Chapter 5

Lead time effects

In Chapter 3 we we introduced a hybrid manufacturing/remanufacturing (HMR) system and an exact analysis to evaluate the associated average long-run costs under PUSH and PULL control. Furthermore, we presented in Chapter 4 a numerical study in which the average system costs are compared under different assumptions with respect to the demand and return processes, and cost structure. This chapter extends the analysis of Chapter 3 and the numerical study of Chapter 4 to allow for *stochastic* lead-times for manufacturing and remanufacturing. In particular we investigate the effects of lead-time *duration* and lead-time *variability*.

The motivations to devote a separate chapter to the analysis of lead-times are, that the analysis requires a non-trivial modification of the cost calculation procedure, and that some of the effects of lead-times on costs need further explanation since they appear somewhat counter-intuitive.

This chapter is further organized as follows. After the introduction of some additional notation and system assumptions needed for the mathematical analysis, Section 5.1 outlines the procedures to numerically evaluate the cost function in presence of stochastic lead-times. Section 5.3 presents the set-up of a numerical study and discusses its outcomes. Finally, concluding remarks are presented in Section 5.3.

5.1 System characteristics and analysis

In presence of stochastic lead-times, the analysis of the cost functions $\overline{C}_{PUSH}(s_m, Q_m, Q_r)$ and $\overline{C}_{PULL}(s_m, Q_m, s_r, S_r)$ only differs from the analysis in Chapter 3 regarding the distribution of net inventory. To do the analysis, we need some additional notation regarding the (re)manufacturing lead-time distributions.

Let the manufacturing lead-time L_m be a *discrete valued* stochastic variable with n_m possible outcomes, say $\ell_m^{(1)}, \ell_m^{(2)}, \ldots, \ell_m^{(n_m)}$. Similarly, let the remanufacturing lead-time L_r be a *discrete valued* stochastic variable with n_r possible outcomes, say $\ell_r^{(1)}, \ell_r^{(2)}, \ldots, \ell_r^{(n_r)}$. Without loss of generality we assume that $\ell_m^{(1)} < \ell_m^{(2)} < \ldots < \ell_m^{(n_m)}$, and $\ell_r^{(1)} < \ell_r^{(2)} < \ldots < \ell_r^{(n_r)}$.

The probability mass function (p.m.f) of L_m and the p.m.f. of L_r are then given by

$$p_{L_m}^{(j)} = \Pr\{L_m = \ell_m^{(j)}\}, \quad 1 \le j \le n_m,$$

and

$$p_{L_r}^{(j)} = \Pr\{L_r = \ell_r^{(j)}\}, \quad 1 \le j \le n_r.$$

Furthermore, define

$$\mathcal{L} = \{\ell^{(1)}, \ell^{(2)}, \ldots \ell^{(n)}\} \stackrel{\text{def}}{=} \{\ell_r^{(1)}, \ell_r^{(2)}, \ldots \ell_r^{(n_r)}\} \bigcup \{\ell_m^{(1)}, \ell_m^{(2)}, \ldots \ell_m^{(n_m)}\}$$

with

$$\ell^{(1)} < \ell^{(2)} < \ldots < \ell^{(n)}.$$

Then we have for $1 \le k \le n$

$$P_{L_m}^{(k)} = \Pr\{L_m < \ell^{(k)}\} = \sum_{i=1}^{max\{j|\ell_m^{(j)} < \ell^{(k)}\}} p_{L_m}^{(i)}, \tag{5.1}$$

and

$$P_{L_r}^{(k)} = \Pr\{L_r < \ell^{(k)}\} = \sum_{i=1}^{max\{j|\ell_r^{(j)} < \ell^{(k)}\}} p_{L_r}^{(i)}. \tag{5.2}$$

We need the cumulative probabilities in (5.1) and (5.2) to adapt the transient analysis that was introduced in Section 3.2 for deterministic lead-times, to the case of stochastic lead-times.

The procedure to calculate the distribution of net inventory is outlined in Section 5.1.1 for the PUSH strategy and in Section 5.1.2 for the PULL strategy.

5.1.1 Analysis of the PUSH strategy

This section outlines a numerical procedure to calculate the distribution of the net serviceable inventory in the steady-state situation, $\Pr\{I_s^{net} = i_s^{net}\}$. From this distribution, the time-average on-hand serviceable inventory and the time-average backordering position are derived as

$$\overline{I}_s^{OH} = \sum_{i_s^{net}>0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}, \tag{5.3}$$

and

$$\overline{B} = - \sum_{i_s^{net}<0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}. \tag{5.4}$$

To calculate $\Pr\{I_s^{net} = i_s^{net}\}$, we use relations (3.9) and (3.12) which still apply, i.e.,

$$\begin{aligned} I_s^{net}(t) = \;& I_s(t - L^{max}) \\ & + Z(t - L^{max}, t - L^{min}) \\ & - D(t - L^{min}, t). \end{aligned}$$

and

$$\begin{aligned} &\lim_{t\to\infty} \Pr\{I_s^{net}(t) = i_s^{net}\} \\ &\quad = \sum_{\Omega} \; \exp^{-\lambda_D L^{min}} \frac{(\lambda_D L^{min})^d}{d!} \\ &\qquad\quad \times \pi_1(i_s, i_r^{OH}) \times h_{z|(i_s,i_r^{OH})}(\Delta L). \end{aligned} \tag{5.5}$$

Note that here $L^{min} = \ell^{(1)}$, $L^{max} = \ell^{(n)}$, and $\Delta L = \ell^{(n)} - \ell^{(1)}$. The conditional probability $h_{z|(i_s,i_r^{OH})}(\Delta L)$ is calculated as

$$h_{z|(i_s,i_r^{OH})}(\Delta L) = \sum_{k=s_m+1}^{\infty} \sum_{\ell=0}^{Q_r-1} q_{(k,\ell,z)|(i_s,i_r^{OH},0)}(\Delta L) \tag{5.6}$$

where $q_{s^{(1)}|s^{(0)}}(\Delta L)$ is the conditional probability that during the interval $(t - \ell^{(n)}, t - \ell^{(1)}]$ the initial system state changes from state $s^{(0)} \in \mathcal{S}_1'$ into state $s^{(1)} \in \mathcal{S}_1'$.

Unfortunately, probabilities $q_{s^{(1)}|s^{(0)}}(\ell^{(n)} - \ell^{(1)})$ cannot be obtained directly from a Markov chain model, because the transitions of $Z(t-\ell^{(n)}, t-\ell^{(1)}]$ are time-dependent: a change in $Z(t-\ell^{(n)}, t-\ell^{(1)}]$ due to a (re)manufacturing order depends on the probability that that particular order will arrive before time t, and this probability depends on the difference between the time of ordering and time t. However, the transitions in each sub-interval $(t-\ell^{(k)}, t-\ell^{(k-1)}]$ for $k=2,\ldots,n$ are time-independent. For instance, every manufacturing order that is ordered in the interval $(t-\ell^{(k)}, t-\ell^{(k-1)}]$ has probability $\Pr\{L_m < \ell^{(k)}\}$ that the order will arrive *before* time t, and a probability $1-\Pr\{L_m < \ell^{(k)}\}$ that the order will arrive *after* time t (see Figure 5.1).

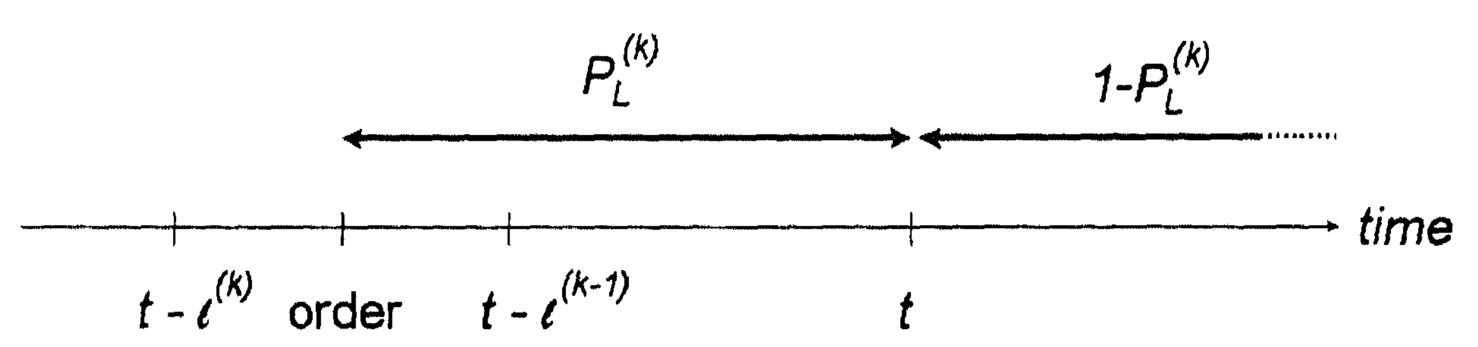


Figure 5.1. *An order placed in the interval $(t-\ell^{(k)}, t-\ell^{(k-1)}]$ with stochastic lead-time L will arrive before or at time t with probability $P_L^{(k)} = \Pr\{L < \ell^{(k)}\}$, and will arrive after time t with probability $1-P_L^{(k)}$.*

Thus, we define for each sub-interval $(t-\ell^{(k)}, t-\ell^{(k-1)}]$, $1 < k \le n$, a Markov chain $\mathcal{M}_1^{(k)}$ with state space $\mathcal{S}_1'$ and state variable

$$X_1^{(k)}(\tau) = (I_s, I_r^{OH}, Z(u, u+\tau)).$$

By definition, $X_1^{(k)}(\tau) = (i_s, i_r^{OH}, z)$ whenever $I_s(\tau) = i_s$, *and* $I_r^{OH}(\tau) = i_r^{OH}$, *and* $Z(u,\tau) = z$, with $\tau \in (u, u+\ell^{(k)} - \ell^{(k-1)}]$.

Using expressions (5.1) and (5.2) we arrive at the following time independent transition rates of $\mathcal{M}_1^{(k)}$:

$$\nu^{(k)}_{(i_s,i_r^{OH},z),(i_s,i_r^{OH}+1,z)} = \lambda_R, \qquad i_r^{OH} < Q_r - 1,$$

$$\left.\begin{aligned} \nu^{(k)}_{(i_s,i_r^{OH},z),(i_s+Q_r,0,z+Q_r)} &= P_{L_r}^{(k)}\lambda_R, \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(i_s+Q_r,0,z)} &= (1-P_{L_r}^{(k)})\lambda_R \end{aligned}\right\}, \qquad i_r^{OH} = Q_r - 1,$$

$$\nu^{(k)}_{(i_s,i_r^{OH},z),(i_s-1,i_r^{OH},z-1)} = \lambda_D, \qquad i_s > s_m + 1,$$

$$\left.\begin{aligned} \nu^{(k)}_{(i_s,i_r^{OH},z),(s_m+Q_m,i_r^{OH},z+Q_m-1)} &= P_{L_m}^{(k)}\lambda_D, \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(s_m+Q_m,i_r^{OH},z-1)} &= (1-P_{L_m}^{(k)})\lambda_D \end{aligned}\right\}, \qquad i_s = s_m + 1.$$

Related to the abovementioned Markov chain we define the conditional probability $q^{(k)}_{s^{(2)}|s^{(0)}}(\ell^{(k)}-\ell^{(k-1)})$, which is the probability that during the interval $(t-\ell^{(k)}, t-\ell^{(k-1)}]$ the initial system state of $X_1^{(k)}$ changes from state $s^{(0)}$ into state $s^{(1)}$. Then, the probabilities $q_{s^{(1)}|s^{(0)}}(\ell^{(n)}-\ell^{(1)})$ can be expressed in terms of the recursive relation $(k=2,\ldots,n)$

$$q_{s^{(1)}|s^{(0)}}(\ell^{(k)}-\ell^{(1)}) = \sum_{s\in\mathcal{S}_1'} q_{s^{(1)}|s}(\ell^{(k-1)}-\ell^{(1)}) q^{(k)}_{s|s^{(0)}}(\ell^{(k)}-\ell^{(k-1)}). \quad (5.7)$$

To calculate $q^{(k)}_{s^{(2)}|s^{(0)}}(\ell^{(k)}-\ell^{(k-1)})$ we evaluate the *transient* behaviour of the Markov chain $\mathcal{M}_1^{(k)}$. To do so we use the discretization technique, which is discussed in more detail in Section 3.2.1. This technique transforms $\mathcal{M}_1^{(k)}$ into an equivalent discrete time Markov chain $\overline{\mathcal{M}}_1^{(k)}$. The one-step transition probabilities $\overline{q}^{(1,k)}_{s^{(1)}|s^{(0)}}$ in $\overline{\mathcal{M}}_1^{(k)}$ are as follows.

$$\overline{q}^{(1,k)}_{s^{(1)}|s^{(0)}} = \begin{cases} \dfrac{\nu^{(k)}_{s^{(0)},s^{(1)}}}{\nu^{(k)}} & s^{(0)} \neq s^{(1)}, \\ \\ 1-\dfrac{\nu^{(k)}_{s^{(0)}}}{\nu^{(k)}}, & s^{(0)} = s^{(1)}, \end{cases} \quad (5.8)$$

where $\nu^{(k)}_{s^{(0)}} = \sum_{s^{(1)}\in\mathcal{S}_1'} \nu^{(k)}_{s^{(0)},s^{(1)}}$, and the constant $\nu^{(k)}$ is chosen such that $\nu^{(k)} = \max_{s^{(0)}\in\mathcal{S}_1'}\{\nu^{(k)}_{s^{(0)}}\}$.

The m-step transition probabilities $\overline{q}^{(m,k)}_{s^{(1)}|s^{(0)}}$ are derived from the recursive relation

$$\overline{q}^{(m,k)}_{s^{(1)}|s^{(0)}} = \sum_{s^{(2)}\in\mathcal{S}_1'} \overline{q}^{(m-1,k)}_{s^{(2)}|s^{(0)}}\ \overline{q}^{(1,k)}_{s^{(1)}|s^{(2)}}. \quad (5.9)$$

Finally, the conditional probabilities $q^{(k)}_{s^{(1)}|s^{(0)}}(\ell^{(k)}-\ell^{(k-1)})$ are calculated as

$$\begin{aligned} &q^{(k)}_{s^{(1)}|s^{(0)}}(\ell^{(k)}-\ell^{(k-1)}) \\ &\quad = \sum_{m=0}^{\infty} \exp^{-\nu^{(k)}(\ell^{(k)}-\ell^{(k-1)})} \frac{(\nu^{(k)}(\ell^{(k)}-\ell^{(k-1)}))^m}{m!} \overline{q}^{(m,k)}_{s^{(1)}|s^{(0)}}. \end{aligned} \quad (5.10)$$

Relations (5.5)–(5.10) now enable to calculate the average net inventory (5.3) and the average backorder position (5.4), which concludes our analysis for the PUSH strategy.

5.1.2 Analysis of the PULL strategy

To evaluate the distribution of the net serviceable inventory under the PULL strategy, the procedure in Section 5.1.1 can also be applied. In doing so, we have to replace the Markov chain $\mathcal{M}_2$ defined in Section 3.2.2 by the family of Markov chains $\mathcal{M}_2^{(k)}$, $1 < k \leq n$. The Markov chain $\mathcal{M}_2^{(k)}$ has state space

$$\mathcal{S}_2' = \{s_m+1, \ldots, U\} \times \{0, \ldots, \infty\} \times \{-\infty, \ldots, \infty\},$$

and state-vector $X_2^{(k)}(\tau) = (i_s, i_r^{OH}, z)$, indicating that at time $\tau \in (t - \ell^{(k)}, t - \ell^{(k-1)}]$ $I_s = i_s$, $I_r^{OH} = i_r^{OH}$, and $Z = z$.

The transition rates of the Markov chain $\mathcal{M}_2^{(k)}$ are as follows.

$$\nu^{(k)}_{(i_s,i_r^{OH},z),(i_s,i_r^{OH}+1,z)} = \lambda_R \ , \qquad \begin{array}{l} i_s > s_r \text{ or} \\ i_r^{OH} < S_r - i_s - 1, \end{array}$$

$$\left.\begin{array}{l} \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,0,z+S_r-i_s)} = P_{L_r}^{(k)}\lambda_R, \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,0,z)} = (1-P_{L_r}^{(k)})\lambda_R \end{array}\right\}, \qquad \begin{array}{l} i_s \leq s_r \text{ and} \\ i_r^{OH} = S_r - i_s - 1, \end{array}$$

$$\nu^{(k)}_{(i_s,i_r^{OH},z),(i_s-1,i_r^{OH},z-1)} = \lambda_D \ , \qquad \begin{array}{l} i_s > s_r + 1 \text{ or} \\ \{i_s > s_m + 1 \text{ and} \\ i_r^{OH} \leq S_r - i_s\Big\}, \end{array}$$

$$\left.\begin{array}{l} \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-x,z+x-1)} = P_{L_r}^{(k)}\lambda_D, \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-x,z-1)} = (1-P_{L_r}^{(k)})\lambda_D \end{array}\right\}, \qquad \begin{array}{l} i_s = s_r + 1 \text{ and} \\ i_r^{OH} \geq x, \end{array}$$

$$\left.\begin{array}{l} \nu^{(k)}_{(i_s,i_r^{OH},z),(s_m+Q_m,i_r^{OH},z+Q_m-1)} = P_{L_m}^{(k)}\lambda_D \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(s_m+Q_m,i_r^{OH},z)} = (1-P_{L_m}^{(k)})\lambda_D \end{array}\right\}, \qquad \begin{array}{l} i_s = s_m + 1 \text{ and} \\ s_r \geq s_m + Q_m \text{ and} \\ i_r^{OH} \geq y, \end{array}$$

$$\left.\begin{array}{l} \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-y,z+S_r-s_m-1)} = P_{11}\lambda_D \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-y,z+Q_m-1)} = P_{12}\lambda_D \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-y,z+y-1)} = P_{21}\lambda_D \\ \nu^{(k)}_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-y,z-1)} = P_{22}\lambda_D \end{array}\right\}, \qquad \begin{array}{l} i_s = s_m + 1 \text{ and} \\ i_r^{OH} < S_r - s_m \text{ and} \\ \Big\{i_r^{OH} < y \\ \text{or } s_r < s_m + Q_m\} . \end{array}$$

where $x = S_r - s_r$, $y = S_r - (s_m + Q_m)$, $P_{11} = P_{L_m}^{(k)} P_{L_r}^{(k)}$, $P_{12} = P_{L_m}^{(k)}(1 - P_{L_r}^{(k)})$, $P_{21} = (1 - P_{L_m}^{(k)}) P_{L_r}^{(k)}$, and $P_{22} = (1 - P_{L_m}^{(k)})(1 - P_{L_r}^{(k)})$.

5.1.3 Optimization with respect to control parameters

For the case with deterministic lead-times we reported in Chapter 3 that we did not succeed in identifying a 'nice' structure in the cost functions

$\overline{C}_{PUSH}(s_m, Q_m, Q_r)$ and $\overline{C}_{PULL}(s_m, Q_m, s_r, S_r)$. Unfortunately, the cost function in case of stochastic lead-times is even more complex than with deterministic lead-times. Therefore, as in the numerical study of Chapter 4 we are committed to apply an enumerative search procedure to calculate

$$\overline{C}^*_{PUSH} = \min_{(s_m, Q_m, Q_r)} \overline{C}_{PUSH}(s_m, Q_m, Q_r)$$

and

$$\overline{C}^*_{PULL} = \min_{(s_m, Q_m, s_r, S_r)} \overline{C}_{PULL}(s_m, Q_m, s_r, S_r).$$

5.2 Numerical study

In a recent study, Song [51] investigated the effect of manufacturing lead-times on total expected costs in a stochastic production/inventory system. The system differs from ours, in that no product returns occur ($\lambda_R = 0$) and, consequently, no remanufacturing operation exists. Furthermore, demands are modeled by a *compound* Poisson process, which is a generalization of the Poisson demand process considered in our system. Finally, manufacturing costs consist of a linear component only, i.e., fixed manufacturing cost are zero ($c^f_m = 0$).

Under these assumptions, a base-stock control policy is optimal (see [5, page 604]). Under base-stock control, Song showed the following two results to hold:

- An increase (decrease) in the *duration* of the manufacturing lead-time results in higher (lower) total expected costs,
- An increase (decrease) in the *variability* of manufacturing lead-times results in higher (lower) total expected costs.

To express the variability of stochastic variables, we follow the definition used by Song:

Definition 5.1 (Song, page 609 in [51]) *Consider two random variables X and Y with equal mean $E(X) = E(Y)$, having distributions F and G with densities f and g. Suppose X and Y are either both continuous or both discrete. We say, X is more variable than Y, denoted by $X \geq_{var} Y$, if*

Sign change $(f - g) = 2$, with sign sequence +, -, +

That is, f crosses g exactly twice, first from above and then from below.

Note that when $\lambda_R = 0$ and $c_m^f = 0$, the above results apply to our PUSH and PULL control strategies as well, since both strategies are in this case equivalent to a base-stock control policy with base-stock level $s_m + 1$.

Main objective in this numerical study is to investigate whether the above results also apply to our PUSH and PULL strategies when $\lambda_R > 0$. For this purpose we have subdivided our numerical study in two parts. In part one (Section 5.2.1) we analyze the influence of the manufacturing and remanufacturing lead-time *duration* on total expected costs, whereas in part two (Section 5.2.2) we investigate the influence of *variability* in the manufacturing and remanufacturing lead-times on total expected costs.

The experiments in both sections start out from a base-case scenario defined in Table 5.1. The influence of alternative assumptions regarding lead-times and other input parameters are briefly discussed in Section 5.2.3.

component	*value*	*unit of measure*
fixed remanufacturing costs (c_r^f)	0	\$/remanufacturing batch
variable remanufacturing costs (c_r^v)	0	\$/product remanufactured
fixed manufacturing costs (c_m^f)	0	\$/manufacturing batch
variable manufacturing costs (c_m^v)	0	\$/product manufactured
remanufacturable inventory costs (c_r^h)	0.5	\$/product per time unit
serviceable inventory costs (c_s^h)	1.0	\$/product per time unit
backordering costs (c_b)	50.0	\$/product per time unit
remanufacturing lead-time		
mean (μ_{L_r})	2	time units
variance ($\sigma^2_{L_r}$)	0	
manufacturing lead-time		
mean (μ_{L_m})	2	time units
variance ($\sigma^2_{L_m}$)	0	
demand process	Poisson	
demand intensity (λ_D)	1.0	product per time unit
return process	Poisson	
return intensity (λ_R)	0.7	product per time unit

Table 5.1. *Base-case scenario.*

5.2.1 The effects of lead-time duration

To analyze the influence of the lead-time duration on total expected costs, we have conducted a set of computational experiments in which either μ_{L_m} or μ_{L_r} is varied, while keeping the other lead-time characteristics fixed. The parameter settings in our experiments are summarized in Table 5.2.

Experiment	*Control strategy*	*Lead-times*	*Reference*
1	PUSH	μ_{L_m} has been varied; $\mu_{L_r} = 0$	Figure 5.2a
2	PUSH	μ_{L_r} has been varied; $\mu_{L_m} = 0$	Figure 5.2b
3	PULL	μ_{L_m} has been varied; $\mu_{L_r} = 0$	Figure 5.3a
4	PULL	μ_{L_r} has been varied; $\mu_{L_m} = 0$	Figure 5.3b
5	PUSH	μ_{L_m} has been varied; $\mu_{L_r} = 0$, or $\mu_{L_r} = 2$, or $\mu_{L_r} = 4$	Figure 5.4a
6	PUSH	μ_{L_r} has been varied; $\mu_{L_m} = 0$, or $\mu_{L_m} = 2$, or $\mu_{L_m} = 4$	Figure 5.4b

Table 5.2. *Summary of computational experiments.*

The computational experiments led to the following observations.

Observation 5.1 *An increase in the manufacturing lead-time results for both strategies in a much larger costs increase than an equivalent increase in the remanufacturing lead-time (see Figures 5.2 and 5.3).*

The reason why this effect occurs is, that our PUSH and PULL strategies give priority to remanufacturing and use manufacturing as a last resort. Therefore, the serviceable inventory position level at which manufacturing orders are placed is lower than or equal to the serviceable inventory position level at which remanufacturing orders are placed. Consequently, when $\mu_{L_r} \approx \mu_{L_m}$, the probability that during an outstanding *manufacturing* order a backordering position occurs is larger than the probability that during an outstanding *remanufacturing* order a backordering position occurs. This causes that longer manufacturing lead-times require higher safety stocks to protect against costly stock-outs events than equivalently longer remanufacturing lead-times. As a consequence, it can be expected that the difference between the effects of increasing the manufacturing lead-time and the effects of increasing the remanufacturing lead-time should increase with increasing backordering costs. Additional experiments have indeed confirmed this.

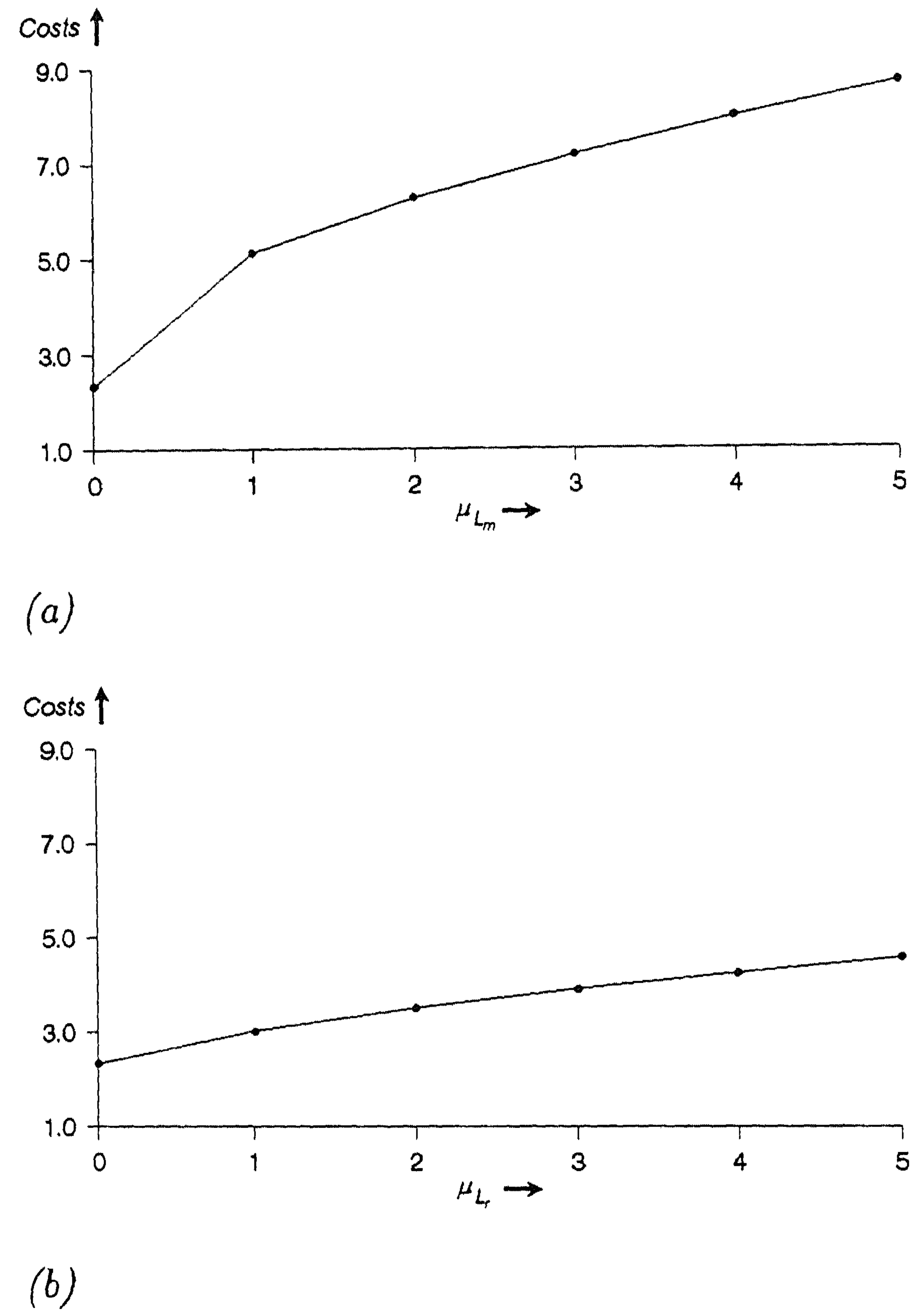
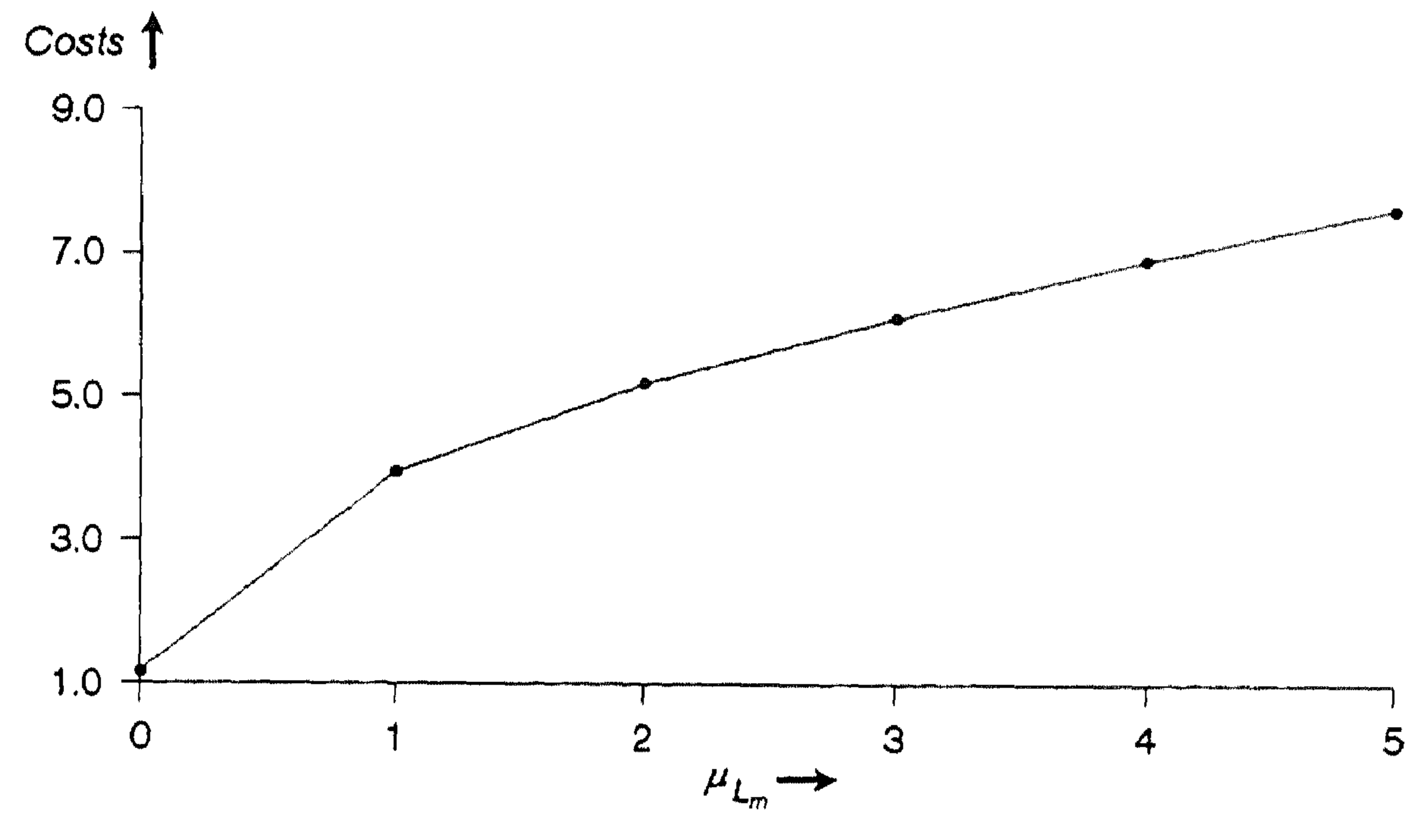


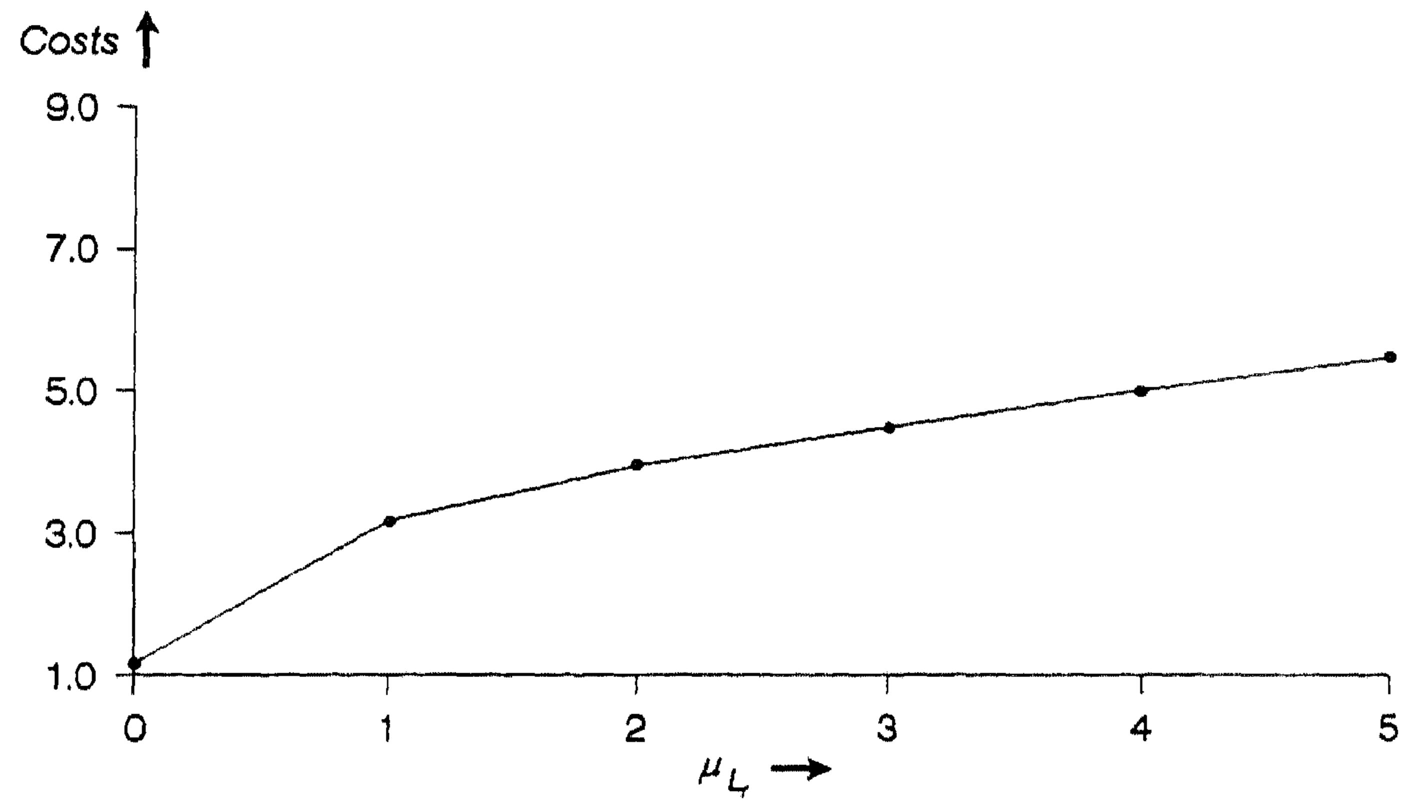
Figure 5.2. *$\overline{C}^*_{PUSH}$ as function of the manufacturing lead-time with $\mu_{L_r} = 0$ (a), and as function of the remanufacturing lead-time with $\mu_{L_m} = 0$ (b).*

Observation 5.2 *For both strategies an increase in the manufacturing lead-time results in a costs increase, while an increase in the remanufacturing lead-time in some cases results in a costs decrease.*

These effects are visualized in Figure 5.4 for the PUSH strategy. Note that the cost decreasing effect of an increase in remanufacturing lead-times occurs in particular when μ_{L_r} is small compared to μ_{L_m}. The arguments on why this effect occurs are identical to the arguments used in Observation 5.1. Since our PUSH and PULL strategies give priority to remanufacturing, the serviceable inventory position level at which a remanufacturing order is placed is higher than or equal to the level at which a manufacturing order is placed. If the lead-time of a remanufacturing order is much shorter



(a)



(b)

Figure 5.3. *$\overline{C}^*_{PULL}$ as function of the manufacturing lead-time with $\mu_{L_r} = 0$ (a), and as function of the remanufacturing lead-time with $\mu_{L_m} = 0$ (b).*

than the lead-time of a manufacturing order, manufacturing orders will arrive too late (which leads to additional stockout costs) or remanufacturing orders will arrive too early (which leads to excessive holding costs). If the remanufacturing lead-time becomes as large as the manufacturing lead-time (i.e., $\mu_{L_r} \approx \mu_{L_m}$), the abovementioned problem does not occur and costs are decreased. Finally, if μ_{L_r} becomes larger than μ_{L_m} expected costs start to increase again, since higher safety stocks must be kept to protect against costly stock-out events that may occur during the (longer) remanufacturing lead-time.

It should be noted that in the above argumentation we left out any holding costs associated with work in process in the remanufacturing facility. If remanufacturing lead-times increase, holding costs of work in process also increase. This negative cost effect partly neutralizes the positive cost effect described above. However, even then it is interesting to observe that an increase in remanufacturing lead-times may have a smaller impact than expected.

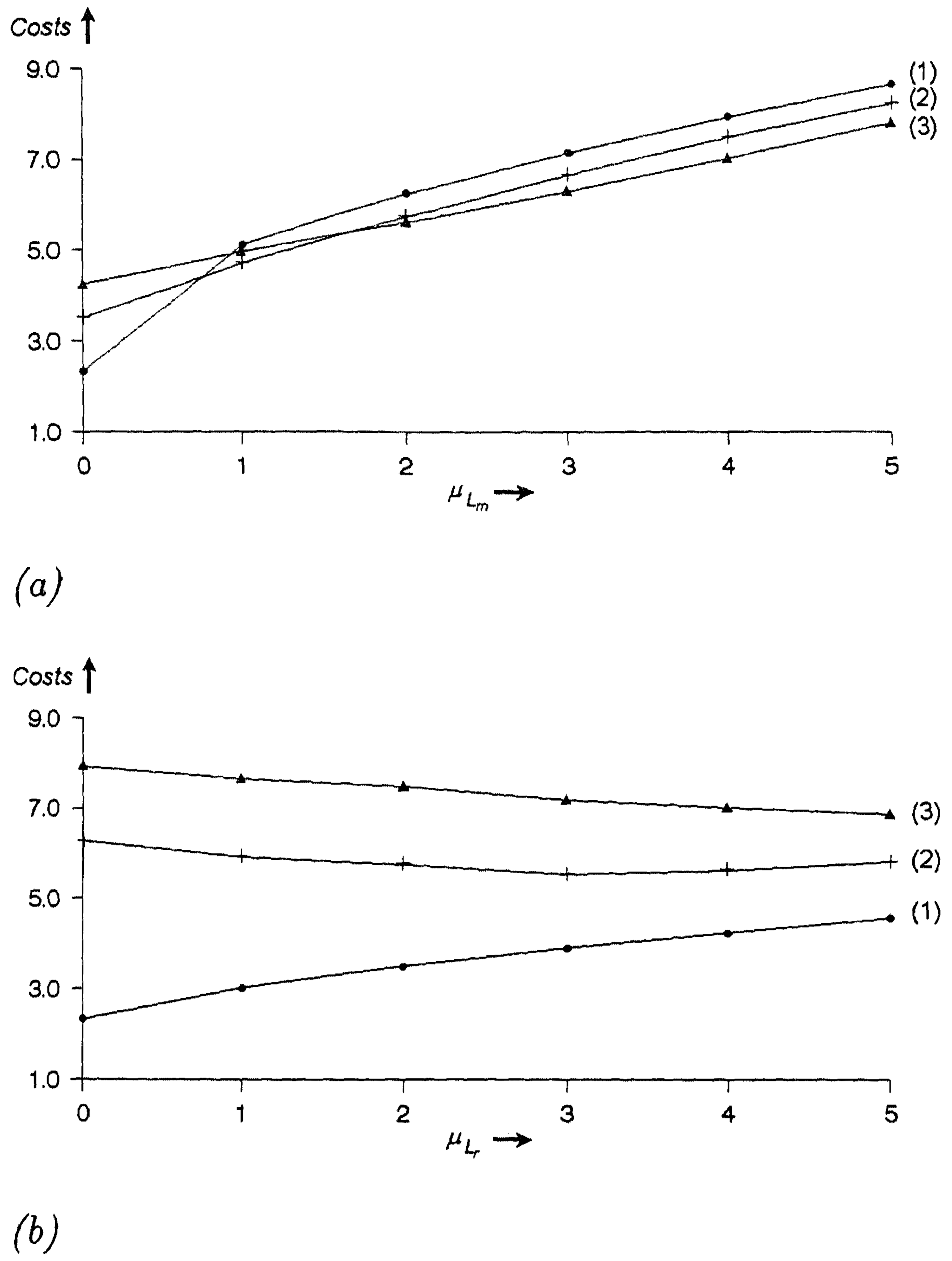


Figure 5.4. $\overline{C}^*_{PUSH}$ *as function of the manufacturing lead-time (a) with* $\mu_{L_r} = 0$ *(curve 1),* $\mu_{L_r} = 2$ *(curve 2), and* $\mu_{L_r} = 4$ *(curve 3), and as function of the remanufacturing lead-time (b) with* $\mu_{L_m} = 0$ *(curve 1)* $\mu_{L_m} = 2$ *(curve 2), and* $\mu_{L_m} = 4$ *(curve 3).*

5.2.2 The effects of lead-time *variability*

To investigate the effects of lead-time variability, we assume in the initial set of experiments Bernoulli distributed lead-times. We have chosen for

Bernoulli distributed lead-times, since the Bernoulli distribution is one of the few commonly used discrete probability distributions of which the parameters provide sufficient freedom to allow for a sensitivity analysis on lead-time variability (variance) at a constant average lead-time $E(L)$. Associated to the Bernoulli distribution, we define the parameters ℓ and P_ℓ, reflecting the situation that either an ℓ-period lead-time will occur (with probability P_ℓ), *or* an instantaneous delivery will occur (with probability $1 - P_\ell$). The parameters ℓ and P_ℓ have been determined, such that the average lead-time $E(L)$ is kept constant at the base-case level of 2 time-units, whereas the variance σ_L^2 has been varied (note that for Bernoulli distributed random variables, a higher variance is equivalent to a higher variability according to Definition 5.2.1).

Figure 5.5 shows the relation between lead-time variability (expressed in terms of σ_L^2) and total expected costs, for the PUSH strategy (Figure 5.5a) and for the PULL strategy (Figure 5.5b). The curves labeled with (1) correspond to Bernoulli distributed *manufacturing* lead-times and deterministic remanufacturing lead-times, whereas the curves indicated by label (2) correspond to Bernoulli distributed *remanufacturing* lead-times and deterministic manufacturing lead-times. From Figure 5.5 the following observations can be made:

Observation 5.3 *An increase in the variability of manufacturing lead-times results for the base-case scenario in lower total expected costs, both under the PUSH strategy and under the PULL strategy (curves* (1) *in Figure 5.5 are decreasing).*

In this context, it should be noted that Song proved that this effect will never occur with $\lambda_R = 0$. To explain why the effect still may occur with $\lambda_R > 0$, note that with Bernoulli distributed manufacturing lead-times a more variable lead-time (given a fixed average lead-time) implies, *(i)* an increase in ℓ, and *(ii)* a decrease in P_ℓ and, consequently, an increase in the probability of a zero lead-time occurrence.

Apparently, if $\lambda_R > 0$ and remanufacturing lead-times are deterministic, the cost increasing effect of *(i)* is partly neutralized by a frequent arrival of remanufacturing orderings, compensating for manufacturing orderings with a long ℓ-period lead-time. Consequently, the impact of the cost increasing effect of *(i)* is dominated by the cost decreasing effects of *(ii)*, resulting in the observed net cost decrease. as a consequence of the arguments above, the magnitude of the effect will not only depend on σ_L^2, but also on the return intensity. If the return intensity is low the cost increasing effect of

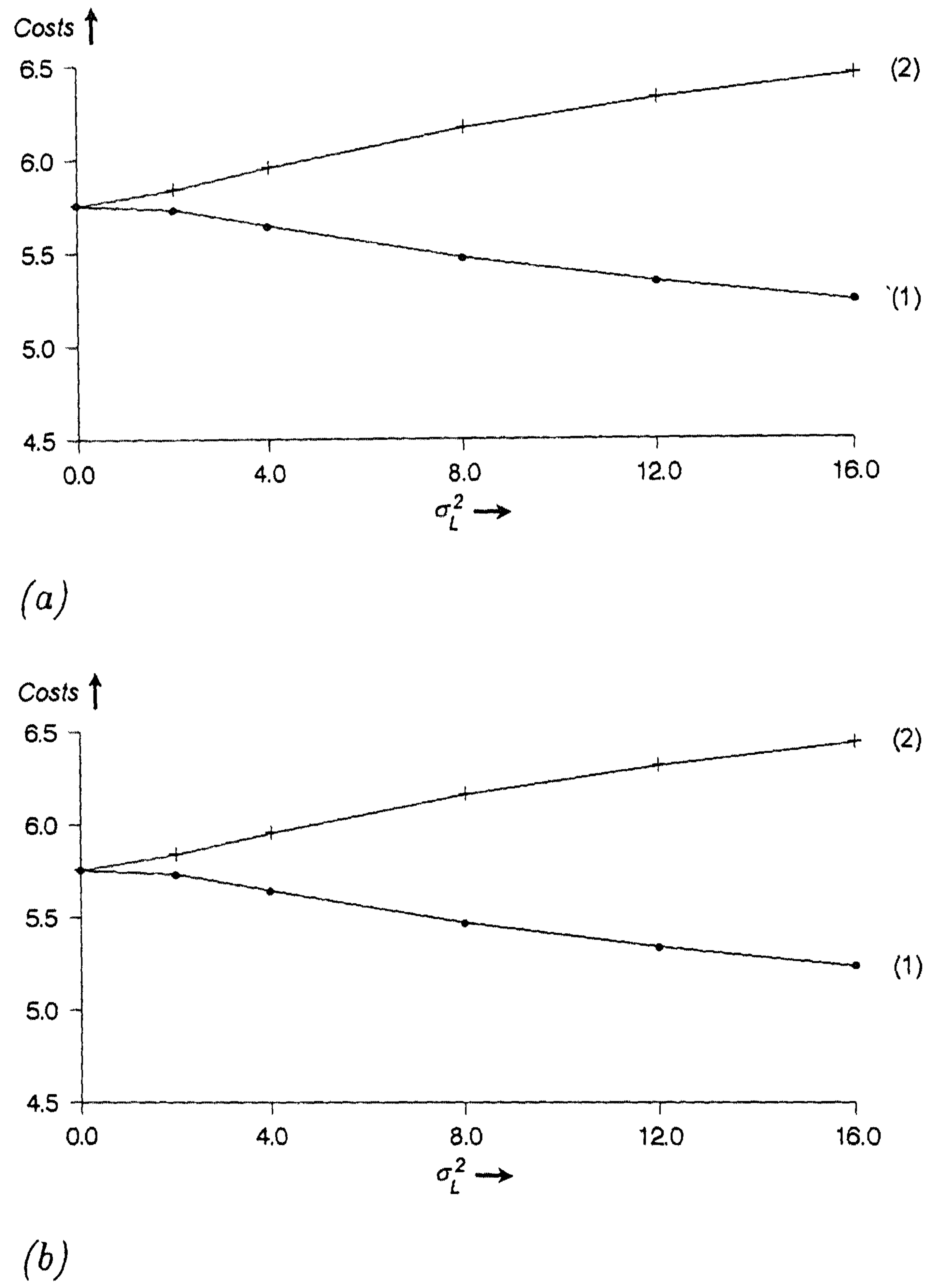


Figure 5.5. *$\overline{C}^*_{PUSH}$ (a) and $\overline{C}^*_{PULL}$ (b) as function of the variance in manufacturing lead-times (curves 1), and as function of the variance in remanufacturing lead-times (curves 2).*

(i) is more dominant, since not all long (ℓ-period) manufacturing lead-times are automatically compensated by the arrival of remanufacturing orderings. On the other hand, if the return intensity is high, there will only be a small number of manufacturing orders and the manufacturing lead-time will not have much influence (see Section 5.3.3).

Since the cost decreasing effect of manufacturing lead-times only occurs for $\lambda_R > 0$ one may intuitively conclude that this effect will only occur in systems with multiple sources to satisfy demands. However in [43] it is shown that a similar effect occurs in the newsvendor problem, which is a *single* source model.

Observation 5.4 *An increase in the variability of remanufacturing lead-times results in an increase in total expected costs, both under a PUSH strategy and under a PULL strategy (curves (2) in Figure 5.5 are increasing).*

Although, we can not analytically prove the observed effect, we were unable to find any dataset for which the opposite effect occurs.

The experimental results also indicate that lead-time variability has not much influence on the relative performance of PUSH and PULL strategy. Analogous to the situation with deterministic lead-times (see Figures 5.2 and 5.3, and Section 4.3.2) the PULL strategy outperforms the PUSH strategy in the base-case scenario, where $c_r^h < c_s^h$.

5.2.3 Sensitivity analysis

To investigate the extent to which the above observations are specific to the assumptions and parameter settings in the base-case scenario, we also study the sensitivity of the total expected costs for changes in demand and return intensity, cost structure, and lead-time distributions. Here we briefly discuss the outcomes of this study.

• *Demands and returns.* As can be expected, the results of the sensitivity analysis indicate that when λ_R is *small* compared to λ_D, the influence of changes in the duration and variability of *remanufacturing* lead-times on costs are limited, since most demands are satisfied by manufacturing rather than by remanufacturing. Also, if $\lambda_R \approx \lambda_D$, costs are rather insensitive to changes in the variability and duration of *manufacturing* lead-times, since most of the demands are satisfied by remanufacturing. Generally, the magnitude of the cost decreasing effect related to an increase in the duration of remanufacturing lead-times and the magnitude of the cost decreasing effect related to an increase in the variability of manufacturing lead-times both increase when increased *coordination* is required between the remanufacturing and the manufacturing processes. The coordination needs to be maximal if the time-average number of remanufactured products is close to the time-average number of manufactured products, i.e., when $\lambda_R \approx (\lambda_D - \lambda_R)$.

• *Cost structure and cost parameters.* The occurrence and magnitude of the observed effects depend very much on the cost structure and cost parameters. For instance, *work-in-process costs* have been disregarded in our study. Taking into account these costs would imply that an additional cost component (which is usually linear in the average lead-time duration), must be added to the cost function. Depending on the numerical value of the work-in-process costs, the cost decreasing effect of an increase in remanufacturing lead-times will be reduced, or even completely disappear. It should however be noted that the observations related to lead-time variability will *not* be affected by the inclusion of work-in-process costs.

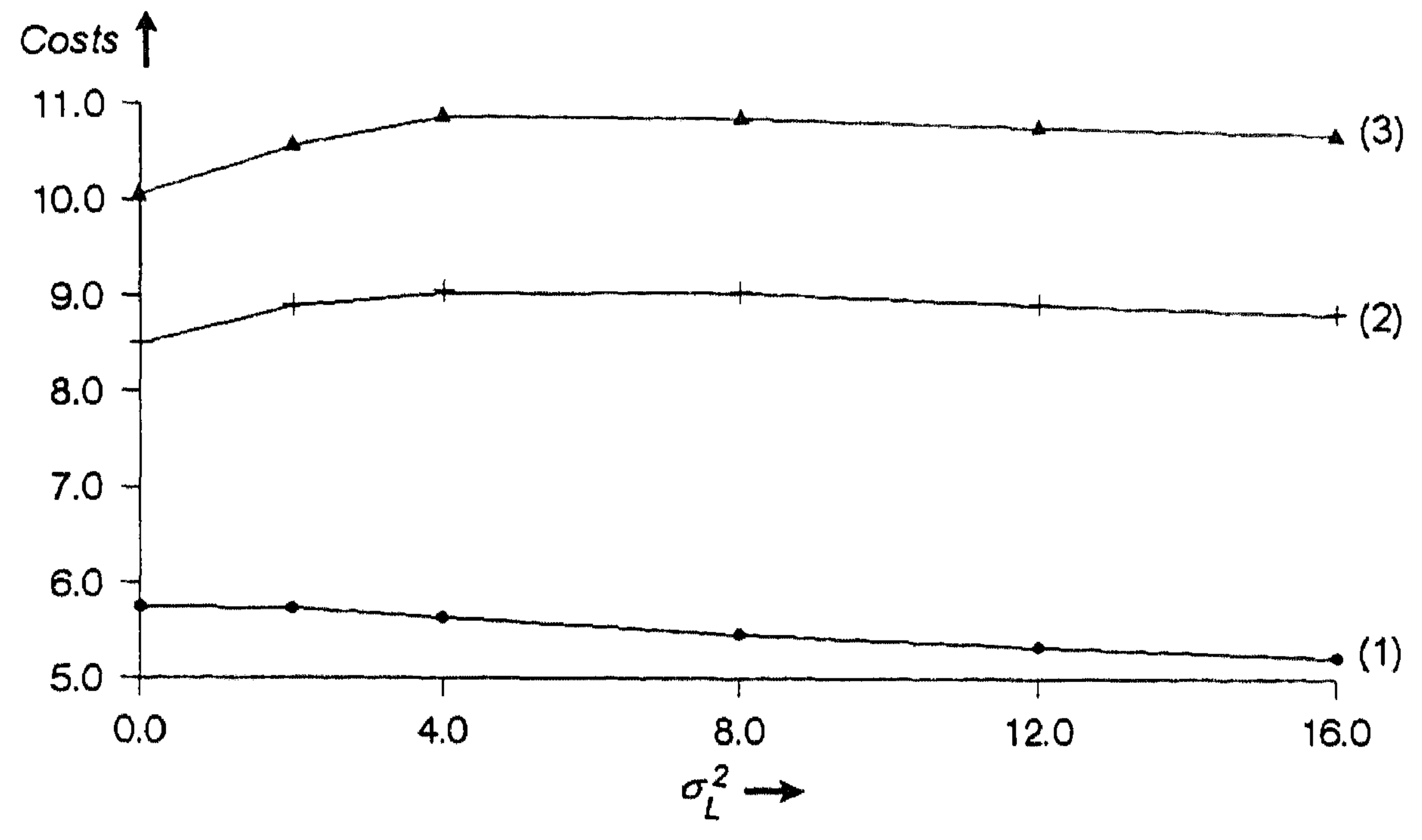
Non-zero *fixed costs* may also influence the total expected operating costs, since non-zero fixed manufacturing (remanufacturing) costs imply larger manufacturing (remanufacturing) batches. We observed that a larger manufacturing or remanufacturing batch will *reduce* the cost decreasing effect of an increase in the variability of manufacturing lead-times (Figure 5.6).

The reason why this effect occurs is, that with larger remanufacturing batches the inter-arrival time between subsequent remanufacturing batches will become more erratic. Consequently, long manufacturing lead-times may not be compensated immediately by the arrival of remanufacturing orderings. Similarly, larger manufacturing batches imply that a long manufacturing lead-time can not be compensated by the arrival of a single remanufacturing ordering.

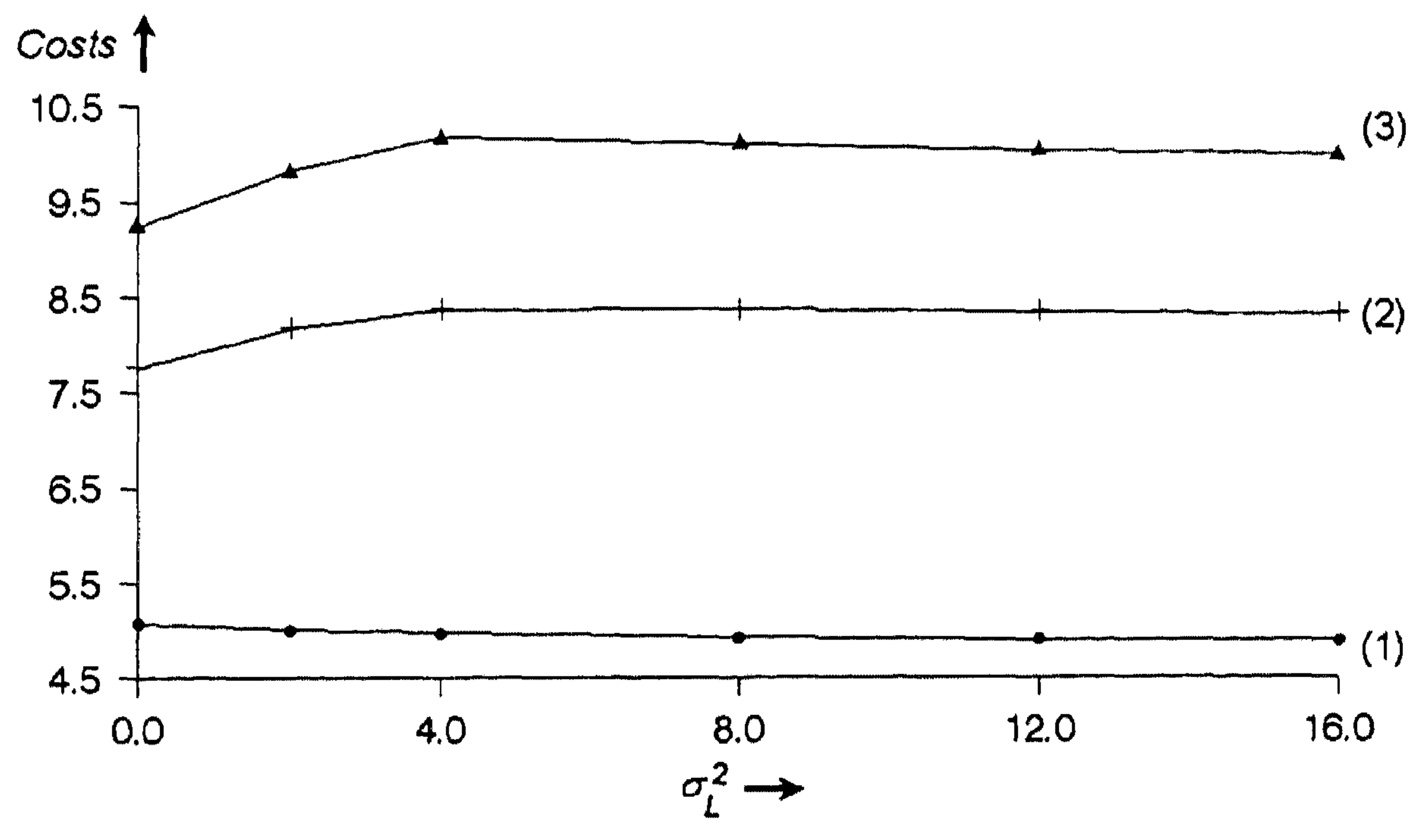
Finally, analogous to the situation with deterministic lead-times the experiments for the case with stochastic lead-times indicate that when remanufacturable inventory is valued *lower* than serviceable inventory ($c_r^h < c_s^h$), the PULL strategy outperforms in most cases the PUSH strategy. Otherwise, the PUSH strategy yields in most cases lower total expected costs. This is consistent with the results of Inderfurth [26] who proved for a similar but periodic system, that a PULL strategy is optimal if $c_r^h < c_s^h$.

• *Lead-time distributions.* The results of a simulation study by Van der Kruk [57] indicate that Observations 5.3–5.4 are *not* specific to *Bernoulli* distributed lead-times only. The effects related to lead-time variability also occur for continuously distributed lead-times, such as for lognormal distributed lead-times (Figure 5.7a), and for Coxian-2 distributed lead-times (Figure 5.7b).[1]

[1] For Lognormal and Coxian-2 distributed random variables a larger variance (with constant expectation) is equivalent to a higher variability according to the definition of $\geq_{var}$ used by Song [51].



(a)



(b)

Figure 5.6. $\overline{C}^*_{PUSH}$ *(a) and* $\overline{C}^*_{PULL}$ *(b) as function of the variance in manufacturing lead-times for* $c^f_m = c^f_r = 0$ *(curves 1),* $c^f_m = c^f_r = 5$ *(curves 2), and* $c^f_m = c^f_r = 10$ *(curves 3).*

5.3 Summary and conclusions

We have investigated the influence of lead-time *duration* and lead-time *variability* on total expected costs in a system with manufacturing and remanufacturing operations. To control the system, we have applied simple PUSH and PULL strategies. Although these strategies are non-optimal they have the advantage that they are easy to implement and actually used in practice. Also, we have introduced a novel and non-trivial procedure to include stochastic lead-times in our analysis.

A numerical study shows that changes in the duration of *manufacturing*

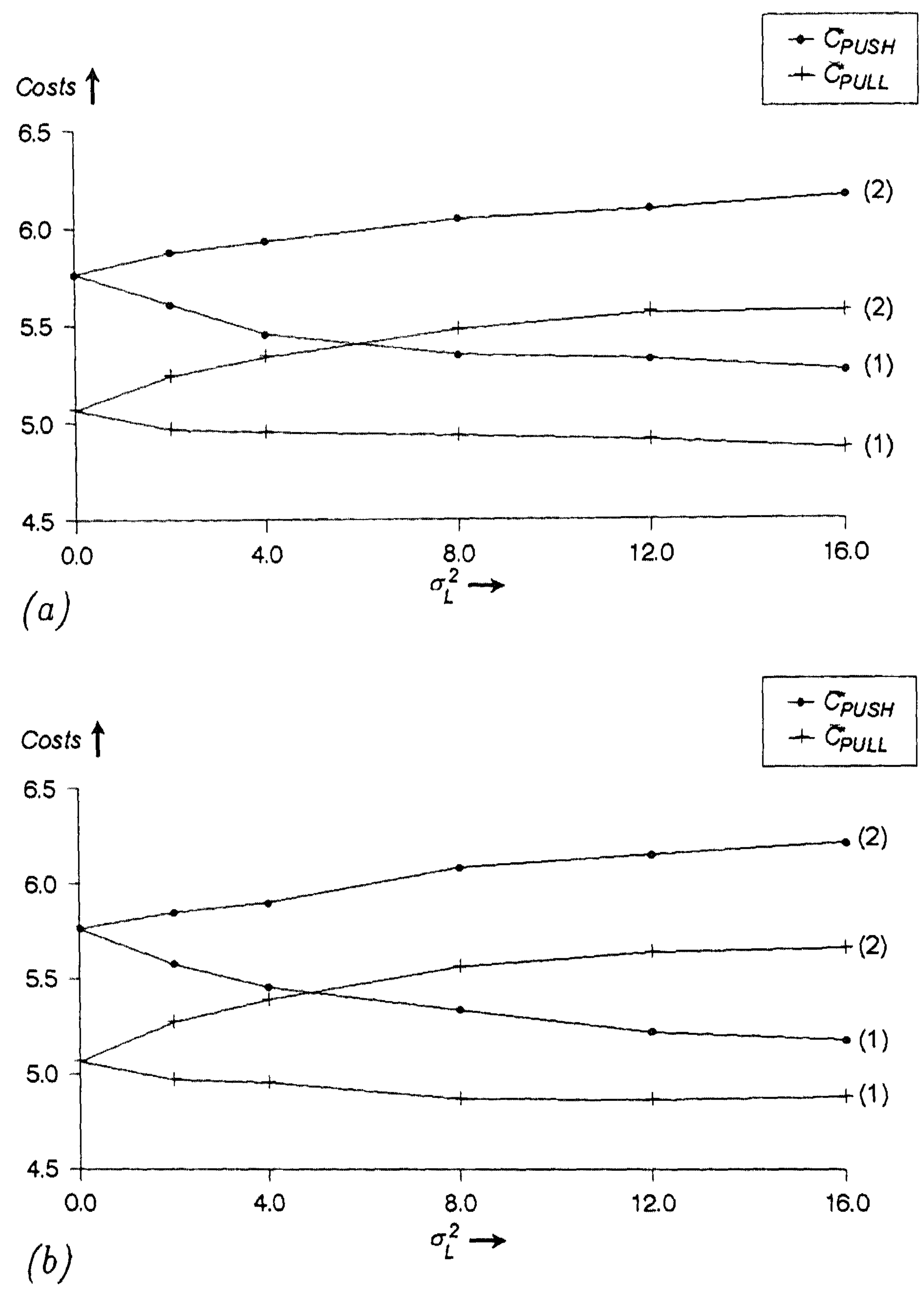


Figure 5.7. *Costs as function of the variance in manufacturing lead-times (curves 1), and remanufacturing lead-times (curves 2), while lead-times follow a Lognormal distribution (a) and a Coxian distribution (b).*

lead-times have larger influences on total expected costs than changes in the duration of *remanufacturing* lead-times. In some cases, an increase in remanufacturing lead-times may even result in cost decreases. In this context, it has been stressed that one should strive for a situation in which the remanufacturing lead-time and the manufacturing lead-time do not differ too much in expectation. Also, the numerical study indicates that cost increases are more sensitive to a larger variability in remanufacturing lead-times than to a larger variability in manufacturing lead-times. In some cases, a higher variability in manufacturing lead-times may even result in cost decreases.

The experiments also indicate that with *stochastic* lead-times the PULL strategy in most cases outperforms the PUSH strategy when c_r^h is smaller then c_s^h. However, further experiments also indicate that when remanufacturable inventory is valued as high as serviceable inventory, the PUSH strategy may outperform the PULL strategy. These observations correspond to the observations made in Section 4.3.2 in relation to deterministic lead-times.