, the slope of the CMV kernel is -/-0.052, while the slope of the CMS kernel in the loss segment is -/-0.130. Further, the conditional kernels increase with the dividend yield, reflecting that marginal utility during bad states is higher than in during good states. Hence, a loss experienced during good states may be assigned a lower weight than a gain experienced during bad states. For example, in October 1974, the excess return on the market portfolio was 16.1 percent. The stock market strongly recovered from a prolonged bear market during which the dividend yield had increased to 5.27. The weight assigned to this scenario is 0.87 in the CMV model. By contrast, the October 1987 crash, with an excess return of -/-23.1 on the market, followed a prolonged bull market during which the dividend yield had fallen to 2.72. The CMV weight assigned to this scenario is 0.99, only marginally higher than the weight of 0.87 for the October 1974 gain of 16.1 percent. In an unconditional model, such small differences in the weights are possible only in case of near-risk-neutrality. However, conditional models recognize that marginal utility is higher during bad states than during good states.

The fixed kernels are well-behaved, that is, they obey nonsatiation and risk aversion over the sample range of the market return and the dividend yield. Imposing these regularity conditions is the key motivation for fixing the kernels. Selecting the kernel to optimize the statistical fit can result in very ill-behaved kernels; see Section IIIA.

[Insert Figure 2 about here]

B. Regular-beta-sorted Portfolios

Panel A of Table I shows the descriptives and results for the regular-beta portfolios. Low (high) beta portfolios have low (high) returns, low (high) variance, negative (positive) skewness and low (high) kurtosis. Especially the pattern in skewness across the portfolios is meaningful in addition to the pattern in variance. Consistent with other studies employing beta-sorted portfolios low-beta stocks are underpriced and high-beta stocks are overpriced in the UMV CAPM. The lowest-beta portfolio and the highest-beta portfolio have regular betas of 0.63 and 1.74 respectively, a beta spread of 1.11. Given the market risk premium of 0.64 percent, this beta spread is too large compared to the mean spread of 0.29 (0.89-/-0.60).

Consistent with Price, Price and Nantell (1982), the downside betas of the low-beta portfolios are higher than the regular betas, while the downside betas of the high-beta portfolios are smaller than the regular betas. For example, the downside beta of the lowest-beta portfolio is 0.66, while the highest-beta portfolio has a downside beta of 1.68. The beta spread decreases from 1.11 to 1.02. As a result the alphas are reduced and the UMS model increases the overall p-value from 0.14 to 0.25.

Apart from downside risk, time-variation also helps to explain the returns of the beta portfolios. Specifically, the betas of the low (high)-beta stocks increase (decrease) during bad times, when the market risk premium is high. This translates into an increase of the conditional betas relative to the unconditional betas. For example, the conditional market beta of the lowest-beta portfolio is 0.72, an increase of 0.11 relative to the unconditional model, while the conditional market beta of the highest-beta portfolio is 1.55, a decrease of 0.19. Overall, the conditional model gives a substantially better fit than the unconditional model; the p-value increases from 0.14 to 0.83.

The best fit is obtained with the CMS model, which combines the two explanations of downside risk and time-variation. Low beta (high beta) stocks are substantially riskier (less risky) than the regular unconditional beta suggests. For example, the conditional downside beta of the lowest-beta portfolio is 0.78 (0.63), and of the highest-beta portfolio is 1.41 (1.74). The beta spread is sharply reduced from 1.11 to 0.83. Compared with the UMV model, the alphas show substantial reductions. The largest pricing error drops from 0.18 to 0.08 and the lowest pricing error improves from -/-0.27 to -/-0.06. Overall, the CMS model gives a near-perfect fit, with a p-value of 0.98.

In brief, while the UMV model performs poorly for beta-sorted portfolios, accounting for downside risk and for time-variation substantially improves the fit. In fact, the combined effect is strikingly good.

[Insert Table I about here]

Panel A of Figure 3 further illustrates the role of conditional downside risk. The figure shows the regular beta and the downside beta for the lowest-regular-beta portfolio and the highest-regular-beta portfolio as a function of the dividend yield, our proxy for the state-of-the-world. In the figure, we clearly see a substantial narrowing of the beta spread during the bad states (high dividend yield). This narrowing is most pronounced for the downside betas.

[Insert Figure 3 about here]

C. Downside-beta-sorted Portfolios

Panel B of Table I shows the results for the downside-beta portfolios. As in Ang, Chen and Xing (2004), the variation in (value-weighted) returns of the downside-beta portfolios increases relative to the regular-beta portfolios. Specifically, the mean spread increases from 0.29 to 0.46 percent per month, while the downside beta spread slightly decreases (1.02 vs. 0.97). The UMS pricing errors are closer to zero, which we already showed in Panel B of Figure 1. We formally test UMS model, and cannot reject the hypothesis of zero pricing errors (p=0.52).

As for the regular-beta portfolios, time-variation and downside risk lead to substantial reductions of the alphas. However, time-variation becomes less important, while downside more important. Panel B of Figure 3 illustrates this finding. Due to the betas of low-downside-beta stocks being higher than those for the low-regular-beta stocks in good states (low dividend yield), there no longer is a clear narrowing of the beta spread; the regular-beta spread increases slightly, while the downside-beta spread decreases slightly. This illustrates the limited role of time variation for the downside-beta portfolios. By contrast, the differences between regular beta and downside beta are more pronounced, especially during the bad states. Again, the CMS model gives a near-perfect fit, with a p-value of 0.96.

D. Double-sorted portfolios

The above results for regular-beta decile portfolios and downside-beta deciles prove strong evidence that downside beta, rather than regular beta, drives returns. Still, regular beta and downside beta are highly correlated (0.999 and 0.997). To disentangle the effect of the two risk measures, we apply a double-sorting routine. We sort stocks first into five quintile portfolios based on regular-beta and then, we subdivide each regular-beta quintile into five portfolios based on downside beta. In addition, we first sort on downside beta and then sort on regular beta. The two resulting datasets of 25 portfolios isolate the separate effects of downside beta and regular beta on average returns.

Table II unambiguously shows that downside beta rather than regular beta explains average returns. In Panel A we see that average return is positively related with downside beta within each regular-beta quintile. In general, the average return of low downside-beta portfolios is 0.69 percent compared to 0.90 for the high downside-beta portfolios. Thus controlled for regular-beta, the positive relation between mean and downside beta remains intact. By contrast, Panel B shows that the positive relation between average return and regular beta disappears (becomes flat/negative) within the downside-beta quintiles. Controlled for downside beta, the average return of low regular-beta portfolios is 0.87 percent compared to 0.77 for the high regular-beta portfolios. Apparently, the positive relation between regular beta and mean returns in Table I and panel A of Table II is due to the fact that regular beta and downside beta are so highly correlated. Separation of the two betas shows that downside beta drives average returns.

[Insert Table II about here]

E. Further Analysis

Table III shows how robust the results are for (1) the specific sample period and (2) the conditioning variable. The conditional models are not included for the subsamples, because splitting the sample greatly reduces the variation in the conditioning variables, hence reducing the value added of these models. Panel A shows the split sample results. The sample is divided into subsamples of equal length based on time period and dividend yield. Clearly, the role of downside risk is most pronounced in the first subsample (1931-1966) and the bad-state subsample. Both subsamples include the bear market of the 1930s. This illustrates the importance of including this specific period when analyzing downside risk. In the more recent subsample (1967-2002) the UMS and UMV models show similar performance for both datasets. For this period, the portfolio-sorting variable is more important for model test results than the pricing kernel. The role of (downside) risk is more important during bad-times than during good-times. The UMS model cannot be rejected relative to the downside-beta sorted portfolios in both time periods ($p>0.50$) and both states of the word ($p>0.38$).

We further investigate how the results are affected by the choice for the specific conditioning variable (z). The dividend price ratio is possibly affected by a change in dividend policy (Fama and French (2002)). Nowadays, firms use share repurchases as a way of returning earnings to stockholders, which structurally lowers the dividend yield. Therefore we also employ the earnings yield (EY) and Credit Spread (CS) as conditioning variables. For the conditional models, the utopia state (z^*) is characterized by a zero value for the earnings yield and the credit spread. Again, the conditional kernels are well-behaved to ensure economic meaningful results. In brief, we find that using the earnings yield leads to a lesser fit and using the credit spread leads to a better fit. Where, the CMV model depends heavily on the choice for the specific conditional variable, the CMS model gives a good fit ($p>0.83$) for all conditioning variables.

[Insert Table III about here]

IV. Discussion

When discussing our findings with colleagues, we have often encountered various questions. Below, we have tried to briefly summarize the most common questions and our attempt to answer these questions.

A. In your study, you have fixed the kernel to impose the regularity conditions of nonsatiation and risk aversion. Do fitted kernels really exhibit strong violations of the regularity conditions?

Yes. To illustrate the need to impose economic structure, Figure 4 shows the kernels that are obtained if the parameters are selected to optimize the statistical fit (JT). The kernels take negative values (violating nonsatiation) and are positively sloped (violating risk aversion) for a large fraction of the observations. Such kernels are economically irrelevant. A statistically good fit for such kernels doesn't mean that we have found a good economic explanation – it only means that we have found a good statistical description.

[Insert Figure 4 about here]

B. The difference between variance and semi-variance is especially important for skewed return distributions. Does the mean-variance-skewness model of Kraus and Litzenberger (1976) give the same results as the mean-semivariance model?

No. The three-moment (3M) CAPM of Kraus and Litzenberger replaces the traditional linear pricing kernel with a quadratic pricing kernel. Unfortunately, the explanatory power of skewness is very limited if we require the kernel to obey risk aversion (see for example Dittmar (2002), Section IIID). Specifically, it follows from the theoretical analysis of Tsiang (1972) that a linear kernel gives a good approximation for any continuously differentiable and decreasing kernel over the typical sample range of asset returns, and that a quadratic kernel is unlikely to improve the fit. Interestingly, this argument does not apply to semi -variance, because this risk measure is associated with a two-piece linear kernel that is not continuously differentiable.

Figure 5 illustrates this point using our data set of regular-beta portfolios. Panel A shows the cubic kernel $m(r_M) = b_0 + b_1 r_M + b_2 r_M^2$ with the parameters selected to optimize the fit (JT) under conditions (3) and (4). The resulting kernel clearly is ill-behaved, as it severely violates risk aversion. Simkowitz and Beedles (1978) already made the point that with risk seeking there is no need for diversification (hold the market portfolio). Panel B shows the results that are found if we require the kernel to obey nonsatiation and risk aversion over the sample range of market return. The resulting kernel comes very close to UMV CAPM kernel. Clearly, a cubic kernel is not sufficiently flexible to capture downside risk aversion if the kernel is required to also be economically well behaved. Indeed, the 3M CAPM gives a worse fit than the UMS CAPM (JT=12.5 vs. JT=11.3), even though the model has one additional parameter that is calibrated to optimize the fit.

[Insert Figure 5 about here]

C. The MS CAPM uses semi-variance, which is the second-order lower partial moment (LPM) with the riskless rate for the target rate of return. Since there exist few prior arguments for selecting the order or the target rate, it would be interesting to see the results for other LPMs.

The general mean-LPM (MLPM) CAPM can be represented by the pricing kernel $m(r_M) = b_0 + b_1 \min(r_M - c)^{k-1}$ with c for the target rate of return and k for the relevant order of the LPM norm. The MS CAPM model is the special case with c=0 and k=2; the MV CAPM is the special case with c>max{r_M} and k=2. Table IV shows the *JT* statistic for various combinations of *c* and *k*. For the regular-beta portfolios, the best fit is obtained for c=-/-10 and k=2, that is, the variance below a return level of -/-10 percent. This suggests that tail risk rather than downside risk even better captures the returns of the regular-beta portfolios. By contrast, for the downside-beta portfolios, the semi-variance (c=0, k=2) is the optimal LPM. In brief, the beta portfolios are best described by a MLPM CAPM with a low target rate, but the MS CAPM gives the best fit for the downside-beta portfolios.

[Insert Table IV about here]

D. The risk of stocks is related to market capitalization (ME). Fama and French (1992) convincingly show that the MV CAPM fails within different size deciles. Does the MS CAPM do any better in this respect?

Yes. Figure 6 shows how the risk-return relation is recovered within the different size deciles. In Panel A of the figure we observe how the risk-return relation is flat within the smallest and largest size deciles. Although Fama and French (1992) employ a shorter sample (1941-1990) and exclude the 1930s, we find similar results in our extended sample (1931-2002). Panel B shows how the beta spread decreases and the mean spread increases when regular beta is replaced by downside beta as the relevant risk measure. This pattern is most pronounced within the smallest size deciles. Finally, panel C shows a further improvement in the risk-return relationship during bad-states of the world. As can be clearly seen from the figures, a residual size effect remains. We emphasize that the MV and MS models assume a perfect capital market and ignore transaction/trading costs. Therefore we do not address the empirical issues typical for the small market segment. Still, the CMS model cannot be rejected within the smallest (p=0.57) and largest (p=0.96) size deciles. Again, we find that downside risk drives asset prices, both within the small and the large market segment.

[Insert Figure 6 about here]

E. The most successful competitor of the MV CAPM is the three-factor model (TFM) of Fama and French (1993). Maybe you also pick up distress risk. How does this model perform relative to the MS CAPM in explaining beta portfolio returns?

Not so good. To answer this question, the three-factor model can be represented by the kernel $b_0 + b_1 r_M + b_2 smb + b_3 hml$, where SMB and HML stand for "small (cap) minus big" and "high (book/market) minus low". We fix the model parameters following the multifactor generalization of the GRS methodology by Fama (1996). Table V compares the fit of the three-factor model with that of the conditional downside risk model. As in Equation (2), the multi-factor betas can be reduced to a single kernel beta (also see Cochrane (2001), Section 8.4). The high-beta portfolios have higher TFM betas than UMV betas, thus leading to larger pricing errors. In fact, the TFM model exhibits a rather weak performance relative to the beta portfolios (p=0.01 and p=0.10). Thus, distress risk does not help to explain the underpricing/overpricing of low/high beta stocks. For beta portfolios, the CMS model does better than the TFM.

We are aware of the fact that the three-factor model certainly helps to explain average returns of stocks within the smallest market segment, most notably small value stocks. However, there are two reasons why we focus on the regular-beta portfolios and downside-beta portfolios in our analysis of the MV and MS models. First, the risk of erroneously rejecting a model due to sampling error increases if we analyze portfolios that are formed on stocks characteristics that are only weakly correlated with the characteristics that are priced according the model. Second, the MS and MV models assume a perfect capital market and we do not expect our models to completely explain the returns of all possible investment strategies. This is especially true for small cap investing, or strategies that involve a high turnover and correspondingly high transactions costs, such as momentum strategies (Lesmond, Schill and Zhou (2004)).

[Insert Table V about here]

F. Your study focuses on explaining the beta-effect. All this is very interesting. However, the 'anomaly-du-jour' is the momentum effect (Jegadeesh and Titman (2001)). Does the MS model fare any better in explaining the returns of momentum strategies than the MV model?

Yes. Momentum profits are related to conditional downside risk. Momentum portfolios are an interesting test case for comparing the MV and MS models, because the returns to momentum strategies generally are characterized by asymmetry (and hence the regular betas and downside betas can be expected to differ substantially). For this reason, we applied all five tests (UMV, UMS, CMV, CMS, and TFM) to ten momentum decile portfolios. Table VI reports the results. The momentum-effect is strongly present; the portfolio of past losers has the lowest mean (0.01% per month) and the highest UMV beta (1.57), while the portfolio of past winners has the highest mean (1.33% per month) and one of the lowest UMV betas (1.00). Interestingly, downside risk and conditioning lead to substantial improvements in the fit. Most notably, in the CMS model, which combines the two explanations, the beta of the loser portfolio falls from 1.57 to 1.15, while the beta of the winner portfolio rises from 1.00 to 1.20. Apparently, past losers involve less downside risk in bad states than suggested by their unconditional regular betas and for past winners the opposite is true. While the improvements are not large enough to rationalize the entire momentum effect and both models have to be rejected, the sizeable reductions of the alphas again confirm our conclusion that the MS CAPM strongly outperforms the MV CAPM – especially during bad states.

[Insert Table VI about here]

G. How do your results compare with those of Ang, Chen and Xing (ACX, 2004)?

As our study, ACX conclude that downside risk is important to explain the cross-section of stock returns. However, the underlying empirical evidence actually is very different. The data and methodology of ACX differ fundamentally from ours. In fact, using our data and methodology, we find no evidence favoring the UMS model over the UMV model in the ACX sample (1963-2001). This confirms our finding that the role of downside risk in the second half of the 20th century is limited (see Table II). The different conclusion of ACX relies on them using (1) equal-weighted portfolio returns and (2) the Fama and MacBeth (1973) cross-sectional methodology. Using equal-weighted portfolio returns rather than value-weighted returns has the effect of placing greater weight on the small cap segment. As illustrated in Figure 6 above, downside risk is relatively more important for the small caps than for the large caps. The cross-sectional methodology further inflates the of downside risk by allowing a high intercept (far above the historical riskless rate) and a low slope (far below the historical equity premium). These two factors explain why the evidence of ACX disappears in our approach that uses value-weighted returns and fixes the model parameters in the spirit of the GRS time-series methodology. In contrast to ACX, our case for the MS CAPM rests of the pattern of downside risk in the earlier years and the bad states-of-the-world. This pattern occurs also for the large caps and if the intercept and slope are fixed.

V. Conclusions

Surprisingly, despite the theoretical limitations of variance, the differences between regular beta and downside beta, and the empirical problems of the mean-variance (MV) CAPM, the mean-semivariance (MS) CAPM has not been subjected to rigorous empirical testing thus far. In an extended sample (1926-2002) we employ unconditional MV and MS tests as well as conditional tests that account for the economic state-of-the-world.

We find that the MS CAPM strongly outperforms the traditional MV CAPM in terms of its ability to explain the cross-section of US stock returns. Especially during bad-states of the world we find a near-perfect relation between risk and return. Downside beta is both theoretically and empirically a better risk measure than regular beta. Further, conditional downside risk (1) explains average returns within the size deciles, (2) is not related to distress risk and (3) can partly explain the momentum effect. In sum, our results provide evidence in favor of market portfolio efficiency, provided we account for conditional downside risk.

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Table I

Descriptives, alphas and betas for beta portfolios

This table shows descriptive statistics for the monthly excess returns of the ten regular-beta portfolios (N=10) and the ten downside-beta portfolios. The sample period is from January 1926 to December 2002 of which the first five years are used for beta estimation. Portfolio returns cover the January 1931 to December 2002 period (T=864 months). In December of each year stocks are sorted in ten decile portfolios based on historical betas. The portfolios are constructed such that each portfolio contains an equal number of stocks. In Panel A and B, the results for the regular-beta and downside-beta portfolios are showed respectively. The alphas ($\hat{a}$) and betas ($\hat{b}$) for each of the following four models are shown: (1) unconditional mean-variance CAPM, (2) conditional mean-variance CAPM, (3) unconditional mean-semivariance CAPM, and finally the (4) conditional mean-semivariance CAPM. The last two columns show the test results for the joint hypothesis that the alphas equal zero. The optimally weighted alphas (JT) are chi-squared distributed with 9 (N-1) degrees of freedom.

Panel A: Regular-beta portfolios

		Low	2	3	4	5	6	7	8	9	High	JT	p
Stats.	Mean	0.60	0.62	0.67	0.63	0.85	0.72	0.77	0.77	0.85	0.89		
	Stdev	4.26	4.64	5.23	5.49	6.34	6.77	7.19	8.22	9.15	10.39		
	Skewness	-0.15	0.40	0.55	0.56	0.81	0.77	0.77	1.45	1.48	1.19		
	Kurtosis	6.1	8.0	9.4	8.9	10.2	10.4	10.5	14.4	14.7	11.0		
Alphas	UMV	0.18	0.11	0.07	0.00	0.12	-0.07	-0.07	-0.18	-0.19	-0.27	13.5	0.14
	UMS	0.16	0.11	0.08	0.01	0.14	-0.05	-0.06	-0.13	-0.13	-0.23	11.3	0.25
	CMV	0.12	0.05	0.08	0.03	0.06	-0.04	-0.06	-0.13	-0.11	-0.15	5.0	0.83
	CMS	0.08	0.05	0.08	0.04	0.07	-0.01	-0.04	-0.03	0.02	-0.06	2.4	0.98
Betas	UMV	0.63	0.77	0.89	0.95	1.10	1.18	1.26	1.42	1.56	1.74		
	UMS	0.66	0.77	0.88	0.93	1.07	1.15	1.23	1.35	1.48	1.68		
	CMV	0.72	0.85	0.88	0.91	1.19	1.14	1.24	1.35	1.44	1.55		
	CMS	0.78	0.85	0.88	0.89	1.17	1.09	1.21	1.19	1.25	1.41		

Panel B: Downside-beta portfolios

		Low	2	3	4	5	6	7	8	9	High	JT	p
Stats.	Mean	0.56	0.65	0.67	0.76	0.83	0.72	0.79	0.83	0.84	1.01		
	Stdev	4.30	4.70	5.16	5.82	6.55	7.13	7.28	8.26	9.79	10.84		
	Skewness	-0.35	0.30	0.04	0.92	0.80	1.66	0.87	1.55	1.97	1.41		
	Kurtosis	5.0	6.7	6.3	12.1	10.2	17.5	10.3	15.4	19.6	12.0		
Alphas	UMV	0.11	0.12	0.07	0.08	0.06	-0.10	-0.05	-0.11	-0.27	-0.17	13.7	0.13
	UMS	0.08	0.11	0.06	0.10	0.08	-0.05	-0.03	-0.06	-0.19	-0.12	8.1	0.52
	CMV	0.11	0.06	0.08	0.08	0.07	-0.08	0.00	-0.06	-0.24	-0.04	9.9	0.36
	CMS	0.05	0.05	0.05	0.11	0.10	0.01	0.04	0.05	-0.06	0.06	3.1	0.96
Betas	UMV	0.67	0.80	0.90	1.02	1.15	1.24	1.26	1.42	1.66	1.76		
	UMS	0.72	0.81	0.92	0.99	1.12	1.16	1.22	1.35	1.54	1.69		
	CMV	0.67	0.88	0.88	1.02	1.14	1.19	1.19	1.35	1.61	1.57		
	CMS	0.77	0.90	0.93	0.98	1.09	1.06	1.12	1.17	1.33	1.42		

Table II
Double-sorted beta portfolios

This table shows the average monthly excess returns of 25 regular-beta/downside-beta portfolios and 25 downside-beta/regular-beta portfolios. The sample period (T=864 months), data requirements, and sorting frequency are identical to the beta portfolios. The portfolios are constructed such that each portfolio contains an equal number of stocks. The last column and row show the average returns across the portfolios. In Panel A, the stocks are sorted into five regular-beta quintile portfolios first and then into five downside-beta quintile portfolios. In Panel B the stocks are sorted into five downside-beta quintile portfolios first and then into five regular-beta portfolios.

Panel A: Regular beta / Downside beta

		Downside beta					
		Low	2	3	4	High	Avg.
Reg. beta	Low	0.57	0.64	0.56	0.79	0.75	0.66
	2	0.54	0.72	0.76	0.79	0.77	0.72
	3	0.70	0.75	0.85	0.96	0.96	0.84
	4	0.86	0.75	0.73	0.87	0.97	0.84
	High	0.77	0.88	0.77	1.04	1.04	0.90
	Avg.	0.69	0.75	0.74	0.89	0.90	0.79

Panel B: Downside beta / Regular beta

		Regular beta					
		Low	2	3	4	High	Avg.
Down. beta	Low	0.56	0.66	0.62	0.60	0.64	0.61
	2	0.73	0.75	0.78	0.73	0.65	0.73
	3	0.93	0.81	0.87	0.82	0.72	0.83
	4	0.99	0.91	0.78	0.83	0.83	0.87
	High	1.14	0.74	0.96	0.90	1.00	0.95
	Avg.	0.87	0.77	0.80	0.78	0.77	0.80

Table III

Robustness analysis

This table shows the split sample results for the regular-beta and downside-beta sorted portfolios, as well as results for different conditional variables. The total sample is divided into subsamples of equal length (*T*=432 months) based on time period and dividend yield. In Panel A results are shown for the unconditional mean-variance (UMV) and mean-semivariance (UMS) models. Panel B shows the results for the conditional mean variance (CMV) and mean semivariance (CMS) if dividend yield is replaced with the one-month lagged earnings yield (EY) and credit spread (CS). The optimally weighted alphas (*JT*) and levels of significance (*p*) are reported for the different models.

Panel A: Split samples

Sample	1931-2002		1931-1966		1967-2002		Bad state		Good state	
	JT	*p*	*JT*	*p*	*JT*	*p*	*JT*	*p*	*JT*	*p*
Regular Beta portfolios										
UMV	13.5	0.14	7.7	0.56	13.3	0.15	3.23	0.95	18.5	0.03
UMS	11.3	0.25	4.4	0.88	13.5	0.14	1.52	1.00	19.3	0.02
Downside Beta portfolios										
UMV	13.7	0.13	12.6	0.18	8.7	0.47	14.2	0.12	9.3	0.41
UMS	8.1	0.52	7.5	0.56	8.3	0.50	6.3	0.71	9.6	0.38

Panel B: Other conditional variables

Variable	DY		EY		CS	
	JT	*p*	*JT*	*p*	*JT*	*p*
Regular Beta portfolios						
CMV	5.0	0.83	11.6	0.24	4.3	0.89
CMS	2.4	0.98	6.6	0.68	3.8	0.92
Downside-beta portfolios						
CMV	9.9	0.36	16.3	0.06	4.9	0.85
CMS	3.1	0.96	4.6	0.87	2.3	0.99

Table IV
Sensitivity LPM model

This table shows the models results for different LPM norms. The general MLPM CAPM is represented by the pricing kernel $m(r_M) = b_0 + b_1 \min(r_M - c)^{k-1}$. The thresholds (c) vary from -15 percent to +15 percent with 5 percent steps. We let the LPM order (*k*) vary from 1 (=expected loss, for c=0) to 4. Each cell contains the optimally weighted alphas (*JT*) and the values in italics correspond with the outcomes of the UMS model. Panel A and B show the results for regular-beta and downside-beta portfolios separately.

		A: Regular beta				**B: Downside beta**			
	Order (*k*)	1	2	3	4	1	2	3	4
Threshold (*c*)	-15	9.0	9.2	10.4	12.6	8.3	8.8	10.3	12.0
	-10	8.8	7.7	9.1	10.3	11.4	9.0	9.3	10.2
	-5	11.1	9.0	8.5	9.1	8.1	8.6	9.1	9.5
	0	13.9	*11.3*	9.5	9.0	8.1	*8.1*	8.6	9.1
	5	13.5	12.1	10.6	9.6	13.5	8.5	8.4	8.7
	10	13.4	12.4	11.3	10.3	25.9	9.6	8.7	8.6
	15	16.7	12.5	11.6	10.8	26.8	10.7	9.1	8.7

Table V

Three-factor model and conditional downside risk

This table shows the alphas ($\hat{a}$) and betas ($\hat{b}$) for the conditional mean-semivariance (CMS) model and the Fama and French three-factor model (TFM). The data are described in Table I. The last two columns show the test results for the joint hypothesis that the alphas equal zero. Panel A shows the results for the regular-beta portfolios and Panel B shows the results for the downside-beta portfolios. The optimally weighted alphas (*JT*) are chi-squared distributed with 9 (*N*-1) degrees of freedom.

Panel A: Regular-beta portfolios

		Low	2	3	4	5	6	7	8	9	High	*JT*	*p*
$\hat{a}$	CMS	0.08	0.05	0.08	0.04	0.07	-0.01	-0.04	-0.03	0.02	-0.06	2.4	0.98
	TFM	0.15	0.11	0.05	-0.02	0.07	-0.13	-0.12	-0.27	-0.30	-0.39	16.0	0.10
$\hat{b}$	CMS	0.78	0.85	0.88	0.89	1.17	1.09	1.21	1.19	1.25	1.41		
	TFM	0.68	0.78	0.94	1.00	1.19	1.29	1.35	1.59	1.76	1.94		

Panel B: Downside-beta portfolios

		Low	2	3	4	5	6	7	8	9	High	*JT*	*p*
$\hat{a}$	CMS	0.05	0.05	0.05	0.11	0.10	0.01	0.04	0.05	-0.06	0.06	3.1	0.96
	TFM	0.03	0.07	0.04	-0.12	0.02	-0.06	-0.10	0.01	-0.11	-0.26	14.9	0.01
$\hat{b}$	CMS	0.77	0.90	0.93	0.98	1.09	1.06	1.12	1.17	1.33	1.42		
	TFM	0.81	0.89	0.90	1.15	1.10	1.28	1.46	1.53	1.86	2.14		

Table VI
Momentum and conditional downside risk

This table shows descriptive statistics for the monthly excess returns of the ten momentum portfolios (N=10). The sample period (T=864 months) and data requirements are identical to the beta portfolios. Each month stocks are sorted in ten portfolios based on 12-month price momentum (cumulative past 12-1 month returns). The portfolios are constructed such that each portfolio contains an equal number of stocks. The alphas ($\hat{a}$) and betas ($\hat{b}$) for each of the following five models are shown: (1) unconditional mean-variance CAPM, (2) conditional mean-variance CAPM, (3) unconditional mean-semivariance CAPM, (4) conditional mean-semivariance CAPM and finally (5) Fama and French three-factor model. The last two columns show the test results for the joint hypothesis that the alphas equal zero. The optimally weighted alphas (JT) are chi-squared distributed with 9 (N-1) degrees of freedom.

		Momentum Portfolios											
		Loser	2	3	4	5	6	7	8	9	Winner	JT	p
Stats.	Mean	0.01	0.41	0.44	0.62	0.57	0.60	0.72	0.92	0.94	1.33		
	Stdev	10.20	8.61	7.25	6.62	6.32	5.98	5.69	5.68	5.96	6.70		
	Skewness	1.92	2.03	1.66	1.41	1.34	0.66	0.01	0.40	-0.17	-0.28		
	Kurtosis	16.2	21.1	18.4	14.9	16.7	11.1	6.7	6.3	4.4	2.2		
Alphas	UMV	-1.03	-0.51	-0.36	-0.13	-0.15	-0.09	0.05	0.27	0.27	0.66	61.5	0.00
	UMS	-0.95	-0.43	-0.30	-0.09	-0.12	-0.09	0.04	0.27	0.23	0.59	49.2	0.00
	CMV	-0.94	-0.52	-0.30	-0.15	-0.17	-0.15	0.04	0.31	0.34	0.65	48.8	0.00
	CMS	-0.75	-0.36	-0.17	-0.07	-0.10	-0.15	0.01	0.29	0.25	0.53	29.4	0.00
	FF	-1.21	-0.63	-0.42	-0.18	-0.20	-0.11	0.07	0.29	0.31	0.72	73.7	0.00
Betas	UMV	1.57	1.38	1.20	1.12	1.09	1.04	0.99	0.98	1.00	1.00		
	UMS	1.44	1.27	1.12	1.06	1.04	1.04	1.02	0.98	1.05	1.11		
	CMV	1.43	1.40	1.12	1.15	1.11	1.13	1.01	0.92	0.90	1.01		
	CMS	1.15	1.16	0.93	1.04	1.00	1.13	1.06	0.94	1.03	1.20		
	FF	1.87	1.59	1.32	1.22	1.17	1.09	0.99	0.97	0.97	0.93		

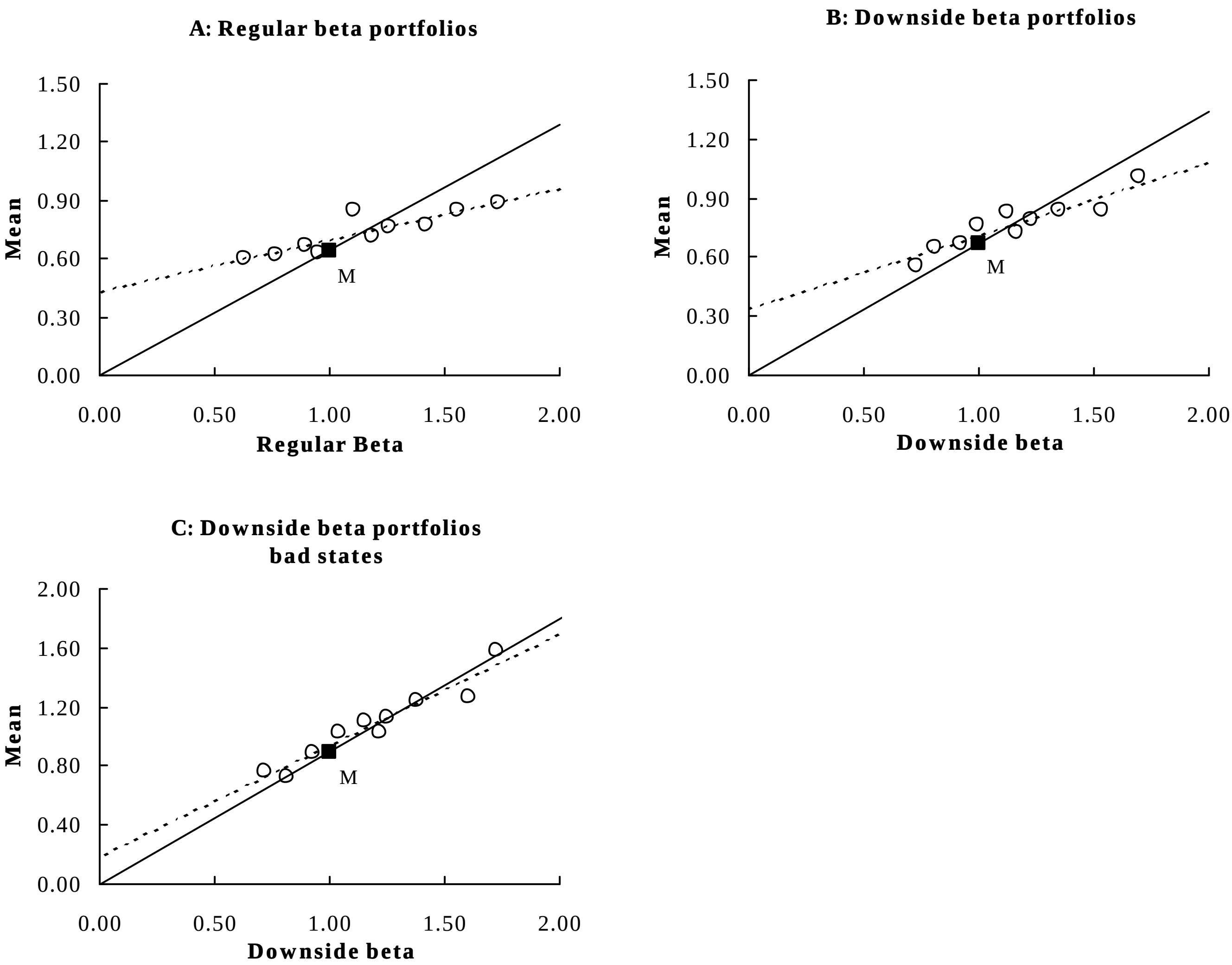


Figure 1: Risk-return relationship of beta portfolios 1931-2002. This figure shows the mean-beta relation of ten stock portfolios (clear dots) and the value-weighted stock market portfolio (filled square). The straight line through the origin and the market portfolio represents the equilibrium condition. The dotted line shows the best fit (OLS). Panel A shows the traditionally weak relationship between regular beta and mean returns. Panel B shows the corresponding relationship when regular beta is replaced with downside beta. Finally, Panel C shows that downside beta and mean return come very close to the equilibrium relationship during bad states-of-the-world (defined here as states with a dividend yield below its median value). For a detailed description of the data and the portfolio formation procedure, see Section II.

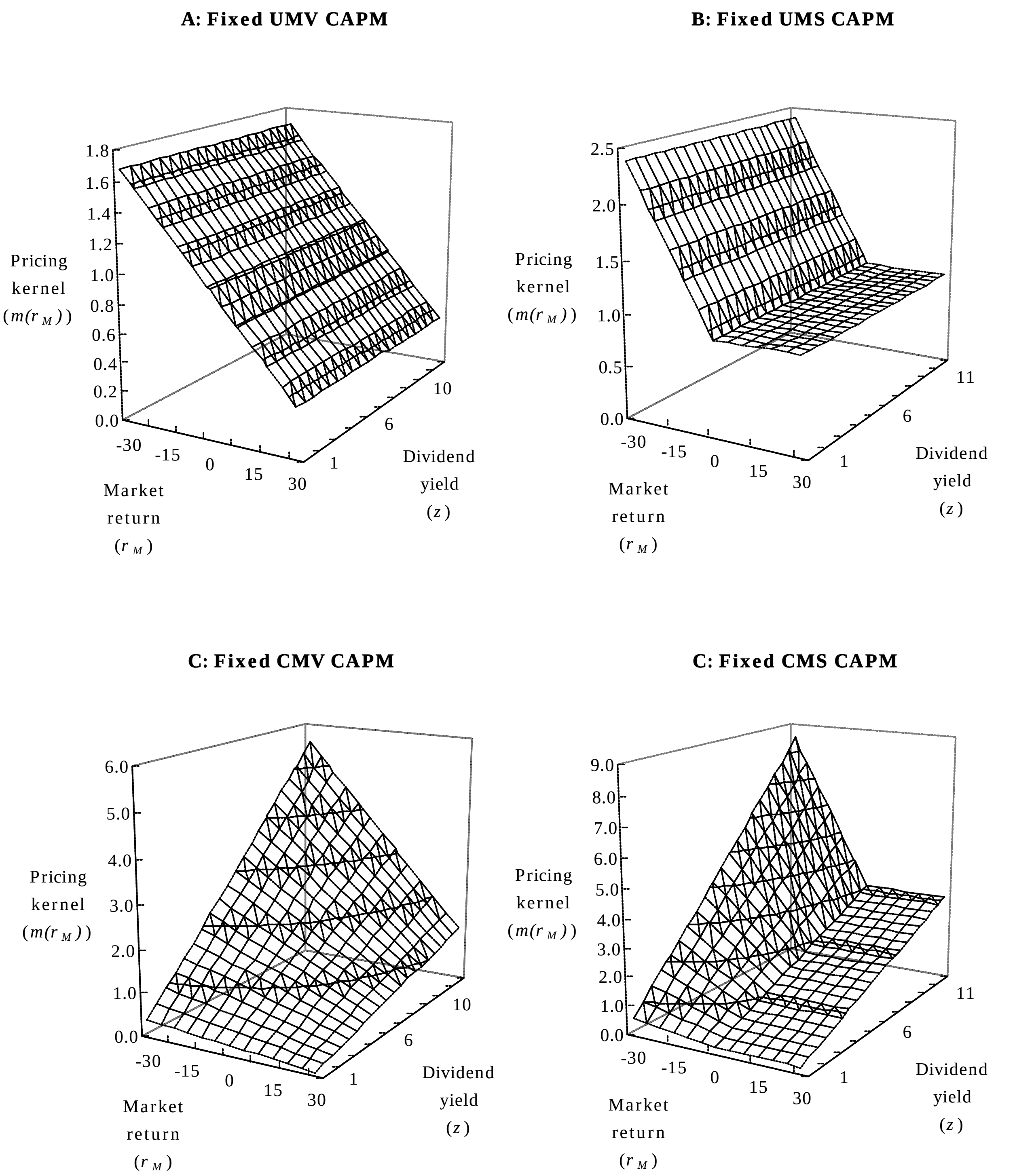


Figure 2: Fixed pricing kernels. The figure shows unconditional and conditional pricing kernels for the MV CAPM and MS CAPM in the full sample (January 1931 - December 2002). The unconditional kernels are found by solving the equalities (3) and (4) for the known parameters. The conditional kernels are obtained by using the one-month lagged dividend yield as the conditional variable and solving the equalities (3)-(6) for the known parameters.

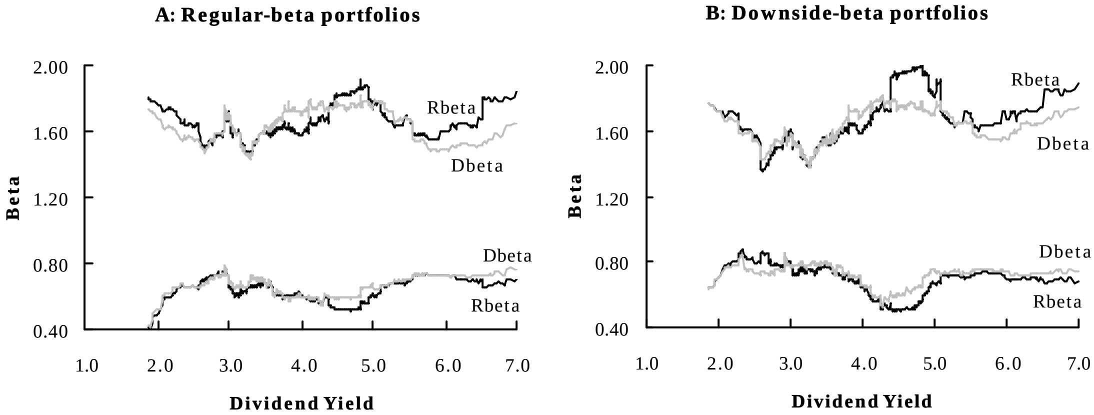
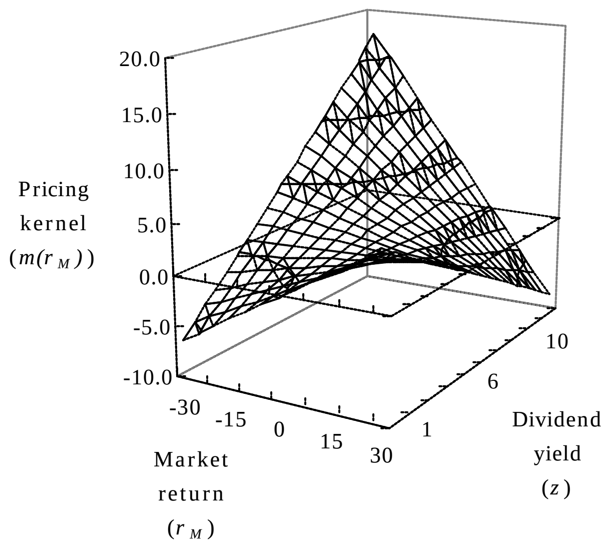


Figure 3: Conditional MV and MS betas. This figure shows the regular beta (black) and the downside beta (grey) of the lowest-beta portfolio and the highest-beta portfolio. We use a rolling 120-months period (1-month steps) after sorting the data based on the one-month-lagged dividend yield. Panel A shows the results for the regular-beta portfolios and Panel B shows the results for the downside-beta portfolios.

A: Fitted CMV CAPM



B: Fitted CMS CAPM

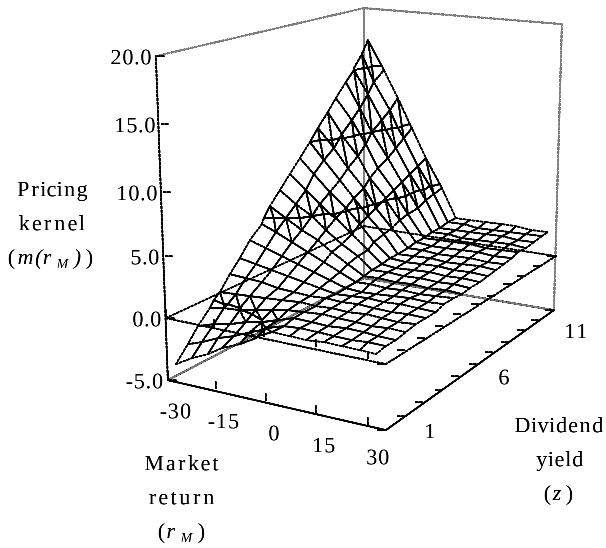


Figure 4: Fitted conditional pricing kernels. The figure shows the fitted CMV and CMS pricing kernels for the full sample (January 1931 - December 2002) and with the one-month lagged dividend yield as the conditioning variable. The fitted kernels are determined by maximizing the empirical fit (*JT*) relative to the ten regular beta-sorted portfolio, while maintaining conditions (3) and (4).

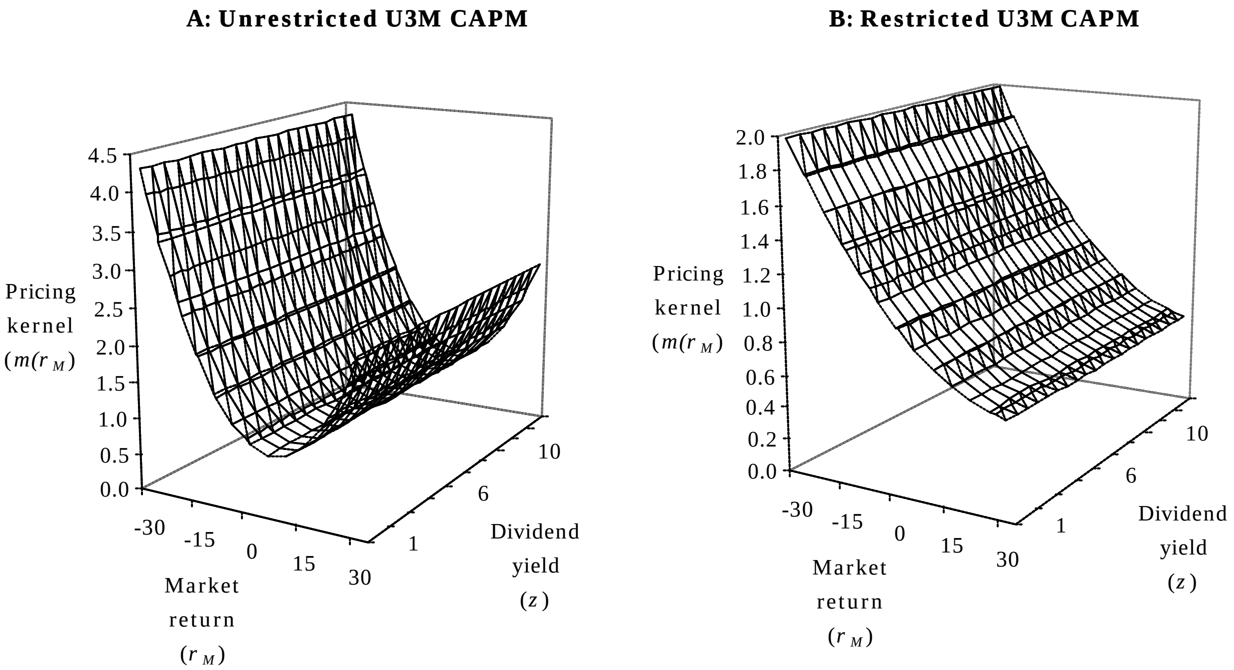


Figure 5: Quadratic pricing kernels. The figure shows the unconditional cubic pricing kernels for the three-moment (3M) CAPM using the full sample (January 1931 - December 2002). The kernel in Panel A is determined by maximizing the empirical fit (JT) relative to the ten beta-sorted portfolios, while maintaining conditions (3)-(4). Panel B shows the results obtained if we add the restrictions of nonsatiation and risk aversion for the sample range of market return and the dividend yield.

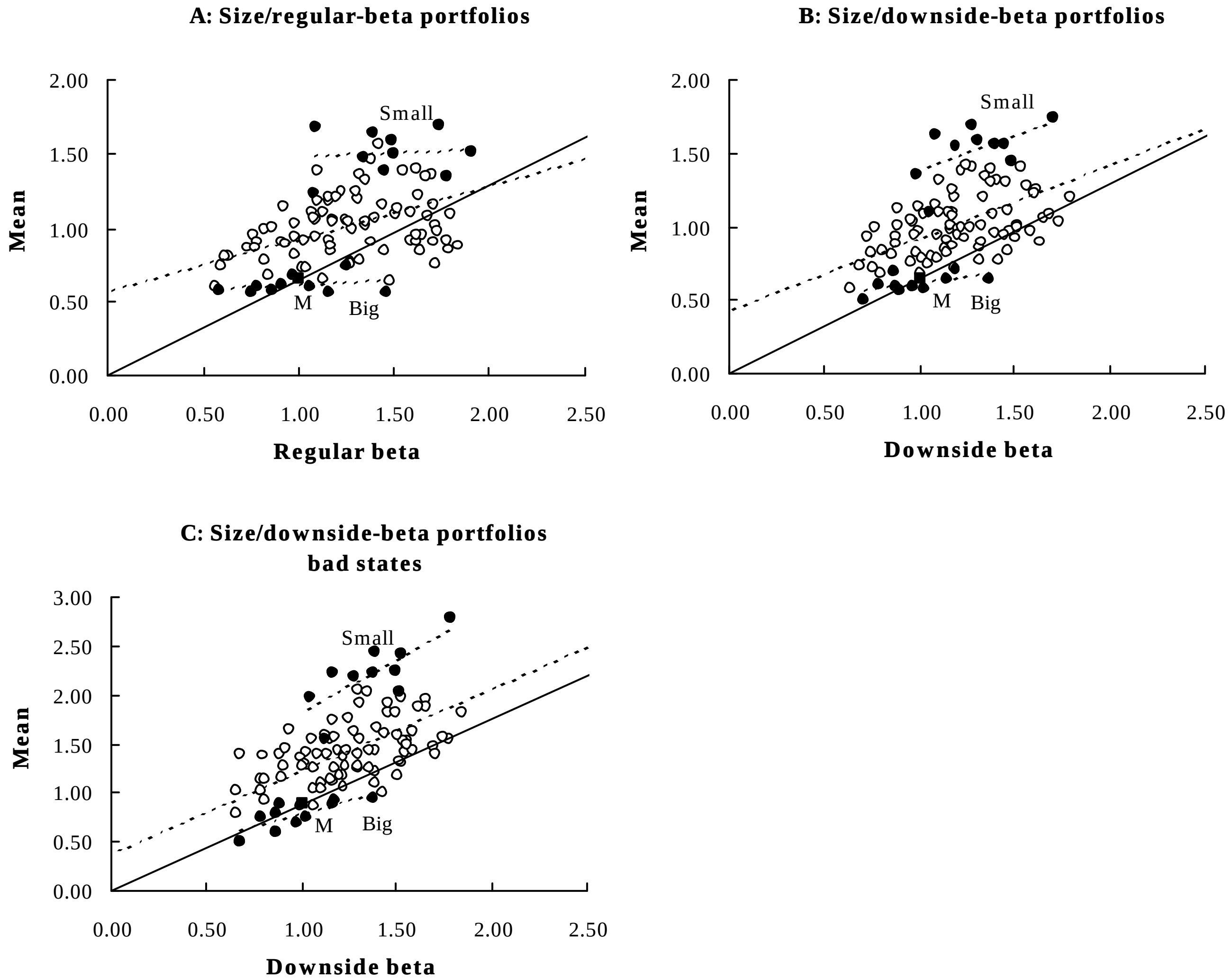


Figure 6: Risk-return relationship of size/beta portfolios 1931-2002. This figure shows the mean-beta relation of 100 double size/beta-sorted stock portfolios (clear dots). The smallest and largest decile portfolios (filled dots) and the value-weighted stock market portfolio (filled square) are labeled separately. All stocks included in this study are sorted based on NYSE size decile breakpoints first and then into ten beta portfolios. The straight line through the origin and the market portfolio represents the equilibrium condition. The dotted lines give the best fit (OLS) for (1) all portfolios and (2) the portfolios within the largest and smallest size deciles. Panel A shows how mean and beta are not related within the different size deciles. Panel B shows the corresponding relationship when regular beta is replaced with downside beta. Finally, Panel C shows that within the NYSE size deciles, mean and downside beta are positively related during bad states-of-the-world (defined here as states with a dividend yield below its median value).

Footnotes

[1] The downside beta is the special case of the second-order LPM beta with the riskless rate as target rate of return.

[2] Using the notation developed in the main text, Harlow and Rao (1989, Eq. 10) use the following regression model to estimate the downside beta (second-order LPM beta with a zero excess return as the target rate):

$$r_i = a_i + b_i(\min(r_M,0) + E[\max(r_M,0)]) + g_i(\max(r_M,0) - E[\max(r_M,0)]) + e_i$$

It is straightforward to show that b_i is *not* the downside beta of Harlow and Rao (1989, Eq. 9):

$$b_{Down,i} \equiv E[r_i \min(r_M,0)]E[r_M \min(r_M,0)]^{-1}$$

and that b_i is not an economically meaningful risk measure. This can be demonstrated by means of the following example with four states-of-the-world:

State	Prob.	Market	Stock *i*	50/50
1	25%	-10%	-5%	-7.5%
2	25%	-5%	-10%	-7.5%
3	25%	10%	10%	10%
4	25%	10%	10%	10%

In this case, $b_i = -1$, reflecting the perfect negative correlation when the market falls. This suggests that we can construct a riskless portfolio by investing 50% in the market portfolio and 50% in Stock *i*. However, the resulting 50/50 portfolio is not riskless, because there is a perfect positive correlation when the market rises. Indeed, in this case, the true downside beta equals $b_{Down,i} = 0.83$. The flaw in the argumentation of Harlow and Rao occurs in their Footnote 15, where the necessary condition $E[r_i] = b_i[r_M]$ is mistakenly treated as a sufficient condition.

[3] Throughout the text, we will use $\Re^N$ for an *N*-dimensional Euclidean space. Further, to distinguish between vectors and scalars, we use a bold font for vectors and a regular font for scalars. Finally, all vectors are column vectors and we use $\boldsymbol{r}^{\mathrm{T}}$ for the transpose of $\boldsymbol{r}$.

[4] The results are not affected by the sorting frequency. When sorting takes place on a monthly basis (instead of in December of each year) we find similar portfolio characteristics and model test results.

[5] We prefer value-weighted returns equal-weighted returns because using equal-weighted returns one implicitly assumes continuous portfolio updating and hence involves a lot of trading/transaction costs.

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