THE VALUE OF INFORMATION IN REVERSE LOGISTICS

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The Value of Information in Reverse Logistics

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Abstract

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1 Introduction

As a general topic, the value of information (VOI) for inventory management has been extensively explored, with references and exercises in the earliest operations research textbooks (e.g., Wagner 1969, ch.16). Recently, there has been renewed interest in this topic by both practitioners and academics that has paralleled the rise of e-commerce and the development of new information technologies. These new technologies promise more timely and accurate information to reduce uncertainties with regard to supply and demand and thereby improve coordination and financial performance. Indeed, much of the more recent literature on the VOI focuses on how information can be used to improve supply chain performance and the conditions in which information is most valuable (e.g. Gavirneni et al. 1999, Cachon and Fisher 2000, Lee et al. 2000, and Moinzadeh 2002). Yet, there has been little research on the VOI in the context of reverse (remanufacturing) supply chains or supply chains with product returns.

Remanufacturing has received increasing attention in the US (Guide 2000) because of its economic benefits, as well as regulatory and consumer demands for more environmentally friendly operations. There are over 73,000 firms engaged in remanufacturing in the US that employ over 350,000 people (Lund 1998). Remanufacturing provides a foundation for the development of closed-loop supply chains and focuses on value added product recovery. Closed-loop supply chains explicitly consider the reverse flows of materials in addition to the traditional forward flows of materials (Guide and Van Wassenhove 2003).

Remanufacturing product returns provides a reuse alternative that may be value-creating for many products, but there are a number of complicating characteristics (Guide 2000) that require close managerial attention if operations are to be competitive. One difference between remanufacturing and other forms of production is the coordination required between two supply functions: new parts, usually procured from an external supplier, and recovered parts, usually obtained from a dedicated remanufacturing shop. Assuming that remanufactured product and new product are substitutes, the natural question arises as to how to plan the mix in order to satisfy demand.

The coordination challenge is amplified by the considerable uncertainty regarding timing and quantity that typically characterize product return flows. In addition, remanufacturing is often subject to stochastic yield. The less that is known about the outcome of the recovery process, the harder it is to coordinate the procurement of new parts and the remanufacturing of returned products to meet demand. The cumulative effect of these characteristics is greater uncertainty inherent in remanufacturing operations. Managers must take actions to reduce uncertainty in the timing and quantity of returns, balance return rates with demand rates, and make material recovery more predictable. In essence, there appears to be significant potential for information to reduce the inherent uncertainties for a firm operating in an environment with product returns. The question is how much is information worth? Perhaps more importantly, which type of information is most valuable and what are the conditions that give rise to the VOI?

In this research we explore the VOI in the context of a remanufacturer that faces uncertainty with respect to demand, product returns, and product recovery (yield loss). We model demand and product returns as independent random variables. Product recovery is uncertain in that each returned unit can be successfully remanufactured to as good as new with a known probability and otherwise it is discarded without cost. We assume a single period model in which the operational decision of interest is the quantity of new product to order.

Our objective is to evaluate the absolute and relative value of the different types of information that such a firm may choose to invest in order to reduce the uncertainty it experiences in matching supply with demand. The different types of information include demand, return, and yield loss. We consider five separate cases that are distinguished by the types of information that are known in each case. In the base case, the firm knows no more than the distributions of the random variables and its cost structure. Each of the other four cases considers information that fully explains one or more sources of uncertainty. The VOI for each case is then measured as the improvement in total expected cost that a firm achieves with the given information set, relative to the base case.

Our results are extensive and reveal that the VOI for any specific type of information depends both on the overall level of uncertainty and the level of uncertainty that is attributed to the information for which it explains (e.g. demand information explains demand uncertainty). We find that there is no dominance in value amongst the different type of information. There are conditions in which demand information may be more (less) than return information and in which yield information may be more (less) than the other two types. We develop and test a theoretical model that is predictive of 1) the value of each type of information, 2) sensitivity of the VOI, and 3) the relative value for each type of information.

The rest of this paper is organized as follows. In §2, we provide a review of the literature on the VOI for inventory management and position our contribution with respect to them. In §3, we introduce an analytic model in which we evaluate the VOI with specific underlying distributional assumptions and then develop and test a generalized theoretical model on the VOI. §4 concludes our study with future research directions.

2 Literature Review

Recently, a few articles have emerged that provide literature reviews and taxonomies addressing the VOI for supply chain management. Sahin and Robinson (2002) and Huang et al. (2003) are representative examples and each of the reviews provides a very broad overview of the literature and uses its own classification scheme. Chen (2002) is notable for its depth of analysis by exploring the different types of information sharing and then explaining and comparing the analytical results among several key contributions to the field. Collectively, the literature reviews indicate that a preponderance of the research in this area focuses on the value of demand information to improve supply chain performance. Bourland et al. (1996), Gavirneni et al. (1999), and Lee et al. (2000) are representative contributions that explore the value of demand information in serial supply chains. Cachon and Fisher (2000) and Moinzadeh (2002) are examples that explore the value of demand information in distribution systems.

There are a few papers that explore the value of supply information. Some of these consider cases where information such as available supplier capacity and lead-time is shared forward in the supply chain so that customers can reduce supply uncertainty (Van der Duyn Schouten et al. 1994 and Chen and Yu 2002). Another form of supply uncertainty arises in perishable systems, where there may be uncertainty with regard to the age of the product that is used for replenishment. Ketzenberg and Ferguson (2004) address the value of a supplier sharing the age of its inventory with a retailer to improve replenishment decisions for a perishable product. Even so, none of these contributions address uncertainty with respect to product returns and remanufacturing yield nor do they provide a comparative assessment of the VOI with respect to demand information as we do in this study.

This is not to say that there is not a wealth of research that address reverse logistics issues like those observed in a remanufacturing facility in which there are uncertainties with respect to demand, return, and yield. This literature falls under the general umbrella of closed-loop supply chain management. For a fairly comprehensive discussion of the field see Fleischmann (2000), Guide and Van Wassenhove (2003), and Dekker et al. (2003). The dissertation and books also contain extensive references to research dealing with inventory management and production, planning, and control in reverse logistics.

While both the literature on the VOI and reverse logistics has grown considerably over the past decade, not many bridge these fields. Ferrer and Ketzenberg (2004) address a remanufacturer that faces a tradeoff between limited information regarding remanufacturing yield and potentially long supplier lead-time. The authors develop four decision–making models to evaluate the impact of yield information and supplier lead-time on manufacturing costs. They identify the operating conditions under which these capabilities are valuable, along with their relative impact on financial performance. Their results indicate that the yield information is generally quite valuable, while investments in supplier responsiveness provide trivial returns to products with few parts. In their model, however, the only uncertainty is with respect to yield since their models assume an infinite supply of product returns and deterministic demand. We differentiate our work here by evaluating the VOI in the context of uncertainties with respect to demand, return, and yield.

Ketzenberg et al. (2004) also address the value of advanced yield information in the context of a mixed assembly-disassembly operation for remanufacturing. The principal focus of this work is in determining the best line configuration. Under a *parallel* configuration, there exist two separate dedicated lines, one for assembly and one for disassembly that are decoupled by inventory buffers. Under a *mixed* configuration, the same station is used for both disassembly and assembly of a specific part. The authors investigate the value of advanced yield information on these two different configurations and find that this information generally improves flow-time. However, there are some instances where information lengthens flow time. Although the authors model an environment with uncertainty in product returns and demands, they do not explore the VOI to explain these other sources of uncertainty.

There are a few other studies that while they do not provide a *specific* treatment of the VOI, they do explore the impact of misinformation or accuracy of information. For example, De Brito and van der Laan (2002) examine the value of misinformation regarding product returns. Souza and Ketzenberg (2002) investigate the impact of inaccuracies in grading the quality of product returns on flow times in a remanufacturing job shop.

Our research bridges and builds on the literature by providing a treatment on the VOI in which there are different types of information to address different sources of uncertainty. In the next section, we introduce our model and provide both exact and approximate analysis on the VOI and the relative VOI between different types of information.

3 Model

The general setting is a remanufacturer that can satisfy demand with new product, remanufactured product, or a mix of both types. This assumes that the quality and reliability of the remanufactured product allow the interchange. While it is less costly to remanufacture than procure new, on average, the rate of returns is less than demand so that at least some portion of demand will be satisfied with new product. Figure 3.1 shows the material flow for both remanufactured and new product. This material flow is well studied in the literature on repairable inventory and is predicated on a model originally developed by Simpson (1978).

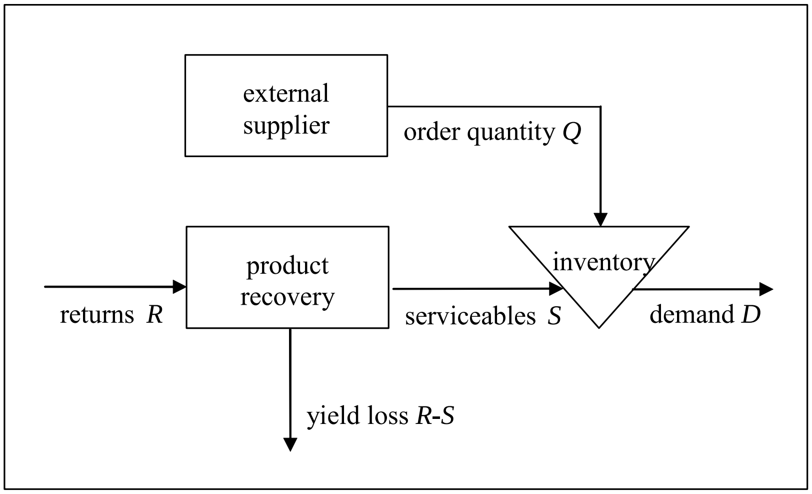

Figure 3.1: Material Flow

We assume a single period model where the operational decision of interest is the quantity of new product Q to order from a perfectly reliable external supplier with the objective to minimize total expected cost. The decision is complicated by uncertainties with respect to the number of demands, the number of returns, and yield loss from the remanufacturing process.

Let D denote the random demand variable with mean μ_D, variance σ_D^2, and let $\bar{d}$ denote its realization. Likewise, define R to be the random returns variable with mean μ_R, variance σ_R^2, and let $\bar{r}$ denote its realization. We assume that D and R are independent.

Remanufacturing is not capacitated and all product returns are remanufactured although the process is subject to stochastic yield. That is, each product return can be successfully remanufactured and brought to a good as new state with probability γ known as the recovery yield. With probability $1-\gamma$, remanufacturing is not successful and the returned unit is disposed. Let Y denote the random variable that indicates if the repair of a product return is successful. Hence, $Y \sim$ Bernoulli (γ). We call those units that are successfully remanufactured serviceable returns. Now, let S denote the serviceable returns random variable where

$S = (Y \circ R)$ with mean μ_S, variance σ_S^2, and let $\bar{s}$ denote its realization. The number of serviceable returns and the quantity of newly procured units are used to satisfy demand to the extent possible. Any unsatisfied demands are lost and are assessed a shortage cost p per unit. Excess ending inventory is charged an overage cost h per unit. We summarize our main notation within a single table in Appendix 1.

Below we introduce a set of five information cases that differ with respect to the information that is known prior to the ordering decision. The base case considers the scenario where the ordering decision is made prior to realizing demand, returns, and yield loss. The only known information includes the sufficient statistics for each random variable and the relevant costs. The other four cases represent an improvement on the base case, where one or more additional items of information are available prior to the ordering decision. Let $\mathcal{I}$ denote an information case, where $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{R}, \mathcal{DR}, \mathcal{S}\}$ as defined in Table 3.1 and let $\bar{i}$ denote the additional information relative to the base case that is known prior to the ordering decision, where $\bar{i} \subset \{\bar{d}, \bar{r}, \bar{s}\}$ as specified in the right-most column of Table 3.1. We define the VOI for information case $\mathcal{I}$ as $\psi_{\mathcal{I}} = (C_{\mathcal{B}} - C_{\mathcal{I}})/C_{\mathcal{B}}$. Consequently, $\psi_{\mathcal{I}}$ is the cost improvement of knowing additional information $\bar{i}$ relative to the base case.

Case $(\mathcal{I})$	Type of Information	Additional Information $(\bar{i})$
$\mathcal{B}$	Base	No information $\varnothing$
$\mathcal{D}$	Demand	Number of demands $\bar{d}$
$\mathcal{R}$	Return	Number of returns $\bar{r}$
$\mathcal{DR}$	Demand and Return	Number of demands $\bar{d}$ and number of returns $\bar{r}$
$\mathcal{S}$	Serviceable Return	Number of serviceable returns $\bar{s}$

Table 3.1: Information Cases

We model the one period decision as a generalized newsvendor problem. See Mostard and Teunter (2002) and Vlachos and Dekker (2003) for seminal research on the newsvendor problem with product returns. Define $N \mid \bar{i}$ as the net demand ($D-S$) over the period given information $\bar{i}$ with CDF $F_{N|\bar{i}}(\cdot)$, mean $\mu_{N|\bar{i}}$, and standard deviation $\sigma_{N|\bar{i}}$. The total expected cost $C(Q \mid \bar{i})$ given order size Q and conditioned on the information $\bar{i}$ is

$$\begin{aligned} C(Q \mid \bar{i}) &= p\mathrm{E}_{N|\bar{i}}\left[N-Q\right]^{+} + h\mathrm{E}_{N|\bar{i}}\left[Q-N\right]^{+} \\ &= p\int_{Q}^{\infty}(N-Q)\,\mathrm{d}F_{N|\bar{i}}(z) + h\int_{-\infty}^{Q}(Q-N)\,\mathrm{d}F_{N|\bar{i}}(z) \\ &= p(\mu_{N|\bar{i}}-Q) + (h+p)\int_{-\infty}^{Q}F_{N|\bar{i}}(z)\,\mathrm{d}z \end{aligned} \tag{3.1}$$

which is optimized for

$$Q_{\bar{i}}^{*} = F_{N|\bar{i}}^{-1}\left(\frac{p}{h+p}\right) \tag{3.2}$$

(see e.g. Silver et al. 1998)[1]. Although equation (3.1) is formulated in terms of continuous distributions, we get an analogous discrete formulation by replacing the integral with an appropriate summation. The total unconditional costs $C_{\mathcal{I}}$ are obtained by integrating over all possible realizations $\bar{i}$. Therefore $C_{\mathcal{I}} = \int_{-\infty}^{\infty} C(Q_{\bar{i}}^{*} \mid \bar{i})\,\mathrm{d}\bar{i}$.

The rest of this section is organized as follows. In §3.1, we provide *exact* analysis on the VOI when the demand and return processes are uniformly distributed. Since the complexity of even this simplified model precludes analysis with uncertain yield, in §3.2 we provide *approximate* analysis for all information cases when demand and return processes are normally distributed. In §3.3 we introduce and evaluate a generalized theoretical model on the VOI and then demonstrate the explanatory power of the model in §3.4.

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[1]To simplify notation, if $\mathcal{I} = \mathcal{B}$ we drop the notation with respect to the conditioning on $\bar{i}$ (which is the empty set). Then, $N \mid \bar{i}$ reduces to N, $Q_{\bar{i}}^{*}$ reduces to Q^{*}, etc.

3.1 *Exact analysis: uniform demands and returns*

Assume that demands D are uniformly distributed on $[a_D, b_D]$, $a_D \geq 0$ and returns are uniformly distributed on $[0, c_R]$, $c_R \leq b_D$. That is, the number of returns cannot exceed the maximum possible number of demands (see Figure 3.1.1). Without prior information, $\mu_D = a_D + c_D / 2$, $\sigma_D = c_D / \sqrt{12}$ (with $c_d = b_D - a_D$), $\mu_R = c_R / 2$, $\sigma_R = c_R / \sqrt{12}$, and net demand has a symmetric, trapezoid-shaped distribution on $[a_D - c_R, b_D]$ with mean $\mu_N = a_D + (c_D - c_R)/2$ and standard deviation $\sqrt{(c_D^2 + c_R^2)/12}$. These assumptions result in a closed form solution for the inverse of net demand, provided that there is no yield loss in the remanufacturing process. Including even the simplest yield process[2] seriously complicates any exact analysis. Hence, we assume perfect yield so that $S \equiv R$. We use continuous uniform distributions rather than the discrete versions for mathematical tractability.

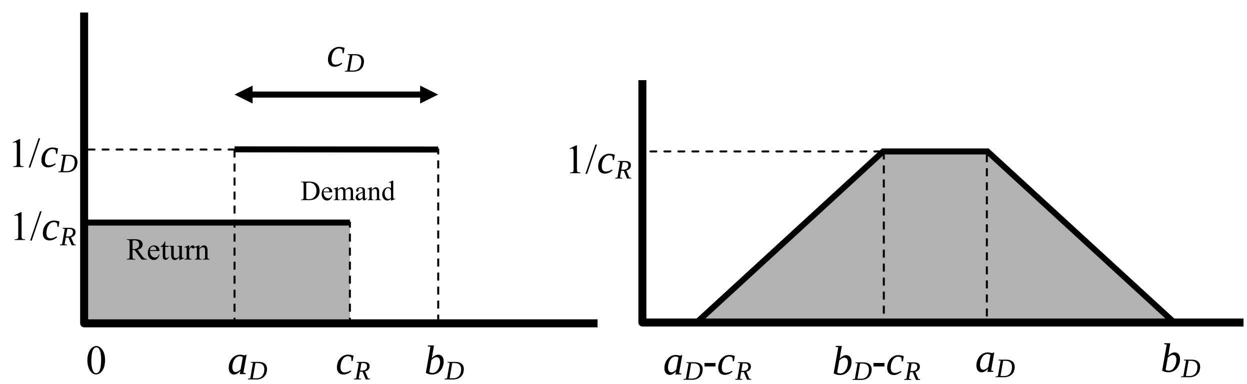

Figure 3.1.1: Illustrative example of a return and demand distribution (left) and the corresponding net demand distribution (right).

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[2]A yield process that still enables mathematical tractability and even has some practical relevance is the following: The quality of *all* the returns during the period is modeled through a Bernoulli process. That is, with probability γ all returns are suitable for remanufacturing and with probability $(1-\gamma)$ all returns have to be disposed. Yet, even this very simple yield process considerably complicates analysis, so we will not explore this venue here.

We assess the VOI exactly by deriving exact expressions for the optimal costs for each information case $\mathcal{I}$, $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{R}\}$. Despite our simplifying assumptions an exact analysis is still not completely straightforward. Consider that the generalized newsvendor equation (3.2) does not guarantee non-negative order quantities. A negative value may occur if (expected) returns are very high compared to (expected) demand. We therefore restrict the parameter settings to those values for which $Q^*_{N|\bar{i}} \geq 0$.

Another complication arises from the form of the net demand distribution without prior information $F_N^{-1}(\cdot)$. As Figure 3.1.1 illustrates, the density function consists of three separate piecewise continuous function and hence, so is the CDF. The specific parameter settings determine exactly which of the three functions defines the optimal solution. However, if the fraction $\frac{h}{h+p}$, which can be interpreted as the probability of running out of stock, is sufficiently small, the optimal solution will be defined by the right hand tail. Since in practice one often desires high service levels, this fraction is typically close to zero. We therefore restrict ourselves to those parameter settings for which this condition holds, thereby simplifying the analysis.

Lemma 1 *If* $\dfrac{h}{h+p} \leq \min\left\{\dfrac{c_R}{2c_D}, \dfrac{c_D}{2c_R}, \dfrac{b_D - c_R}{c_D}, \dfrac{a_D}{c_R}\right\}$, *then*

$$C_{\mathcal{B}} = h\left(\frac{c_D + c_R}{2} - \frac{2}{3}\sqrt{\frac{2c_D c_R h}{h+p}}\right),\ C_{\mathcal{D}} = \frac{1}{2}\frac{hp}{h+p}c_R,\ \text{and}\ C_{\mathcal{R}} = \frac{1}{2}\frac{hp}{h+p}c_D.$$

Proof *see Appendix 2.*

Now, define $\rho = \frac{\sigma_R}{\sigma_D}$. Under the condition of Lemma 1 the following propositions hold.

Proposition 1 *If demands and returns are uniformally distributed, then the value of demand information and the value of return information are given by*

$$\psi_{\mathcal{D}} = 1 - \left(\frac{p}{h+p}\right)\left(\frac{\rho}{1+\rho-\frac{4}{3}\sqrt{\frac{2\rho h}{h+p}}}\right) \text{ and } \psi_{\mathcal{R}} = 1 - \left(\frac{p}{h+p}\right)\left(\frac{1}{1+\rho-\frac{4}{3}\sqrt{\frac{2\rho h}{h+p}}}\right).$$

Proposition 2 *If demands and returns are uniformally distributed, then the value of demand information versus the value of return information is characterized as follows*:

$$\psi_{\mathcal{D}} = \psi_{\mathcal{R}} \text{ if and only if } \sigma_{\mathcal{D}} = \sigma_{\mathcal{R}} \text{ and } \psi_{\mathcal{D}} > \psi_{\mathcal{R}} \text{ if and only if } \sigma_{\mathcal{D}} > \sigma_{\mathcal{R}}.$$

Proposition 3 *If demands and returns are uniformally distributed, then the limiting behavior of the VOI with respect to penalty cost p is as follows*:

$$\lim_{p\to\infty} \psi_{\mathcal{D}} = \frac{1}{1+\rho} = \frac{\sigma_D}{\sigma_D+\sigma_R} \text{ and } \lim_{p\to\infty} \psi_{\mathcal{R}} = \frac{\rho}{1+\rho} = \frac{\sigma_R}{\sigma_D+\sigma_R}.$$

That is, for high service levels, the value of either demand or return information are completely determined through ρ, the ratio of the return and demand standard deviations. Furthermore, at the limit, the value of demand (return) information is monotonously decreasing (increasing) in ρ. The proofs of Propositions 1-3 follow directly from the definition of VOI combined with Lemma 1. In the following section, we explore whether the propositions also hold for normally distributed demand and return distributions.

3.2 Approximate analysis: normal demands and returns

We now assume that demands and returns are both normally distributed. We will derive approximate expressions for the expected cost for all five information cases and use those to determine the VOI. We start by deriving the mean and variance of net demand conditional on the available information for each case. In doing so, we ignore the fact that the distribution of the number of returns R is continuous rather than discrete, and apply the following well-known result from statistical theory. If X is a non-negative discrete random variable with mean μ and variance σ^2, then the binomial distribution with the number of repetitions equal to the outcome of X and probability of success γ, $0 \le \gamma \le 1$ has mean $\gamma\mu$ and variance $\gamma^2\sigma^2 + \gamma(1-\gamma)\mu$ (Bain and Englehardt 1987). The resulting means and variances for all cases are given in Table 3.2.1.

Case	Mean μ_N	Variance σ_N^2
Base	$\mu_D - \mu_R$	$\sigma_D^2 + \gamma^2\sigma_R^2 + \gamma(1-\gamma)\mu_R$
Demand	$\bar{d} - \mu_R$	$\gamma^2\sigma_R^2 + \gamma(1-\gamma)\mu_R$
Return	$\mu_D - \gamma\bar{r}$	$\sigma_D^2 + \gamma(1-\gamma)\bar{r}$
Demand and Return	$\bar{d} - \gamma\bar{r}$	$\gamma(1-\gamma)\bar{r}$
Serviceable Return	$\mu_D - \bar{s}$	σ_D^2

Table 3.2.1: Conditional distribution of net demand

The distribution of net demand is clearly normal for the case with information on the number of serviceable returns. For the other cases, the exact distribution of net demand is unclear. However, combining the two well-known results that (i) the difference of two normally distributed variables is again normal and (ii) according to the Central Limit Theorem the binomial distribution is asymptotically normal for large numbers of repetitions, it follows that net demand is approximately normally distributed (with mean and variance given in Table 3.2.1).

For a representative example, Figure 3.2.1 confirms that the distribution of net demand is indeed approximately normal for all information cases. In this example, the model parameters are $\mu_D = 30$, $\sigma_D = 10$, $\mu_R = 15$, $\sigma_R = 5$, and $\gamma = 0.6$. The realizations are $\bar{d} = 38$, $\bar{r} = 13$, and $\bar{s} = 11$. Other examples show similar results. In Figure 3.2.1, the estimated distribution function based on 500 drawings of net demand is compared to the normal distribution function. The drawings of net demand are conditional on the known realizations. For example, in the Return information case with $\overline{r} = 13$, a drawing of net demand results from drawing demand from $N(30,10)$ and drawing serviceable return from $B(13, 0.6)$. The estimated distribution function is obtained by plotting the 500 drawings of net demand in ascending order against $1/500$, $2/500$, …, $499/500$, and 1 respectively.

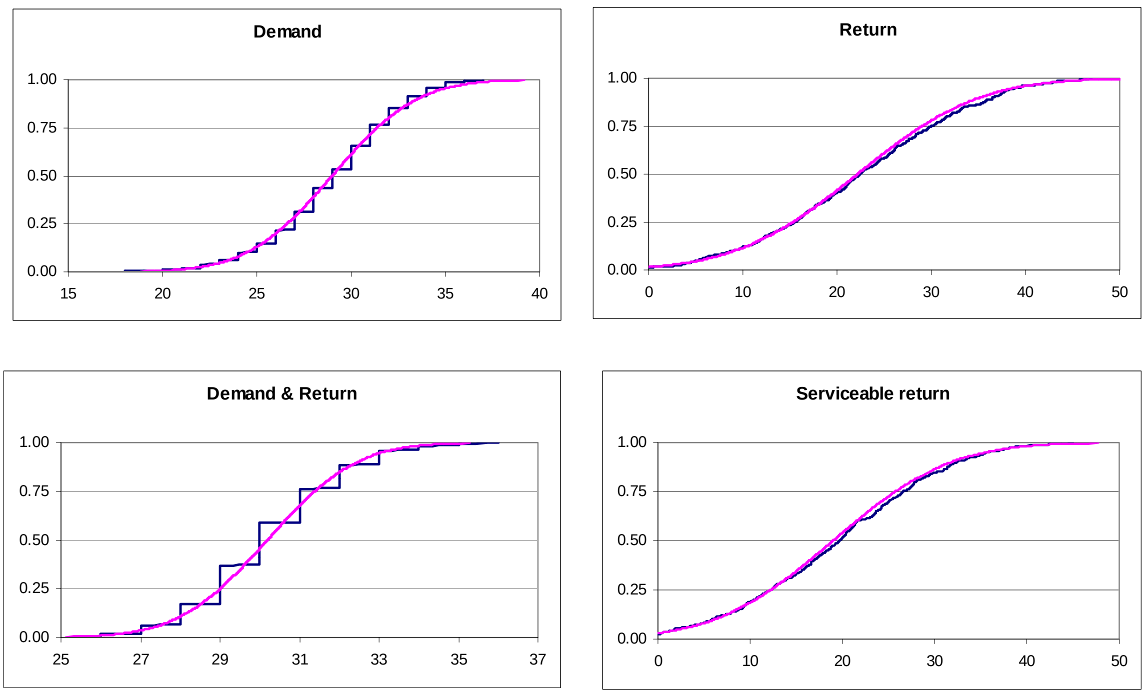

Figure 3.2.1: Comparison of the distribution function of net demand based on 500 random drawings to the normal distribution function with mean and standard deviation as given in Table 3.2.2 $\left(\mu_D = 30, \sigma_D = 10, \mu_R = 15, \sigma_R = 5, \overline{d} = 38, \overline{r} = 13, \overline{s} = 11\right)$

3.2.1 Expressions for the expected total cost

The approximate closed-form expressions that we will derive for the expected total cost for all information cases are based on two assumptions. First, based on the above results, the distribution of net demand is assumed to be normal for all cases. Second, it is assumed that the optimal order quantity is strictly positive. This is justifiable, since the expected demand is usually larger than the expected return in practical situations. Without either of these two assumptions, the cost analysis would be far more complex.

Based on the assumption of normal net demand, it easily follows that the newsvendor equation (3.2) can be rewritten as

$$Q_{\bar{i}}^* = \mu_{N|\bar{i}} + k\sigma_{N|\bar{i}} \tag{3.3}$$

where the *safety factor* k is defined as $k = \Phi^{-1}\left(\frac{p}{h+p}\right)$ and Φ denotes the standard normal distribution function. Combining equation (3.1) and equation (3.3) yields

$$C(Q_{\bar{i}}^*) = hk\sigma_{N|\bar{i}} + (h+p)\sigma_{N|\bar{i}}G(k) = \big(hk + (h+p)G(k)\big)\sigma_{N|\bar{i}}$$

where the loss function $G(v)$ is defined as $G(v) = \int_v^\infty (x-v)\phi(x)dx$ and ϕ denotes the standard normal density function. Note that the expected total cost $C(Q_{\bar{i}}^* \mid \bar{i})$ is linear in the standard deviation of net demand $\sigma_{N|\bar{i}}$. Hence, for each information case it holds that

$$C_{\mathcal{I}} = E_{\bar{i}}C(Q_{\bar{i}}^* \mid \bar{i}) \;=\; \big(hk + (h+p)G(k)\big)E_{\bar{i}}[\sigma_{N|\bar{i}}]. \tag{3.4}$$

Expressions for $E_{\bar{i}}[\sigma_{N|\bar{i}}]$ in the different cases can easily be determined using the results in Table 3.2.1 and are given in Table 3.2.2. The expressions are closed-form for the Base, Demand, and Serviceable return cases, but not the other two cases since for those cases the standard deviation of net demand depends on the realized number of returns. However, it is straightforward to determine a closed-form approximation and upper bound for these two cases by using Jensen's inequality. This inequality states that for any concave function f and stochastic variable X it holds that $E[f(X)] \le f(E[X])$ (Krantz 1999). These upper bounds are also given in Table 3.2.2. We provide insight into the tightness of the upper bounds in Appendix 3 and they are demonstrated numerically in §3.4.

Case	$E_{\bar{i}}[\sigma_{N\mid\bar{i}}]$
Base	$\sqrt{\sigma_D^2+\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}$
Demand	$\sqrt{\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}$
Return	$\int_{\bar{r}}\phi\left(\left(\bar{r}-\mu_R\right)/\sigma_R\right)\sqrt{\sigma_D^2+\gamma(1-\gamma)\bar{r}}\,d\bar{r} \quad \leq \quad \sqrt{\sigma_D^2+\gamma(1-\gamma)\mu_R}$
Demand and Return	$\int_{\bar{r}}\phi\left(\left(\bar{r}-\mu_R\right)/\sigma_R\right)\sqrt{\gamma(1-\gamma)\bar{r}}\,d\bar{r} \quad \leq \quad \sqrt{\gamma(1-\gamma)\mu_R}$
Serviceable Return	σ_D

Table 3.2.2: Expectation $E_{\bar{i}}[\sigma_{N\mid\bar{i}}]$ of the standard deviation of net demand

Combining equation (3.4) and the expectation for (the upper bounds of) $E_{\bar{i}}[\sigma_{N\mid\bar{i}}]$ in Table 3.2.2 leads to the following proposition.

Proposition 4 *If demands and returns are normally distributed, then approximations for the value of information are*

$$\psi_{\mathcal{D}}\approx 1-\frac{\sqrt{\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}}{\sqrt{\sigma_D^2+\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}} \qquad \psi_{\mathcal{R}}\approx 1-\frac{\sqrt{\sigma_D^2+\gamma(1-\gamma)\mu_R}}{\sqrt{\sigma_D^2+\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}}$$

$$\psi_{\mathcal{DR}}\approx 1-\frac{\sqrt{\gamma(1-\gamma)\mu_R}}{\sqrt{\sigma_D^2+\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}} \qquad \psi_{\mathcal{S}}\approx 1-\frac{\sigma_D}{\sqrt{\sigma_D^2+\gamma^2\sigma_R^2+\gamma(1-\gamma)\mu_R}}$$

For the special case with no yield loss ($\gamma=1$) this gives

$$\psi_{\mathcal{D}}\approx 1-\frac{\rho}{\sqrt{1+\rho^2}} \quad \text{and} \quad \psi_{\mathcal{R}}\approx 1-\frac{1}{\sqrt{1+\rho^2}}, \text{ where } \rho=\sigma_R/\sigma_D.$$

As for the situation with uniform demands and returns (see Propositions 2 and 3 of §3.1), it turns out that the (approximate) value of demand, as well as return information, is completely determined through ρ, and that the value of demand information is larger if and only if ρ is less than one. The exact expressions for the VOI are not the same though.

3.3 Generalized model

In this section, we build upon the results from §3.1 and §3.2 and develop a generalized model on the VOI. This model enables a complete investigation into the conditions in which each type of information (demand, return, and yield) is most valuable and also enables a relative comparison of value between different types of information. We begin by restating a key result from §3.2 that the VOI can be expressed, approximately, in terms of a reduction in the standard deviation of net demand, which we consider to be a proxy for the level of uncertainty. From this perspective, we proceed in our analysis on the VOI by using the approximations as *estimators* for the VOI. By doing so, we enable a comprehensive and holistic evaluation of the VOI. We will later show in §3.4 that the theoretical insights we obtain here largely explain the behavior we observe in numerical results.

In this section, we use the notation $\hat{x}$ to denote an estimator of x. Therefore, let $\hat{\psi}_{\mathcal{I}}$ denote an estimator of $\psi_{\mathcal{I}}$ as set forth in Proposition 4 of §3.2. Furthermore, let $\hat{\sigma}_S$ and $\hat{\sigma}_N$ denote estimators for σ_S and σ_N respectively, where $\hat{\sigma}_S = \sqrt{\gamma^2\sigma_R^2 + \gamma(1-\gamma)\mu_R}$ and $\hat{\sigma}_N = \sqrt{\sigma_D^2 + \gamma^2\sigma_R^2 + \gamma(1-\gamma)\mu_R}$.

3.3.1 On the value of information

From our definitions for the estimators of the VOI, we find the VOI depends on both the amount of uncertainty that information explains and the overall level of uncertainty. In Table 3.3.1, we summarize the functional relationship between $\hat{\psi}_{\mathcal{I}}$ and the parameters σ_D, σ_R, μ_R, and γ that influence $\hat{\sigma}_N$.

Value of Information	Parameter			
	σ_D	σ_R	μ_R	γ
$\hat{\psi}_{\mathcal{D}}$	increasing	decreasing	decreasing	convex
$\hat{\psi}_{\mathcal{R}}$	decreasing	increasing	decreasing	increasing
$\hat{\psi}_{\mathcal{DR}}$	increasing	increasing	decreasing	convex
$\hat{\psi}_{\mathcal{S}}$	decreasing	increasing	increasing	concave

Table 3.3.1: Sensitivity of $\hat{\psi}_{\mathcal{I}}$ with respect to each parameter

Note that $\hat{\sigma}_N$ is strictly increasing with respect to each parameter, except γ. Consequently, we find that $\hat{\psi}_{\mathcal{I}}$ is either monitonically increasing or decreasing with respect to each parameter, except γ as shown in Table 3.3.1. Now, with respect to γ, $\hat{\sigma}_N$ is largest at $\gamma = 0.5$ and smallest at $\gamma = 0.0$ and $\gamma = 1.0$. In turn, this relationship gives rise to the convex and concave functional relationships between γ and $\hat{\psi}_{\mathcal{I}}$ for each case as listed in Table 3.3.1 and we illustrate these in Figure 3.3.1 with an example where $\sigma_D = \sigma_R = 2$ and $\mu_R = 25$

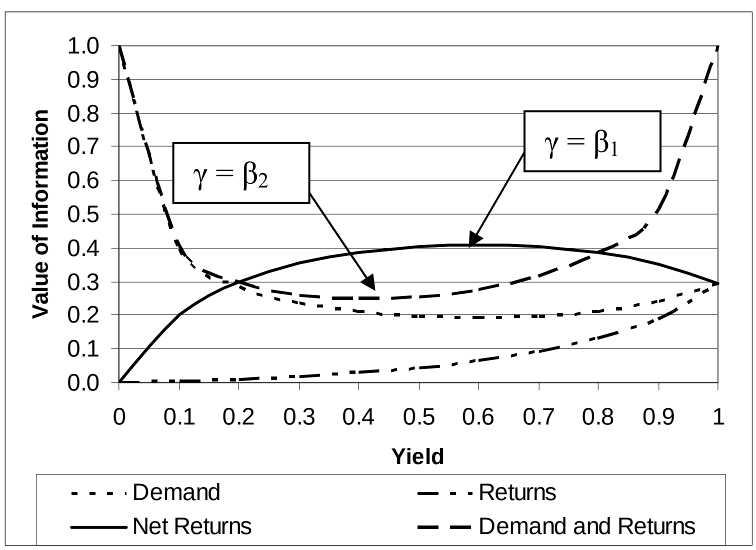

Figure 3.3.1: $\hat{\psi}_{\mathcal{I}}$ as a function of yield where $\sigma_D = \sigma_R = 2$ and $\mu_R = 25$.

Note that only for $\hat{\psi}_{\mathcal{R}}$ do we find that the VOI is strictly increasing with respect to γ. We observe that both $\hat{\psi}_{\mathcal{D}}$ and $\hat{\psi}_{\mathcal{DR}}$ are convex with respect to γ, with minimum values at β_1 and

β_2 respectively. Conversely, $\hat{\psi}_{\mathcal{S}}$ is concave with respect to γ, with a maximum value at β_1. The analytic expressions for both β_1 and β_2 are easily derived by solving the derivative of $\hat{\psi}_{\mathcal{I}}$ with respect to γ for the corresponding information case. For example, we find

$$\beta_1 = -\frac{\mu_R}{2\left(\sigma_R^2 - \mu_R\right)} \text{ for } 0 \le \frac{\mu_R}{2\left(\sigma_R^2 - \mu_R\right)} \le 1 \text{ and } \sigma_R^2 \ne \mu_R \text{, otherwise } \beta_1 = 1.$$

Further examination of Figure 3.3.1 also provides us with some intuitively appealing results. We observe for example that the value of demand information is greatest at $\gamma = 0$. Under this condition, there is no uncertainty with respect to returns or yield – demand is only met with newly purchased items. Therefore, $\hat{\psi}_{\mathcal{R}} = \hat{\psi}_{\mathcal{S}} = 0$ and since all of the uncertainty in net demand is solely attributed to demand, $\hat{\psi}_{\mathcal{D}} = \hat{\psi}_{\mathcal{DR}} = 1.0$. In other words, there is no residual uncertainty once demand is explained.

As yield increases from zero, there is increasing uncertainty with respect to yield and returns so that $\hat{\psi}_{\mathcal{R}}$ and $\hat{\psi}_{\mathcal{S}}$ are both increasing while $\hat{\psi}_{\mathcal{D}}$ is decreasing. These results demonstrate that the value of any type of information is proportional to the portion of total uncertainty it seeks to explain. As yield increases from zero, σ_D represents a smaller portion of $\hat{\sigma}_N$ and consequently, $\hat{\psi}_{\mathcal{D}}$ must decrease.

Now, consider the VOI for each case on the right hand side of Figure 3.3.1 where $\gamma = 1$. As we find for $\gamma = 0$, there is no uncertainty with respect to yield (since all units are serviceable) so that all the uncertainty that arises from serviceable returns arises from the uncertainty in the returns process itself. Hence, $\hat{\psi}_{\mathcal{R}} = \hat{\psi}_{\mathcal{S}}$ and $\hat{\psi}_{\mathcal{DR}} = 1.0$. Moreover, since $\sigma_D = \sigma_R$ in this particular example, then $\hat{\psi}_{\mathcal{D}} = \hat{\psi}_{\mathcal{R}}$.

As yield decreases from one, the uncertainty with respect to yield increases and at the same time, uncertainty with respect to the return process decreases since it is clear that $\gamma^2\sigma_R^2$ is also decreasing. Therefore, we observe a decrease in $\hat{\psi}_{\mathcal{R}}$. In our particular example, the increase in yield uncertainty is greater than the decrease in return uncertainty so that the *overall* level of uncertainty with respect to serviceable returns is greater. Hence we observe an increase in $\hat{\psi}_{\mathcal{S}}$ and a corresponding decrease in $\hat{\psi}_{\mathcal{D}}$. This will always occur if $\frac{\mu_R}{\sigma_R^2} > 2$ and we find that for γ, $\beta_1 \leq \gamma \leq 1$, $\hat{\psi}_{\mathcal{S}}$ is decreasing with respect to γ while $\hat{\psi}_{\mathcal{D}}$ is increasing with respect γ. However, if $\frac{\mu_R}{\sigma_R^2} \leq 2$, then for γ, $0 \leq \gamma \leq 1$, $\hat{\psi}_{\mathcal{S}}$ is increasing with respect to γ and $\hat{\psi}_{\mathcal{D}}$ is decreasing with respect to γ. We illustrate this latter scenario in Figure 3.3.2 with a numerical example where $\sigma_D = \sigma_R = 2$ and $\mu_R = 3$.

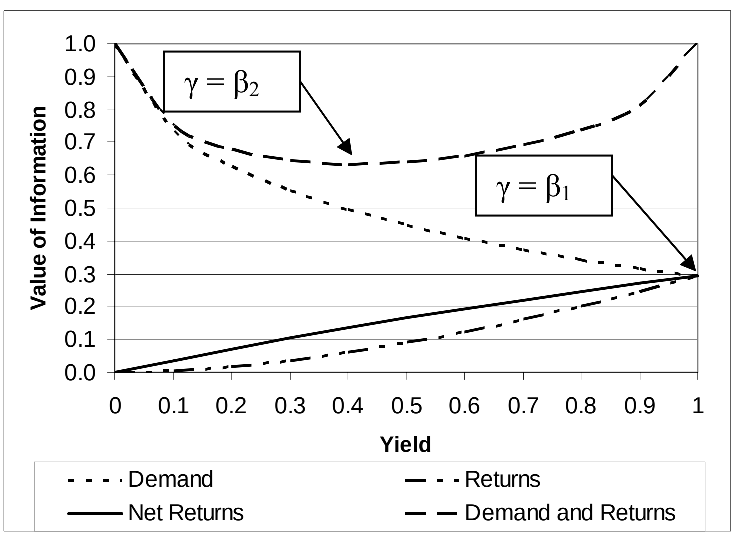

Figure 3.3.2: $\hat{\psi}_{\mathcal{I}}$ as a function of yield where $\sigma_D = \sigma_R = 2$ and $\mu_R = 3$.

We can also see from just the two illustrative examples provided in figures 3.3.1 and 3.3.2 that there is no strict dominance relationship between $\hat{\psi}_{\mathcal{D}}$ and $\hat{\psi}_{\mathcal{S}}$, between $\hat{\psi}_{\mathcal{D}}$ and $\hat{\psi}_{\mathcal{R}}$, and between $\hat{\psi}_{\mathcal{DR}}$ and $\hat{\psi}_{\mathcal{S}}$. Clearly, $\hat{\psi}_{\mathcal{DR}}$ will always be equal to or greater than either $\hat{\psi}_{\mathcal{D}}$ or $\hat{\psi}_{\mathcal{R}}$. Likewise, $\hat{\psi}_{\mathcal{S}}$ will always be equal to or greater than $\hat{\psi}_{\mathcal{R}}$. In the next section, we clarify the relationships with respect to the relative VOI between the other information cases.

3.3.2 On the relative value of information

In §3.1, we found that $\psi_{\mathcal{D}} = \psi_{\mathcal{R}}$ for $\sigma_D = \sigma_R$ and $\gamma = 1$. This result also indicates that $\psi_{\mathcal{D}} = \psi_{\mathcal{S}}$ for $\sigma_D = \sigma_S$ and $\gamma = 1$. Moreover, we need to look no further than the equations for the estimators of the VOI to know that $\hat{\psi}_{\mathcal{D}} = \hat{\psi}_{\mathcal{S}}$ for $\sigma_D = \hat{\sigma}_S$, independent of γ. In fact, the VOI for a given information case will equal the VOI for another information case whenever the proportion of total uncertainty they respectively explain is the same. Consequently, to find the conditions when the VOI for two information cases are equal (or different), it is simply a matter of 1) formulating the difference between the respective VOI estimators, 2) setting the difference to zero, and 3) solving with respect to a parameter of interest.

To demonstrate, we consider the example of determining when $\hat{\psi}_{\mathcal{D}} = \hat{\psi}_{\mathcal{S}}$ with respect to μ_R. Here, we find $\mu_R = \dfrac{\sigma_D^2 - \gamma^2 \sigma_R^2}{\gamma(1-\gamma)}$ and illustrate this point of equality in Figure 3.3.3 for a numerical example where $\sigma_D = \sigma_R = 2$ so that $\mu_R = \dfrac{\sigma_D^2 - \gamma^2 \sigma_R^2}{\gamma(1-\gamma)} = \dfrac{0.8^2 \cdot 2^2 - 2^2}{0.8(0.8-1)} = 9$ and for $\mu_R > 9$, $\hat{\psi}_{\mathcal{S}} > \hat{\psi}_{\mathcal{D}}$. The same approach can be taken in determining the relative VOI between $\hat{\psi}_{\mathcal{S}}$ and $\hat{\psi}_{\mathcal{DR}}$ with respect to μ_R or, for that matter, the relative VOI between any two information cases and with respect to any of the parameters.

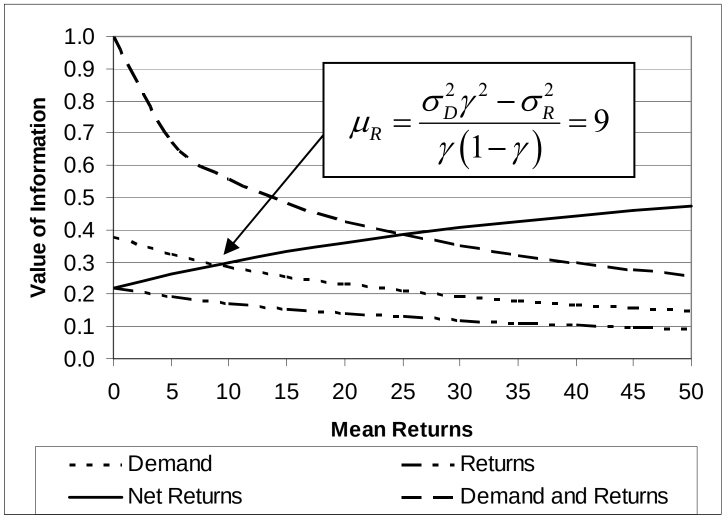

Figure 3.3.3: $\hat{\psi}_{\mathcal{I}}$ as a function of mean returns where $\sigma_D = \sigma_R = 2$ and $\gamma = 0.80$

3.4 Numerical examples

We now proceed to demonstrate the explanatory power of the model presented in §3.3 using a set of numerical examples. In these examples, we compare the estimated VOI from our model that arises from our approximations to the exact VOI that arises from an exhaustive search for the optimal solution in each information case. We find that our numerical results for the VOI demonstrate a high degree of correspondence with the theoretical model in terms of 1) the VOI for each information case, 2) sensitivity with respect to the parameters, and 3) the relative VOI among the information cases.

From a practical perspective, any numerical test will require certain distributional assumptions. Both of the analyses in §3.1 and §3.2 assume continuous demand and return distributions. Yet, our interest in demonstrating the generality, and more importantly, the relevance, of our model to practice lead us to consider discrete distributions. Moreover, there is no other practical or meaningful way to accommodate yield loss. Hence, we test the model with discrete uniform and discrete normal distributions. The flatness of the uniform distribution as compared to the bell-shaped symmetry of the normal distribution provides a test of the model's robustness with respect to distributional assumptions.

Clearly, a discrete uniform distribution is straight-forward to implement numerically. A discrete normal distribution, however, requires further consideration because there is no standardized discrete version and it is appropriate to avoid the realization of negative demands. Our approach is as follows. Let $\phi(x)$, $x = 0, 1, 2...$ denote the probability mass function of a discrete normal random variable with mean μ and variance σ^2 and let $\Phi(\cdot)$ denote the cumulative density function for a continuous normal random variable with the same mean and variance. Furthermore, we truncate the distribution between zero and a value z such that $\Phi(z) \geq 0.999$. Consequently,

$$\phi(x) = \begin{cases} \Phi(0.5) & x = 0 \\ \Phi(x+0.5) - \Phi(x-0.5) & 0 < x < z \\ 1 - \Phi(z-0.5) & x = z \end{cases}.$$

Through a series of numerical experimentation, we find that the transformation results in an approximately normal random variable, with a mean that is within 0.1% of μ and a variance that is within 2% of σ^2, so long as the density below zero of the continuous random variable is negligible (<0.005). We conclude that the approximation is sufficient for our test purposes and proceed below with both qualitative and quantitative assessments of the theoretical VOI.

3.4.1 Qualitative assessment

We choose two quite different examples to qualitatively illustrate how well the results from the theoretical model explain the behavior we observe in numerical tests on the VOI. The first example considers the case $\sigma_D = \sigma_R = 2$, $\mu_D = 50$, $\mu_R = 20$, $p = 10$, and $h = 1.0$. Figure

3.4.1 displays $\psi_{\mathcal{I}}$ as a function of γ. Figure 3.4.1 (and later Figure 3.4.2) is actually composed of three separate charts. The left-most chart presents the numerical results for the discrete normal distribution, while the center chart presents the theoretical results, and the right-most chart presents the numerical results for the discrete uniform distribution.

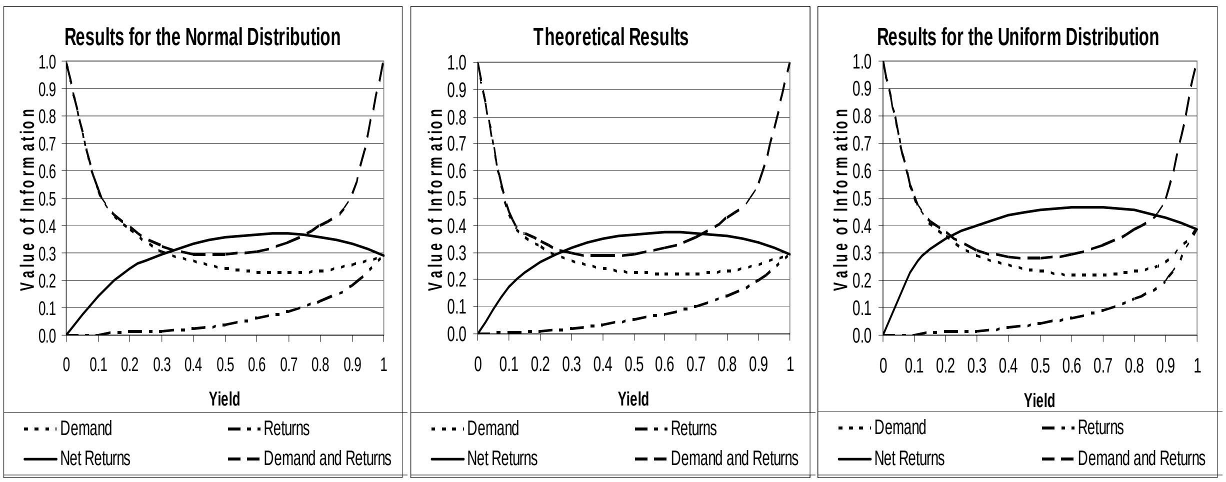

Figure 3.4.1: Comparison of numerical results for uniform and normal distributions with theoretical results where $\sigma_D = \sigma_R = 2$, $\mu_D = 50$ $\mu_R = 20$, $p = 10$, and $h = 1.0$.

A comparison among the three charts in Figure 3.4.1 shows the high degree of correspondence between the numerical results and the theoretical results. The VOI reported at each value of γ for the discrete normal distribution is nearly identical to that of the theoretical results. Moreover, while the results for the uniform distribution are not quite as close, we qualitatively observe the same relationships.

In Figure 3.4.2 we extend the comparisons to the case where $\sigma_D = 2$, $\sigma_R = 4$, and $\mu_R = 30$, while all other parameter values the same as before. Here, we have a very different picture than observed in Figure 3.4.1. In this case, the much greater uncertainty of the returns process relative to the demand process significantly alters the relative VOI among the information cases. Even so, we find a very high degree of correspondence between the numerical results for both distributions and the theoretical results.

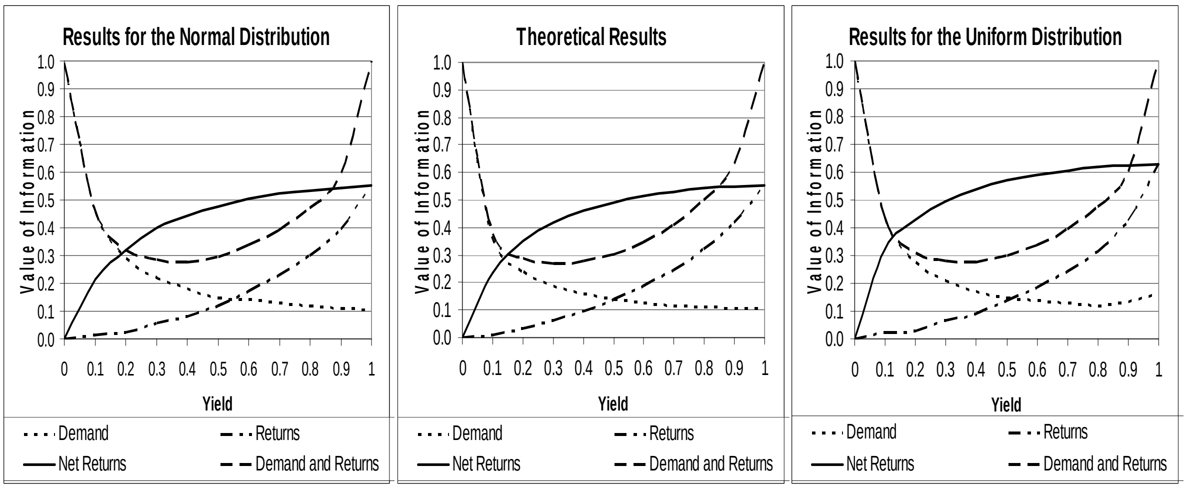

Figure 3.4.2: Comparison of numerical results for uniform and normal distributions with theoretical results where $\sigma_D = 2$, $\sigma_R = 4$, $\mu_D = 50$, $\mu_R = 30$, $p = 10$, and $h = 1.0$.

Collectively, the comparisons made between the numerical results and the theoretical results in these two illustrative examples are representative of the more robust set of tests we explore in the next section.

3.4.2 Quantitative assessment

We employ a factorial design on the set of parameter values listed in Table 3.4.1. The values chosen for σ_D and σ_R enable an exact and equivalent comparison between uniform and normal distributions where we can explore the behavior exhibited by the VOI for cases in which the uncertainty in demand is equal to, less than, or greater than that in the returns process. The values for yield are limited to those that are generally observed to be economical in remanufacturing practice (Ferrer and Ketzenberg 2004). We vary the values of the penalty cost over a range to ensure a high service level and fix the holding cost at $1.00. The values for the mean return rate correspond to fractions $(0.2, 0.4, 0.6)$ of the mean demand rate where $\mu_D = 50$.

Parameter	Values
σ_D	1.41, 2.00, 3.16
σ_R	1.41, 2.00, 3.16
μ_R	10, 20, 30
γ	0.7, 0.8, 0.9, 1.0
p	5, 10, 20

Table 3.4.1: Factorial Design

With a full factorial design for the parameters as specified in Table 3.4.1, there are 324 numerical examples for each distribution and these are duplicated for the theoretical model in order to provide a comparative analysis. It is also appropriate to mention that the set of numerical examples is, at least partially, constrained by our interest in comparing the results of the uniform distribution and the normal distribution. Clearly, there is less flexibility with choosing a value for the standard deviation of a uniform distribution. Another limitation arises with respect to the discrete normal distribution in that we are restricted to selected values of μ and σ^2 to ensure that there is little density below zero in the corresponding continuous case.

We report the results of the VOI for each information case in Table 3.4.2. The values for the VOI are transformed to percentages for readability. In this table, the VOI for each information case is reported in rows and the value reported in a column corresponds to the average VOI across all examples for a fixed parameter value as indicated by the column header. We have omitted the results with respect to the penalty cost as we have not observed any meaningful sensitivity with respect to this parameter – exactly as predicted by the theoretical model. Note also that there are three rows of results exhibited for each information case: one each for the results of the normal distribution, theoretical model, and uniform distribution.

		Parameter												
		σ_D			σ_R			μ_R			γ			
VOI	Case	1.4	2.0	3.2	1.4	2.0	3.2	10	20	30	0.7	0.8	0.9	1.0
$\psi_{\mathcal{D}}$	N	15.8	25.1	40.8	34.1	28.1	19.3	31.0	26.8	23.8	25.2	25.4	26.9	31.4
	T	15.5	24.7	40.4	34.3	27.8	18.6	30.1	26.5	24.0	24.6	25.2	26.9	30.9
	U	18.5	27.6	42.2	35.4	30.3	22.5	33.8	28.8	25.6	24.8	25.5	28.0	39.3
$\psi_{\mathcal{R}}$	N	26.0	19.9	12.1	10.5	17.3	30.2	21.3	19.1	17.6	10.7	14.7	20.6	31.4
	T	27.4	20.8	12.5	11.1	18.2	31.4	22.2	20.0	18.5	12.1	16.1	21.8	30.9
	U	27.9	22.5	15.2	13.2	20.0	32.4	24.4	21.5	19.8	11.1	15.3	21.8	39.3
$\psi_{\mathcal{DR}}$	N	53.8	58.7	67.3	56.1	58.9	64.8	67.6	58.7	53.4	39.2	44.9	55.5	100.0
	T	56.4	61.0	69.1	58.4	61.2	66.9	69.9	61.0	55.5	41.2	47.9	59.5	100.0
	U	52.6	57.6	65.9	54.8	57.7	63.6	66.2	57.5	52.4	37.7	43.3	53.8	100.0
$\psi_{\mathcal{S}}$	N	48.5	35.8	20.9	27.7	33.4	44.1	31.3	35.4	38.5	37.1	36.7	35.1	31.4
	T	47.8	35.3	20.4	26.8	32.9	43.9	31.0	34.8	37.8	36.8	36.1	34.4	30.9
	U	57.2	44.5	28.4	36.7	42.0	51.3	39.7	43.7	46.6	45.7	45.1	43.4	39.3

Table 3.4.2: Sensitivity analysis of the VOI for discrete normal (N), theoretical results (T), and discrete uniform (U). The values for $\psi_{\mathcal{I}}$ have been transformed into percentages for readability.

The results reported in Table 3.4.2 build on the illustrative examples reported earlier. Clearly, the VOI reported for the normal distribution across all parametric settings are quite close to the VOI we obtain for the theoretical model. Further, the VOI reported for the uniform distribution, while not as close, are approximately the same as the model. Note that the sensitivity of the VOI to each parameter is the same for each distribution and is consistent with the model. There are, however, differences, between the theoretical results and the numerical results for each distribution that warrant closer examination. For example, note that the theoretical model appears to under-estimate $\psi_{\mathcal{D}}$, $\psi_{\mathcal{R}}$, and $\psi_{\mathcal{S}}$, while over-estimate $\psi_{\mathcal{DR}}$ for the uniform distribution. We explore this observation in Table 3.4.3.

Table 3.4.3 reports the difference in the VOI between the theoretical results and the numerical results $\left(\hat{\psi}_{\mathcal{I}} - \psi_{\mathcal{I}}\right)$ for each distribution according to percentiles within the set of 324 numerical examples. The zero percentile reports the smallest difference between the theoretical results and the numerical results for a given information case as specified by the column header. The 0.50 percentile reports the median difference and the 1.00 percentile reports the largest difference. Note that negative values indicate the theoretical model under-estimates the VOI, while positive values indicate the theoretical model over-estimates the VOI.

Percentile	Difference between Theoretical Results and Normal Distribution Results				Difference between Theoretical Results and Uniform Distribution Results			
	$\psi_{\mathcal{D}}$	$\psi_{\mathcal{R}}$	$\psi_{\mathcal{DR}}$	$\psi_{\mathcal{S}}$	$\psi_{\mathcal{D}}$	$\psi_{\mathcal{R}}$	$\psi_{\mathcal{DR}}$	$\psi_{\mathcal{S}}$
0.00	-5.2	-5.2	0.0	-5.5	-18.6	-18.6	0.0	-20.1
0.05	-2.1	-1.0	0.0	-3.1	-13.2	-3.2	0.0	-17.2
0.10	-1.4	-0.6	0.0	-2.3	-10.3	-0.3	0.0	-15.9
0.25	-0.8	-0.1	0.3	-1.3	-4.2	-2.7	0.7	-13.2
0.50	-0.3	0.8	2.3	-0.6	-1.5	-0.2	3.0	-7.6
0.75	0.3	1.8	3.4	-0.2	.4	1.3	5.0	-3.0
0.90	0.1	2.8	4.5	0.2	1.7	2.6	6.6	-2.2
0.95	1.9	3.5	5.2	0.5	2.9	3.6	7.5	-2.0
1.00	3.8	4.9	9.0	1.5	6.4	6.1	9.6	-0.7

Table 3.4.3: Difference (x 100) in the VOI reported between the normal distribution and the theoretical model (left) and between the uniform distribution and the theoretical model (right).

The differences reported for the normal distribution are mostly as expected. For all information cases, except $\psi_{\mathcal{DR}}$ even the largest absolute differences (zero and one percentiles) between the theoretical model and the numerical results are quite small. Even for $\psi_{\mathcal{DR}}$ the differences are quite small for a clear majority of the examples. However, there appears to be a bias in over-estimating $\psi_{\mathcal{DR}}$. We believe over-estimation arises because the theoretical model does not account for the costs associated with returns in excess of demand. Hence, even full information will not necessarily reduce cost to zero as predicted by the model. Even so, we note that the bias exceeds 0.05 for less than 5% of the examples and the largest difference is 0.09.

As for the comparison of the VOI between the theoretical model and the uniform distribution, the difference between the two, while quite small for a preponderance of the cases, can be large – particularly in terms of under-estimating the VOI. Consider for example that across all numerical examples, the theoretical model under-estimates $\psi_{\mathcal{S}}$. We believe that at least a partial explanation arises from the fact that under the uniform distribution, the VOI is proportional to a decrease in the sum of the standard deviations for demand and serviceable returns, rather than the standard deviation of net demand as explained in §3.1. For any example, $\sqrt{\sigma_D^2 + \sigma_R^2} > \sigma_D + \sigma_R$. Accordingly, we should find that the uncertainty regarding a process will be under-estimated (since the denominator in $\hat{\psi}_{\mathcal{I}}$ is always larger) and it follows that we should also expect the VOI to be under-estimated.

4 Conclusion

In this research, we have studied the VOI in the context of a firm that can satisfy demand with either new product, remanufactured product, or a mix of both types. There are three potential sources of uncertainty: demand, return, and yield. We developed a theoretical model that provides complete, albeit approximate analysis on the VOI. A clear result is that the value of any type of information is proportional to the amount of uncertainty that it seeks to explain, where the standard deviation of net demand is a proxy for uncertainty. Indeed, our model and results both formalize and verify our intuition.

There are several contributions that arise from our analysis. The model, as encapsulated in Proposition 4, can be used to determine potential gains from information on demand, return, and yield. This can be seen more directly by a simple comparison among σ_D^2, $\gamma^2\sigma_R^2$, and $\gamma(1-\lambda)\mu_R$. The larger the value expressed by a term, the larger the potential gain from reducing the corresponding type of uncertainty. Moreover, Proposition 4, and as numerically demonstrated in Table 3.4.2, indicates that $\psi_{\mathcal{D}} + \psi_{\mathcal{R}} \le \psi_{\mathcal{DR}}$ and similarly, $\psi_{\mathcal{D}} + \psi_{\mathcal{S}} \le 1.0$. The message is to invest in more than one type of information, even if one type of uncertainty dominates, since the return on investment will be greater.

While our analysis is squarely focused on the value of information, the model also generalizes to determine the value of *any* investments to reduce uncertainty or otherwise influence the model parameters since value is implicit in the reduction of the standard deviation of net demand. For example, the model enables an evaluation of the value to improve the quality of returns (and hence yield), perhaps through investments in product durability. Naturally, the model extends to enable a comparison of the relative value among alternative investments and can also be used to provide a sensitivity analysis.

Just as the model generalizes to the value of investments other than information, we believe that the model should also generalize to a framework for evaluating the value of other types of information that we have not explicitly incorporated into the model. Indeed, this represents one important avenue for future research. Consider that since value is implicit in uncertainty and the information that can reduce it, then theoretically the model should also be able to address other types of uncertainty and hence other types of information. For example, we have not addressed uncertainty with respect to supplier service which is another potential source of uncertainty that may influence the ordering decision.

There are several other important directions for future research. First, our newsvendor solution and the optimality condition developed in this paper provide a good starting point for research on the multi-period case. Generally, a newsvendor solution can be used to determine an approximately optimal order quantity in a multi-period setting without a fixed setup or ordering cost. This suggests that our results are also indicative for the multi-period case. It should also be interesting to study the case where the return rate is not independent of the demand rate.

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References

Bain, L. and M. Engelhardt 1987. *Introduction to Probability and Mathematical Statistics*, Bost, MA, PWS Publishers.

Bourland, K. E., S. G. Powell, and D. F. Pyke 1996. Exploiting timely demand information to reduce inventories, *European Journal of Operational Research*, **92**, 239–253.

Cachon, G. P. and M. Fisher 2000. Supply chain inventory management and the value of shared information, *Management Science* **46**(8), 1032–48.

Chen, F., 2002, "Information Sharing and Supply Chain Coordination," To appear in the Handbook of Operations Research and Management Science: Supply Chain Management, by T. de Kok and S. Graves (eds.), North-Holland.

Chen, F. and B. Yu., 2001. Quantifying the Value of Leadtime Information in a Single-Location Inventory System, Working Paper, Columbia Business School Working Paper, New York, NY.

De Brito, M.P. and E. van der Laan, 2002. Inventory Control with Product Returns: The Impact of (Mis)information, Erasmus University, Econometric Institute Report EI 2002-29, 2002.

De Brito, M.P. and R. Dekker, 2004. "A framework for Reverse Logistics" in R. Dekker, K. Inderfurth, L. van Wassenhove and M. Fleischmann (eds.), Quantitative Approaches to Reverse Logistics, Springer, Berlin Heidelberg.

Dekker, K. Inderfurth, L. van Wassenhove and M. Fleischmann (eds.), 2004. Quantitative Approaches to Reverse Logistics, Springer, Berlin Heidelberg.

Ferrer, G. and M. Ketzenberg, 2004. Value of Information in Remanufacturing Complex Products, *IIE Transactions*, **36**(3), 265-278.

Fleischmann, M., 2000. *Quantitative Models for Reverse Logistics*, Rotterdam, The Netherlands: Erasmus University Rotterdam Ph.D. Thesis.

Gavirneni, S., R. Kapuscinski and S. Tayur, 1999. Value of information in capacitated supply chains, *Management Science*, **45**(1), 16–24.

Guide, Jr., V.D.R., 2000. Production Planning and Control for Remanufacturing: Industry Practice and Research Needs, *Journal of Operations Management*, **18**, 467-483.

Guide Jr., V. D. and L. N. Van Wassenhove (eds.), 2003. "Business Perspectives in Full Cycle Supply Chains", Carnegie Bosch Institute, Pittsburgh, PA, 2003.

Huang, G., J. Lau, and K. Mak 2003. The impacts of sharing production information on supply chain dynamics: a review of the literature, *International Journal of Production Research*, **41**(7), 1483-1518.

Lund, R., 1983. *Remanufacturing: United States Experience and Implications for Developing Nations*. The World Bank, Washington, DC.

Ketzenberg, M. and M. Ferguson 2004, Information Sharing to Improve Retail Product Freshness of Perishables, Colorado State University Working Paper, College of Business.

Ketzenberg, M., G. Souza, and D. Guide 2004. Mixed Assembly and Disassembly Operations for Remanufacturing, forthcoming in *Production and Operations Management*.

Krantz, S. G. 1999. Jensen's Inequality, §9.1.3 in Handbook of Complex Variables. Boston, MA: Birkhäuser, p. 118, 1999.

Moinzadeh, K., 2002. A Multi–Echelon Inventory System with Information Exchange, *Management Science*, **48**(3), 414–426.

Mostard, J. and R. Teunter 2002. The Newsboy Problem with Resalable Returns, Erasmus Report Series in Management, ERS-2002-89-LIS, Erasmus Research Institute of Management (ERIM), Erasmus University Rotterdam.

Rogers, D. S. and R. S. Tibben-Lembke,1999. "Going Backwards: Reverse Logistics Trends and Practices", Reverse Logistics Executive Council (RLEC), Pittsburgh, PA.

Sahin, F. and E. P. Robinson 2002. Flow Coordination and Information Sharing in Supply Chains: Review, Implications, and Directions for Future Research, *Decision Sciences*, **33**(4), 1-32.

Silver, E.A., D.F. Pyke and R. Peterson 1998. *Inventory Management and Production Planning and Scheduling,* Third Edition, New York, NY: John Wiley and Sons, Inc.

Simpson, V.P. 1978. Optimum solution structure for a repairable inventory problem, *Operations Research*, **26**, 270-281.

Souza, G., M. Ketzenberg, and D. Guide, 2002. Capacitated Remanufacturing with Service Level Constraints, *Production and Operations Management*, **11**, Summer 2002, 231-248.

Van der Duyn Schouten, F. A., M. J. G. van Eijs, and R. M. J. Heuts 1994. The value of supplier information to improve management of a retailer's inventory, *Decision Sciences,* **25**(1), 1–14.

Vlachos, D. and R. Dekker 2003. Return Handling Options and Order Quantities for Single Period Product, *European Journal of Operational Research*, **151**(1), 38-52.

Wagner, H.M. 1969. *Principles of Operations Research*, Englewood Cliffs, NJ: Prentice-Hall, Inc.

Appendix 1
Table of Notation

D	number of demands (stochastic variable)
$\bar{d}$	number of demands (realization)
μ_D	expected number of demands
σ_D	standard deviation of the number of demands
R	number of returns (stochastic variable)
$\bar{r}$	number of returns (realization)
μ_R	expected number of returns
σ_R	standard deviation of the number of returns
S	number of serviceable returns (stochastic variable)
$\bar{s}$	number of serviceable returns (realization)
μ_S	expected number of serviceable returns
σ_S	standard deviation of the number of serviceable returns
N	net demand $N = (D - S)$
$\mathcal{I}$	information case, where $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{R}, \mathcal{DR}, \mathcal{S}\}$
$\bar{i}$	additional information relative to the base case, $\bar{i} \subset \{\bar{d}, \bar{r}, \bar{s}\}$
$\mu_{N\|\bar{i}}$	mean of net demand given information $\bar{i}$
$\sigma_{N\|\bar{i}}$	standard deviation of net demand given information $\bar{i}$
γ	probability that a returned item can be remanufactured
p	penalty cost
h	holding cost
Q	order quantity
$Q_{\bar{i}}^*$	optimal order quantity given information $\bar{i}$
$C(Q \mid \bar{i})$	total expected cost for order quantity Q given information $\bar{i}$
$\psi_{\mathcal{I}}$	value of information

Appendix 2
Proof of Lemma 1

No information

Assume $c_R \le c_D$. The CDF of net demand is then given as follows.

$$F_N(z) = \text{Prob}(D-R<z) = \begin{cases} \frac{(z-a_D+c_R)^2}{2c_Dc_R}, & a_D - c_R \le z \le a_D \\ \frac{(c_R+2(z-a_D))}{2c_D}, & a_D \le z \le b_D - c_R \\ 1-\frac{(z-b_D)^2}{2c_Dc_R}, & b_D - c_R \le z \le b_D \end{cases}$$

The total expected costs $C(Q)$ are optimized for $Q^* = F_N^{-1}(\frac{p}{h+p})$. If $\frac{p}{h+p} \ge F_N(b_d - c_r) = 1 - \frac{c_R}{2c_D}$ (condition 1a)**,** then Q^* is determined by the inverse of $F_N(z)$ that corresponds with the right hand tail, i.e., $b_D - c_R \le z \le b_D$:

$$Q^* = F_N^{-1}(\frac{p}{h+p}) = b_D - \sqrt{\frac{2c_Dc_Rh}{h+p}}$$

Note that under condition 1a, which can be rephrased as $\frac{h}{h+p} \le \frac{c_R}{2c_D}$, we have $Q^* \ge b_D - c_r \ge 0$.

Inserting Q^* in the cost function gives

$$\begin{aligned} C(Q^*) &= p(\mu_N - Q^*) + (h+p)\int_{a_D-c_R}^{Q^*} F_N(z)\,\mathrm{d}z \\ &= p(\mu_N - Q^*) + (h+p)\left(b_D - \mu_N - \int_{Q^*}^{b_D}\left(1-\frac{(z-b_D)^2}{2c_Dc_R}\right)\mathrm{d}z\right) \\ &= p(\mu_N - Q^*) + (h+p)\left(Q^* - \mu_N - \frac{(Q^*-b_D)^3}{6c_Dc_R}\right) \\ &= h\left(\frac{c_D+c_R}{2} - \frac{2}{3}\sqrt{\frac{2c_Dc_Rh}{h+p}}\right) \end{aligned}$$

where we used that $\int_{a_D-c_R}^{b_D} F_N(z)\mathrm{dz} = b_D - \mu_N$.

Now assume $c_R \ge c_D$. The CDF of net demand is then given as follows.

$$F_N(z)=\text{Prob}(D-R<z)=\begin{cases}\frac{(z-a_D+c_R)^2}{2c_Dc_R}, & a_D-c_R\le z\le b_D-c_R\\ \frac{2(z+c_R-a_D)-c_D}{2c_R}, & b_D-c_R\le z\le a_D\\ 1-\frac{(z-b_D)^2}{2c_Dc_R}, & a_D\le z\le b_D\end{cases}$$

If $\frac{p}{h+p}\ge F_N(a_D)=1-\frac{c_D}{2c_R}$ (condition 1b)**,** then the optimal value of Q is determined by the inverse of $F_N(z)$ that corresponds to the right hand tail, i.e., $a_D\le z\le b_D$. This function is exactly the same as for the previous case, $c_R\le c_D$. The characterization of the optimal order quantity and the optimal costs are therefore exactly the same for both cases. Note that under condition 1b, which can be rephrased as $\frac{h}{h+p}\le\frac{c_D}{2c_R}$, we have $Q^*\ge a_D\ge 0$.

Demand information

The CDF of net demand given that the number of demands equals $\overline{d}$ reads

$$F_{N|\overline{d}}(z)=\text{Prob}(N<z\,|\,D=\overline{d})=\frac{z+c_R-\overline{d}}{c_R},\quad \overline{d}-c_R\le z\le\overline{d}.$$

The order quantity that minimizes $C(Q\,|\,\overline{d})$ equals

$$Q^*_{\overline{d}}=F^{-1}_{N|\overline{d}}\left(\frac{p}{h+p}\right)=\overline{d}-\frac{h}{h+p}c_R\ .$$

Note that $Q^*_{\overline{d}}\ge 0$ as long as $\overline{d}-\frac{h}{h+p}c_R\ge 0$. In that case, inserting $Q^*_{\overline{d}}$ in the cost function gives

$$\begin{aligned}C(Q^*_{\overline{d}}\,|\,\overline{d}) &= p(\overline{d}-\mu_R-Q^*_{\overline{d}})+(h+p)\int_{\overline{d}-c_R}^{Q^*_{\overline{d}}}\frac{z+c_R-\overline{d}}{c_R}\,\mathrm{d}\,z\\ &= p(\overline{d}-\mu_R-Q^*_{\overline{d}})+(h+p)\frac{(Q^*_{\overline{d}}+c_R-\overline{d})^2}{2c_D}\\ &= \frac{1}{2}\frac{ph}{h+p}c_R\end{aligned}$$

Clearly, $C(Q^*_{\overline{d}}\,|\,\overline{d})$ does not depend on demand information $\overline{d}$, so if for all $_{\overline{d}}\left\{\overline{d}-\frac{h}{h+p}c_R\ge 0\right\}$ (condition 2)**,** then $C_{\mathcal{D}}\equiv C(Q^*_{\overline{d}})=\frac{1}{2}\frac{ph}{h+pi}c_R$. Condition 2 can be rephrased as

$\frac{h}{h+p} \le \frac{a_D}{c_R}$.

Return information

The CDF of net demand given that the number of returns equal $\bar{r}$ reads

$$F_{N|\bar{r}}(z) = \text{Prob}(D-\bar{r} < z) = \frac{z-a_D+\bar{r}}{c_D}, \quad a_D-\bar{r} \le z \le b_D-\bar{r}.$$

The order quantity that minimizes $C(Q\,|\,\bar{r})$ equals

$$Q^*_{\bar{r}} = F^{-1}_{N|\bar{r}}\left(\frac{p}{h+p}\right) = a_D - \bar{r} + \frac{p}{h+p}c_D .$$

Note that $Q^*_{\bar{r}} \ge 0$ as long as $a_D - \bar{r} + \frac{p}{h+p}c_D \ge 0$. In that case, the optimal costs are

$$\begin{aligned} C(Q^*_{\bar{r}}|\bar{r}) &= p(\mu_D - \bar{r} - Q^*_{\bar{r}}) + (h+p)\int_{a_D-\bar{r}}^{Q^*_{\bar{r}}} \frac{z-a_D+\bar{r}}{c_D}\,\mathrm{d}\,z \\ &= p(\mu_D - \bar{r} - Q^*_{\bar{r}}) + (h+p)\frac{(Q^*_{\bar{r}}-a_D+\bar{r})^2}{2c_D} \\ &= \frac{1}{2}\frac{ph}{h+p}c_D \end{aligned}$$

If for all $_{\bar{r}}\left\{a_D - \bar{r} + \frac{p}{h+p}c_D \ge 0\right\}$ (condition 3), then $C_{\mathcal{R}} \equiv C(Q^*_{\bar{r}}\,|\,\bar{r}) = \frac{1}{2}\frac{ph}{h+p}c_D$. Condition 3 can be rephrased as $\frac{h}{h+p} \le \frac{b_D-b_R}{c_D}$.

Grouping conditions 1-3, the above cost functions $C_{\mathcal{B}}$, $C_{\mathcal{D}}$, and $C_{\mathcal{R}}$ hold if

$\frac{h}{h+p} \le \min\left\{\frac{c_R}{2c_D}, \frac{c_D}{2c_R}, \frac{b_D-c_R}{c_D}, \frac{a_D}{c_R}\right\}$.

Appendix 3
Upper Bounds on the Standard Deviation of Net Demand

To get insight into the tightness of the upper bounds, they can be seen as approximations based on Taylor series approximations (see Bain and Engelhardt 1987). The two term Taylor series approximation of any function $f(x)$ around the value μ is

$$f(x) \approx f(\mu) + f'(\mu)(x-\mu) + \frac{1}{2} f''(\mu)(x-\mu)^2.$$

So, if X is a stochastic variable with mean μ and standard deviation σ then

$$E[f(X)] \approx f(\mu) + \frac{1}{2} f''(\mu)\sigma^2.$$

Applying this approximation for the Return case ($f(\overline{r}) = \sqrt{\sigma_D^2 + \gamma(1-\gamma)\overline{r}}$) gives

$$E\left[\sqrt{\sigma_D^2 + \gamma(1-\gamma)R}\right] \approx \sqrt{\sigma_D^2 + \gamma(1-\gamma)\mu_R}\left(1 - \frac{1}{8}\gamma^2(1-\gamma)^2 \frac{\sigma_R^2}{\left(\sigma_D^2 + \gamma(1-\gamma)\mu_R\right)^2}\right).$$

For the Demand and Return case ($f(\overline{r}) = \sqrt{\gamma(1-\gamma)\overline{r}}$) we get

$$E\left[\sqrt{\gamma(1-\gamma)R}\right] \approx \sqrt{\gamma(1-\gamma)\mu_R}\left(1 - \frac{1}{8}\gamma^2(1-\gamma)^2 \frac{\sigma_R^2}{\left(\gamma(1-\gamma)\mu_R\right)^2}\right) = \sqrt{\gamma(1-\gamma)\mu_R}\left(1 - \frac{1}{8}\left(\frac{\sigma_R}{\mu_R}\right)^2\right).$$

Note that the term $\frac{1}{8}\left(\frac{\sigma_R}{\mu_R}\right)^2$ is at most 0.03 if μ_R is at least twice as large as σ_R, a threshold which is often used to decide whether the Normal distribution is suitable in the first place. Hence, the upper bound in Table 3.2.2 for the Demand and Return case is accurate for all relevant situations. The upper bound is even tighter for the Return case.

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