An Improved Estimator For Black-Scholes-Merton Implied Volatility

WINFRIED G. HALLERBACH

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Winfried G. Hallerbach *)

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Key words: implied volatility, options, approximation methods

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*) Department of Finance, and Erasmus Research Institute of Management, Erasmus University Rotterdam, POB 1738, NL-3000 DR Rotterdam, The Netherlands.
Phone: +31.10.408-1290, facsimile: +31.10.408-9165. E-mail: hallerbach@few.eur.nl ,
homepage: http://www.few.eur.nl/few/people/hallerbach/.

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1. Introduction

Without doubt, the European call option pricing formulas developed by Black & Scholes [1973] and Merton [1973] (henceforth BSM) mark a huge success in the history of financial modeling. Black [1975, p.64], however, was the first to observe volatility biases displayed by option market prices with respect to the BSM-formula. Out-of-the-money put options tend to be overpriced (giving rise to a high volatility implied by the BSM-model) and in-the-money put options tend to be underpriced (so the BSM-model implies a low volatility).[1] This volatility "snear" or "skew" is quite common in equity derivatives markets, while foreign exchange derivatives exhibit volatility smiles in the sense that both in- and out-of-the-money options tend to have higher implied volatilities than at-the-money options.

The implied volatility smile effect is a well-documented empirical phenomenon.[2] To uncover volatility smile patterns with respect to the BSM-model it is of great theoretical and practical importance to calculate volatilities implied by option market prices. Implied volatilities can be obtained either exactly by applying numerical methods[3] or approximately by using approximation formulas. Two widely known and used examples of the latter category are the formulas derived by Brenner & Subrahmanyam [1988] and Corrado & Miller [1996b].[4,5] The Brenner &

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[1] From Put-Call parity it follows that European out-of-the-money calls (puts) exhibit the same price biases as in-the-money puts (calls). The seminal paper on implied volatilities is by Latané & Rendleman [1976]. For extensive overviews, we refer to Mayhew [1995] and Corrado & Miller [1996a].

[2] In his presidential address, Rubinstein [1994, pp.774ff] discusses the notable emergence of (stock index) volatility smiles after the stock market crash of October 1987.

[3] Notably the well-known Newton-Raphson procedure which has quadratic convergence, or Halley's method which is more stable and has cubic convergence. See Press et al. [1992] and http://mathworld.wolfram.com/HalleysMethod.html. Manaster & Koehler [1982] present a starting value that guarantees convergence (except for the case where the option is exactly at-the-money-forward).

[4] A summary of Corrado & Miller [1996b] appeared as Corrado & Miller [1996c].

Subrahmanyam [1988] formula only applies for at-the-money-forward options and is a special case of the more general Corrado & Miller formula. In effect, the latter approximation formula is the best currently available.

In this paper we derive an alternative implied volatility estimator that compared to the conventional Corrado & Miller [1996b] estimator exhibits substantially higher approximation accuracy and extends over a wider region of moneyness. Our formula is derived from a quadratic approximation of the option price around the at-the-money spanning point. Since first and second derivatives of an at-the-money option price with respect to the strike price are (approximate) functions of the at-the-money option price, we can next use the Brenner & Subrahmanyam [1988] approximation to solve for the implied volatility.

The structure of the paper is as follows. In section 2 we summarize the Brenner & Subrahmanyam [1988] and Corrado & Miller [1996b] approximations. In section 3 we derive our approximation, both in raw (unadjusted) form and in tweaked form to further enhance approximation accuracy. We use the Corrado & Miller [1996b] approximation as a benchmark. Section 4 further investigates approximation accuracy by means of root (weighted) mean squared approximation errors. Section 5 concludes and provides suggestions for future research.

2. Implied volatility estimators

The Black & Scholes [1973] price of a European call option C on a non-dividend paying stock S with strike price K and remaining maturity of T years is given by:

(1) $$C = S \cdot N\left(d_1\right) - X \cdot N\left(d_2\right)$$

with $d_1 = \dfrac{\ln\left(S/X\right)}{\sigma\sqrt{T}} + ½\sigma\sqrt{T}$ and $d_2 = d_1 - \sigma\sqrt{T}$

where: S = the current stock price,

[5] Other approximation formulas have been derived by Chance [1993] and Bharadia et al. [1996]. However, the former requires evaluation of the (inverse) cumulative normal distribution function and the latter is only fairly accurate for close at-the-money options.

$X \equiv Ke^{-rT}$ is the discounted strike price K, with riskfree interest rate r[6],

σ = the volatility of the underlying stock per annum,

and where $N(\cdot)$ is the cumulative standard normal distribution function.

The call option formula can be generalized as follows.[7] When cash dividends are paid on the underlying stock, the current stock price S is replaced by the current stock price less the present value of the dividends paid during the life of the option. For a European call option on a stock index paying a continuous dividend yield at rate q per annum, the current stock price S is replaced by $S\exp(-qT)$. For a European option on a foreign currency, q is replaced by the foreign riskfree interest rate r_f. For a European commodity option, q is replaced by y, the continuous compounded net convenience yield per annum. For a European futures option, finally, q is replaced by the riskfree rate r per annum and S is replaced by the current futures price. When appropriate, these adjustments can be made throughout the rest of the paper.

For a call option that is at-the-money in the forward sense (henceforth denoted as ATM),

$$(2) \qquad S = X$$

eq.(1) reduces to:

$$(3) \qquad \begin{aligned} C &= S \cdot N\left(½\sigma\sqrt{T}\right) - X \cdot N\left(-½\sigma\sqrt{T}\right) \\ &= S\left[1 - 2 \cdot N\left(-½\sigma\sqrt{T}\right)\right] \end{aligned}$$

The Maclaurin series expansion of the cumulative normal distribution function is:[8]

$$(4) \qquad N(z) = ½ + \frac{1}{\sqrt{2\pi}}\left[z + \mathrm{O}(z^3)\right]$$

[6] A more general representation of the discount factor $\exp(-rT)$ is the current price of a riskfree zero-coupon bond with face value 1, maturing at time T.

[7] See Hull [2003], e.g.

[8] See for example Stuart & Ord [1987, p.184].

Using the expansion (4) truncated after the first linear term in (3), Brenner & Subrahmanyam [1988] derive the following approximation to the implied BSM volatility of an ATM call option:

$$(5) \qquad \sigma\sqrt{T} \approx \sqrt{2\pi}\,\frac{C}{S} \qquad \text{for } X = S$$

In the same spirit, Corrado & Miller [1996b] derive a more general formula, which extends the range of accuracy to a wider range of option moneyness. For non-ATM options, the application of the linear normal distribution approximation (4) in (1) yields a quadratic equation in $\sigma\sqrt{T}$. After some manipulations, the relevant root of this equation is:

$$(6) \qquad \sigma\sqrt{T} \approx \sqrt{2\pi}\left(\frac{C-½(S-X)}{S+X}\right)+\sqrt{2\pi\left(\frac{C-½(S-X)}{S+X}\right)^2-\alpha\left(\frac{S-X}{S+X}\right)^2}$$

with $\alpha = 4$. For ATM options, (6) reduces to the Brenner & Subrahmanyam [1988] approximation (5). Corrado & Miller [1996b] next use the parameter α to minimize the concavity of (6) with respect to the stock price for $S = X$ (note that α is only relevant when $S \neq X$). Choosing $\alpha = 2$, (6) reduces to their improved quadratic formula:[9]

$$(7) \qquad \sigma\sqrt{T} \approx \frac{\sqrt{2\pi}}{S+X}\left(C-½(S-X)+\sqrt{\left(C-½(S-X)\right)^2-\frac{(S-X)^2}{\pi}}\right)$$

3. An improved implied volatility estimator

In this section, we derive an alternative implied volatility estimator. We start with a straddle, indicating the call *C* and the put *P* as explicit functions of the discounted strike price *X*:

[9] Tweaking this formula further cannot improve approximation accuracy.

$$(8) \qquad C(X) + P(X) = 2 \cdot C(X) + X - S$$

where the equality is implied by European Put-Call parity.[10] A second order Taylor series approximation of the straddle around $X = S$ takes the form:

$$(9) \qquad C(X) + P(X) \approx 2 \cdot C(S) + (X - S)\left[C'(S) + P'(S)\right] + ½(X - S)^2\left[C''(S) + P''(S)\right]$$

where the primes indicate first and second order derivatives of the option prices with respect to the discounted strike price, evaluated at the argument between parentheses. Since $C'(X) = -N(d_2)$ we have:

$$(10) \qquad C'(S) = -N\left(-½\sigma\sqrt{T}\right)$$

so we can express the ATM call price as:

$$(11) \qquad C(S) = S\left[1 + 2 \cdot C'(S)\right]$$

Combining this expression for the ATM call price with the Brenner & Subrahmanyam [1988] approximation eq.(5) yields:

$$(12) \qquad \sigma\sqrt{T} \approx \sqrt{2\pi}\left[1 + 2 \cdot C'(S)\right]$$

Since $C''(X) = \dfrac{N'(d_2)}{X\sigma\sqrt{T}}$ we have:

$$(13) \qquad C''(S) = \frac{N'(½\sigma\sqrt{T})}{S\sigma\sqrt{T}}$$

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[10] Using a straddle only simplifies the derivation of the approximation. A second order approximation of a separate call option yields exactly the same implied volatility estimator.

where $N'(\cdot)$ is the standard normal density function. From (4) we have:

$$(14) \qquad N'\left(\tfrac{1}{2}\sigma\sqrt{T}\right) = \frac{1}{\sqrt{2\pi}}\left[1 + \mathrm{O}\left(\sigma^2\right)\right] \approx \frac{1}{\sqrt{2\pi}}$$

Hence, using (12) and (14) in (13) gives:

$$(15) \qquad C''(S) \approx \frac{1}{2\pi S\left[1 + 2\cdot C'(S)\right]}$$

This simple ATM approximation holds very well for a wide range of volatilities.

Finally, from Put-Call parity we have:

$$(16) \qquad P'(S) = C'(S) + 1 \qquad \text{and} \qquad P''(S) = C''(S)$$

Plugging (8), (11), (15) and (16) in (9) gives:

$$(17) \qquad 2\cdot C(X) + X - S \approx \left(S + X\right)\left[1 + 2\cdot C'(S)\right] + \frac{\left(X - S\right)^2}{2\pi S\left[1 + 2\cdot C'(S)\right]}$$

Multiplying both sides with $1 + 2\cdot C'(S)$ yields:

$$(18) \qquad \left(S + X\right)\left[1 + 2\cdot C'(S)\right]^2 - \left(2\cdot C(X) + X - S\right)\left[1 + 2\cdot C'(S)\right] + \frac{\left(X - S\right)^2}{2\pi S} \approx 0$$

Treating this expression as exact, (18) is a quadratic form in $1 + 2\cdot C'(S)$. Its largest root is:

$$(19) \qquad 1 + 2\cdot C'(S) = \frac{2C(X) + X - S + \sqrt{\left(2C(X) + X - S\right)^2 - 2\left(S + X\right)\dfrac{\left(X - S\right)^2}{\pi S}}}{2(S + X)}$$

Only the largest root is consistent with (11) when $S = X$ (the smallest root gives zero in that case). Using (12), we get, with $C = C(X)$:

$$\text{(20)} \qquad \sigma\sqrt{T} = \frac{\sqrt{2\pi}}{2(S+X)}\left[2C + X - S + \sqrt{\left(2C + X - S\right)^2 - 2\left(S+X\right)\frac{\left(X-S\right)^2}{\pi S}}\;\right]$$

Note that for ATM options, (20) reduces to the Brenner & Subrahmanyam [1988] approximation (5). Rewriting (20) as:

$$\text{(21)} \qquad \sigma\sqrt{T} = \sqrt{2\pi}\,\frac{C + ½(X - S)}{S+X} + \sqrt{2\pi\left(\frac{C + ½(X - S)}{S+X}\right)^2 - \frac{\left(X-S\right)^2}{S(S+X)}}$$

and comparing it to the unadjusted formula of Corrado & Miller [1996b] in eq.(6) with $\alpha = 4$, their last term under the square root is $4\left(\frac{S-X}{S+X}\right)^2$ whereas in our eq.(21) this term can be expressed as $4\left(\frac{S-X}{S+X}\right)^2\left[\frac{S+X}{4S}\right]$, thus incorporating the extra term between square brackets.

We first compare the accuracy of eq.(20) with the unadjusted Corrado & Miller [1996b] formula eq.(6), where $\alpha = 4$. Since the BSM option is linearly homogeneous in the underlying and the (discounted) strike price, we normalize with respect to the current stock price.[11] The discounted strike divided by the current stock price, $M \equiv X / S$, indicates the moneyness of the option.

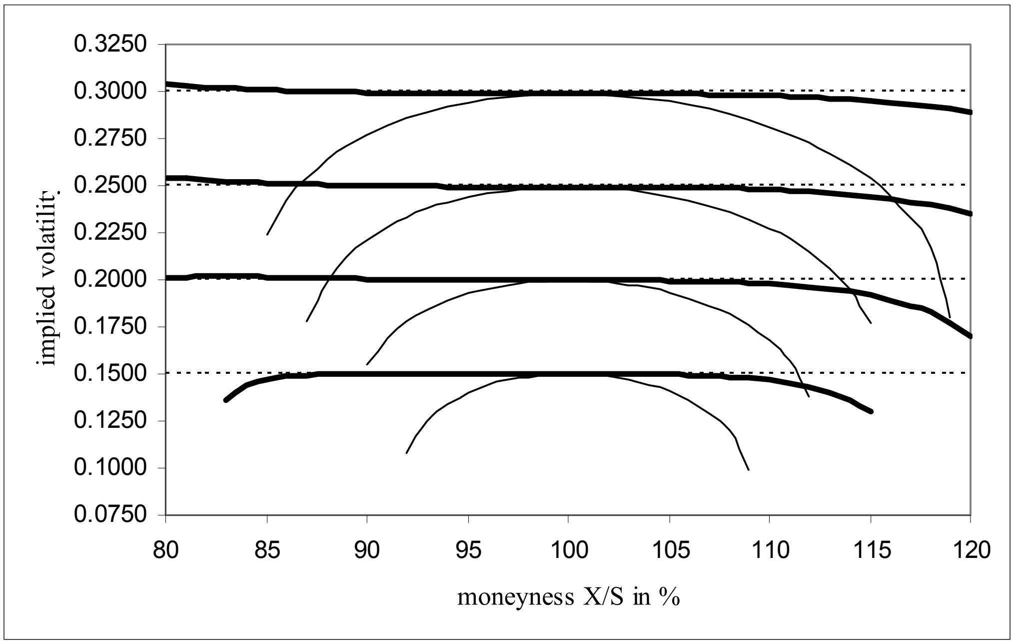
In Figure 1, we plot implied volatilities ($\sigma\sqrt{T}$) for moneyness ranging from 80% to 120%. This range covers the degree of moneyness encountered in practice very well. We consider volatilities $\sigma\sqrt{T}$ over the range from 3% to 30%. A volatility of 30% (50%) per annum, for example, roughly translates into a volatility of 3% over one month (two weeks). A volatility of just over 25% per annum means a volatility of

[11] This linear homogeneity was first noticed by Merton [1973] as a special characteristic. It was generalized by Hoogland & Neumann [2001a,b] who recognized it as a fundamental property and even termed it "the relativity principle of finance" (p.6).

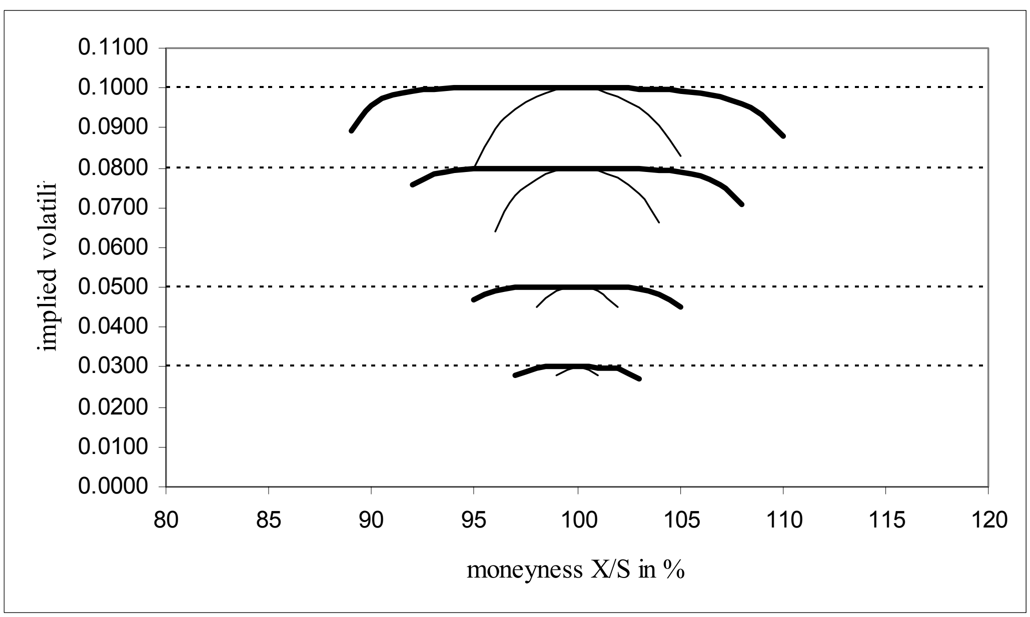
Figure 1: Accuracy of the unadjusted implied volatility estimators.

Panel A covers the implied volatilities $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$ and panel B shows $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$, according to unadjusted estimates of Corrado & Miller's [1996b] formula eq.(6) and our eq.(20). The bold (flatter top) curves represent our estimates.

Panel A: $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$



Panel B: $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$



8% over one month. In panel A, implied volatilities $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$ are covered whereas panel B depicts $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$. When implied volatilities are not shown in the graph, the corresponding formula yields no real solution.[12] The fat top curves, fairly straight, indicate our unadjusted approximation according to eq.(20) whereas the unadjusted Corrado & Miller [1996b] formula eq.(6) yields the more concave curves. Even without further adjustment, approximation accuracy of (20) is already very good. It also yields estimates over a wider range of moneyness. The higher the volatility, the better the approximation accuracy of (20); therefore, we do not show results for $\sigma\sqrt{T} > 0.30$.

Next we tweak our approximant (20) to further improve approximation accuracy. We start rewriting it as:

$$(22) \qquad \sigma\sqrt{T} = \frac{\sqrt{2\pi}}{2(S+X)}\left[2C + X - S + \sqrt{\left(2C + X - S\right)^2 - \gamma\left(S+X\right)\frac{\left(X-S\right)^2}{\pi S}}\right]$$

where $\gamma = 2$. By adjusting the tweaking parameter γ we can change the curvature of (22) without affecting the approximation accuracy for ATM options. Evaluating the approximation accuracy for a range of $\sigma\sqrt{T} = 0.03, ..., 0.30$ we find that a somewhat lower value of $\gamma = 1.85$ performs best. However, the implied volatility estimates show a tendency to overshoot for low moneyness *M* and undershoot for high *M*. This is also observed in Figure 1 Panel A for the unadjusted approximation in the higher implied volatility range. We therefore also include a moneyness correction in the tweaking factor to further enhance approximation accuracy:[13]

$$(23) \qquad \gamma = 1.85\sqrt{\frac{S}{X}}$$

[12] The discriminant of eq.(18) becomes negative and hence its roots are imaginary.
[13] We admit that this is more art than science, but our only goal is to obtain accurate implied volatility estimates for a wide range of moneyness.

For $M = 80\% (120\%)$ we have $\sqrt{S/X} = 0.894 (1.095)$, so depending on the degree of moneyness we have a tilt of -10% to +10%. Hence, our tweaked formula finally becomes:

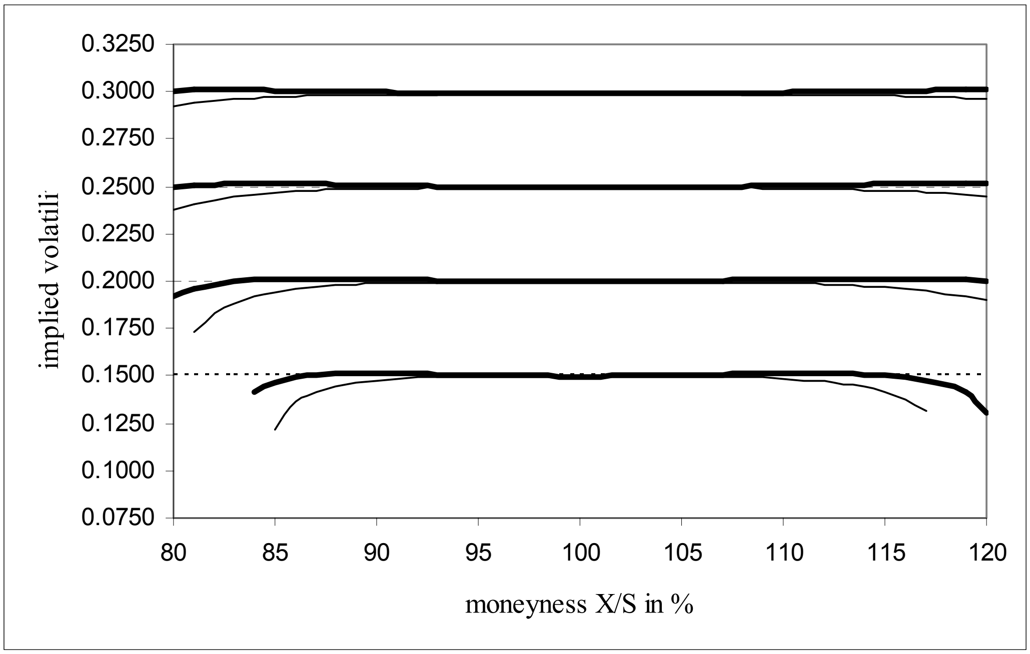
$$(24) \qquad \sigma\sqrt{T} = \frac{\sqrt{2\pi}}{2(S+X)}\left[2C + X - S + \sqrt{(2C + X - S)^2 - 1.85\frac{(S+X)(X-S)^2}{\pi\sqrt{XS}}}\right]$$

Figure 2 compares Corrado & Miller's [1996b] improved (adjusted) quadratic formula eq.(7) with our tweaked approximant eq.(24). We again plot implied volatilities against the range of moneyness. Panel A again covers implied volatilities $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$ and panel B shows the estimates for the selection $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$. The almost straight fat top curves indicate our tweaked estimator eq.(24). The improved Corrado & Miller [1996b] formula eq.(7) does a fair job, but it yields more concave curves and extends over a more narrow range of moneyness. Our approximation is outstanding over a wide range of moneyness both in absolute sense and when compared to the Corrado & Miller [1996b] formula. This not only applies to low implied volatilities $\sigma\sqrt{T}$ which are relevant for short maturity options, but also to higher implied volatilities which are relevant for longer maturity options. Especially for the latter options the approximation accuracy in the lower moneyness region is relevant. After all, for longer maturity options which are at-the-money in the conventional sense (i.e. $K = S$), we expect that $X << S$ and hence $M << 100\%$.

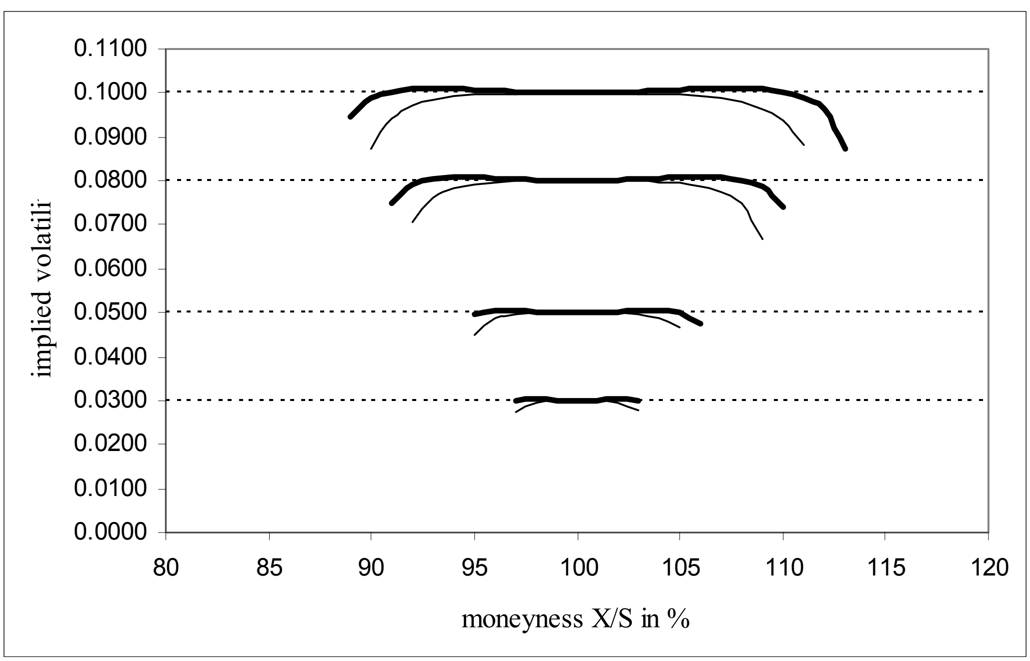
Figure 2: Accuracy of the adjusted implied volatility estimators.

Panel A covers the implied volatilities $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$ and panel B depicts $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$, according to adjusted estimates of Corrado & Miller's [1996b] formula eq.(7) and our tweaked approximant eq.(24). The bold (flatter top) curves represent our estimates.

Panel A: $\sigma\sqrt{T} = 0.15, 0.20, 0.25, 0.30$



Panel B: $\sigma\sqrt{T} = 0.03, 0.05, 0.08, 0.10$



4. A closer look at approximation accuracy

To gain further insight into the approximation accuracy, we compute root mean squared approximation errors. We consider moneyness $M = X/S$ over the maximum range of 80% to 120% in steps of 1%. Since the Corrado & Miller [1996b] approximation is our benchmark, we only consider the range of moneyness $\{M^*\}$ for which their formula yields an implied volatility estimate. The number of percentage levels of moneyness considered is the cardinality of $\{M^*\}$, indicated as $|M^*|$.

The root mean squared error (RMSE) of an approximation is computed as:

$$(25) \qquad RMSE\left(\sigma\sqrt{T}; M^*\right) = \sqrt{\frac{1}{|M^*|} \sum_{i \in M^*} \left(\hat{\sigma}\sqrt{T} - \sigma\sqrt{T}\right)^2}$$

where $\hat{\sigma}\sqrt{T}$ is the implied volatility from the corresponding approximation. This is the unweighted RMSE. Since vega is highest for ATM options and decreases when the option moves further in- or out-of-the-money, it makes sense to weigh the squared approximation errors with vega.[14] After all, the more an option is in- or out-of-the-money, the lower its sensitivity to changes in volatility and hence the less important the approximation error. The vega of a European call, normalized to the current stock price, C/S, is:

$$(26) \qquad \frac{\partial C/S}{\partial \sigma\sqrt{T}} = N'\left(d_1\right)$$

where d_1 is as defined in (1). Hence, for each level *i* of moneyness in $\{M^*\}$ we define the weight:

$$(27) \qquad w_i = \frac{N'\left(d_1\right)_i}{\sum_{j \in M^*} N'\left(d_1\right)_j} \qquad \forall i \in M^*$$

[14] A weighting scheme on the basis of the options' vegas was first applied by Latané & Rendleman [1976].

where $N'(d_1)_i$ indicates $N'(d_1)$ evaluated at moneyness level *i*. We have normalized the weights to sum to unity for each $\{M^*\}$. The root weighted-mean squared error (RWMSE) is then computed as:

$$(28) \qquad RWMSE\left(\sigma\sqrt{T}; M^*\right) = \sqrt{\sum_{i \in M^*} w_i \left(\hat{\sigma}\sqrt{T} - \sigma\sqrt{T}\right)^2}$$

Table 1: Approximation accuracy, as measured by RMSE (unweighted, see eq.(25)) and RWMSE (vega-weighted, see eq.(28)). Comparison is between the improved Corrado & Miller [1996b] (C&M) formula eq.(7) and our tweaked approximation eq.(24). The R(W)MSEs are expressed in percentage terms. Below the R(W)MSE of our approximation is the R(W)MSE of our approximation expressed as a percentage of the corresponding C&M's R(W)MSE.

	moneyness	RMSE		RWMSE	
$\sigma\sqrt{T}$	M in %	C&M	our eq.(24)	C&M	our eq.(24)
3%	97-103	0.1368	0.0189	0.1179	0.0188
			14%		16%
5%	95-105	0.1936	0.0341	0.1654	0.0331
			18%		20%
8%	92-109	0.4179	0.0620	0.3537	0.0579
			15%		16%
10%	90-111	0.4304	0.0700	0.3649	0.0661
			16%		18%
15%	85-117	0.7208	0.1089	0.6005	0.0983
			15%		16%
20%	81-120	0.6247	0.1120	0.5178	0.1007
			18%		19%
25%	80-120	0.3408	0.0786	0.3006	0.0765
			23%		25%
30%	80-120	0.2618	0.0712	0.2450	0.0719
			27%		29%

Table 1 shows the weighted and unweighted RMSEs. Almost without exception, RWMSE is lower than RMSE: although approximation accuracy decreases when moving to lower and higher levels of moneyness, also the corresponding vega-weight decreases. Also, R(W)MSEs seem to be highest in the 15% to 20% volatility range. Corrado & Miller's [1996b] unweighted RMSE ranges from 14 to 72 basis points (bps), as compared to only 2 to 11 bps for our approximant (in relative terms this is 14 to 27%). Switching to the weighted case, the RWMSE of our approximation is 16 to 29% of Corrado & Miller's [1996b] RWMSE, ranging from 2 to 10 bps. So in all cases where Corrado & Miller's [1996b] approximant yields an implied volatility estimate, our estimator is expected to reduce approximation error with at least 70%.

Since the range of moneyness M^* is truncated at the points where the Corrado & Miller [1996b] estimator fails to deliver an implied volatility estimate, we also evaluated the approximation accuracy of our estimator over its own relevant range of moneyness. Table 2 shows the details. Comparing with Table 1, the RMSE and RWMSE is substantially lower than for the Corrado & Miller [1996b] estimator for all levels of volatility considered. For the unweighted case, approximation error ranges from 2 to 40 bps, and for the weighted case from 2 to 32 bps. This signifies an important improvement over the Corrado & Miller [1996b] estimator.

Table 2: Approximation accuracy of our tweaked approximation eq.(24), as measured by RMSE (unweighted, see eq.(25)) and RWMSE (vega-weighted, see eq.(28)), over the whole relevant range of moneyness. The R(W)MSEs are expressed in percentage terms.

$\sigma\sqrt{T}$	moneyness M in %	RMSE	RWMSE
3%	97-103	0.0189	0.0188
5%	95-106	0.0797	0.0668
8%	91-110	0.1809	0.1469
10%	89-113	0.2936	0.2366
15%	84-120	0.3957	0.3226
20%	80-120	0.1717	0.1406
25%	80-120	0.0786	0.0765
30%	80-120	0.0712	0.0719

5. Summary and conclusions

In this paper, we derived an alternative estimator for implied volatility in the standard Black-Scholes-Merton framework. This estimator provides accurate implied volatility estimates over a wide range of moneyness and significantly improves on the familiar Corrado & Miller [1996b] approximation formula. Especially in spread sheet applications where closed-form approximants are favored, the higher approximation accuracy paired with the wider range of moneyness are very welcome features of our proposed estimator. Whereas the Corrado & Miller [1996b] formula signified an important step forward in closed-form implied volatility estimations, our results imply that further improvements can be achieved. We therefore would like to encourage further research in this area.

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