Chapter 6

PUSH and PULL control with disposal

Although the system implemented at the copier manufacturer may be ecologically sound due to module remanufacturing and material reuse, production and inventories are more difficult to plan and control than in traditional systems without remanufacturing. It has been experienced that these difficulties are mainly due to the many interactions that exist between the manufacturing, remanufacturing and disposal operations. Interactions between manufacturing and remanufacturing processes occur for instance when the output of the remanufacturing process is too low to satisfy all the demands for new modules adequately. In this case manufacturing orders must be placed regularly to avoid shortages.

To plan and control manufacturing, remanufacturing, and disposal operations simultaneously, the copier remanufacturer implemented a PUSH strategy. By this strategy returned modules are *'pushed'* through the remanufacturing process as soon as a sufficient amount of modules becomes available from the disassembly plant. If the joint inventory of new and remanufactured modules appears to be too low to satisfy the future expected demands adequately, a manufacturing order is placed to produce new modules. In principle, disposal of a disassembled module occurs only when the quality of the module is insufficient for remanufacturing.

The copier manufacturer had the impression that the efficiency of their system could be improved by the introduction of a control strategy which offers a higher level of coordination between the manufacturing, remanufacturing and disposal operations. For this purpose, the change to a PULL strategy had been investigated. Under this strategy disassembled modules

are *'pulled'* through the remanufacturing process only when they are actually needed to satisfy the demand for new modules. If the output of the remanufacturing process appears to be too low to cover the future expected demands, a manufacturing order is placed.

Interactions between remanufacturing and disposal processes occur when the number of returned modules is higher than the demand for new modules. In this case some of the returned modules must be disposed of instead of being remanufactured to avoid too high, and therefore too expensive, stocking positions. Therefore, another way that had been considered to improve the system efficiency is to not only dispose of returned products when their quality is too low for remanufacturing, but also when the system inventories become too high.

The contributions of this chapter are as follows. First, we formulate a general manufacturing/remanufacturing system with disposal operations, which allows for correlated demands and returns, Coxian-2 distributed demand and return inter-arrival time distributions, and non-zero manufacturing and remanufacturing lead times (Section 6.2). The cost structure consists of fixed and variable manufacturing and remanufacturing costs, (possibly different) holding costs for remanufacturables and serviceables, backordering costs, and variable disposal costs. For this system we extend the PUSH and PULL strategies proposed in Chapter 3, to the PUSH- and PULL-disposal strategies. The procedures to calculate the total expected costs (Section 6.3) are exact. In Section 6.4 we provide numerical examples to show *(i)* when and why systems with *planned* disposals (i.e., disposals of returned products that are in principle remanufacturable) are economically more efficient than systems in which no planned disposals occur, *(ii)* to show the difference between the PUSH- and PULL-disposal strategies, and *(iii)* to investigate the robustness of the control parameters over the different stages of a remanufacturable product life-cycle. Finally, Section 6.5 presents our conclusions.

6.1 System characteristics

The system that will be considered in the sequel of this chapter is shown in Figure 6.1. Here we assume that all returned products satisfy the quality requirements for remanufacturing. Consequently, only *planned* disposals need to occur. For all the other system characteristics we refer to section 3.1.

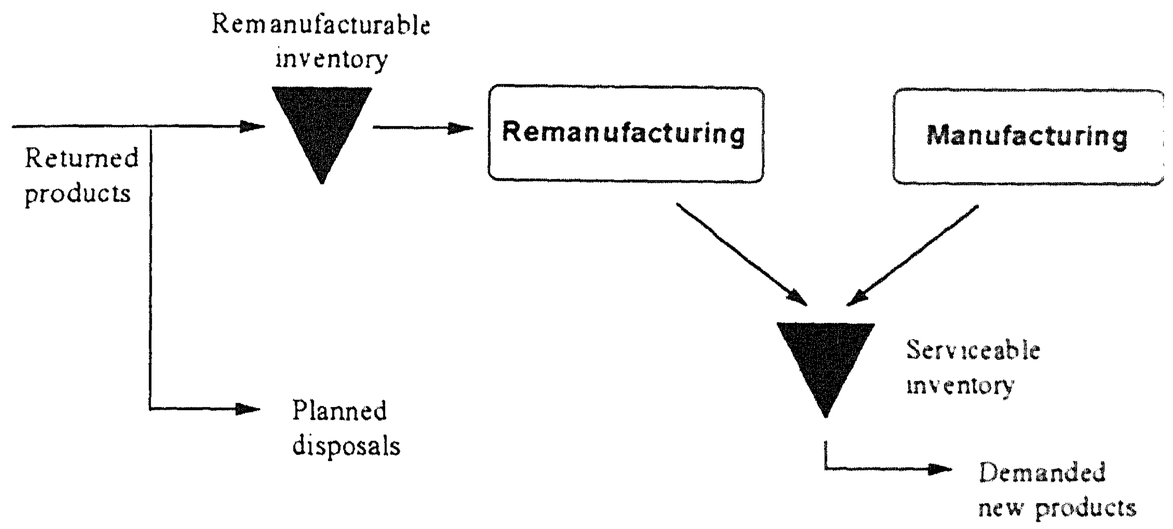


Figure 6.1. *A hybrid system with manufacturing, remanufacturing and disposal operations, and stocking of remanufacturable and serviceable products.*

The long-run average system costs per unit of time under PUSH-disposal control,

$$\overline{C}_{PUSH-d}(s_m, Q_m, Q_r, s_d),$$

and under PULL-disposal control,

$$\overline{C}_{PULL-d}(s_m, Q_m, s_r, S_r, s_d),$$

consist of the following components:

c_s^h	$\times$	average serviceable inventory per time unit
c_r^h	$\times$	average remanufacturable inventory per time unit
c_r^v	$\times$	average number of remanufactured products per time unit
c_r^f	$\times$	average number of remanufacturing batches per time unit
c_m^v	$\times$	average number of manufactured products per time unit
c_m^f	$\times$	average number of manufacturing batches per time unit
c_b	$\times$	average backordering position per time unit
c_d	$\times$	average number of disposed products per time unit

where c_s^h are the inventory holding costs per product in serviceable inventory per unit of time, c_r^h are the inventory holding costs per product in remanufacturable inventory per unit of time, c_m^v are the variable remanufacturing costs per product, c_r^f are the fixed remanufacturing costs per

batch, c_m^v are the variable manufacturing costs per product (including material costs), c_m^f are the fixed manufacturing costs per batch, c_d are the disposal costs per product, and c_b are the backordering costs per product per unit of time. All cost factors are non-negative, except the disposal costs c_d, which are negative when the returned products have a positive salvage value.

In the next section we formulate the PUSH- and PULL-disposal strategy, that will be used to control the system above.

6.2 Strategy definitions and analysis

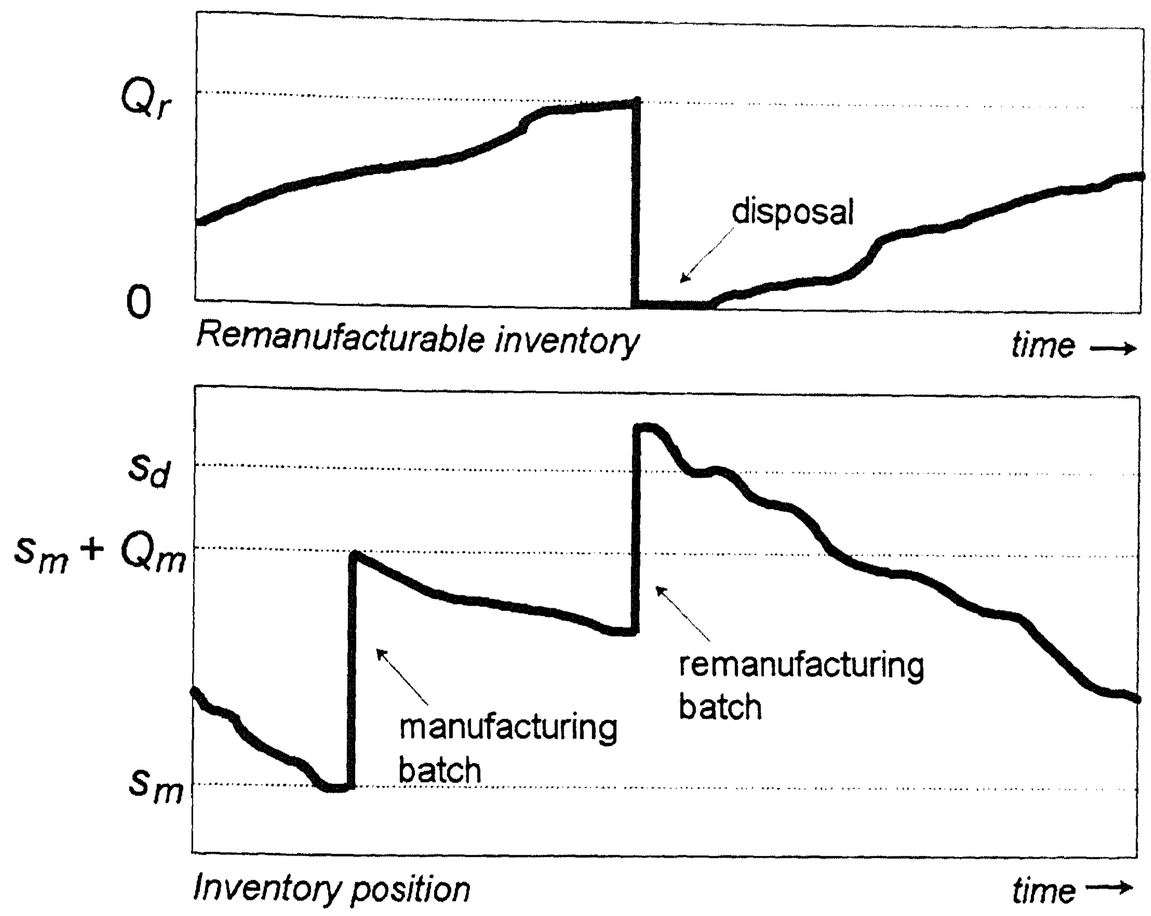
As an extension to the PUSH and PULL strategy that have been introduced in Chapter 3, we define the PUSH-disposal and PULL-disposal strategy as follows:

- **The (s_m, Q_m, Q_r, s_d) PUSH-disposal strategy** (Figure 6.2a). Under this strategy remanufacturing starts whenever the remanufacturable inventory contains exactly Q_r products. Manufacturing starts whenever the inventory position (which is defined as the number of products in on-hand serviceable inventory minus the number of products in backorder plus the total number of products that are currently manufactured and remanufactured) reaches the level s_m. The manufacturing batch contains Q_m products. Returned products are disposed of if upon arrival the inventory position is at or above the disposal level $s_d (\geq 0)$.

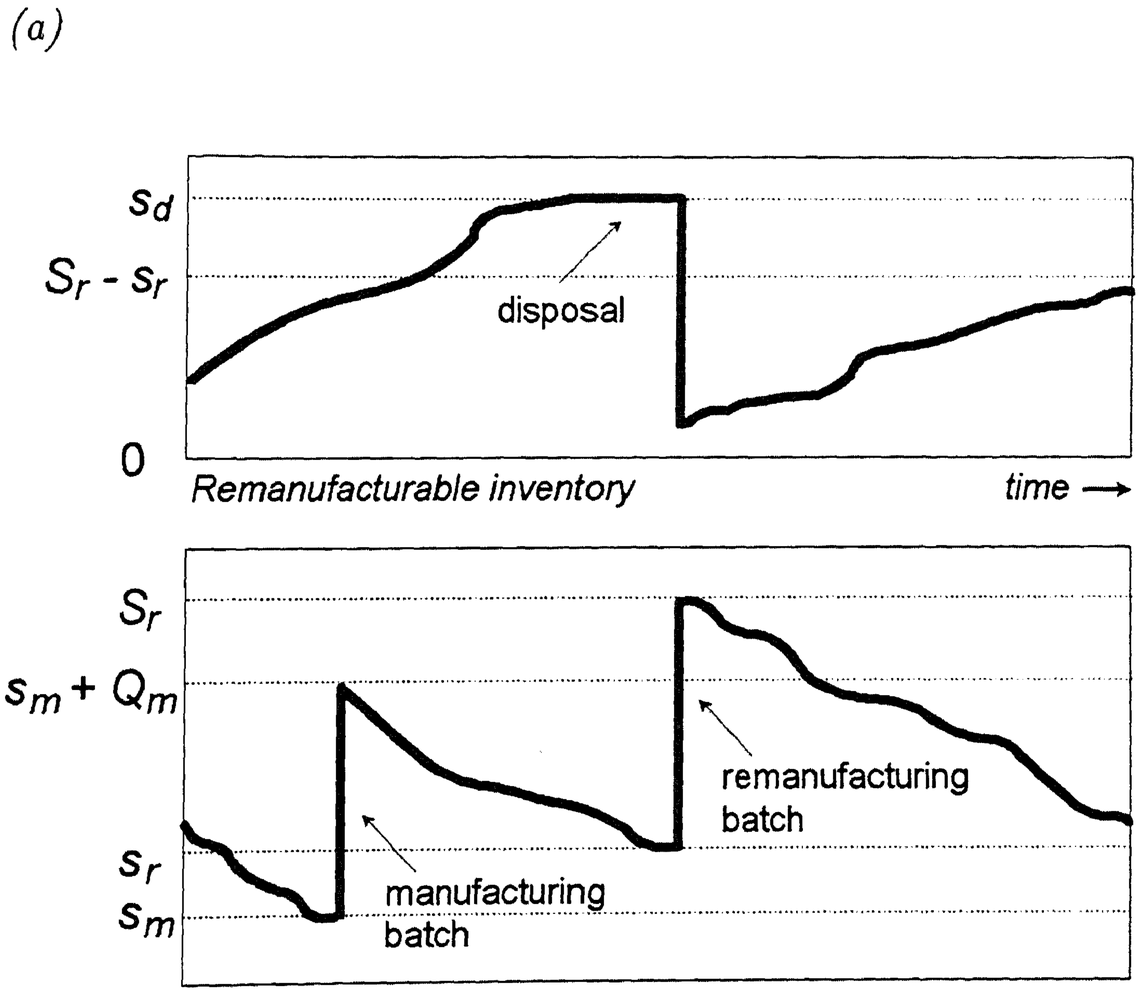
  Note that this strategy implies that the remanufacturable inventory never exceeds $Q_r - 1$ and the inventory position is bounded by $s_d + Q_r - 1$. Furthermore, for $s_d = \infty$ the PUSH-disposal strategy is equivalent to the PUSH strategy without disposals.

- **The $(s_m, Q_m, s_r, S_r, s_d)$ PULL-disposal strategy** (Figure 6.2b). Under this strategy remanufacturing starts whenever the inventory position is at or below the level s_r and the remanufacturable inventory contains sufficient products to raise the inventory position level to S_r. Manufacturing starts whenever the inventory position drops to the level s_m $(\leq s_r)$. The manufacturing batch size is Q_m. Disposal occurs to every returned product that arrives when the remanufacturable inventory equals the disposal level s_d (≥ 0).

  Note that for $s_d = \infty$ this strategy is equivalent to the PULL strategy without disposals.



(a)



(b)

Figure 6.2. *A schematic representation of the PUSH-disposal strategy (a), and the PULL-disposal strategy (b).*

Remark 6.1 *Alternatively to the PULL-disposal strategy we have also investigated a variant with a fixed remanufacturing batch size. Numerical experiments showed that the difference between the two strategies is however small. Therefore, we restrict the discussion in this chapter to the above implementation.*

As can be concluded from the above, the most important difference between PUSH and PULL control is the timing of remanufacturing and disposal operations. With PUSH control the start of the remanufacturing operation is solely based on the number of products in remanufacturable inventory, whereas under PULL control the start depends both on the inventory position and on the number of products in remanufacturable inventory. Furthermore, under the PUSH-disposal strategy the disposal decision depends on the inventory position, whereas it depends under the PULL-disposal strategy on the on-hand remanufacturable inventory.

The reason why in these two strategies the disposal decision is based on different inventories is that under PUSH control without planned disposals the *inventory position* (and therefore the serviceable inventory) is unbounded, i.e. may grow uncontrollably high, whereas under PULL control without planned disposals the *remanufacturable inventory* is unbounded. By defining s_d the way we did for the PUSH- and PULL-disposal strategy, all inventories are controllable.

In Section 3.1 and Section 3.2 we outline a procedure to calculate the total expected costs $\overline{C}_{PUSH-d}(s_m, Q_m, Q_r, s_d)$ for the PUSH-disposal strategy, and $\overline{C}_{PULL-d}(s_m, Q_m, s_r, S_r, s_d)$ for the PULL-disposal strategy. The notation that we use in this outline is identical to the notation in Table 3.1. For ease of explanation we have restricted the scope of the outline to the situation with uncorrelated and exponentially distributed demand and return inter-occurrence times. For the modifications required to model correlations and Coxian-2 distributed demand and return inter-occurrence times we refer to Section 4.1–4.2.

To find

$$\overline{C}^*_{PUSH-d} = \min \overline{C}_{PUSH-d}(s_m, Q_m, Q_r, s_d),$$

the minimal system costs under PUSH-disposal control, and

$$\overline{C}^*_{PULL-d} = \min \overline{C}_{PULL-d}(s_m, Q_m, s_r, S_r, s_d),$$

the minimal system costs under PULL-disposal control, we implemented an enumerative search procedure.

6.2.1 Analysis of the PUSH-disposal strategy

The state transitions of the manufacturing/remanufacturing system defined in Section 6.1 under the (s_m, Q_m, Q_r, s_d) PUSH-disposal strategy can be formulated as a continuous time Markov chain. This Markov chain, $\mathcal{M}_{dis1}$ say, has two-dimensional state variable

$$X_{dis1}(t) = \left(I_s(t), I_r^{OH}(t)\right)$$

and a two-dimensional state space

$$\mathcal{S}_{dis1} = \{s_m + 1, s_m + 2, \ldots, U_1\} \times \{0, 1, \ldots, Q_r - 1\}, \tag{6.1}$$

where

$$U_1 = \max\{s_d + Q_r - 1, s_m + Q_m\}.$$

To explain (6.1) we make the following observations:

- The inventory position $I_s(t)$ can never be smaller than $s_m + 1$ since in this state a demand would trigger a manufacturing operation and the inventory position is immediately raised to $s_m + Q_m$. Also, if the inventory position is at or above s_d all incoming returns will be disposed off. As a consequence, a remanufacturing batch of size Q_r will only be triggered by a return if the inventory position is less than s_d. Then the inventory position will never be larger than $\max\{s_d + Q_r - 1, s_m + Q_m\}$. So, all possible realizations of $I_s(t)$ are elements of the set $\{s_m + 1, s_m + 2, \ldots, U_1\}$.

- The remanufacturable inventory $I_r^{OH}(t)$ can never be larger than $Q_r - 1$ since in this state, a product return would trigger a remanufacturing operation and the inventory is decreased to zero. So, all possible realizations of $I_r^{OH}(t)$ are elements of the set $\{0, 1, \ldots, Q_r - 1\}$.

The transition rates $\nu_{s^{(1)},s^{(2)}}$ related to a transition from state $s^{(1)} \in \mathcal{S}_{dis1}$ to $s^{(2)} \in \mathcal{S}_{dis1}$ are defined as

$$\begin{aligned}
\nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} &= \lambda_R, && i_s < s_d \text{ and } i_r^{OH} < Q_r - 1,\\
\nu_{(i_s,i_r^{OH}),(i_s+Q_r,0)} &= \lambda_R, && i_s < s_d \text{ and } i_r^{OH} = Q_r - 1,\\
\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} &= \lambda_D, && i_s > s_m + 1,\\
\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} &= \lambda_D, && i_s = s_m + 1.
\end{aligned}$$

average remanufacturable inventory per time unit	$= \sum_{i_r^{OH}=1}^{Q_r-1} i_r^{OH} \pi_{dis1}^{(r)}(i_r^{OH})$
average number of remanufactured products per time unit	$= (1-f_1)\lambda_R$
average number of remanufacturing batches per time unit	$= \frac{(1-f_1)\lambda_R}{Q_r}$
average number of manufactured products per time unit	$= \lambda_D - (1-f_1)\lambda_R$
average number of manufacturing batches per time unit	$= \frac{\lambda_D-(1-f_1)\lambda_R}{Q_m}$
average number of disposals per time unit	$= f_1\lambda_R$

Table 6.1. *Cost components of* $\overline{C}_{PUSH-d}(s_m, Q_m, Q_r, s_d)$, *apart from the average serviceable inventory and average backorder position.*

The limiting joint probability distribution

$$\pi_{dis1}(i_s, i_r^{OH}) = \lim_{t\to\infty} \Pr\{I_s(t) = i_s, I_r^{OH}(t) = i_r^{OH}\}, \tag{6.2}$$

is obtained by solving the associated balance equations. Based on (6.2) we derive the marginal distribution of remanufacturable inventory, $\pi_{dis1}^{(r)}$, as

$$\pi_{dis1}^{(r)}(i_r^{OH}) = \sum_{i_s=s_m+1}^{\infty} \pi_{dis1}(i_s, i_r^{OH}). \tag{6.3}$$

The time average fraction of products that are disposed of, f_1, equals the fraction of time that the inventory position is equal to or exceeds s_d, or

$$f_1 = \Pr\{I_s \geq s_d\} = \sum_{i_s=s_d}^{U_1} \sum_{i_r^{OH}=0}^{Q_r-1} \pi_{dis1}(i_s, i_r^{OH}). \tag{6.4}$$

The cost components of $\overline{C}_{PUSH-d}(s_m, Q_m, Q_r, s_d)$ that are shown in Table 6.1, are now easily obtained using (6.3) and (6.4).

The calculation of the average on-hand serviceable inventory and the average backorder position, i.e.,

$$\overline{I}_s^{OH} = \sum_{i_s^{net}>0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}, \tag{6.5}$$

and

$$\overline{B} = - \sum_{i_s^{net}<0} i_s^{net} \Pr\{I_s^{net} = i_s^{net}\}, \tag{6.6}$$

proceeds analogously to the analysis in section 3.2.1. We only have to redefine the Markov chain $\mathcal{M}_1'$ in the following way. We define a Markov chain $\mathcal{M}_{dis1}'$ with state vector

$$X_{dis1}(\tau) = \left(I_s(\tau), I_r^{OH}(\tau), Z(u,\tau)\right)$$

and state space

$$\mathcal{S}_{dis1}' = \{s_m+1, s_m+2, \ldots, U_1\} \times \{0, 1, \ldots, Q_r - 1\} \times \{-\infty, \ldots, \infty\}.$$

By definition, $X_{dis1}(\tau) = (i_s, i_r^{OH}, z)$ whenever $I_s(\tau) = i_s$, *and* $I_r^{OH}(\tau) = i_r^{OH}$, *and* $Z(u,\tau) = z$, with $\tau \in (u, u+\Delta L]$.

If the manufacturing lead time is *larger* than the remanufacturing lead time, *all* of the remanufacturing batches and *none* of the manufacturing batches that were ordered during time $(t - L_{max}, t - L_{min}]$, will arrive before or at time t. We model this by defining the binary variable δ, which is assigned the value 1 if the manufacturing lead time is larger than the remanufacturing lead time, and 0 otherwise. The transition rates of $\mathcal{M}_{dis1}'$ are then as follows.

$$\begin{aligned}
&\nu_{(i_s,i_r^{OH},z),(i_s+1,i_r^{OH}+1,z)} = \lambda_R, && i_s < s_d \text{ and } i_r^{OH} < Q_r - 1,\\
&\nu_{(i_s,i_r^{OH},z),(i_s+Q_r,0,i_r^{OH}+Q_r,z+\delta Q_r)} = \lambda_R, && i_s < s_d \text{ and } i_r^{OH} = Q_r - 1,\\
&\nu_{(i_s,i_r^{OH},z),(i_s-1,i_r^{OH},z-1)} = \lambda_D, && i_s > s_m + 1,\\
&\nu_{(i_s,i_r^{OH},z),(S_m,i_r^{OH},z+(1-\delta)(S_m-s_m)-1)} = \lambda_D, && i_s = s_m + 1.
\end{aligned}$$

6.2.2 Analysis of the PULL-disposal strategy

The analysis of the $(s_m, Q_m, s_r, S_r, s_d)$ PULL-disposal strategy proceeds analogously to the analysis of the (s_m, Q_m, Q_r, s_d) PUSH-disposal strategy if we replace the Markov chains $\mathcal{M}_{dis1}$ and $\mathcal{M}_{dis1}'$ with the Markov chains $\mathcal{M}_{dis2}$ and $\mathcal{M}_{dis2}'$.

The Markov chain $\mathcal{M}_{dis2}$ is defined by the state variable

$$X_{dis2}(t) = \left(I_s(t), I_r^{OH}(t)\right)$$

with state space

$$\mathcal{S}_{dis2} = \{s_m + 1, s_m + 2, \ldots, U_2\} \times \{0, 1, \ldots, s_d\}, \tag{6.7}$$

where

$$U_2 = \max\{s_m + Q_m, S_r\}.$$

To explain (6.7) we make the following observations:

- The inventory position $I_s(t)$ can never be smaller than $s_m + 1$ since in this state a demand would trigger a manufacturing operation and the inventory position is immediately raised to $s_m + Q_m$. Remanufacturing operations may eventually raise the inventory position to S_r. So, all possible realizations of $I_s(t)$ are elements of the set $\{s_m + 1, s_m + 2, \ldots, U_2\}$.

- The remanufacturable inventory is limited to the level s_d, so all possible realizations of $I_r^{OH}(t)$ are elements of the set $\{0, 1, \ldots, s_d\}$.

The transition rates $\nu_{s^{(1)},s^{(2)}}$ related to a transition from state $s^{(1)} \in \mathcal{S}_{dis2}$ to $s^{(2)} \in \mathcal{S}_{dis2}$, are defined as

$$\begin{aligned}
&\nu_{(i_s,i_r^{OH}),(i_s,i_r^{OH}+1)} = \lambda_R, && i_r^{OH} < s_d \text{ and} \\
&&& \left\{i_s > s_r \text{ or } i_r^{OH} < S_r - i_s - 1\right\}, \\
&\nu_{(i_s,i_r^{OH}),(S_r,0)} = \lambda_R, && i_r^{OH} < s_d \text{ and } i_s \leq s_r \\
&&& \text{and } i_r^{OH} = S_r - i_s - 1, \\
&\nu_{(i_s,i_r^{OH}),(i_s-1,i_r^{OH})} = \lambda_D, && i_s > s_r + 1 \text{ or} \\
&&& \left\{i_s > s_m + 1 \text{ and } i_r^{OH} \leq S_r - i_s\right\}, \\
&\nu_{(i_s,i_r^{OH}),(S_r,i_r^{OH}-(S_r-s_r))} = \lambda_D, && i_s = s_r + 1 \text{ and } i_r^{OH} \geq S_r - s_r, \\
&\nu_{(i_s,i_r^{OH}),(S_r,i_r^{OH}-(S_r-s_m-Q_m))} = \lambda_D, && i_s = s_m + 1 \text{ and } s_r \geq s_m + Q_m \\
&&& \text{and } i_r^{OH} \geq S_r - s_m - Q_m, \\
&\nu_{(i_s,i_r^{OH}),(s_m+Q_m,i_r^{OH})} = \lambda_D, && i_s = s_m + 1 \text{ and } i_r^{OH} < S_r - s_m \\
&&& \text{and } \left\{i_r^{OH} < S_r - s_m - Q_m\right. \\
&&& \quad \left.\text{or } s_r < s_m + Q_m\right\}.
\end{aligned}$$

The limiting joint probability distribution $\pi_{dis2}(i_s, i_r^{OH})$ is obtained by solving the associated balance equations. Based on this distribution we derive the marginal distribution of remanufacturable inventory, i.e.,

$$\pi_{dis2}^{(r)}(i_r^{OH}) = \sum_{i_s=s_m+1}^{U_2} \pi_{dis2}(i_s, i_r^{OH}). \tag{6.8}$$

Using (6.8) we derive the average number of remanufacturing batches as

$$\begin{aligned}\overline{O}_r &= \lambda_R \times \Pr\{\text{a return triggers a remanufacturing batch}\} \\ &\quad + \lambda_D \times \Pr\{\text{a demand triggers a remanufacturing batch}\} \\ &= \sum_{i_s=s_m+1}^{s_r} \pi_{dis2}(i_s, S_r - i_s - 1)\lambda_R \\ &\quad + \sum_{i_r^{OH}=S_r-s_r}^{s_d} \pi_{dis2}(s_r+1, i_r^{OH})\lambda_D \\ &\quad + \begin{cases} \displaystyle\sum_{i_r^{OH}=S_r-s_m-Q_m}^{S_r-s_m} \pi_{dis2}(s_m+1, i_r^{OH})\lambda_D, & s_r \geq s_m + Q_m \\ 0, & \text{otherwise.} \end{cases}\end{aligned} \tag{6.9}$$

The time average fraction of products that are disposed of under the PULL-disposal strategy, f_d, is equivalent to the probability that the remanufacturable inventory equals s_d, i.e.,

$$f_2 = \Pr\{I_r^{OH} = s_d\} = \sum_{i_s=s_m+1}^{U_2} \pi_{dis2}(i_s, s_d), \tag{6.10}$$

Using (6.8) and (6.10), all components of $\overline{C}_{PULL-d}(s_m, Q_m, s_r, S_r, s_d)$ apart from (6.5), (6.6), and (6.9) are obtained from Table 6.2.

The Markov chain $\mathcal{M}'_{dis2}$ has state space

$$\mathcal{S}'_{dis2} = \{s_m+1, s_m+2, \ldots, U_2\} \times \{0, 1, \ldots, s_d\} \times \{-\infty, \ldots, \infty\},$$

and state variable $X_2(\tau)'$. The transition rates of $\mathcal{M}'_{dis2}$ are

$$\nu_{(i_s,i_r^{OH},z),(i_s,i_r^{OH}+1,z)} = \lambda_R, \qquad \begin{array}{l} i_r^{OH} < s_d \text{ and} \\ \{i_s > s_r \text{ or} \\ i_r^{OH} < S_r - i_s - 1\}, \end{array}$$

$$\nu_{(i_s,i_r^{OH},z),(S_r,0,z+\delta(S_r-i_s))} = \lambda_R, \quad \begin{array}{l} i_r^{OH} < s_d \text{ and } i_s \leq s_r \text{ and} \\ i_r^{OH} = S_r - i_s - 1, \end{array}$$

$$\nu_{(i_s,i_r^{OH},z),(i_s-1,i_r^{OH},z-1)} = \lambda_D, \qquad \begin{array}{l} i_s > s_r + 1 \text{ or} \\ \{i_s > s_m + 1 \text{ and} \\ i_r^{OH} \leq S_r - i_s\}, \end{array}$$

average remanufacturable inventory per time unit	$= \sum_{i_r^{OH}=1}^{s_d} i_r^{OH} \pi_{dis2}^{(r)}(i_r^{OH})$
average number of remanufactured products per time unit	$= (1-f_2)\lambda_R$
average number of manufactured products per time unit	$= \lambda_D - (1-f_2)\lambda_R$
average number of manufacturing batches per time unit	$= \frac{\lambda_D-(1-f_2)\lambda_R}{Q_m}$
average number of disposals per time unit	$= f_2\lambda_R$

Table 6.2. *Cost components of* $\overline{C}_{PULL-d}(s_m, Q_m, s_r, S_r, s_d)$, *apart from the average number of remanufacturing batches per time unit, the average serviceable inventory and the average backorder position.*

$$\nu_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-(S_r-s_r),z+\delta(S_r-s_r)-1)} = \lambda_D, \quad i_s = s_r+1 \text{ and } i_r^{OH} \geq S_r - s_r,$$

$$\nu_{(i_s,i_r^{OH},z),(S_r,i_r^{OH}-S_r+s_m+Q_m,z+c(\delta))} = \lambda_D, \quad i_s = s_m+1 \text{ and } s_r \geq s_m+Q_m \text{ and } i_r^{OH} \geq S_r - s_m - Q_m,$$

$$\nu_{(i_s,i_r^{OH},z),(s_m+Q_m,i_r^{OH},z+(1-\delta)Q_m-1)} = \lambda_D, \quad i_s = s_m+1 \text{ and } i_r^{OH} < S_r - s_m \text{ and } \left\{ i_r^{OH} < S_r - s_m - Q_m \text{ or } s_r < s_m + Q_m \right\}.$$

where $c(\delta) = (1-\delta)Q_m + \delta(S_r - s_m - Q_m)$.

6.2.3 PUSH- and PULL-disposal with Coxian-2 arrivals and correlation between returns and demands

The foregoing analysis is only valid for exponentially distributed inter-occurrence times of returns and demands, and under the assumption of zero correlation between the return and demand process. However, as in Chapter 4, we can adapt the analysis to allow Coxian-2 distributed demand and return inter-occurrence times, and non-zero correlation. The procedure necessary for this adaptation proceeds analogously to the procedures outlined in Section 4.1 and 4.2.

6.3 Numerical study

This section reports on a numerical study in which a number of issues were investigated. As in the other numerical studies, we start out from a base-case scenario, which is defined in Table 6.3. The scenarios that have been investigated for Sections 6.4.1–4.3 were obtained by varying the parameter values in this base-case scenario.

component	*value*	*unit of measure*
fixed remanufacturing costs (c_r^f)	0	\$/remanufacturing batch
variable remanufacturing costs (c_r^v)	5	\$/product remanufactured
fixed manufacturing costs (c_m^f)	0	\$/manufacturing batch
variable manufacturing costs (c_m^v)	10	\$/product manufactured
remanufacturable inventory costs (c_r^h)	0.5	\$/product per time unit
serviceable inventory costs (c_s^h)	1.0	\$/product per time unit
backordering costs (c_b)	50.0	\$/product per time unit
remanufacturing lead-time		
mean (μ_{L_r})	2	time units
variance ($\sigma^2_{L_r}$)	0	
manufacturing lead-time		
mean (μ_{L_m})	2	time units
variance ($\sigma^2_{L_m}$)	0	
demand process	Poisson	
demand intensity (λ_D)	1.0	product per time unit
demand uncertainty (cv_D^2)	1.0	
return process	Poisson	
return intensity (λ_R)	0.8	product per time unit
return uncertainty (cv_R^2)	1.0	
correlation (ρ_{RD})	0	

Table 6.3. *Base-case scenario.*

The issues investigated are:

- *A comparison between systems* with *and systems* without *product disposals.*

Section 4.3.1 indicated that it is not always cost-efficient to remanufacture all returns, especially when the return intensity is high. In section 6.3.1 we investigate the effect of a disposal strategy on cost-efficiency.

• *A comparison between the* PUSH- *and the* PULL-*disposal strategy.*

Section 4.3.2 indicated a cost-domination of the PULL strategy over the PUSH strategy if the remanufacturable holding costs are valued lower than the serviceable holding costs. In Section 6.4.2 we investigate whether this result also holds for the PUSH-disposal and PULL-disposal strategy.

• *A study on the robustness of the control parameters over the life-cycle of a remanufacturable product.*

In practice, the return and demand intensities will not remain constant over time. While the return rate may be small at the introduction of a product, it may well exceed the product demand in the declining stage of a product's life-cycle. Section 6.4.3 is to investigate the robustness of our control strategies under changing demand and return rate combinations.

6.3.1 Systems *with* versus systems *without* disposals

In Section 4.3.1 it has been visualized for a system without product disposals that when λ_R increases, the total expected system costs first decrease, but later on start to increase. It has been argued that the decrease in total expected system costs is a direct consequence of the cost structure in the base-case scenario, where variable remanufacturing costs are lower than variable manufacturing costs. Furthermore, the increase in total expected system costs is due to a higher variability in the output of the remanufacturing process, which causes an increase in the sum of inventory holding costs and backordering costs.

Finally, it has been shown that the return rate at which the cost decrease changes into a cost increase mainly depends on how manufacturing costs, remanufacturing costs, and inventory holding costs relate to each other. Intuitively, one would expect this point to be close to λ_D. However, Figures 6.3 show that, due to the system variability, the total expected costs may start to increase a long time before the return rate equals the demand rate.

Figure 6.3a (Figure 6.3b) shows a comparison between the total expected costs under a PUSH (PULL) strategy with and a PUSH (PULL) strategy without product disposals. The figures indicate that when λ_R increases, the control strategies that allow for product disposals yield lower costs than the strategies that do not allow for product disposals. The magnitude of the cost differences depends of course on the disposal costs c_d.

Figure 6.4 and 6.5 show that the cost differences occur due to the fact that the strategies with product disposal succeed in reducing the variability in

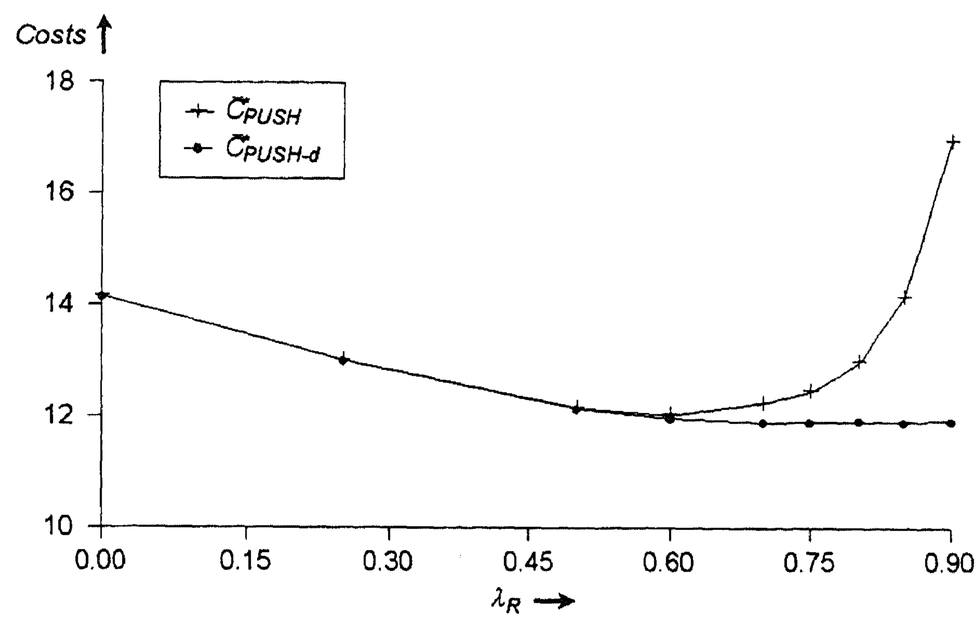


Figure 6.3. *Costs as function of the return rate for the PUSH (-disposal) strategy (a) and the PULL (-disposal) strategy (b).*

the inventories. In particular, the PUSH-disposal strategy helps to reduce the variability of the net serviceable inventory (Figure 6.4), whereas the PULL-disposal strategy reduces the variability of the on-hand remanufacturable inventory (Figure 6.5).

The difference between strategies with and without product disposal becomes larger when the system uncertainty increases. For example, Figure 6.6 shows that the cost differences increase when the variability in the product return inter-occurrence times increases. Here, the variability is measured in terms of the squared coefficient of variation cv_R^2 of a Coxian-2 distribution.

Figure 6.7 shows that the cost differences increases when the correlation

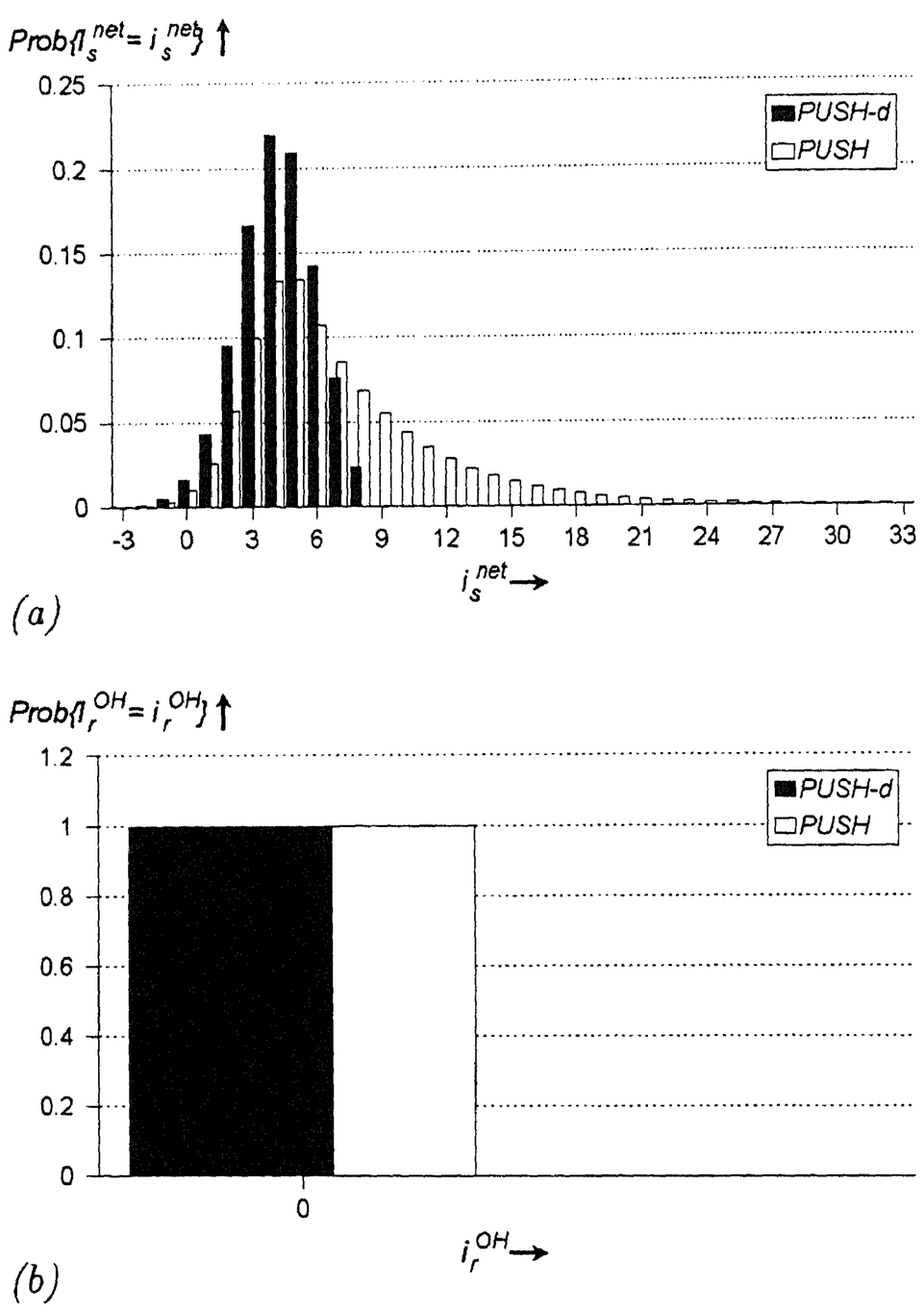


Figure 6.4. *Distribution of net inventory (a) and remanufacturable inventory (b) for the PUSH strategy and the PUSH-disposal strategy.*

between demands and returns decreases. Here, the correlation is measured in terms of the coefficient $\rho_R D$, which is the probability that a product return instantaneously generates a product demand.

6.3.2 PUSH-disposal versus PULL-disposal

In Section 4.3.2 it has been demonstrated for the case without product disposals, that the PULL strategy outperforms the PUSH strategy with respect to the total expected system costs when remanufacturable inventory is valued *lower* than serviceable inventory. Additional experiments and Figure 6.7 confirm that the cost dominance relation between the PUSH and the PULL strategies is not much influenced by the occurrence of product

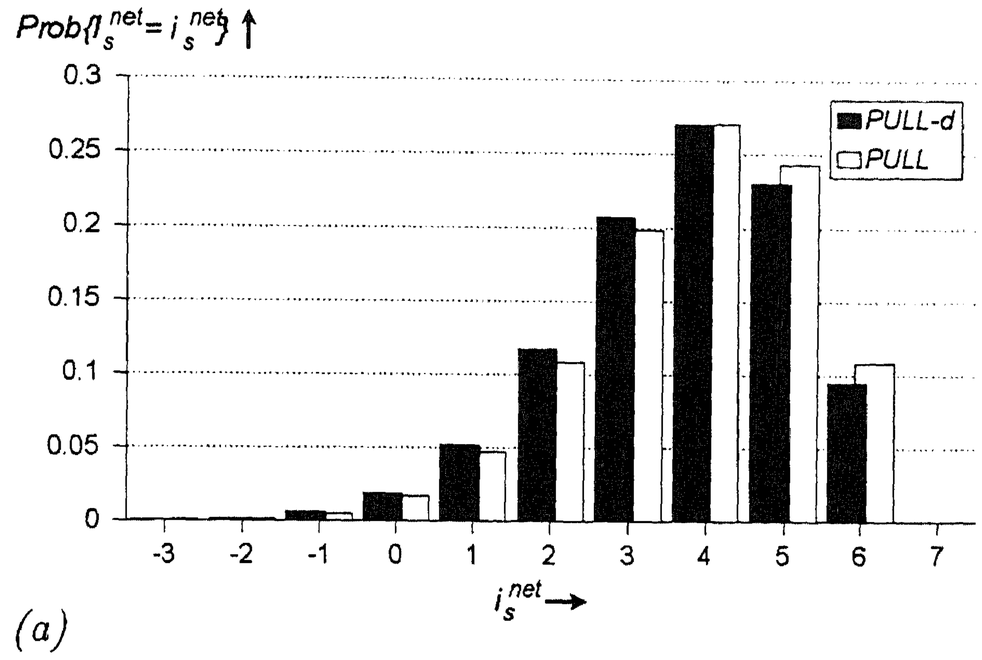


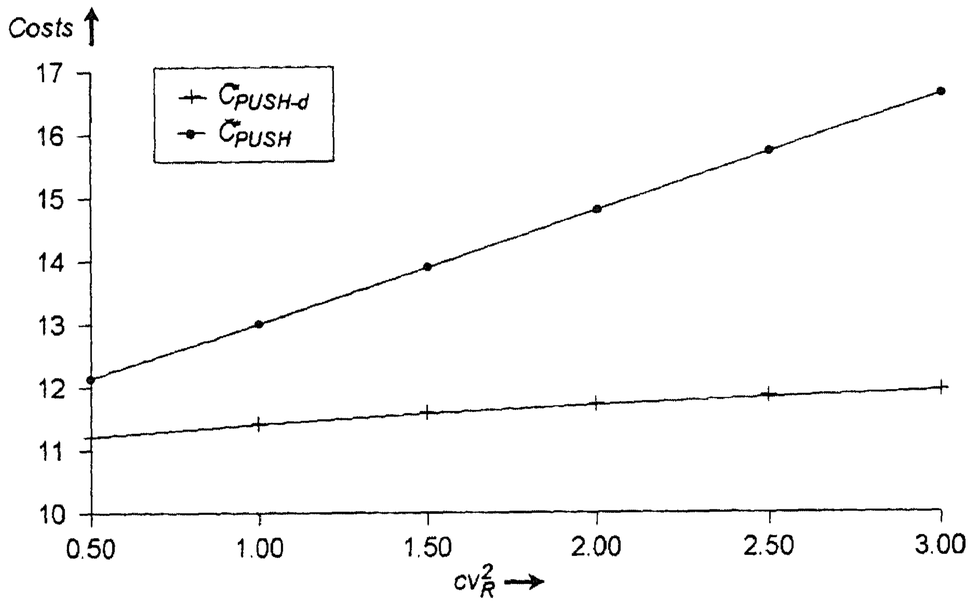
Figure 6.5. *Distribution of net inventory (a) and remanufacturable inventory (b) for the PULL strategy and the PULL-disposal strategy.*

disposals.

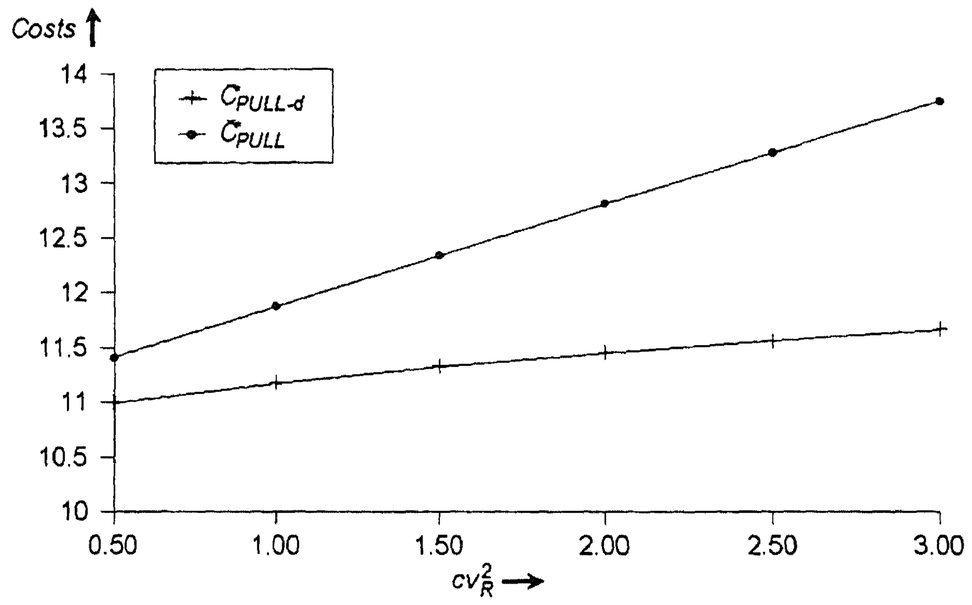
6.3.3 Robustness of control strategies during the product life-cycle

The life-cycle of a remanufacturable product is more complex to model than the life-cycle of a traditional, non-remanufacturable product. The reason is, that during the life-cycle of a remanufacturable product not only the demand rate λ_D varies over time, but also the return rate λ_R.

Here we investigate the influence of changes in the demand and return rates on the control variables and the system costs. To do so we introduce two



(a)

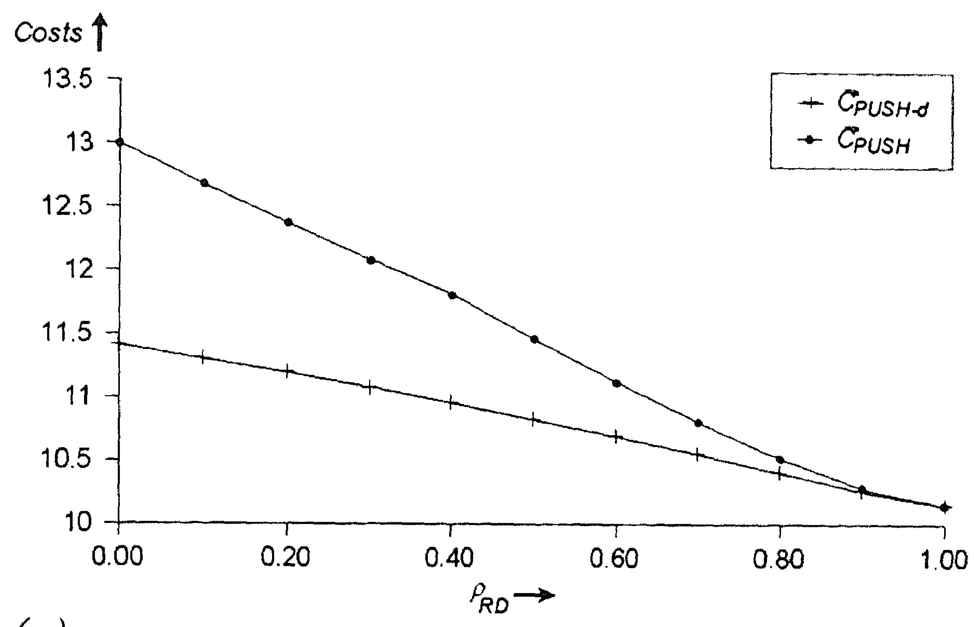


(b)

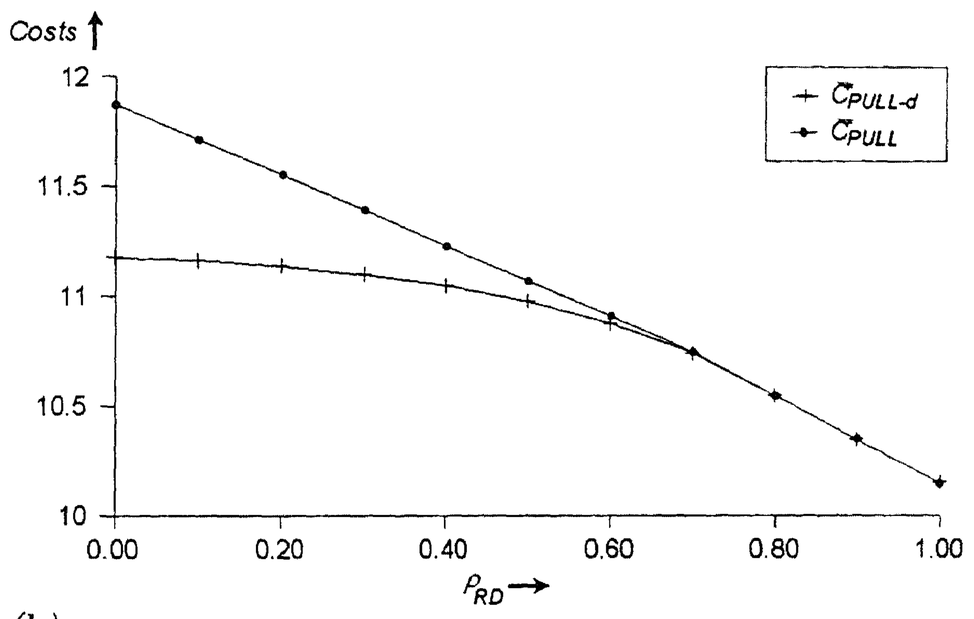
Figure 6.6. *Costs as function of the return uncertainty for the PUSH (-disposal) strategy (a) and the PULL (-disposal) strategy (b).*

types of robustness, i.e., *(i) parameter robustness*, and *(ii) cost robustness.* We consider a strategy parameter robust, if the optimal control parameters are relatively insensitive to changes in the demand and return rates. Furthermore, we consider a strategy cost robust, if the costs in a 'neighborhood' of the optimal control parameters do not differ very much from the minimal costs. Of course, both parameter and cost robustness are very beneficial from a practical (implementational) point of view, since the control parameters need not be revised very frequently during the subsequent stages of the product life-cycle.

To investigate the robustness of the strategies, we define the following life-



(a)



(b)

Figure 6.7. *Costs as function of the correlation coefficient for the PUSH (-disposal) strategy (a) and the PULL (-disposal) strategy (b).*

cycle stages.[1]

I The *introduction* stage. During this stage the demand rate increases, whereas the return rate is almost zero.

II The *growth* stage. During this stage the demand rate further increases, while the return rate slowly starts to increase.

III The *maturity* stage. In this stage the demand and return rate have become in a stable situation.

[1] Our definition of the life-cycle stages for remanufacturables relies on the definition of the life-cycle stages that is commonly used for non-remanufacturables (see e.g. Kotler [31]).

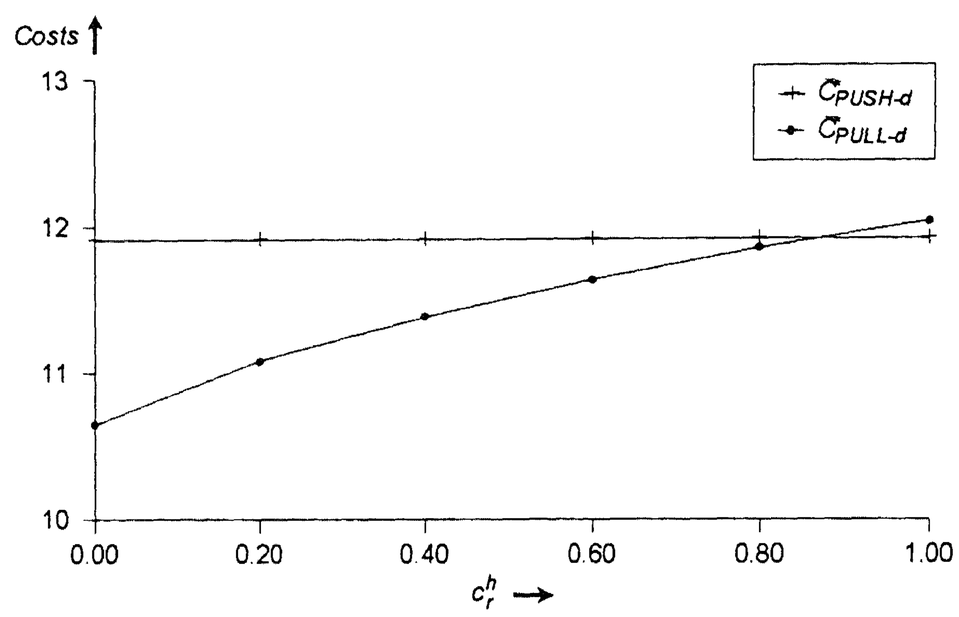


Figure 6.8. *Costs as function of the remanufacturing holding costs for the PUSH-disposal strategy and the PULL-disposal strategy.*

IV The *decline* stage. In this stage the demand rate decreases rapidly, whereas the return rate may slowly start to decrease.

V The *terminal* stage. In this stage no demands occur anymore, whereas the return rate may still be positive.

The above five stages and the typical return and demand rate patterns are schematically represented in Figure 6.9. In the numerical study below we assume a similar pattern of demand and return rate combinations by choosing a number of combinations, labeled 1 to 11, from Figure 6.9 (see Figure 6.10).

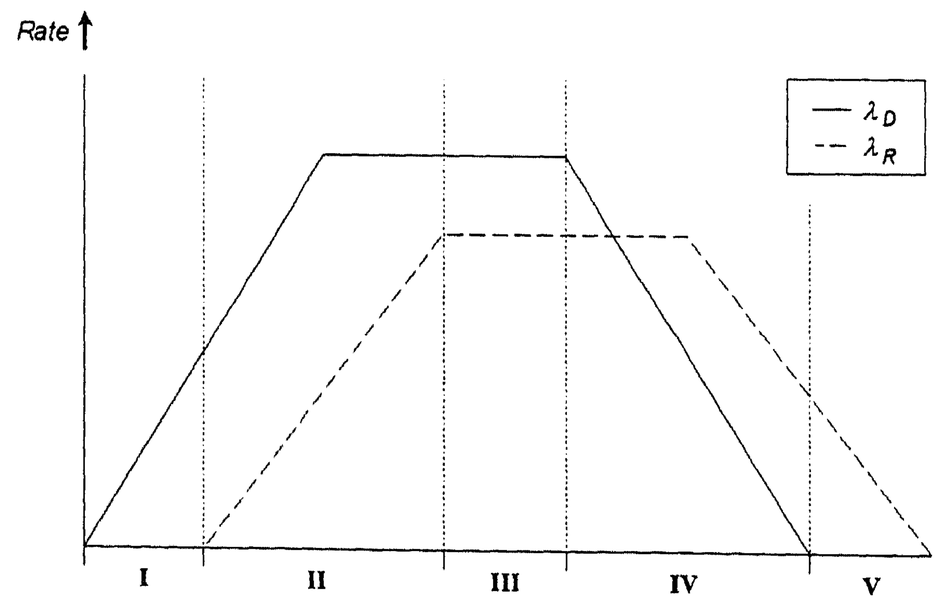


Figure 6.9. *Demand and return rates during a typical life cycle of a remanufacturable product.*

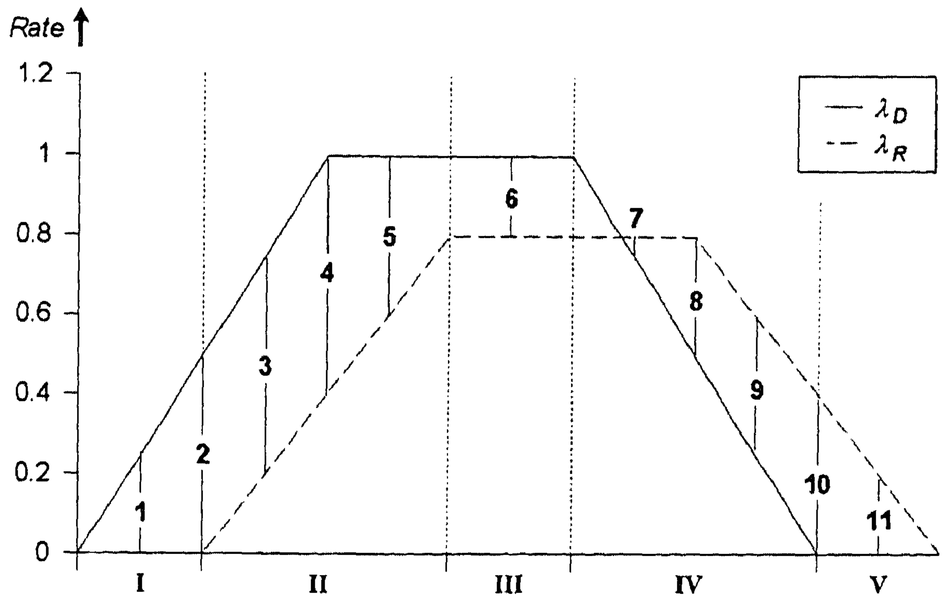


Figure 6.10. *Demand and return rates during a typical life cycle of a remanufacturable product.*

The second and third column of Table 6.3 (PUSH-disposal strategy) and Table 6.4 (PULL-disposal strategy) present a set of demand and return rate combinations that correspond to the five stages of the product life-cycle (see also Figure 6.9). Each row represents a demand and return rate combination.

Stage		λ_D	λ_R	s_m	Q_m	Q_r	s_d	C^*_{PUSH-d}
Stage I:	1	0.25	0	1	1	–	–	4.83
Introduction	2	0.50	0	2	1	–	–	8.19
Stage II:	3	0.75	0.20	3	1	1	8	10.32
Growth	4	1.00	0.40	4	1	1	9	12.42
	5	1.00	0.60	4	1	1	8	11.83
Stage III:	6	1.00	0.80	4	1	1	8	11.41
Maturity								
Stage IV:	7	0.75	0.80	3	1	1	6	8.96
Decline	8	0.50	0.80	2	1	1	5	6.60
	9	0.25	0.60	1	1	1	3	4.14
Stage V:	10	0	0.40	–	–	–	–	0
Terminal	11	0	0.20	–	–	–	–	0

Table 6.3. *Strategy parameters and costs associated with frequent parameter revision of the PUSH-disposal strategy*

For each of these combinations we calculate the long run optimal control

Stage		λ_D	λ_R	s_m	Q_m	s_r	S_r	s_d	C^*_{PULL-d}
Stage I:	1	0.25	0	1	1	–	–	–	4.83
Introduction	2	0.50	0	2	1	–	–	–	8.19
Stage II:	3	0.75	0.20	3	1	4	5	5	10.29
Growth	4	1.00	0.40	4	1	5	6	6	12.34
	5	1.00	0.60	4	1	5	6	4	11.65
Stage III:	6	1.00	0.80	4	1	5	6	3	11.17
Maturity									
Stage IV:	7	0.75	0.80	3	1	4	5	2	8.78
Decline	8	0.50	0.80	2	1	3	4	1	6.45
	9	0.25	0.60	1	1	2	3	1	4.27
Stage V:	10	0	0.40	–	–	–	–	–	0
Terminal	11	0	0.20	–	–	–	–	–	0

Table 6.4. *Strategy parameters and costs associated with frequent parameter revision of the PULL-disposal strategy*

parameters and the associated minimal costs (columns 1–11 under *Frequent parameter revision*). Unfortunately, from the tables it must be concluded that both strategies are *not* very parameter robust.

Remark 6.2 *During stage I no returns occur. As a consequence we do not need to calculate Q_r, s_r, S_r, and s_d. For stage V we did not calculate any parameter value, since in this degenerated case we have no demand occurrences. The optimal policy here is to dispose every returned product, because keeping stocks makes no sense. Since in the base case scenario disposal costs are zero, the optimal costs in stage V are also zero.*

Next, we investigate the effects on system costs if we would *not* regularly revise the control parameters, but fix the control parameters during the complete life-cycle. To this end we take three distinct return and demand rate combinations corresponding to the life-cycle stages, for instance the ones represented by the rows numbered 3, 6, and 9 in Table 6.3 (PUSH-disposal strategy) and the rows numbered 3, 6, and 9 in Table 6.4 (PULL-disposal strategy).

For each fixed policy we calculate the optimal long run costs associated with all the listed demand and return rate combinations (rows 1–11 in Table 6.5 and 6.6). A cost comparison between frequent parameter revision and fixed

parameters clearly indicates that both strategies are cost robust only in a small neighborhood of the optimal parameter combinations. The tables indicate that a fixed parameter combination may perform well during one or two stages, but never during all stages of the product life-cycle. As a result, frequent parameter monitoring and revision is necessary in practice.

Stage		λ_D	λ_R	(s_m, Q_m, Q_r, s_d) (1,1,1,3)	(3,1,1,8)	(4,1,1,8)
Stage I:	1	0.25	0	4.83	6.01	7.00
Introduction	2	0.50	0	11.29	8.22	9.04
Stage II:	3	0.75	0.20	19.70	10.32	10.58
Growth	4	1.00	0.40	31.75	13.28	12.43
	5	1.00	0.60	29.92	12.26	11.83
Stage III:	6	1.00	0.80	28.50	11.53	11.41
Maturity						
Stage IV:	7	0.75	0.80	15.88	9.32	9.76
Decline	8	0.50	0.80	7.84	8.54	8.83
	9	0.25	0.60	4.14	8.12	8.21
Stage V:	10	0	0.40	3.00	8.00	8.00
Terminal	11	0	0.20	3.00	8.00	8.00

Table 6.5. *Costs associated with fixed parameters, i.e. no parameter revision of the PUSH-disposal strategy*

6.4 Summary and conclusions

In this chapter we have extended the PUSH and PULL strategies defined in Chapter 3 to include the option of product disposal. A numerical study indicated that disposal may be effective, since it reduces the variability in the systems' inventories. As a result, the total expected system costs will be lower than in a system without disposals. This cost reducing effect may already be present when the return rate is much lower than the demand rate.

The decision whether to implement the PUSH-disposal strategy or the PULL-disposal strategy mainly depends on the cost dominance relation between stocks. Only if remanufacturable inventory is valued sufficiently lower than serviceable inventory, the PULL-disposal strategy is favorable

Stage		λ_D	λ_R	$(s_m, Q_m, s_r, S_r, s_d)$ (1,1,2,3,1)	(3,1,4,5,5)	(4,1,5,6,3)
Stage I:	1	0.25	0	4.83	6.01	7.00
Introduction	2	0.50	0	11.29	8.22	9.04
Stage II:	3	0.75	0.20	19.20	10.29	10.54
Growth	4	1.00	0.40	30.30	13.29	12.35
	5	1.00	0.60	27.66	12.25	11.66
Stage III:	6	1.00	0.80	25.65	11.54	11.17
Maturity						
Stage IV:	7	0.75	0.80	13.91	9.15	9.48
Decline	8	0.50	0.80	7.02	8.39	8.56
	9	0.25	0.60	4.27	7.90	7.94
Stage V:	10	0	0.40	3.50	7.50	7.50
Terminal	11	0	0.20	3.50	7.50	7.50

Table 6.6. *Costs associated with fixed parameters, i.e. no parameter revision of the PULL-disposal strategy*

over the PUSH-disposal strategy, otherwise the PUSH-disposal strategy is more favorable.

Finally, although the PUSH- and PULL-disposal strategy are conceptually rather simple, they are not very robust to changes in the demand and return rates which occur during the successive stages of the product life-cycle. As a consequence, infrequent revision of the control parameters may lead to unnecessarily high system costs in practice.