Chapter 7

Heuristics

As was already noted in Chapter 2, only a few papers consider returns and outside procurement in a manufacturing environment, where the total amount of products or modules may change over time. We categorized the most relevant papers into three groups: cash balance models, periodic review models and continuous review models.

In cash balance models demands and returns are explicitly modeled and various inventory control policies are considered. Yet in all variants there is no lead-time, which makes the analysis much easier. The same holds for the few papers discussing the periodic review case, with the exception of Inderfurth [26]. However, the latter paper focuses on the structure of the optimal policy and not on finding the optimal policy parameters.

What makes the continuous review case so interesting is that it can handle (stochastic) lead-times. Yet also Heyman [23] and Hoadly and Heyman [24] assume zero lead-times and do not take fixed order costs into account. The only paper which does consider lead-times is a paper by Muckstadt and Isaac [37], in which an approximative method is developed for a single product (s_m, Q_m) model with fixed lead-times but no disposal.

In this chapter we will take the model of Muckstadt and Isaac as a starting point to develop heuristics for the HMR systems that were considered in the previous chapters. Next, in Section 7.1, we briefly discuss the model and the approximative method that were introduced in Muckstadt and Isaac. In Section 7.2 we develop two alternative approximation procedures, which we will compare to the Muckstadt and Isaac approach in Section 7.3.

7.1 The Muckstadt & Isaac approach

We consider a single product inventory system similar to the one introduced in Chapter 3, which was first proposed in Muckstadt and Isaac [37]. The only difference with the model of Chapter 3 is that we do not make any assumptions regarding the remanufacturing facility other than independence of processing times. Further, holding costs for remanufacturable inventory and fixed ordering costs for remanufacturing orders are not considered. We assume that the inventory is continuously reviewed and that an $(s_m, Q_m, Q_r = 1)$ PUSH strategy is applied to the inventory position. Our objective is to determine those parameters s_m,Q_m that minimize the total long-run average costs.

For the analysis define the net inventory at time t, $I_s^{net}(t)$, as the number of on-hand serviceables in storage, $I_s^{OH}(t)$, minus the number of outstanding backorders, $B(t)$. The inventory position, $I_s(t)$, is the sum of the net inventory, the number of products in the remanufacturing facility, $R(t)$, and the number of products on order, $P(t)$.

Remark 7.1 *Note that the random variable $R(t)$ includes remanufacturable inventory and work in process. Consequently, remanufacturable products enter inventory position upon arrival. This differs from the model in Chapter 3, in which remanufacturable products enter inventory position only at the moment that they are ordered. Both definitions of inventory position are equal in the special case that the remanufacturing capacity is infinite. In this case every returned product enters the remanufacturing work in process upon arrival. The reason why Muckstadt and Isaac adopt the first definition is that in this way an analytical expression of the mean and variance of inventory position becomes available. In practice, our definition seems more appropriate, since the time of becoming available for serviceable inventory is less uncertain for products already in work in process than for products in remanufacturable inventory.*

Notice now that at time t all the outstanding orders at time $t - \mu_{L_m}$ have arrived. Hence, the net inventory at time t equals the inventory position at time $t - \mu_{L_m}$ minus the number in the remanufacturing shop at that time minus the demand plus the output of the remanufacturing shop during the interval $(t - \mu_{L_m}, t]$. In formula,

$$I_s^{net}(t) = I_s(t - \mu_{L_m}) - R(t - \mu_{L_m}) + Z(t - \mu_{L_m}, t), \tag{7.1}$$

where the latter term indicates the output of the remanufacturing shop during the interval $(t-\mu_{L_m}, t]$, minus the total demand during the interval $(t-\mu_{L_m}, t]$.

For the analysis we are interested in the average number of orders, the average on-hand inventory and the average number of backorders. Since both demand and return inter-arrival times are negative exponentially distributed, we can formulate a continuous-time Markov chain for the inventory position. Demands now decrease the inventory position with one product unless the level s_m is reached in which case a manufacturing order instantaneously increases the inventory position to level $s_m + Q_m$. A product return always increases the inventory position by one product. Since at this point we do not consider disposals we have to assume that $\lambda_R < \lambda_D$.

Using a generating function approach it is easy to find the following expressions for the first and second moment of the inventory position, which read as (see [37])

$$\overline{I}_s := \lim_{t\to\infty} E(I_s(t)) = s_m + 1 + \frac{Q_m - 1}{2} + \frac{\lambda_R}{\lambda_D - \lambda_R},$$

and

$$Var(I_s) := \lim_{t\to\infty} Var(I_s(t)) = \frac{(Q_m)^2 - 1}{12} + \frac{\lambda_D \lambda_R}{(\lambda_D - \lambda_R)^2}. \tag{7.2}$$

Taking limits in (7.1) yields

$$\overline{I}_s^{net} = s_m + 1 + \frac{Q_m - 1}{2} + \frac{\lambda_R}{\lambda_D - \lambda_R} - \overline{R} + (\lambda_D - \lambda_R)\mu_{L_m}, \tag{7.3}$$

since $\lim_{t\to\infty} E(Z(t-\mu_{L_m}, t)) = (\lambda_D - \lambda_R)\mu_{L_m}$.

Using (7.2) and $Var(Z(\mu_{L_m})) := \lim_{t\to\infty} Var(Z(t-\mu_{L_m}, t))$ we have

$$\begin{aligned} Var(I_s^{net}) \approx \tilde{V}(I_s^{net}) \;&:=\; \tfrac{(Q_m)^2-1}{12} + \tfrac{\lambda_D\lambda_R}{(\lambda_D-\lambda_R)^2} \\ &\quad + Var(R) + Var(Z(\mu_{L_m})), \end{aligned}$$

where in the latter we have discarded the covariance terms.

Note that exact expressions and good approximations for $\overline{R}$, $Var(R)$, and $Var(Z(\mu_{L_m}))$ are available for various standard queuing systems. For example, for $M/M/c$ and $M/G/\infty$ queues the output of the remanufacturing facility is a Poisson process (see [37]), so in this case $Var(Z(\mu_{L_m})) = \lambda_D + \lambda_R$.

In order to calculate the average number of backorders we need to get hold of the distribution of the net inventory at an arbitrary point in time. To do so Muckstadt and Isaac propose a normal distribution as an approximation of the distribution of net inventory. This results in the following expression of the average number of backorders, $\overline{B}_{MI}$:[1]

$$\overline{B}_{MI} = \tilde{V}(I_s^{net})\phi\left(\frac{\overline{I}_s}{\tilde{V}(I_s^{net})}\right) - \overline{I}_s\Phi\left(\frac{\overline{I}_s}{\tilde{V}(I_s^{net})}\right), \tag{7.4}$$

where $\phi(.)$ and $\Phi(.)$ are the standard normal density and standard normal distribution functions respectively.

We define the cost function to be optimized, $C_{MI}(s_m, Q_m)$, as

$$\overline{C}_{MI}(s_m, Q_m) = c_m^f\overline{O}_m + c_b\overline{B}_{MI} + h_s\left(\overline{B}_{MI} + \overline{I}_s^{net}\right) \tag{7.5}$$

where $\overline{O}_m$ is the average number of manufacturing orders per unit of time. Substituting (7.3), (7.4), and $\overline{O}_m = (\lambda_D - \lambda_R)/Q_m$, we rewrite (7.5) as

$$\begin{aligned} \overline{C}_{MI}(s_m, Q_m) \;=\; & c_m^f\left(\tfrac{\lambda_D-\lambda_R}{Q_m}\right) + (c_b + h_s)\overline{B}_{MI} \\ & + h_s\Big(s_m + 1 + \tfrac{Q_m-1}{2} + \tfrac{\lambda_R}{\lambda_D-\lambda_R} \\ & \quad - \overline{R} + (\lambda_D - \lambda_R)\mu_{L_m}\Big). \end{aligned} \tag{7.6}$$

Finally, in [37] it is shown that the optimal parameter values s_m^* and Q_m^* have to satisfy

$$\frac{(Q_m^*)^3}{\sqrt{\frac{(Q_m^*)^2}{12} + d}} = \frac{12(\lambda_D - \lambda_R)c_m^f}{\alpha},$$

and

$$s_m^* = \left(\sqrt{\frac{(Q_m^*)^2}{12} + d}\right)\Phi^{-1}\left(\frac{c_b}{c_b + h_s}\right) - \frac{Q_m^*}{2} - c,$$

where

$$c = \frac{\lambda_R}{\lambda_D - \lambda_R} + \frac{1}{2} - \overline{R} + (\lambda_D - \lambda_R)\mu_{L_m},$$

[1] In the remainder of this chapter the subscript '*MI*' refers to the Muckstadt and Isaac approach, whereas the subscripts '*L1*' and '*L2*' refer to the two alternative procedures proposed by the author (Section 7.3).

$$d = \frac{\lambda_D \lambda_R}{(\lambda_D - \lambda_R)^2} - \frac{1}{12} + Var(R) + Var(Z(\mu_{L_m})),$$

and

$$\alpha = (c_b + h_s)\phi\left(\Phi^{-1}\left(\frac{c_b}{c_b + h_s}\right)\right).$$

A disadvantage of the approximation procedure of Muckstadt and Isaac is that the asymptotic properties of the approximation of the net inventory do not correspond to actual behaviour in some cases (see Van der Laan [58], and Table 7.1).

	Muckstadt & Isaac procedure			Actual behaviour		
	s_m^*	Q_m^*	$\overline{B}_{MI}$	s_m^*	Q_m^*	$\overline{B}$
$Q_m \to \infty$	–	–	∞	–	–	0
$\lambda_R \to \lambda_D$	const./∞	const.	∞	const.	1	0
$s_m \to \infty$	–	–	0	–	–	0

Table 7.1. *Asymptotic behaviour of the approximation procedure of Muckstadt and Isaac compared to actual behaviour.*

7.2 An alternative procedure

For the alternative approximations that we will develop in this section we assume that at any point in time there is at most one order outstanding. The output of the remanufacturing facility is now approximated by an independent Poisson process with mean λ_R, which is exact for $M/M/c$ and $M/G/\infty$ queues. In the first approximation we assume that the net demand during the manufacturing lead-time, $Z(\mu_{L_m})$, follows a normal distribution. Furthermore we assume that at the moment of a manufacturing order, the net inventory equals $s_m - \overline{R}$. Hence, the expected number of backorders just before a replenishment, $F(s_m, \mu_{L_m})$, is approximated by the expected surplus net demand over the level $s - \overline{R}$ of a normal distribution with mean $\mu = (\lambda_D - \lambda_R)\mu_{L_m}$, and variance $\sigma^2 = (\lambda_D + \lambda_R)\mu_{L_m}$, or in formula,

$$F(s_m, \mu_{L_m}) = (\mu + \overline{R} - s_m)\Phi\left(\frac{\mu + \overline{R} - s_m}{\sigma}\right) + \sigma\phi\left(\frac{\mu + \overline{R} - s_m}{\sigma}\right)$$

Assuming a linear increase of the number of backorders per time unit it follows that the average number of backorders during the time that net inventory is negative equals $F(s_m, \mu_{L_m})/2$. From the same assumption it also follows that the average time that net inventory is negative equals $F(s_m, \mu_{L_m})/(\lambda_D - \lambda_R)$ divided by the average cycle length $Q_m/(\lambda_D - \lambda_R)$. Hence, the expected number of backorders can be approximated by

$$\overline{B}_{L1} := \frac{F(s_m, \mu_{L_m})^2}{2Q_m} \tag{7.7}$$

Using (7.7) we obtain as total cost function $\overline{C}_{L1}(s_m, Q_m)$:

$$\begin{aligned} \overline{C}_{L1}(s_m, Q_m) &= c_m^f \left(\tfrac{\lambda_D - \lambda_R}{Q_m}\right) + (c_b + h_s)\overline{B}_{L1} \\ &\quad + h_s \Big(s_m + 1 + \tfrac{Q_m - 1}{2} + \tfrac{\lambda_R}{\lambda_D - \lambda_R} \\ &\qquad - \overline{R} + (\lambda_D - \lambda_R)\mu_{L_m}\Big) . \end{aligned} \tag{7.8}$$

In the following lemma we prove that the function $\overline{C}_{L1}(s_m, Q_m)$ is a strictly convex function in s_m and Q_m.

Lemma 7.1 *The function $\overline{C}_{L1}(s_m, Q_m)$ defined in (7.8) is a strictly convex function in the control parameters s_m and Q_m.*

Proof It is quite easy to show that the second derivative with respect to Q_m is positive. Next consider the second derivative with respect to s_m. The main complication is in the term $F(s_m, \mu_{L_m})^2$. Notice now that

$$\begin{aligned} \tfrac{d^2 F(s_m, \mu_{L_m})^2}{ds_m^2} &= 2F(s_m, \mu_{L_m}) \tfrac{d^2 F(s_m, \mu_{L_m})}{ds_m^2} + 2\left(\tfrac{dF(s_m, \mu_{L_m})}{ds_m}\right) \\ &= \tfrac{2}{\sigma} F(s_m, \mu_{L_m}) \phi\left(\tfrac{\mu + \overline{R} - s_m}{\sigma}\right) + 2\left(\tfrac{dF(s_m, \mu_{L_m})}{ds_m}\right)^2 > 0, \end{aligned}$$

which shows that the function is convex in s_m. To finish the proof we apply the following standard lemma on convexity, which we state without proof.

Lemma 7.2 *Let $f(x)$ be a positive valued, decreasing and convex function in x, and let $g(y)$ be a linear positive valued function in y, then $h(x, y) := f(x)/g(y)$ is convex in (x, y).*

Taking $f(s_m) = F(s_m, \mu_{L_m})^2$ and $g(Q_m) = Q_m/(\lambda_D - \lambda_R)$ yields the desired result. □

The optimal value of Q_m given s_m^* is easily found by taking the derivative of (7.8) and equals

$$Q_m^* = \sqrt{\frac{c_b + h_s}{h_s} F(s_m^*, \mu_{L_m})^2 + \frac{2(\lambda_D - \lambda_R) c_m^f}{h_s}}$$

Notice that the second term within the square root resembles the well-known EOQ formula (see e.g. Silver and Peterson [48]) adjusted for returns. Taking the derivative of (7.8) with respect to s_m yields the following equation for the optimal value of s_m given Q_m^*,

$$\frac{F(s_m^*, \mu_{L_m})^2}{Q_m^*} \Phi \left(\frac{\mu + \overline{R} - s_m}{\sigma} \right) = \frac{h_s}{c_b + h_s}$$

which can easily be solved with numerical techniques. Since both s_m and Q_m are integer valued, the final optimal parameter combination is that neighbor which has lowest average costs.

In the second approximation procedure we approximate the difference between the demand and the output process from the remanufacturing facility by a Brownian motion with drift equal to $\mu_t = (\lambda_D - \lambda_R)t$ and a variance $\sigma_t = (\lambda_D + \lambda_R)t$ over t time units. Consequently, the net inventory t time units after the ordering of a replenishment follows a normal distribution with mean $s_m - \overline{R} - (\lambda_D - \lambda_R)t$ and with variance $(\lambda_D + \lambda_R)t$. Hence, the time-average amount of backorders is approximated as

$$\overline{B}_{L2} := \frac{\lambda_D - \lambda_R}{Q_m} \int_0^{\mu_{L_m}} F(s_m, t) dt \tag{7.9}$$

This leads to another total cost function, $\overline{C}_{L2}(s_m, Q_m)$, defined as

$$\begin{aligned} \overline{C}_{L2}(s_m, Q_m) &= c_m^f \left(\tfrac{\lambda_D - \lambda_R}{Q_m} \right) + (c_b + h_s) \overline{B}_{L2} \\ &\quad + h_s \Big(s_m + 1 + \tfrac{Q_m - 1}{2} + \tfrac{\lambda_R}{\lambda_D - \lambda_R} \\ &\quad - \overline{R} + (\lambda_D - \lambda_R) \mu_{L_m} \Big) . \end{aligned} \tag{7.10}$$

Again it is possible to show that this function is convex:

Lemma 7.3 *The function $\overline{C}_{L2}(s_m, Q_m)$ defined in (7.10) is a strictly convex function in the control parameters s_m and Q_m.*

Proof The main complication is in the term $\overline{B}_{L2}$. given by (7.9). We consider the first and second derivative of $\int_0^{\mu_{Lm}} F(s_m,t)dt$ with respect to s_m:

$$\begin{aligned}
\frac{\partial}{\partial s_m}\int_0^{\mu_{Lm}} F(s_m,t)dt &= \int_0^{\mu_{Lm}} \frac{\partial}{\partial s_m}F(s_m,t)dt \\
&= \int_0^{\mu_{Lm}} -\phi\left(\frac{\mu+\overline{R}-s_m}{\sigma}\right) dt < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2}{\partial s_m^2}\int_0^{\mu_{Lm}} F(s_m,t)dt &= \int_0^{\mu_{Lm}} \frac{\partial^2}{\partial s_m^2}F(s_m,t)dt \\
&= \int_0^{\mu_{Lm}} \frac{1}{\sigma_t}\phi\left(\frac{\mu_t+\overline{R}-s_m}{\sigma_t}\right) dt > 0
\end{aligned}$$

Thus, $\int_0^{\mu_{Lm}} F(s_m,t)dt$ is strictly decreasing and strictly convex in s_m. Applying Lemma 7.2 with $f(s_m) = \int_0^{\mu_{Lm}} F(s_m,t)dt$ and $g(Q_m) = Q_m/(\lambda_D - \lambda_R)$ yields the proof. □

Using approximation (7.9), the optimal value of Q_m, is computed as

$$Q_m^* = \sqrt{\left((c_b+h_s)\int_0^{\mu_{Lm}} F(s_m^*,t)dt + c_m^f\right)\frac{2(\lambda_D-\lambda_R)}{h_s}},$$

and the optimal value of s_m must satisfy

$$\frac{\lambda_D-\lambda_R}{Q_m}\int_0^{\mu_{Lm}} \Phi\left(\frac{\mu_t+\overline{R}-s_m}{\sigma_t}\right) dt = \frac{h_s}{c_b+h_s}.$$

	Alternative procedure 1			Alternative procedure 2			Actual behaviour		
	s_m^*	Q_m^*	$\overline{B}_{L1}$	s_m^*	Q_m^*	$\overline{B}_{L2}$	s_m^*	Q_m^*	$\overline{B}$
$Q_m \to \infty$	–	–	0	–	–	0	–	–	0
$\lambda_R \to \lambda_D$	const.	const.	const.	const.	1	0	const.	1	0
$s_m \to \infty$	–	–	0	–	–	0	–	–	0

Table 7.2. *Asymptotic behaviour of the two alternative approximation procedures compared to actual behaviour.*

We summarize the asymptotic properties of the two alternative approximation methods in Table 7.2. Clearly, the asymptotic properties of the second alternative procedure follows the actual behaviour more accurately than the procedure of Muckstadt and Isaac...

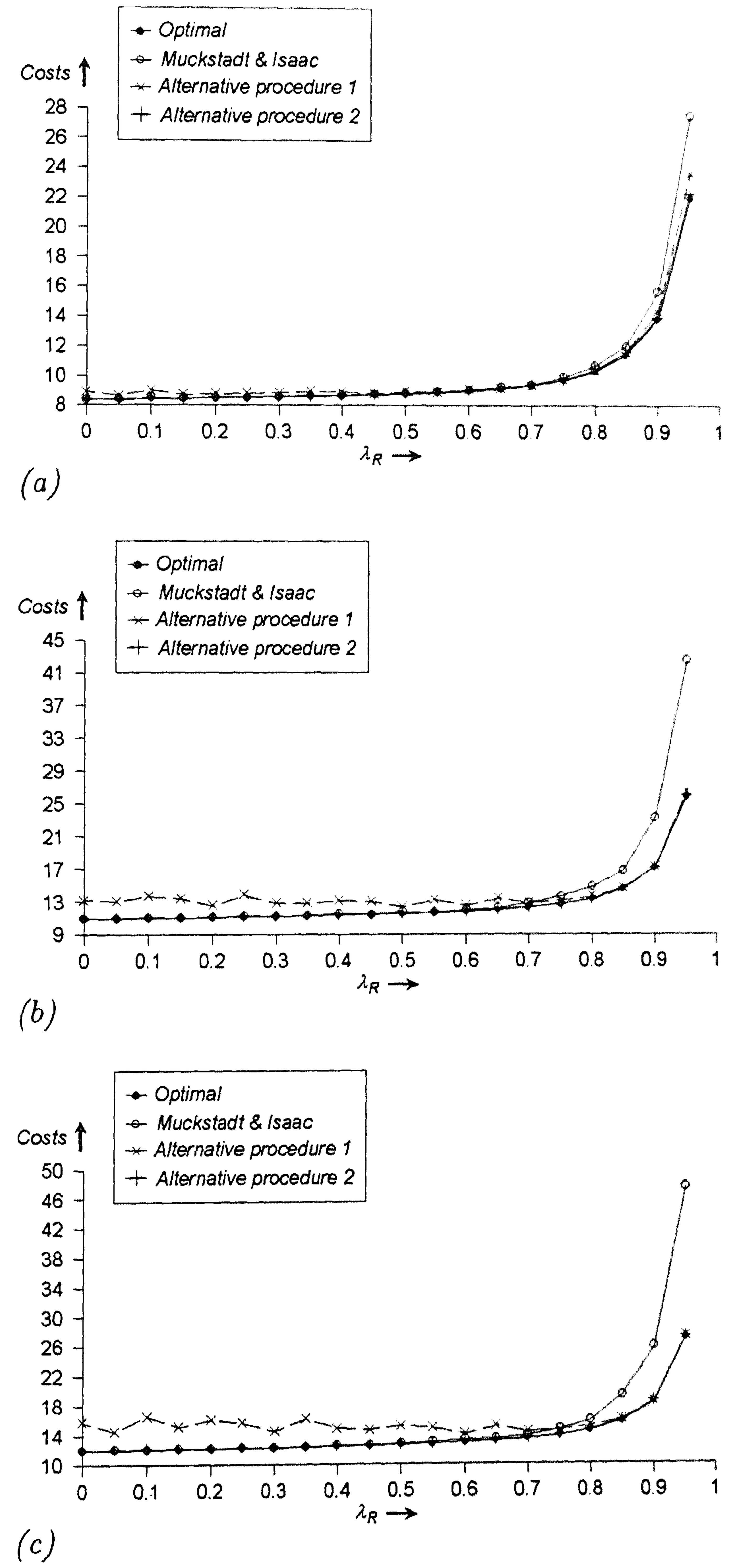

Figure 7.1 *Exact expected total costs as function of the return intensity, under the optimal strategy and under the strategies determined by the approximation procedures, with* $c_b = 10$ *(a),* $c_b = 50$ *(b), and* $c_b = 100$ *(c).*

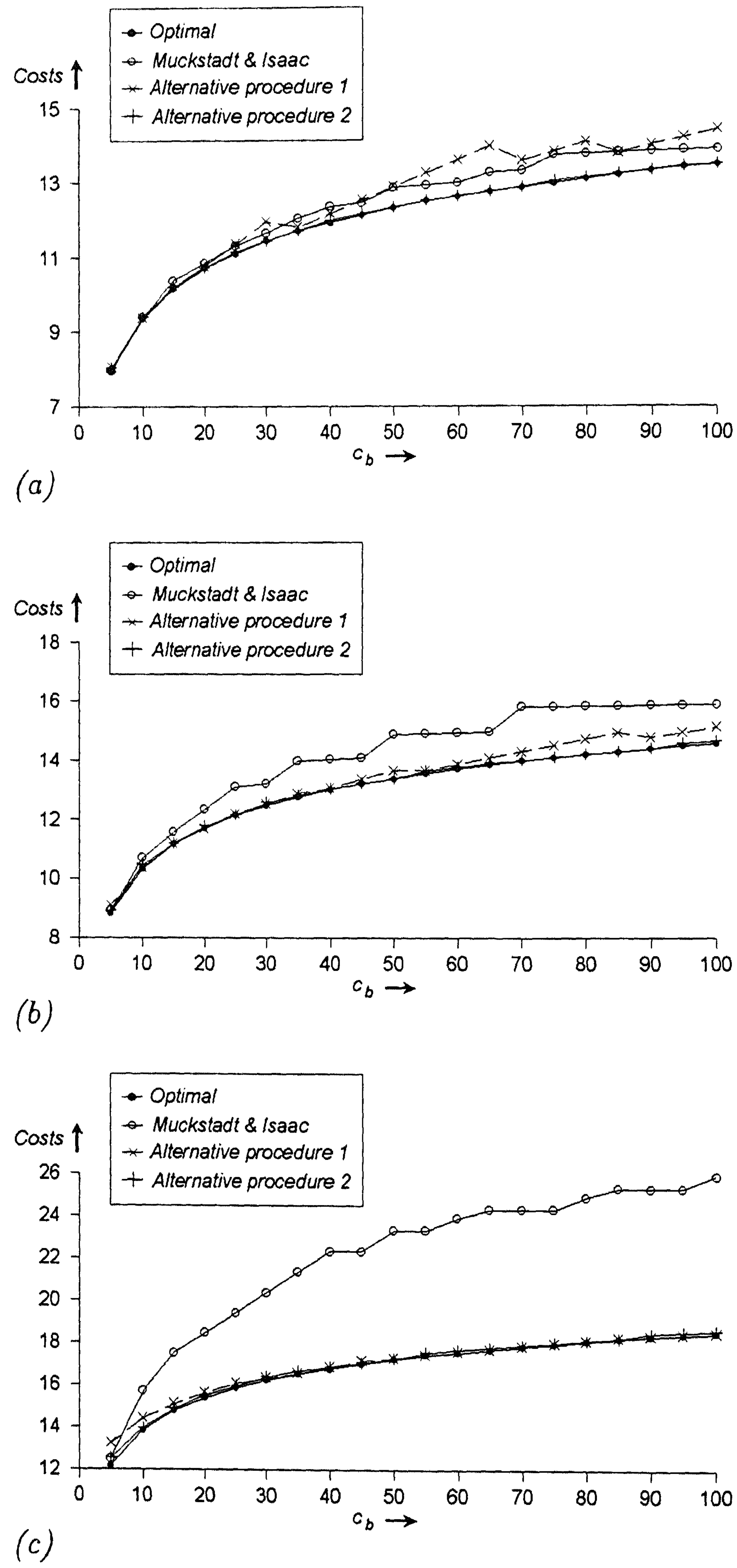

Figure 7.2 *Exact expected total costs as function of the return intensity, under the optimal strategy and under the strategies determined by the approximation procedures, with* $\lambda_R = 0.7$ *(a),* $\lambda_R = 0.8$ *(b), and* $\lambda_R = 0.9$ *(c).*

7.3 Numerical comparison

For the following numerical comparison of the alternative approximation procedures we use the following base case scenario: $\lambda_D = 1.0$, $\lambda_R = 0.7$, $\mu_{L_m} = 10$, $c_m^f = 10$, $h_s = 1.0$, $c_b = 10$. The remanufacturing facility is modeled by an $M/M/\infty$ queue with mean remanufacturing time $\mu_{L_r} = 0.5$. The remanufacturing batch size is always equal to 1. As a consequence we can compute *exact* costs given some combination of the policy parameters, by applying a similar analysis as given in Chapter 3 for the PUSH-strategy. Details can be found in Salomon et al. [45].

Extensive numerical experiments using the optimal values obtained by our two methods and the method of Muckstadt and Isaac, and computing the accompanying *exact* costs, show that the three methods differ only slightly for moderate values of c_b together with high values of λ_R/λ_D (see Figures 7.1(a) and 7.2(a)). For higher values of c_b, alternative procedure 1 does not perform very well for $\lambda_R/\lambda_D < 0.75$, whereas for values of λ_R close to λ_D the Muckstadt and Isaac procedure performs considerably worse for $\lambda_R/\lambda_D > 0.75$ (see Figures 7.1(b)-(c)) and 7.2(a)-(c)). In all cases that we considered our second method generated results that are very close to optimal, and we have to conclude that this procedure is very accurate, and superior to the other methods in almost all cases.

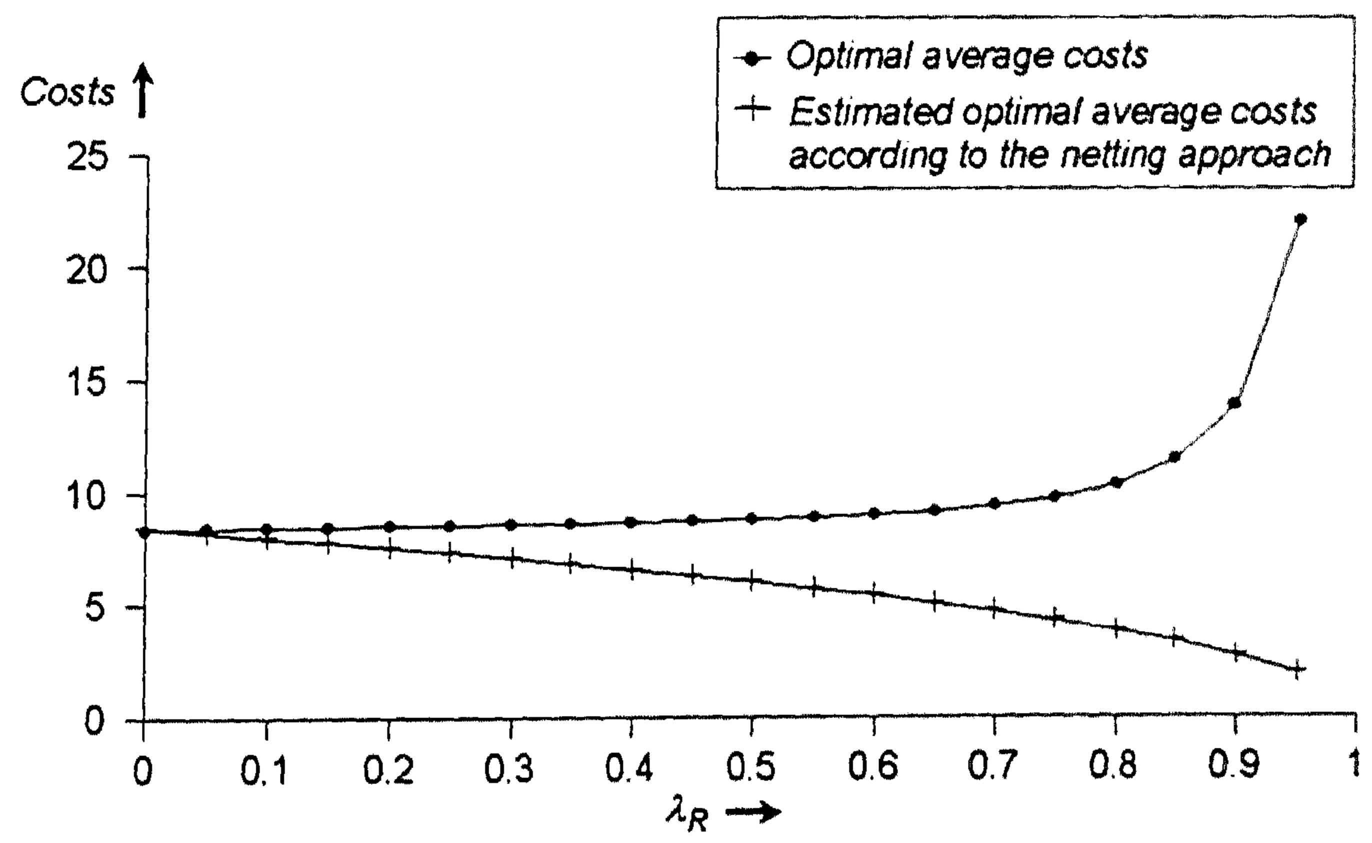

Figure 7.3. *Comparison between the optimal average costs resulting from the explicit modeling of the return flow and the optimal average costs as estimated by the netting approach.*

A popular method to handle return flows in practice is the so-called 'netting approach'. Basically, in this approach the returns are treated as negative demands. The expected demand flow is corrected by subtracting the number of expected returns from the number of expected demands. The resulting expected demand flow after correction, is then treated with the existing planning and control methods that do not allow for return flows.

To show that this approach is fundamentally a bad approximation to the real underlying processes, we present the following example. First we explicitly model the return flow with intensity λ_R and the demand flow with intensity λ_D, and calculate for the base case scenario the optimal average costs. Second, we model only the demand flow corrected for the expected return flow, thus with intensity $\lambda_D - \lambda_R$, and calculate the optimal average costs. These costs can be seen as the costs that would result if the netting approach would be a correct procedure.

Figure 7.3 shows that the netting aproach considerably underestimates the optimal average costs in a remanufacturing environment. This can by explained by the fact that an increasing return rate causes an increase of process uncertainty and eventually in an increase of average costs. The netting approach however predicts the opposite, namely that the uncertainty decreases due to a decreasing net demand flow, $\lambda_D - \lambda_R$. From this simple example it can be seen that the netting approach fundamentally is a poor methodology to handle product returns, since it not only *disregards* the uncertainty related to product returns, it also assumes that the uncertainty *decreases* with an increasing return rate, while the opposite is true.

7.4 Summary and discussion

In this Chapter we introduced some heuristical procedures to find optimal policy parameters to control a system that was first introduced by Muckstadt and Isaac. At least one of these procedures seems to be reasonably accurate for the scenarios that we considered. A disadvantage of these procedures however is that they only apply to PUSH type strategies. If PULL type strategies are considered, it is crucial to take into account the interactions between the remanufacturable inventory and the inventory position, which is quite difficult. The same holds for disposal strategies, although some preliminary ideas for a heuristical approach are given in Van der Laan [58] and Van der Laan et al. [61].

A simple heuristical approach to handle product returns, the 'netting approach', does not capture the increase in uncertainty due to the return flow.

Therefore, optimization methods that are based on a netting approach applied to traditional inventory models are not expected to be very successful. The approximation procedures developed in this chapter seem to provide a better basis for an extension to more realistic HMR systems.