Chapter 2

Literature review

Many articles have appeared in the production planning and inventory control literature in which both the return process and the demand process are explicitly modeled. From the literature in which demand and return processes are considered simultaneously, we will focus the discussion on those inventory models that directly apply to the situation of remanufacturing. As selection criteria we use that in addition to the demand and return process, the models must describe the remanufacturing process either implicitly, or explicitly, and the production or outside procurement process. For a general review of quantitative models in reverse logistics we refer to Fleischmann et al. [17].

We do not consider models in which demands for new products are generated by product failures only, i.e., product demands and product returns are perfectly correlated. These models are typical for the situation of spare part and repair management, but do usually not apply to the situation with remanufacturing. Reviews on spare part and repair management can be found in Pierskalla and Voelker [40], Nahmias [38], Cho and Parlar [10], and Mabini and Gelders [34].

Another difference between models for spare part management and remanufacturing lies in the objective: with spare part management the objective is to determine the fixed number of spare parts in the system, such that the associated long-run average costs are minimized. With remanufacturing the objective is to develop a policy on when and how much to remanufacture, dispose, and produce, such that some cost function is minimized. Essential is that with remanufacturing the number of products in the system may vary over time. Our selection criteria imply that the well-known family of METRIC models described by Sherbrooke [47] for spare-part management
will not be considered here.

Before we review the literature on models that satisfy our selection criteria, we first list the most common assumptions that are made in these models with respect to the processes introduced in Chapter 1.

- **Demand and return process.** To model the demand and return process, assumptions are made on the inter-occurrence times, the demand quantity per occurrence, and the relation between the two processes (i.e., stochastically dependent or independent).

- **Disassembly process.** This process is not considered in any model in the literature. The reason is, that all models apply to the situation of a single product, and each product is assumed to consist of a single component only. Clearly, in this situation it is assumed that no disassembly operations need to occur.

- **Testing process.** This process is modeled by means of a single testing facility, where returned products are tested. Assumptions are made concerning the testing capacity, the testing time, and the variable testing costs.

- **Remanufacturing process.** This process is modeled by means of a single remanufacturing facility, consisting of a number of parallel workcenters that carry out the remanufacturing operations. Assumptions are made on the number of parallel workcenters, the remanufacturing time, and the variable remanufacturing costs.

- **Outside procurement/production process.** This process is modeled in terms of an outside procurement source (external supply) or production resource (internal supply). With respect to this process assumptions are made on fixed and variable costs, on the lead-time in case of outside procurement, and on the production time and production capacity in case of internal production.

- **Assembly process.** See disassembly process.

- **Inventory process.** Two types of inventory are modeled. Type I inventory is the inventory of returned products that have passed the test and are waiting for remanufacturing or planned disposal. Type II inventory is the inventory of all serviceable products, i.e., products that were remanufactured or newly produced. For both types of inventories, assumptions are made on storage capacity and inventory holding costs.
2.1 Periodic review models

- Disposal process. The disposal process is modeled by means of a disposal center. Assumptions are made on fixed and variable disposal costs.

In addition to the above classification of processes, we also make a distinction between two types of customer service, i.e.,

- customer service in terms of backorder costs per product per time unit, or,

- customer service in terms of a service level. In case of periodic review this level, $\delta^{(n)}_b$ say, is defined as the maximum allowable probability of a stock-out occurrence in period $n$ in between two successive outside procurements or internal production runs. In case of continuous review this level is defined as the long-run average maximum allowable probability of a stockout position.

In the following sections we separately discuss models with discrete planning periods (periodic review models, Section 2.1), models with continuous planning opportunity (continuous review models, Section 2.2), and a particular type of financial models that is related to the topic of remanufacturing (cash-balancing models, Section 2.3).

2.1 Periodic review models

In periodic review models the planning horizon is subdivided into a predetermined (in)finite number of planning periods. At the beginning of each planning period $n$, decisions are taken according to the values of the following decision variables.

\[
Q_d^{(n)} = \text{the quantity (batch-size) of remanufacturable products that is disposed of in planning period } n, \]

\[
Q_p^{(n)} = \text{the quantity (batch-size) of products that is procured outside or internally produced in planning period } n, \]

\[
Q_f^{(n)} = \text{the quantity (batch-size) of products that is remanufactured in planning period } n. \]

All decision variables are assumed to be integer.
Objective in the periodic review models is to determine the values for the decision variables, such that the total expected costs over the entire planning horizon are minimized. Some models also take the service level explicitly into account as a constraint.

Within the category of periodic review models, Simpson [49] considers a model, with the following assumptions and characteristics:

- **Demand and return process.** One demand and return occurrence per planning period, demand and return quantities are correlated and specified by means of a period dependent joint probability density function.

- **Testing process.** No testing facility.

- **Remanufacturing process.** No remanufacturing lead-time; the capacity of the remanufacturing facility is infinite.

- **Procurement/production process.** No procurement lead-time; no fixed procurement costs.

- **Inventory process.** Type I and Type II inventory buffers have infinite capacity; Type I and Type II inventory have (different) variable inventory holding costs.

- **Disposal process.** No fixed or variable disposal costs.

- **Customer service.** Modeled in terms of backorder costs,

- **Control strategy:** at the beginning of each period $n$ decisions are taken on $Q_d^{(n)}$, $Q_y^{(n)}$, and $Q_r^{(n)}$, such that the total expected costs over the planning horizon are minimized.

Simpson develops a dynamic programming based algorithm to determine the optimal values for the above decision variables. Also, an interesting structure on the optimal decisions is identified. It is proved that for each period $n$ there exist three constants $\alpha_n$, $\beta_n$, and $\gamma_n$, such that the optimal strategy is as follows.

1. if at the beginning of period $n$ Type II inventory is smaller than $\alpha_n$, as many as possible products from Type I inventory will be remanufactured to increase Type II inventory to $\alpha_n$, 
2. if Type I inventory is insufficient to increase Type II inventory to $\beta_n$ ($< \alpha_n$), also an outside procurement order is placed to increase Type II inventory to $\beta_n$, and

3. if after the previous steps the sum of Type I inventory and Type II inventory is larger than $\alpha_n + \gamma_n$, as many as possible products from Type I inventory are disposed of, such that after disposal the sum of Type I and Type II inventory is no less then $\alpha_n + \gamma_n$.

Kelle and Silver [30] formulate a periodic review model, which differs somewhat from Simpson’s model.

- **Demand and return process.** One demand and return occurrence per planning period, but opposed to Simpson’s model, demand and return quantities in each period are independent stochastic variables.

- **Testing process.** No testing facility.

- **Remanufacturing process.** No remanufacturing lead-time; infinite capacity of the remanufacturing facility; no remanufacturing costs$^1$.

- **Procurement/production process.** No procurement lead-time; fixed and variable production costs.

- **Inventory process.** No Type I inventory; Type II inventory buffer has infinite capacity,

- **Disposal process.** No disposal,

- **Customer service.** Modeled in terms of service level constraint,

- **Control strategy:** at the beginning of each period $n$ a decision is taken on $Q^n_B$, such that the total expected costs over the planning horizon are minimized.

Kelle and Silver formulate their model as a probability constraint integer program. The probability constraints state that the probability on a backlogging position at the end of period $n$ may not be larger then $\delta_B^{(n)}$. They suggest an approximation procedure to solve the chance constraint integer program.

$^1$This model applies to containers, bottles, etc.
First step in the approximation procedure is to replace the stochastic inventory variables by their expectations, and to replace the probabilistic service level constraints by appropriate deterministic constraints on the minimum inventory level at the end of each period. These transformations yield a variant of the well-known (deterministic) Wagner-Whitin model for dynamic lotsizing (see Wagner and Whitin [65]). In this variant positive as well as negative demands are allowed to occur in each period. Second step in the approximation procedure is to transform this variant of the Wagner-Whitin model into an equivalent model in which positive demands occur only. The latter model is then solved to optimality, using an appropriate dynamic programming based technique.

Inderfurth [26] extends the work of Simpson to allow for non-zero remanufacturing lead-times:

- Demand and return process. All returns and demands per period are continuous time-independent random variables. The inter-arrival distributions are arbitrary distribution functions, which may be stochastically dependent.

- Testing process. No testing facility.

- Remanufacturing process. The remanufacturing lead-time $L_r$ is non-stochastic and equal to $\mu_{L_r}$; the remanufacturing facility has infinite capacity and variable remanufacturing costs.

- Procurement/production process. The procurement lead-time $L_m$ is non-stochastic and equal to $\mu_{L_m}$; there are variable production costs.

- Inventory process. Both Type I and Type II inventory buffers have infinite capacity.

- Disposal process. Variable disposal costs.

- Service. Modeled in terms of backorder costs.

- Control strategy: at the beginning of each period $n$ decisions are taken on $Q_d^{(n)}$, $Q_p^{(n)}$, and $Q_r^{(n)}$, such that the total expected costs over the planning horizon are minimized.

Inderfurth considers several special cases, regarding the stocking policy of the returned products, and regarding the values of the manufacturing lead-time $\mu_{L_m}$ and the remanufacturing lead-time $\mu_{L_r}$. For the case that
2.1. Periodic review models

returned items are not allowed to be stocked, for instance because the items are perishable, Inderfurth provides the following results.

- If \( \mu_{L_m} = \mu_L \), the structure of the optimal policy can be formulated as a so-called \((L, U)\) policy:

\[
\begin{align*}
Q_p^{(n)} &= L^{(n)} - x, & Q_r^{(n)} &= x_r, & Q_d^{(n)} &= 0, & \text{for } x < L^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= x_r, & Q_d^{(n)} &= 0, & L^{(n)} \leq x < U^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= x_r - (x - U^{(n)}), & Q_d^{(n)} &= x - U^{(n)}, & x > U^{(n)},
\end{align*}
\]

Here, \( x_r \) is the remanufacturable inventory and \( x_s \) is the inventory position of serviceable products.

- If \( \mu_{L_m} > \mu_L \), the structure of the optimal policy can be formulated as a so-called \((L, U, \hat{U})\) policy (For details we refer to [26]).

- If \( \mu_{L_m} < \mu_L \), the structure of the optimal policy is not of a simple form, even if the manufacturing lead-time and remanufacturing lead-time differ only one period.

Note that if returned products are not allowed to be stocked, all returned products will be remanufactured or disposed off at the end of each period. On the other hand, if returned products are allowed to be stocked there can be more interaction between the production, the remanufacturing and the disposal process. In the latter case Inderfurth derives the following results concerning the structure of the optimal policy.

- If \( \mu_{L_m} = \mu_L \), the structure of the optimal policy can be formulated as a so-called \((L, M, U)\) policy:

\[
\begin{align*}
Q_p^{(n)} &= L^{(n)} - x, & Q_r^{(n)} &= x_r, & Q_d^{(n)} &= 0, & \text{for } x < L^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= x_r, & Q_d^{(n)} &= 0, & L^{(n)} \leq x < M^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= x_r - x_s, & Q_d^{(n)} &= 0, & M^{(n)} \leq x \leq U^{(n)}, & x_s < M^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= 0, & Q_d^{(n)} &= 0, & M^{(n)} \leq x \leq U^{(n)}, & x_s \geq M^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= x_r - x_s, & Q_d^{(n)} &= x - U^{(n)}, & x > U^{(n)}, & x_s < M^{(n)}, \\
Q_p^{(n)} &= 0, & Q_r^{(n)} &= 0, & Q_d^{(n)} &= x - U^{(n)}, & x > U^{(n)}, & x_s \geq M^{(n)}.
\end{align*}
\]
Here, \( x_r \) is the remanufacturable inventory, \( x_s \) is the inventory position of serviceables, and \( x = x_s + x_r \).

- If \( \mu_{Lm} \neq \mu_L \), the structure of the optimal policy is much more difficult to obtain and becomes very complex, even if the manufacturing lead-time and remanufacturing lead-time differ only one period.

Inderfurth’s results show that under general conditions the optimal policy will be very complex and difficult to identify.

\section*{2.2 Continuous review models}

In \textit{continuous review} models the time axis is continuous, and decisions are taken according to some predefined \textit{control policy}. For the control policies considered in literature, the following integer valued decision variables are defined:

\begin{align*}
  s_p &= \text{inventory position (i.e., the sum of Type I and Type II inventory) at which an outside procurement or production order is placed,} \\
  Q_p &= \text{the quantity (batch-size) that is procured outside or produced,} \\
  s_d &= \text{inventory position at which returned products are disposed off,} \\
  Q_d &= \text{quantity (batch-size) of returned products that is disposed off.}
\end{align*}

Objective is to determine values for the decision variables, such that the long-run average costs per unit of time are minimized. In some models also a service level constraint is explicitly taken into account.

Within the category of continuous review models, Heyman [23] analyzes a model with the following assumptions and characteristics:

- \textit{Demand and return process}. Demands and returns are independent. The inter-occurrence times and quantities are distributed according to general distribution functions.

- \textit{Testing process}. No testing facility.
2.2 Continuous review models

- **Remanufacturing process.** No remanufacturing lead-times; the capacity of the remanufacturing facility is infinite; variable remanufacturing costs.

- **Procurement/production process.** No procurement lead-times; variable outside procurement costs.

- **Inventory.** Type I inventory is not modeled; the Type II inventory buffer has infinite capacity; variable holding costs of Type II inventory are explicitly taken into account.

- **Disposal.** Variable disposal costs.

- **Customer service.** The system has perfect service, since backlogging never occurs due to zero remanufacturing and outside procurement lead-times.

- **Control strategy:** the system is controlled by a single parameter $s_d$ strategy: whenever the inventory position equals $s_d$, incoming remanufacturables are disposed of.

Heyman presents an expression for the disposal level $s_d$ such that the sum of inventory holding costs, variable remanufacturing costs, variable outside procurement costs, and variable disposal costs is minimized. In case that the inter-arrival times of demands and returns are exponentially distributed, the exact expression is based on the analogy between this inventory model, and a simple queuing model. Heyman proves that the single parameter control rule dominates all other possible control rules in terms of total expected costs, i.e. no alternative control rule can ever result in lower expected costs.

In case that the inter-arrival times of demands and returns are generally distributed, Heyman derives an approximation procedure to determine the disposal level for which the total expected costs are minimal. The approximation procedure is based on diffusion processes. A small numerical study in his paper shows that the approximation procedure performs rather well.

Muckstadt and Isaac [37] consider a model that extends Heyman's in the sense that a remanufacturing facility is explicitly modeled, and lead-times are non-zero. However disposal decisions are not taken into account:

- **Demand and return process.** Demands and returns are independent; the inter-occurrence times are exponentially distributed; the demand
and return quantities are always equal to one product per occurrence. To avoid unlimited growth of inventories it is assumed that the return rate is smaller than the demand rate.

- **Testing process.** No testing facility.

- **Remanufacturing process.** The remanufacturing lead-time is arbitrarily distributed; the capacity of the remanufacturing facility may be finite.

- **Procurement/production process.** The procurement lead-time is constant; fixed outside procurement costs are considered.

- **Inventory.** Type I and Type II inventory buffers have infinite capacity; inventory holding costs are taken into account for Type II inventory only.

- **Disposal process.** No disposal.

- **Customer service.** Service is considered in terms of backorder costs.

- **Control strategy.** the system is controlled by an \((s_p, Q_p)\) strategy. Whenever the inventory position equals \(s_p\), an outside procurement order of size \(Q_p\) is placed. Returned products are remanufactured as soon as possible.

Muckstadt and Isaac present an approximation procedure to determine the control parameters \(s_p\) and \(Q_p\), such that the sum of fixed outside procurement costs, inventory holding costs, and backordering costs is minimized. Their procedure is based on the fact that, with exponentially distributed demand and return inter-arrival times, the steady-state distribution for the inventory position can be computed exactly, by solving a continuous time Markov-chain model.

From the steady-state distribution of the inventory position an approximation on the distribution of the net inventory is derived. In this approximation the net inventory is assumed to behave as a Normal distribution function, with mean and variance based on an approximation of the first two moments of the steady-state distribution of the net inventory. From the normal approximation of the net inventory expressions on the expected on-hand inventory position and on the expected backordering position are then derived. Using these expressions an approximation on the long run average costs per unit of time as function of the policy parameters \(s_p\) and \(Q_p\)
is obtained. This cost function is then minimized, resulting in an expression for $s_p$ (in closed-form) and an algorithm to determine $Q_p$ numerically.

In the second part of their paper Muckstadt and Isaac consider a two echelon warehouse-retailer model, with an $(s_p, Q_p)$ reorder policy for the warehouse, and an $(S^{(j)} - 1, S^{(j)})$ reorder level policy for the retailers. Here, $S^{(j)}$ is defined as the order up-to level for retailer $j$. Based on the results for the single-echelon case, an approximation procedure is developed to determine values for the policy parameters in the two echelon case, such that long-run average costs per unit of time are minimized. As far as we know, this model is the only model that has appeared in the literature, in which distribution is combined with remanufacturing and production.

An alternative approximation procedure for the same single-echelon model as formulated by Muckstadt and Isaac, was proposed by Van der Laan [58]. The main difference between the Muckstadt and Isaac procedure and the Van der Laan procedure lies in the nature of the approximation. In the latter procedure an approximation is used on the distribution of the net demand during the procurement lead-time, instead of an approximation on the distribution of net inventory.

A numerical comparison in Van der Laan [58] has shown that in many cases this approach results in a more accurate approximation of the expected number of backorders, and hence in a better (lower cost) choice of the policy parameters $s_p$ and $Q_p$. Furthermore, an extension of the single-echelon Muckstadt and Isaac model is given, in which customer service is considered in terms of a service level constraint, instead of backordering costs.

In Van der Laan [58] and Van der Laan et al. [61] two models are formulated in which remanufacturing and disposal decisions are considered simultaneously. The first model, proposed in [61] differs from the single echelon model proposed by Muckstadt and Isaac with respect to the following:

- **Inventory process.** Type I (work-in-process) inventory capacity is limited to $N$; Type II inventory has infinite capacity; inventory holding costs are considered for Type II inventory only,

- **Disposal process.** Variable disposal costs are considered,

- **Control strategy.** The system is controlled by an $(s_p, Q_p, N)$ strategy. This strategy is defined as follows: whenever the inventory position
equals $s_p$, an outside procurement order of size $Q_p$ is placed; whenever the number of products in Type I inventory equals $N$, every incoming remanufacturable product is disposed off before having entered the remanufacturing facility.

In [61] an approximation procedure is described to determine the policy parameters $s_p$, $Q_p$, and $N$ simultaneously. The procedure is an extension of the approximation procedure in [58] for the $(s_p, Q_p)$ model.

The second model, proposed in [58], differs from the first model in that the system is controlled by an $(s_p, Q_p, s_d)$ policy. With this policy, the disposal decision is based on the number of products in inventory position, rather than on the number of products in Type I inventory. The complete policy is as follows: whenever the inventory position equals $s_p$, an outside procurement order of size $Q_p$ is placed; whenever the inventory position equals $s_d$, each additional incoming remanufacturable product is disposed off.

All continuous review models considered so far are more or less extensions or modifications of the so called two-source inventory models. The analysis of multiple source inventory models goes back to Barankin [3] who developed a model for a single planning period only and the two options available are a one-period and a zero period lead-time only. The problem addressed in two-source models is when and how much to order in a situation that one can choose between two supply sources, of which one is cheaper but offers a poorer lead-time.

The most general analysis in the class of periodic review models is due to Whittmore and Saunders [66], who construct a multi-period dynamic model and allow both the long and short lead-times to be of arbitrary length. Fixed ordering costs are not present. The form of the optimal policy in there model is very complex and requires the solution of a multi-state dynamic program.

A continuous review variant of the above model with fixed lead-times in which also fixed ordering costs are taken into account is studied by Moinzadeh and Nahmias [36]. To control the system a double $(s, Q)$ policy is used, which is similar to the PULL strategy that will be introduced in Chapter 3. However, there is a fundamental difference between the two supplier case and a remanufacturing environment. In a two supplier model it is assumed that both sources have an unlimited source of raw materials.
2.3. Cash balancing models

In a remanufacturing environment however, the remanufacturing source is dependent on the return flow. Due to the stochastic nature of the return flow it is uncertain if the remanufacturing source is able to deliver at the right moment. While in the classical two supplier models the decision between the two options is based on balancing the ordering costs against the lead-times, in our HMR system the decision also depends on the time dependent capacity of the remanufacturing source.

If disposals are allowed, the remanufacturing model more closely resembles the classical two supplier model in the sense that in this case also a decision is taken on what fraction to order from which supplier. However, this case is even more complex than remanufacturing without disposal.

Recently, an optimal EOQ-like policy for a deterministic system with manufacturing, repair, and disposal operations and zero lead times has been proposed by Richter [41], [42]. This model only differs from the deterministic model by Schrady [46] in the control policy. The disadvantage of a deterministic model however is that it disregards the stochastic nature of a HMR system.

2.3 Cash balancing models

Alternative models that could serve as a starting point for remanufacturing models stem from the area of finance: the cash-balancing models. The reason why we only briefly discuss these models here, is that many characteristics that are typical for a remanufacturing and production environment, like detailed modeling of the remanufacturing process itself, and non-zero lead-times for remanufacturing and production, are disregarded. Nevertheless, some of these models match our selection criteria, and may very well serve as a starting point from which models for remanufacturing and production could be developed further.

Cash balancing models usually consider a local cash of a bank with incoming money flows stemming from customer deposits, and outgoing money flows, stemming from customer withdrawals. The possibility exists of increasing the cash-level of the local cash by ordering money from the central cash, or decreasing the cash level of the local cash by transferring money to the central cash. Objective in these models is to determine the time and quantity of the cash transactions, such that the sum of fixed and variable transaction costs, backlogging costs, and interest costs related to the local
cash is minimized. There exist continuous review and periodic review cash-balancing models. An interesting result is, that for the continuous review model a four parameter \((s_p, s_d, S_p, S_d)\) strategy is optimal in case that no remanufacturing or procurement lead-times exist.

For example in Constantinides [11] the state of the cash management system at time \(t\) is defined by the cash level \(x(t)\), or simply \(x\). The cost rate of keeping a positive or negative cash balance \(x\) is \(C(x) = \max \{hx, -px\}\), where the constants \(h > 0\) and \(p > 0\) are the holding cost rate and penalty cost rate respectively. Holding costs can be seen as opportunity costs, since a positive cash balance position cannot be invested in interest-bearing bonds, and penalty costs can be seen as interest costs on a negative cash balance position. The cost of changing the cash balance from \(x_0\) to \(x_1\) is given by

\[
B(x_1 - x_0) = \begin{cases} 
K^+ + k^+(x_1 - x_0), & x_1 > x_0, \\
0, & x = x_0, \\
K^- + k^-(x_0 - x_1), & x_1 < x_0,
\end{cases}
\]

where \(k^+, k^-, K^+, K^-\) are positive constants. It is assumed that no costs decrease the cash balance. The cumulative (net) demand in the time interval \([t, s]\) is modeled in terms of a Wiener process \(\omega(t)\) as

\[
D(t, s) = (s - t)\mu + (\omega(s) - \omega(t))\sigma
\]

The random variable \(D(t, s)\) is normally distributed with mean \((s-t)\mu\) and variance \((s-t)\sigma^2\).

In the presence of only proportional transaction cost, it is assumed that the optimal policy is continuous. In the presence of fixed transaction cost, the optimal policy is one of impulse control. Let \(0 = \tau_0 \leq \tau_1 \leq \cdots \leq \tau_N = T\) be a sequence of stopping times and \(\xi_0, \xi_1, \cdots, \xi_N\) a sequence of impulse controls such that the cash balance is changed instantaneously at time \(\tau_i\) due to impulse control \(\xi_i\). Than, if \(p\) is a policy, the expected transaction cost rate is given by

\[
\gamma = \inf_p \lim_{T \to \infty} T^{-1} E_{x(0), 0}^p \left\{ \sum_{i=0}^{N} B(\xi_i) + \int_0^T [C(x(s)) + B(w(s))] ds \right\}
\]

where \(E_{x(0), 0}^p\) is the expectation operator at time zero, cash level \(x(0)\) and policy \(p\).
2.4. Concluding remarks

Assuming that the optimal policy $f : x \rightarrow y(x)$ is of the simple form

\[
y(x) = \begin{cases} 
S_p, & x \leq s_p, \\
x, & s_p < x < s_d, \\
S_d, & x \geq s_d,
\end{cases}
\]

and $s_p \leq S_p \leq S_d \leq s_d$

optimal values for $s_p, S_p, S_d,$ and $s_d$ can be obtained. In general however, no closed form solutions exist. A few years after publication Constantinides and Richard [12] proved the interesting result that the simple form also is an optimal policy.

Summarizing, the optimal strategy is as follows: if the inventory level at the local cash becomes less than $s_p$, an order is placed at the central cash to increase the local cash level to $S_p$. If the local cash level becomes higher than $s_d$, the local cash level is reduced to $S_d$ by transferring money to the central cash. Note that according to our notation $Q_p = S_p - s_p$ and $Q_d = s_d - S_d$ if the demand and return quantities are always equal to one unit per transaction.

An extensive overview of these models is given by Inderfurth [25].

2.4 Concluding remarks

In the literature many articles have appeared which study repair management systems, where product returns are generated by product failures only. There is not much literature however on systems in which there is not a perfect correlation between the demand and return process, and in which the number of products in the system is not constant over time. We have divided these systems into three areas: periodic review models, continuous review models, and cash balancing models. The advantage of continuous review models over periodic review models is that continuous review allows an easier description and understanding of the long-run system behaviour, the modeling of lead-times is less complex, and the predetermined strategies are widely accepted and related to the ones that are used in practice. The drawback of the existing cash-balancing models is that the remanufacturing process is not explicitly modeled, and that they do not consider lead-times.
Among the continuous review models, the model of Muckstadt and Isaac [37] and the extensions of Van der Laan [58] and Van der Laan et al. [61], seem the most appropriate to study the long-run behaviour of a simple HMR system. However, their analysis is approximative while we want to study the real system behaviour. Therefore, in the next chapter, we will introduce an exact analysis of a single product, single component HMR system with non-zero (re)manufacturing lead-times.