Chapter 4

PUSH and PULL control without disposal

In this chapter we will present a numerical study to investigate the effects of remanufacturing on system behaviour (Section 4.3). Before doing so, we will investigate the system defined and analyzed in Chapter 3 under more general assumptions. In particular, we will study the system under Coxian-2 distributed inter-arrival times for demands and returns (Section 4.1), and correlation between the demand and return process (Section 4.2).

4.1 Modeling uncertainty in the demand and return processes

In this section we sketch how the average long-run costs under PUSH control, $\overline{C}_{PUSH}(s_m, Q_m, Q_r)$, and the average long-run costs under PULL control, $\overline{C}_{PULL}(s_m, Q_m, s_r, S_r)$, can be evaluated under Coxian-2 distributed demand and return inter-occurrence times. To do so, we first have to introduce the Coxian-2 distribution function more formally.

A random variable Y is Coxian-2 distributed if

$$Y = \begin{cases} Y_1 & \text{with probability } p \\ Y_1 + Y_2 & \text{with probability } 1-p \end{cases},$$

where Y_1 and Y_2 are independent exponentially distributed random variables with parameters γ_1 and γ_2 respectively. Furthermore, $0 \le p \le 1$, and $\gamma_1, \gamma_2 > 0$. It should be noted that the Coxian-2 distribution reduces to an Erlang-2 distribution if $p = 0$, and to an exponential distribution if $p = 1$.

In principle it is possible to fit the first three moments of an arbitrary distribution using a Coxian-2 distribution (see Altiok [2]), but for our purposes a two-moment fit will suffice. Since the Coxian-2 distribution has three parameters, there does not exist a unique way to do a two-moment fit to a random variable Y for which $cv_Y^2 > \frac{1}{2}$. Under a Gamma normalization, an arbitrary distribution function with first moment $E(Y)$ and squared coefficient of variation $cv_Y^2 \geq \frac{1}{2}$ can be approximated by a Coxian-2 distribution with

$$\begin{aligned}
\gamma_1 &= \tfrac{2}{E(Y)}\left(1+\sqrt{\left(\tfrac{cv_Y^2-\frac{1}{2}}{cv_Y^2+1}\right)}\right), \\
\gamma_2 &= \tfrac{4}{E(Y)} - \gamma_1, \\
p &= (1-\gamma_2 E(Y)) + \tfrac{\gamma_2}{\gamma_1},
\end{aligned} \tag{4.1}$$

and with a third moment equal to the third moment of a Gamma distribution with first moment $E(Y)$ and squared coefficient of variation $cv_Y^2 \geq \frac{1}{2}$ (see Tijms [56], pages 399-400).

Besides that the Coxian-2 distribution enables a two-moment fit of an arbitrary arrival process, another advantage is that inter-arrival times can be formulated as a Markov chain model $\{U(t)|t > 0\}$, with state space $\mathcal{S} = \{1, 2\}$. These states can be interpreted as being the states in a closed queuing network with two serial service stations and a single customer. With probability p the customer requires service from the first station only, and with probability $1-p$ service from both stations is required. The process is cyclical in the sense that after service completion the customer enters the first service station again (see Figure 4.1).

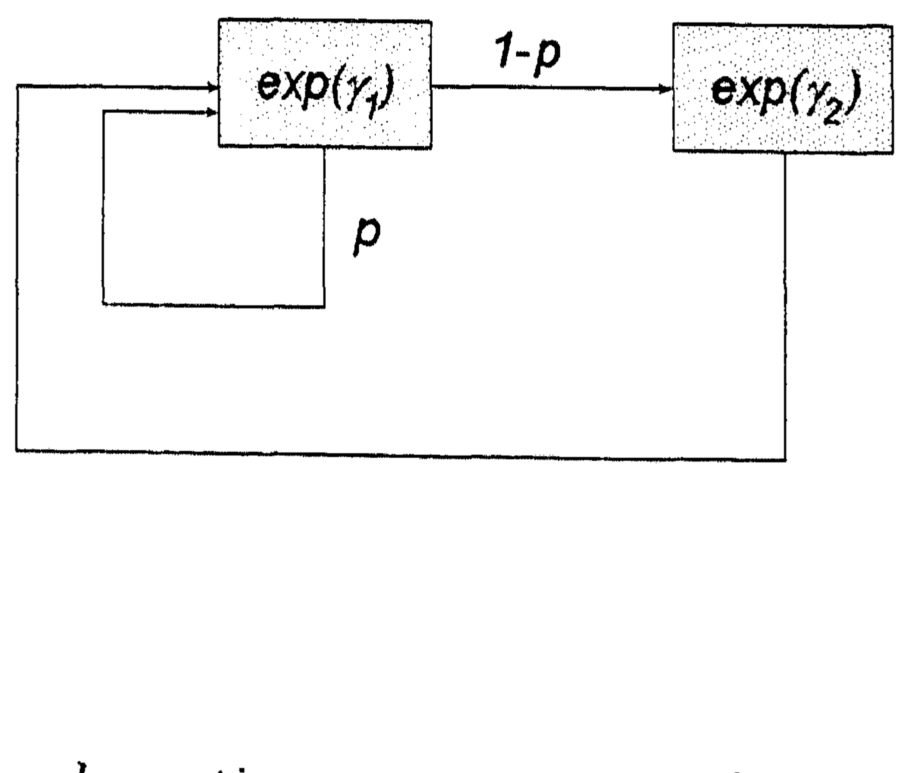

Figure 4.1. *A schematic representation of the Coxian-2 arrival process.*

If state $U(t) = 1$ corresponds to the situation that at time t the customer is being served by station one, and $U(t) = 2$ corresponds to the situation that at time t the customer is being served by station two, then the transition rates for this process are as follows.

$$\begin{aligned} \nu_{1,1} &= p\gamma_1, \\ \nu_{1,2} &= (1-p)\gamma_1, \\ \nu_{2,1} &= \gamma_2. \end{aligned}$$

The analysis of $\overline{C}_{PUSH}(s_m, Q_m, Q_r)$ and $\overline{C}_{PULL}(s_m, Q_m, s_r, S_r)$ under Coxian-2 distributed demand and/or return inter-occurrence times solely requires a modification of the underlying Markov chain models $\mathcal{M}_1$ and $\mathcal{M}_1'$, and $\mathcal{M}_2$ and $\mathcal{M}_2'$ respectively. To demonstrate this modification, we adapt $\mathcal{M}_1$ to account for Coxian-2 distributed return inter-occurrence times, with $E(Y) = \frac{1}{\lambda_R}$ and $cv_Y^2 \geq \frac{1}{2}$. The adapted Markov chain model $\mathcal{M}_{cox1}$ has a three-dimensional state variable

$$X_{cox1}(t) = \left(I_s(t), I_r^{OH}(t), U(t)\right),$$

with state space

$$\mathcal{S}_{cox1} = \{s_m+1, s_m+2, \ldots, \infty\} \times \{0, 1, \ldots, Q_r - 1\} \times \{1, 2\}.$$

Note that in $\mathcal{M}_{cox1}$ every return of a remanufacturable product corresponds to a service completion in the Markov chain model $\{U(t)|t > 0\}$. Furthermore, the transition rates in $\mathcal{M}_{cox1}$ are as follows.

$$\begin{aligned}
&\nu_{(i_s,i_r^{OH},1),(i_s,i_r^{OH}+1,1)} &&= p\gamma_1, && i_r^{OH} < Q_r - 1 \\
&\nu_{(i_s,i_r^{OH},1),(i_s,i_r^{OH},2)} &&= (1-p)\gamma_1, && \\
&\nu_{(i_s,i_r^{OH},2),(i_s,i_r^{OH}+1,1)} &&= \gamma_2, && i_r^{OH} < Q_r - 1 \\
&\nu_{(i_s,i_r^{OH},1),(i_s+Q_r,0,1)} &&= p\gamma_1, && i_r^{OH} = Q_r - 1 \\
&\nu_{(i_s,i_r^{OH},2),(i_s+Q_r,0,1)} &&= \gamma_2, && i_r^{OH} = Q_r - 1 \\
&\nu_{(i_s,i_r^{OH},u),(i_s-1,i_r^{OH},u)} &&= \lambda_D, && i_s > s_m + 1 \\
&\nu_{(i_s,i_r^{OH},u),(s_m+Q_m,i_r^{OH},u)} &&= \lambda_D, && i_s = s_m + 1
\end{aligned}$$

where γ_1, γ_2, and p are calculated according to (4.1), with $E(Y) = \lambda_R$ and $cv_Y^2 = cv_R^2$. The modifications of $\mathcal{M}_1'$, $\mathcal{M}_2$, and $\mathcal{M}_2'$ proceed analogously.

Remark 4.1 *The modifications needed to adapt for Coxian-2 distributed demand inter-occurrence times, are similar to the ones just described for Coxian-2 return inter-occurrence times. However, since the demand process is no longer Poisson, we can not evaluate the distribution of $D(t-L_{min},t)$ in (3.9) directly. We can solve this problem by conditioning on $Z(t-L_{max},t-L_{min})$ and following a transient analysis approach.*

4.2 Modeling correlation between demands and returns

Correlations between the timing of return and demand occurrences are modeled by modifying the Markov chain models $\mathcal{M}_1$ and $\mathcal{M}'_1$ for the PUSH strategy and the Markov chain models $\mathcal{M}_2$ and $\mathcal{M}'_2$ for the PULL strategy. As an example we extend the PUSH strategy for the situation with correlated returns and demands. The modified Markov chain model $\mathcal{M}_{cor1}$ has two-dimensional state-variable $X_1(t)$ and state space $\mathcal{S}_1$. If ρ_{RD} is the probability that a return immediately induces a demand, then the transition rates of $\mathcal{M}_{cor1}$ are as follows.

$$
\begin{array}{llll}
(i) & \nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} = (1-\rho_{RD})\lambda_R, & i_r^{OH} < Q_r - 1, & \\
(ii) & \nu_{(i_s,i_r^{OH}),(i_s+Q_r,0)} = (1-\rho_{RD})\lambda_R, & i_r^{OH} = Q_r - 1, & \\
(iii) & \nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH}+1)} = \rho_{RD}\lambda_R, & i_s > s_m + 1, & i_r^{OH} < Q_r - 1, \\
(iv) & \nu_{(i_s,i_r^{OH}),(i_s+Q_r-1,0)} = \rho_{RD}\lambda_R, & i_s > s_m + 1, & i_r^{OH} = Q_r - 1, \\
(v) & \nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH}+1)} = \rho_{RD}\lambda_R, & i_s = s_m + 1, & i_r^{OH} < Q_r - 1, \\
(vi) & \nu_{(i_s,i_r^{OH}),(s_m+Q_r,0)} = \rho_{RD}\lambda_R, & i_s = s_m + 1, & i_r^{OH} = Q_r - 1, \\
(vii) & \nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} = \lambda_D - \rho_{RD}\lambda_R, & i_s > s_m + 1, & \\
(viii) & \nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} = \lambda_D - \rho_{RD}\lambda_R, & i_s = s_m + 1. &
\end{array}
$$

These rates can be explained as follows:

(i)-(ii): These transition rates are associated with the fraction of returns that do *not* immediately induce a demand.

(iii-vi): These transition rates are associated with the fraction of returns that immediately induce a demand.

(vii–viii): These transition rates are associated with the fraction of demands that are not induced by a return.

The modifications of $\mathcal{M}_1'$, $\mathcal{M}_2$, and $\mathcal{M}_2'$ proceed analogously.

4.3 Numerical study

Since it seems technically infeasible to derive analytical results regarding the behaviour of the cost functions, we have set-up a numerical study. The numerical study starts out from a *base-case* scenario (Table 4.1). Subsequently, additional scenarios have been generated in which elements from the base-case scenario have been varied.

component	*value*	*unit of measure*
fixed remanufacturing costs (c_r^f)	0	\$/remanufacturing batch
variable remanufacturing costs (c_r^v)	0	\$/product remanufactured
fixed manufacturing costs (c_m^f)	0	\$/manufacturing batch
variable manufacturing costs (c_m^v)	0	\$/product manufactured
remanufacturable inventory costs (c_r^h)	0.5	\$/product per time unit
serviceable inventory costs (c_s^h)	1.0	\$/product per time unit
backordering costs (c_b)	50.0	\$/product per time unit
remanufacturing lead-time (μ_{L_r})	2	time units
manufacturing lead-time (μ_{L_m})	2	time units
demand process	Poisson	
demand intensity (λ_D)	1.0	product per time unit
demand uncertainty (cv_D^2)	1.0	
return process	Poisson	
return intensity (λ_R)	0.7	product per time unit
return uncertainty (cv_R^2)	1.0	
correlation (ρ_{RD})	0	

Table 4.1. *Base-case scenario.*

Scenario 1 (Figure 4.2). Scenario 1 is to compare HMR systems with remanufacturing to traditional systems without remanufacturing. We assume that the traditional systems are controlled by (s, S)-policies with associated costs $\overline{C}^*$, indicated by the dotted lines in Figure 4.2.

Scenario 2 (Figure 4.2). This scenario is to study costs at different stages of the product life cycle (represented by different ratios between λ_R and λ_D). The different ratios are obtained by keeping λ_D at the base-case level and varying λ_R between $[0, 0.9]$.

Scenario 3 (Figure 4.2). This scenario is to compare systems in which variable manufacturing costs are lower than variable remanufacturing costs ($c_m^v = 10$ and $c_r^v = 12$) to systems in which variable manufacturing costs are higher than variable remanufacturing costs ($c_m^v = 10$ and $c_r^v = 4$ or $c_r^v = 8$).

Scenario 4 (Figure 4.3). To indicate that the valuation of inventories is an important factor in deciding whether a PUSH strategy or a PULL strategy to implement, we have varied the remanufacturable inventory holding costs c_r^h between [0,1] at different fixed cost structures ($c_m^f = c_r^f = 0$ in Figure 4.3a, and $c_m^f = c_r^f = 10$ in Figure 4.3b).

Scenario 5 (Figure 4.4). This scenario is to investigate the influence of uncertainties in the timing of product returns. For this purpose the squared coefficient of variation of the Coxian-2 distributed return process (cv_R^2) has been varied between $[0.5, 3]$.

Scenario 6 (Figure 4.5). Here, the effect of correlations between returns and demands are investigated. For this purpose, the correlation coefficient ρ_{RD} has been varied over the interval [0,1]. Note that $\rho_{RD} = 0$ ($\rho_{RD} = 1$) corresponds to the extreme situation with zero (perfect) correlation between returns and demands.

Remark 4.2 *We have limited our discussion in this chapter to the above scenarios. Many alternative scenarios (related to e.g. backordering costs, fixed costs and demand uncertainties) are discussed in Van der Kruk ([57]). The influence of lead-times and lead-time uncertainty is investigated in Chapter 5.*

4.3.1 HMR systems versus traditional systems

The total costs in the HMR system implemented at the copier remanufacturer turned out to be *lower* than in their traditional system without remanufacturing, mainly because the reuse of modules saves material costs. These cost savings make it cheaper to remanufacture a used module than to manufacture a completely new module.

However, alternative case-studies and this numerical study have shown that the opposite cost effect may also occur, i.e., the operating costs in PUSH and PULL controlled systems with remanufacturing may become *higher* than in traditional (s, S) controlled systems without remanufacturing, even when the variable costs to remanufacture a used module are lower than the variable costs to manufacture a completely new module (see Figure 4.2, in which all fixed costs are zero).

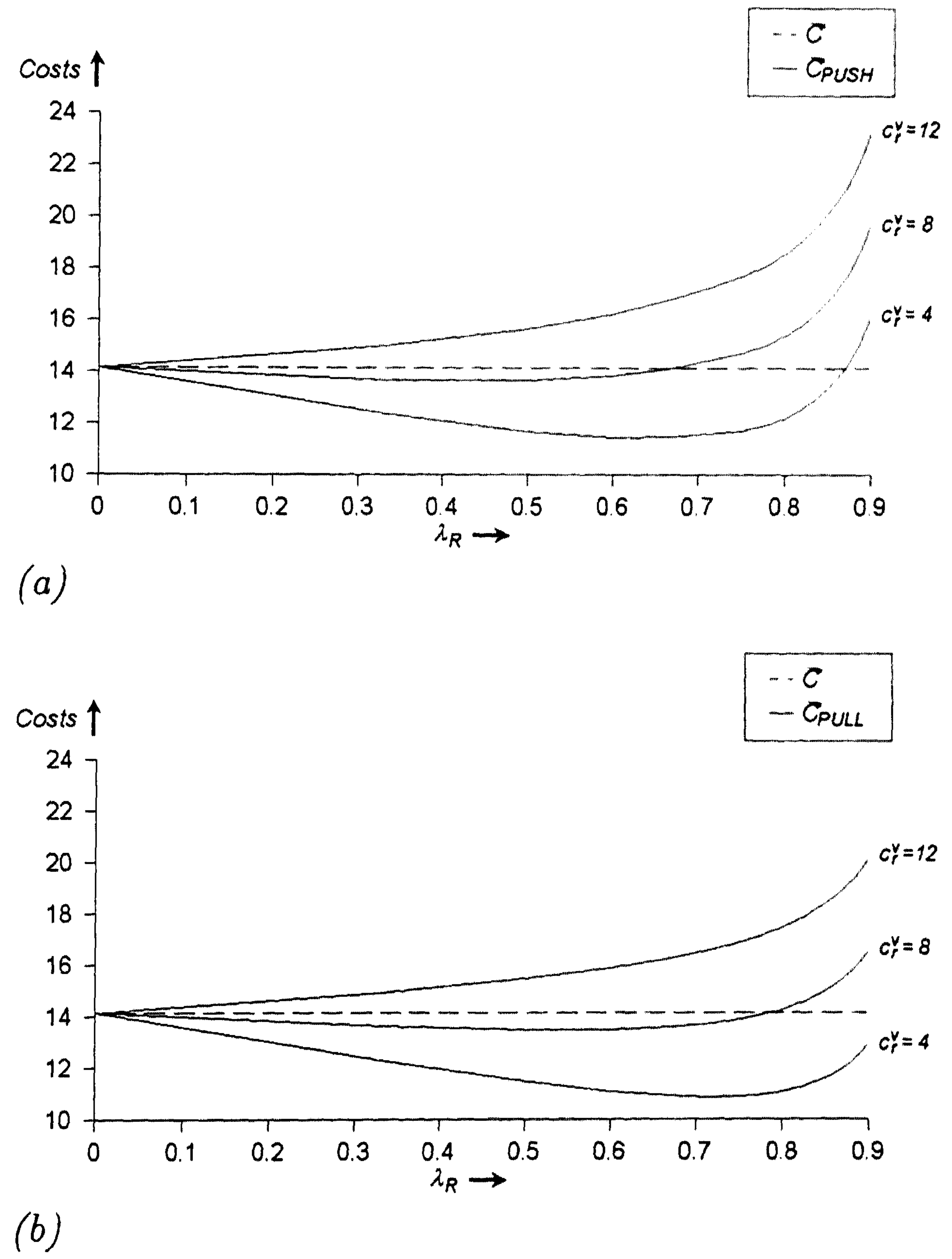

Figure 4.2. *Costs as function of the return intensity for the situation with remanufacturing and for the situation without remanufacturing under PUSH control (a), and under PULL control (b);* $c_m^v = 10$.

This effect occurs due to various sources of uncertainty which are absent in traditional manufacturing systems. These uncertainties (to be discussed in more detailed in Section 4.3) induce a high variability in the output of the remanufacturing process, and cause in this way an increase in the sum of inventory holding costs and backordering costs. Apparently, the increase in inventory holding costs and backordering costs may dominate cost savings from material reuse.

4.3.2 PUSH versus PULL control

In the beginning the HMR system at the copier remanufacturer was controlled by a variant of the PUSH strategy. However, investigations showed that PULL control could be economically favorable, particularly due to savings in inventory holding costs. Therefore, the copier remanufacturer decided to change their PUSH controlled system into a PULL controlled system.

The cost savings related to this change could have been expected in advance, since the copier remanufacturer values modules in remanufacturable inventory much lower than modules in serviceable inventory. More precise, the inventory holding costs of serviceable modules were taken proportional to the manufacturing costs of a new module, independently of whether the serviceable module had been manufactured or remanufactured. The inventory holding costs of a remanufacturable module were taken proportional to the difference between the manufacturing costs of a new module and the remanufacturing costs of a returned module.

Under such an inventory holding cost structure a strategy in which the serviceable inventory is kept low at the extend of a somewhat higher remanufacturable inventory tends to perform better than a strategy in which most of the stock is kept as serviceable inventory. Indeed, Figure 4.3 shows that only in the situation where remanufacturable inventory is valued at least as high as serviceable inventory, a PUSH strategy may be economically favorable over a PULL strategy. In this case a higher serviceable inventory enables to react faster on extreme demand situations, resulting in lower backordering costs.

Nevertheless, although the PULL strategy may have *economical* advantages over the PUSH strategy, the PUSH strategy may from an *organizational* point of view still be preferable, since remanufacturable inventory and serviceable inventory can be controlled independently.

Remark 4.3 *It should be noticed that the cost dominance relation between $\overline{C}_{PUSH}$ and $\overline{C}_{PULL}$ is independent of variable manufacturing or variable remanufacturing costs, since these variable costs are equal under both strategies. Furthermore, experiments have shown that the cost dominance relation is not much influenced by fixed costs (see Figure 4.3 for an example*

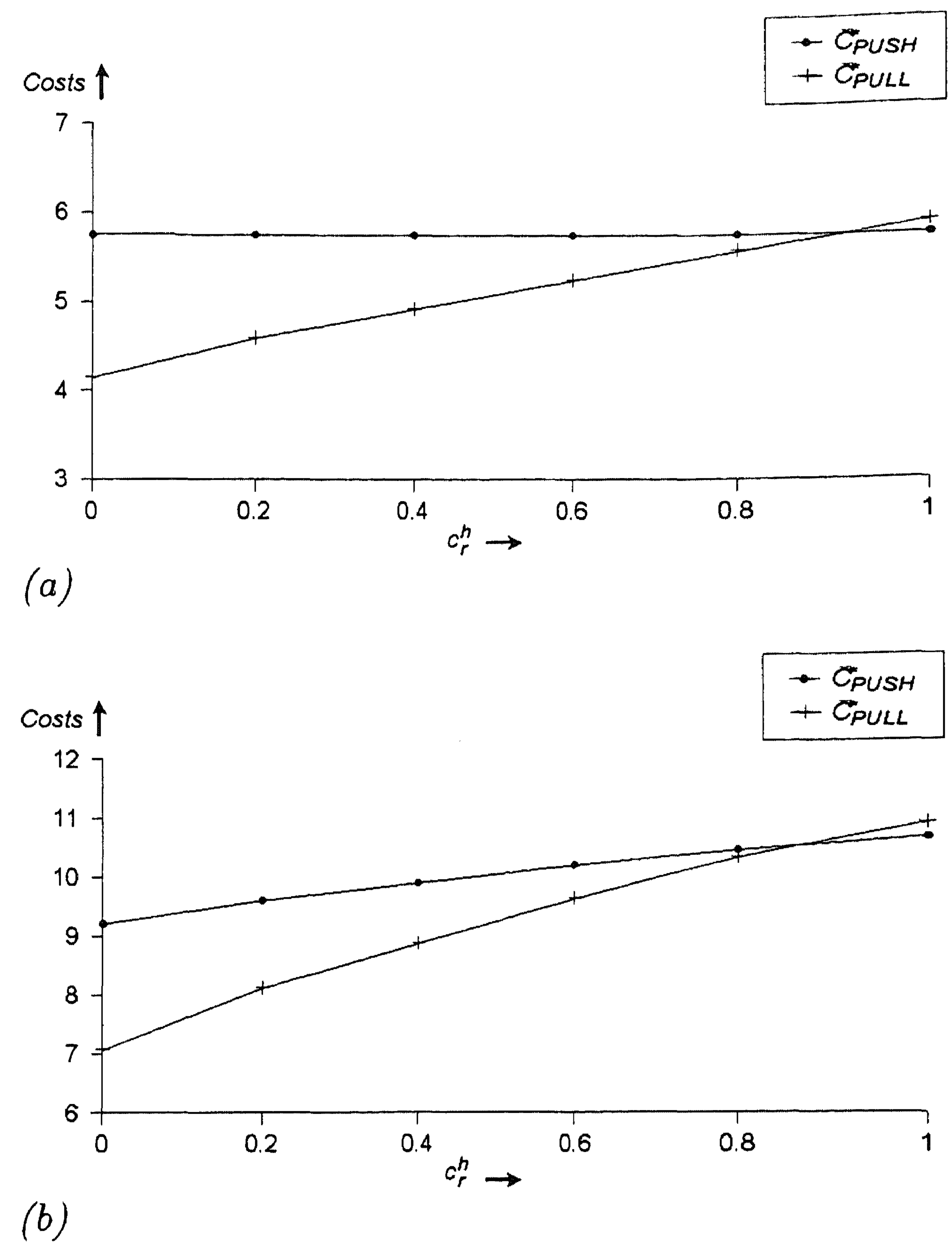

Figure 4.3. *Costs as function of the remanufacturing holding costs with* $c_m^f = c_r^f = 0$ *(a), and* $c_m^f = c_r^f = 10$ *(b).*

and Van der Kruk [57] for a more extensive study) or by the backordering costs (except when the backordering costs become extremely low).

4.3.3 Advises to management

Based on the observation of the numerical study we further provide the following advises to the management of remanufacturing companies:

- *From an economical point of view it is not always profitable to remanufacture all remanufacturables, even when the return intensity is lower than the demand intensity.*

The cost decreases in Figure 4.2 occur since an increase in the return intensity implies that a larger fraction of the demands can be fulfilled by remanufacturing operations instead of by more expensive manufacturing operations. However, when the return intensity further increases, the variability in the output of the remanufacturing process increases, leading again to a higher sum of inventory holding costs and backordering costs (see also above). The return intensity at which $\overline{C}^*_{(.)}$ reaches its minimum depends on the cost structure and other factors, but most importantly, Figure 4.2 shows that $\overline{C}^*_{(.)}$ may reach its minimum far before the average number of product returns equals the average number of demands. This indicates that remanufacturing companies should at *all* stages of the product life-cycle consider which remanufacturables should actually be remanufactured. If alternative options such as disposal exist to handle the remanufacturables, then these may be economically favorable (see Chapter 6 for an extension of the PUSH and PULL strategies with planned product disposals).

• *Remanufacturing companies should attempt to keep the uncertainty in the timing and quality of returned products as low as possible.*

Figure 4.4 shows that total system costs increase with increasing cv_R^2, in particular when the return intensity increases. Uncertainties in the number of remanufacturable products are mainly due to two components, i.e., uncertainties in the *timing* of product returns and uncertainties in the *quality* of returned products. While quality control is usually a subject of continuous concern for all sorts of products, with or without remanufacturing, controlling the timing of product returns is typical for remanufacturing.

A popular instrument to reduce the uncertainty in the timing of product returns are lease contracts with a fixed lease period. Furthermore, maintenance contracts and built-in diagnostic tools to obtain reliable information on the quality of the product components a long time ahead of product return seem valuable instruments.

• *Remanufacturing companies should keep track of correlations between product returns and product demands.*

Figure 4.5 shows that when the correlation between product returns and product demands increases, the total system costs decrease. Furthermore, the magnitude of this effect increases with an increasing return intensity. The cost reduction occurs since correlations between returns and demands reduce total system uncertainty. In this way, the sum of inventory holding costs and backordering costs can be reduced. The observed effect stresses

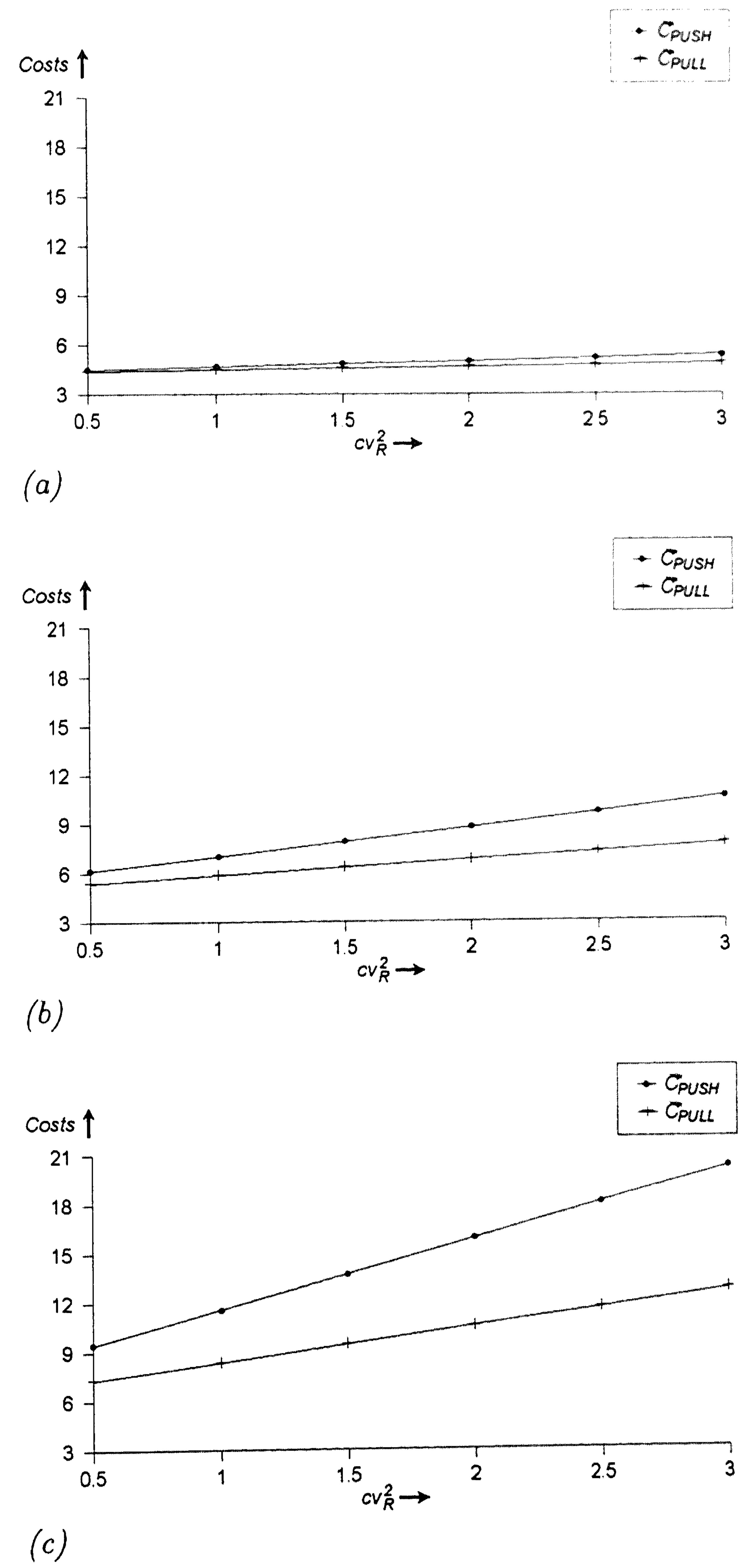

Figure 4.4. *Costs as function of the return uncertainty with* $\lambda_R = 0.5$ *(a),* $\lambda_R = 0.8$ *(b),* $\lambda_R = 0.9$ *(c).*

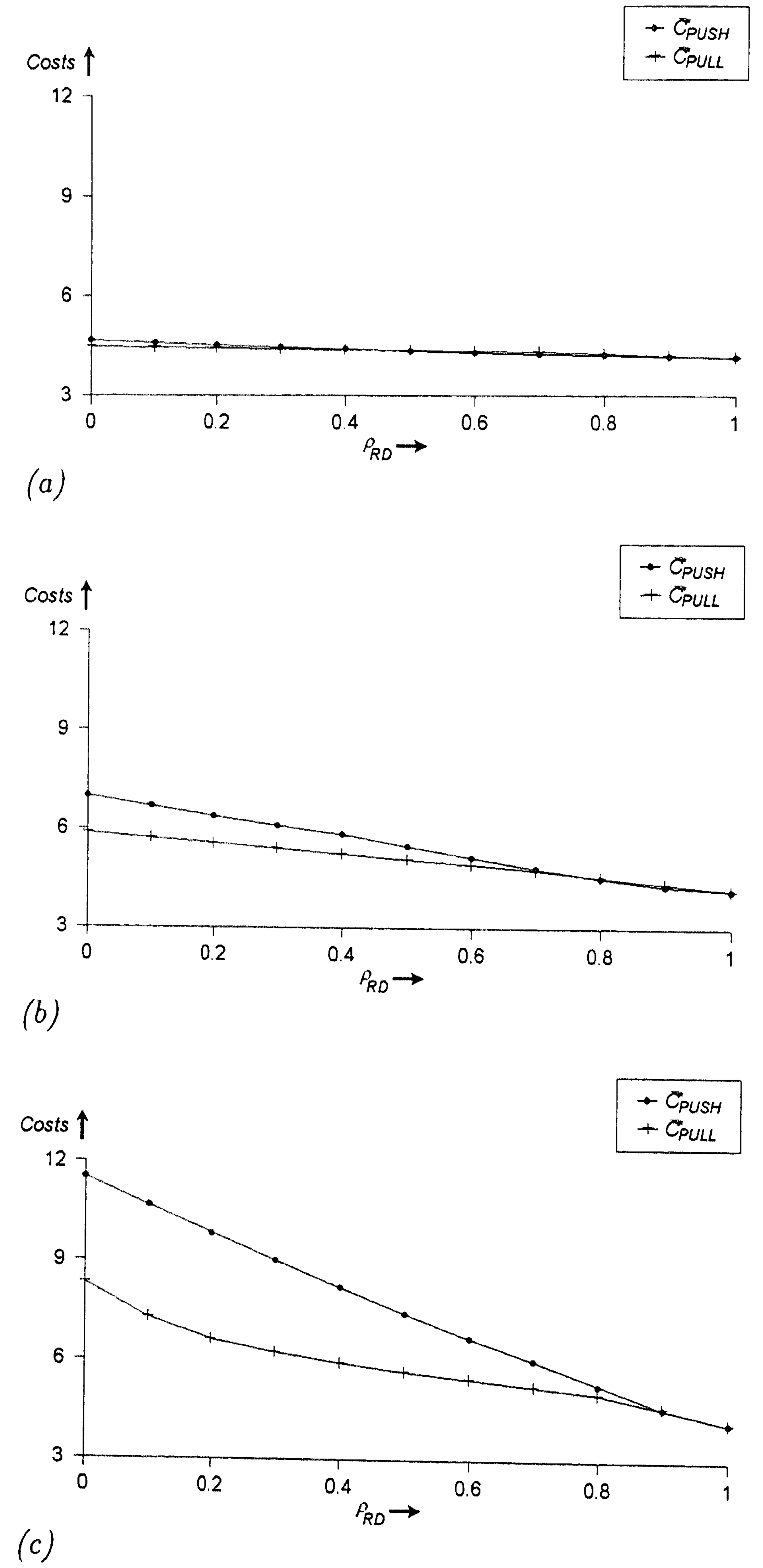

Figure 4.5. *Costs as function of the correlation coefficient with* $\lambda_R = 0.5$ *(a),* $\lambda_R = 0.8$ *(b), and* $\lambda_R = 0.9$ *(c).*

the importance of data collection to estimate ρ_{RD}. Both underestimates and overestimates of ρ_{RD} may lead to unnecessary high costs for remanufacturing companies.

4.4 Summary and conclusions

This chapter presents one of the first attempts to analyze the effects of remanufacturing in PUSH and PULL controlled production/inventory systems. An important conclusion is, that efficient planning and control in these systems tends to be more complex than in traditional systems without remanufacturing.

The numerical study confirms the findings in practice at the remanufacturer of photocopiers, that important factors which are responsible for this complexity include *system interactions*, such as the interaction between the output of the manufacturing and remanufacturing processes, and the correlation between demands and returns, and *uncertainties in the return process*, such as uncertainties in the timing and quality of returned products. Clearly, these factors are not present in traditional systems.

This study has also indicated that management should take the decision to remanufacture only after thorough study, since total expected production and inventory related costs in systems with remanufacturing may become higher than in systems without remanufacturing. Once management has decided to remanufacture, the selection of a suitable control policy in combination with other efficiency improving actions is essential. Examples of such actions include the stimulation of lease contracts instead of regular purchasing contracts (to reduce the uncertainty in the timing of product returns), robust product design, maintenance contracts and diagnostic tools (to reduce the uncertainty in the quality of returned products), and the collection of data on correlations between demands and returns (to reduce total system uncertainty). Finally, the valuation of inventories turns out to be an important factor in deciding between PUSH or PULL control.

From a technical point of view, we conclude that the analysis of control policies in HMR systems with stochastic demands and returns may become mathematically complex, even though the strategies are extensions of seemingly straightforward PUSH and PULL concepts. The existence of a 'simple' model and methodology by means of which the effects that have been observed in our numerical study can be proved analytically seem therefore highly questionable.