A.

The Use of Correlation Analysis in Economic Research*

§ 55. Introductory

This article tries to give a summary of the methods of correlation analysis, developed during the last decennia, and the use that may be made of them in the field of economic research. It does not contain anything new, but may be of some value to economists, when they have to judge results of statistical investigations or to plan such investigations.

The need for quantitative insight in economic relations is now generally felt. There are some simple possibilities of gaining this insight, consisting in the direct measurement of the variables whose relation must be studied. An example is that of family budget statistics for the relation between incomes and expenditures.

In other cases a simple comparison between two series of figures may be sufficient. An example is the relation between the price of certain crop products and the available supply of such products. Here it may be assumed that price fluctuations can be explained by supply fluctuations only and hence a simple scatter diagram will be a useful tool.

As a rule, however, the fluctuations in any variable x will be caused by fluctuations in more than one other variable: y, z, etc. and one of the most important problems therefore arising in statistical analysis of economic phenomena is how then to find the relation between the "variable to be explained" x and the "explanatory variables" y, z, etc.

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* Ekonomisk Tidskrift, 49 (1947), 173-192. Reprinted by the courtesy of the publisher.

§ 56. Multiple Correlation Analysis

In principle a solution is possible by the use of multiple correlation analysis. Under certain conditions this type of analysis

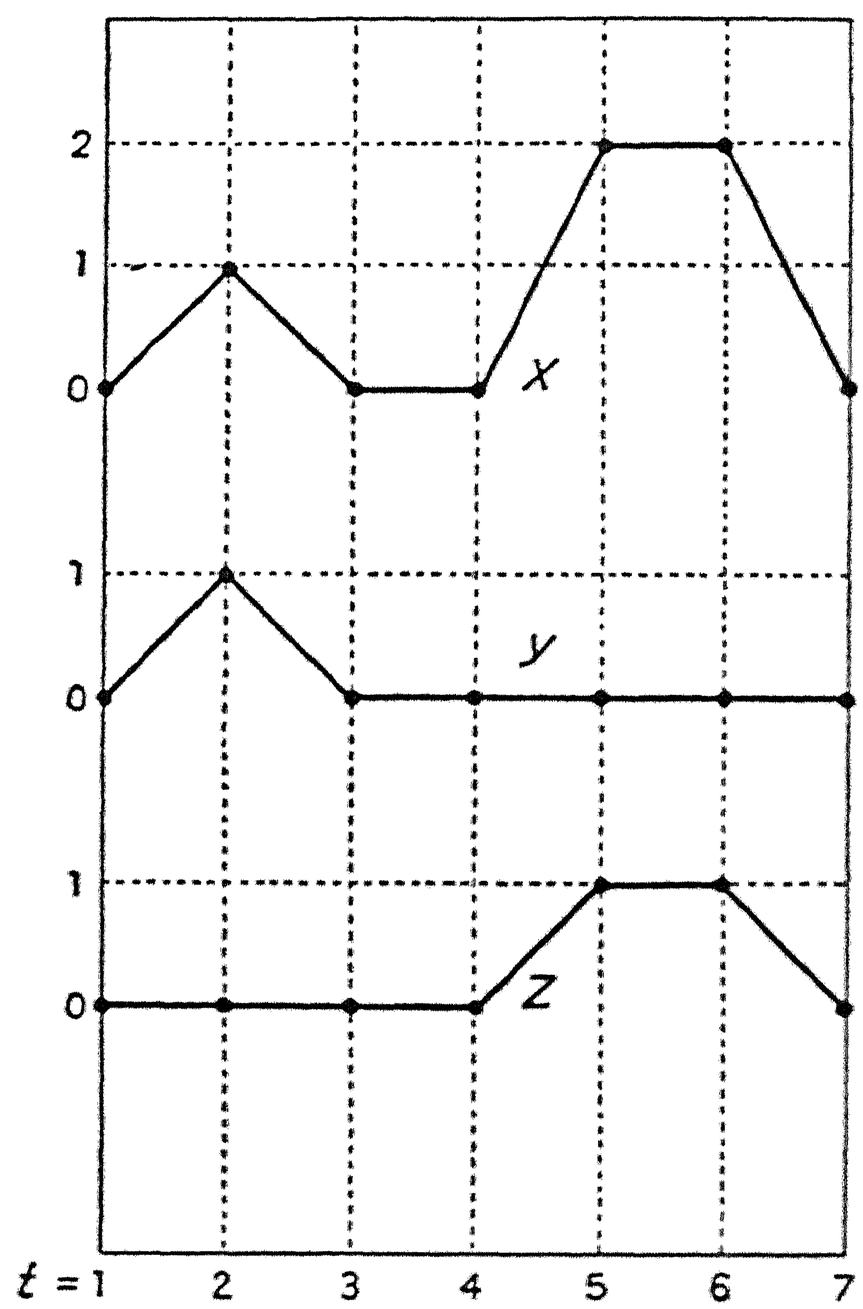

Fig. 17.

is able to tell us what are the most likely values of the coefficients a and b to be used in the expression:

$$x' = ay + bz + \cdots \tag{56.1}$$

which may be called the "explanation" of x.

This equation, which provisionally has been given a linear form, is called a regression equation and a and b are called re-

gression coefficients. It is the intention of this "explanation" that the consecutive values of x' for all the time units in the period considered are as near to the corresponding values of x as possible.

The ideal situation presents itself if the values of x, y, and z are such that it is possible to obtain complete equality of the corresponding values of x' and x. This may be the case without the existence of a parallelism in the movements of y and z (restricting ourselves to the case of two explanatory variables, y and z). If so, there will only be one pair of values a and b which yield us the equality of x' and x (see Fig. 17, where a very simple case has been represented; it will be immediately evident that only for $a = 1$ and $b = 2$ equality of x and x' can be reached).

The choice of the variables y, z, etc. which have to explain the fluctuations in x, must be made by the economist. Once this choice has been made—which means that a theory of the fluctuations of x has been presented—the task of the statistician, when applying multiple correlation analysis, is twofold: He may *test the theory* and if this theory can be accepted he may *measure the coefficients* a and b, which may be called the strength with which the variables y and z act on x.

As to the testing of the theory, two extreme cases may present themselves:

(1) if x and x' are badly correlated, the theory is erroneous or at least incomplete;
(2) if the correlation between x and x' is complete, the theory does not contradict the facts.

If the theory be accepted by the economist, the statistician is now able, under certain conditions, to measure a and b. Generally speaking there will be only one pair of values a and b by which the highest correlation between x and x' is obtained. These values of a and b may be calculated by the method of "least squares," making $\Sigma (x' - x)^2$ a minimum. The regression equation so obtained is called the first (elementary) regression equation. The obtained values are considered as the most likely

ones. If other values of a and b would be assumed to prevail, x and x' would be less well correlated and this is a less probable situation, once that the economist makes us believe that the theory put forward is correct.

From the foregoing it will be clear that the statistician never can give a *proof* of any economic theory.

Measurement of a and b will sometimes be impossible, even if a very good fit between x and x' is found.

The simplest case presents itself if y and z themselves are exactly correlated. In that case there will not be only one pair of values a and b for which a good fit is obtained, but an infinity of them. Any part of the influence of y may be replaced by part of the influence of z. Sometimes this situation is the consequence of the properties of the economic system under consideration: it may be, e.g., that there is a second relation between y and z which is the cause of their correlation.

Example: x represents consumption outlay, y total income, z the price level. The two relations may be:

$$x = ay + bz \text{ (demand equation)} \tag{56.2}$$

and

$$z = cy \text{ (relation determining price level)} \tag{56.3}$$

More complicated situations in which the coefficients a, b, c, etc. are not measurable may arise in more complicated systems. Generally these cases are described as cases of "multicollinearity" (Frisch). In cases where there is not complete correlation between the explanatory variables a, b, etc. are not completely uncertain, but to some extent uncertain (see § 57).

T. Koopmans has summarized the logic of multiple correlation analysis in the following elegant way.* Unconditional conclusions can only be negative: the rejection of some theory. For

* T. Koopmans, The Logic of Econometric Business Cycle Analysis, Journal of Political Economy, April 1941, p. 157.

positive conclusions additional information is needed. This information is subject to the principles of:

(a) statistical censorship: the additional information must not be contradictory to observation;
(b) scientific efficiency: it must not be a consequence of assumptions already made or of data already used;
(c) the solid base: it must be as plausible as possible;
(d) the sufficient base: it must be sufficient to draw the conclusions.

As a rule the following hypotheses will be made when constructing the relation to be tested. The influences *that are not included* in the relation are:

(i) accidental and small or
(ii) included in the trend term or
(iii) present only in individual years which are excluded from the analysis.

As to the *included* variables it will usually be assumed that:

(iv) these act in a given mathematical way, usually represented by a linear formula;
(v) the lags included are given or must at least be positive and
(vi) some of the regression coefficients have a given algebraic sign or a value within a given interval.

§ 57. Determination of Uncertainty in Results

In the above only the extreme cases are mentioned in which the correlation between x and x' is either absent or perfect and in which the correlation between y and z is moderate or perfect.

In reality various types of intermediary cases may be found. This means that it will only very seldom be possible to find exact values for a and b or to conclude that a and b are perfectly uncertain.

In most cases a and b will be only uncertain to some extent. Various methods have been developed for the judgement of the degree of uncertainty:

(A) R. A. Fisher assumed that the lack of complete correlation must be attributed to errors of measurement in x. Each set of observations may be considered as a sample drawn at random from a large population of possibilities: sometimes large errors in x will be made, sometimes smaller. The results will be uncertain because of the errors in the samples. If all the possible samples would be drawn, a frequency distribution of a and b would be found, characterized by a certain standard deviation which is a measure of the uncertainty in a and b. These standard deviations may be calculated provided that the following conditions are fulfilled:

(1) the values of the independent variables y and z are given and do not change from one sample to the other;
(2) the consecutive errors in x are mutually independent;
(3) their frequency distribution is normal.

At present we do not think so much of errors of measurement in x; far more important is the influence on x of the explanatory variables not included in the regression equation. This influence will usually be assumed to behave as an error of measurement.

(B) R. Frisch approached the problem from another angle. He developed his theory since Fisher's formula becomes inexact, if condition (3) is not rigorously fulfilled and if at the same time there is a high correlation within the set of "explanatory" variables: y, z, etc.

Frisch assumes that:

(1) every variable is composed of a systematic part and an error;

(2) there is perfect correlation between the systematic part of x at one side and of y, z, etc. at the other side;
(3) there is no correlation between any error component and the systematic ones;
(4) there is no correlation between the error components mutually.

He shows that under certain conditions the "true regression coefficients" (being the regression coefficients between the systematic parts) are between the regression coefficients one obtains when taking the first, second, third, etc. elementary regression equation and solving for x in the case of the second, third, etc.

By the second regression equation we mean the equation obtained if we try to approximate y by a linear combination of x, z, etc. and use the method of least squares.

One gets the impression of the limit of uncertainty by drawing the so-called "bunch maps" in which the values found for one and the same regression coefficient from the first, second, third, etc. elementary regression equation are plotted. If the limits do not differ very much, the results are considered highly trustworthy. If on the other hand they are rather different, it is difficult to make a decision as long as one does not know what relative weight should be given to the errors in the various variables.

(C) T. Koopmans, developing both theories and using some results obtained by Van Uven, distinguishes between errors of sampling and errors of weighting. He shows that sometimes the limits of Frisch's method give too unfavourable an impression of the degree of uncertainty.

Taking, e.g. the third regression means that the whole discrepancy between the observed and the calculated value of x is ascribed to errors in the variable z. If now the influence of z on x is only weak, this would come to assuming that the errors in z are very large as compared with the fluctuations in z itself. In many cases where z is rather ac-

curately known, this hypothesis may be rejected beforehand. This rejection is made automatically by Koopmans since he introduces a priori a hypothesis on the extreme values of errors to be accepted.

As we already observed, in most economic problems (though not in all) the errors of measurement are far less important than the errors introduced into the relation by the simple fact that the influence of a number of unknown explanatory variables is neglected. This means that as a rule the largest errors must be assumed in x and that they have the significance of the non-specified influences on x. If so, the use of the first elementary regression equation is most appropriate.

§ 58. Simultaneous Relations

All the foregoing methods are based on the hypothesis that there is only one relation between the variables studied.

T. Haavelmo directed attention to the fact that in almost every economic problem more than one relation must be assumed to exist between the variables under consideration. Price and quantity in one market, e.g. have to obey both the demand and the supply equation. In this case Fisher's assumption (1) is no longer fulfilled. This may be exemplified by the following simple case.

Let q be the quantity sold and bought in a certain market and p the price. Then the demand equation may be written as

$$q = \alpha p + u \tag{58.1}$$

and the supply equation as

$$q = \beta p + v \tag{58.2}$$

where u and v are the unexplained parts of q, comparable with Fisher's "errors."

The most appropriate statistical hypothesis to be made now is that u and v are two independent random variables with a

common two-dimensional frequency distribution. Each set of real observations represents one combined sample of u and v.

Solving for p and q we find

$$p = \frac{-u + v}{\alpha - \beta} \tag{58.3}$$

and

$$q = \frac{-\beta u + \alpha v}{\alpha - \beta} \tag{58.4}$$

From these equations we see that p is not independent of u which means that in repeated samples p cannot be assumed to be constant.

It is easily found that the most likely values of the regression coefficients a and b approximating α and β are now no longer found by applying the least squares method to each of the equations separately. On this basis important further methods and theorems have been developed by Haavelmo and others at the University of Chicago.*

In more complicated cases it is very useful to make a distinction between the simultaneous variables which have to be explained by the set of equations and the so-called "predetermined" variables which may also be found in the equations. Predetermined variables may be either data or lagged values of the simultaneous variables. Their common property is that they remain actually the same in repeated samples, since they do not depend on the random variables u and v, etc. In the systems of equations describing dynamic economic systems such lagged values of the variables will often appear among the explanatory variables in the equations.

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* For a very clear and simple summary of the essence of the method see T. Koopmans, Statistical Estimation of Simultaneous Economic Relations, Cowles Commission Papers, New Series, No. 11, Chicago 1945 (reprint from J. A. S. A.).

From the foregoing it will be clear that the new method will give the same results as the least squares method if:

(a) the right hand side of the relations contains only predetermined variables. This is always the case if the relations are given the so-called "reduced form," meaning that they have been solved for the simultaneous variables. In contradistinction, the original relations, representing the most direct economic connections between the variables, will be called the structural relations or equations.

In the case of a single market being described by a demand and a supply relation

$$q = \alpha p + \gamma d + u \tag{58.5}$$
$$q = \beta p + \delta s + v \tag{58.6}$$

where d represents the demand factors and s the supply factors, the reduced forms will be

$$p = \pi_1 s + \pi_2 d + \cdots \tag{58.7}$$
$$q = \varkappa_1 s + \varkappa_2 d + \cdots \tag{58.8}$$

These equations are sometimes described as the "price equation" and the "turn-over equation" or the "sales equation." The price equation expresses the price and the sales equation the quantity sold as a function of demand factors and supply factors only. In the price equation q does not appear; in the sales equation p does not appear. Statistical approximations of these latter equations therefore may be obtained by the old least squares method.

Sometimes it will be possible to determine, e.g. the demand relation from the price and the sales equation. Since the demand equation is a relation in which no term with s appears, we may obtain it if we eliminate s from (58.7) and (58.8). This is possible exactly only if there is one term with s. If there are several variables of s it will generally not be possible or only approximately be possible.

On the other hand, if the number of relations in the system is larger than the number of s's to be eliminated + 1, an infinity of results would be obtained, meaning that the demand equation cannot be determined.

The results obtained by the new simultaneous equation methods are not different either from the results of the least squares method if (b), the case studied by Bentzel and Wold,* presents itself.

(b) In this case the structure of the system of equations is such that the simultaneous variables x, y, z, etc. may be found in succession without the necessity of solving any simultaneous pair of relations. This will be the case if one equation contains only one of the variables, a second equation only one other aside from the one already mentioned, etc. Lagged variables may appear everywhere.

It is interesting to note that this particular type of system of equations just coincides with the most complicated system of equations accessible to literary economists; verbal solution of economic problems only being possible if this successive solution is possible.

(c) The last case to be mentioned where the new method yields the same results as the least squares methods is the case in which all correlations are complete. This means that the results obtained by the new method and those obtained by the old one will not diverge very much if good fits are obtained.

§ 59. Some Further Remarks on the Reduced Form Method

From the foregoing it is seen that the chief advantage of the reduced form method is that the least squares calculation may be applied which is much simpler than the simultaneous equations method.

* R. Bentzel and H. Wold, On Statistical Demand Analysis from the Viewpoint of Simultaneous Equations, Skandinavisk Aktuarietidskrift 1946, p. 95.

The chief disadvantage of the reduced form method is that the relations as a rule contain more terms than the structural relations and that not so much is known beforehand of the values of the coefficients.

It is therefore more difficult to test economically the results obtained. In the case of the demand equation it is certain beforehand that no supply factors have to occur in that relation; in the supply function no demand factor should occur; in the price or the sales equation, however, both types of factors are coming in.

Sometimes (but not always) the sign of some of the terms in the reduced form relations will not be known beforehand.

The reduced form equations are not the same as the "final equations" of a dynamic system. The final equation for y, e.g. expresses y as a function only of lagged values of y, whereas the reduced form equation for y includes lagged variables of any kind. In the final equation longer lags will be included. If the disturbances of each structural equation are nonautocorrelated, the reduced form will also not have autocorrelated disturbances but the disturbances in the final equation may be autocorrelated (J. Marschak).

§ 60. General Remarks on Application of Correlation Method

(1) Immeasurability.

A general objection often made against statistical analysis of economic relations which in fact applies to all quantitative formulations of economic relations, is the immeasurability of a number of economic phenomena. It must be admitted that there do exist immeasurable economic phenomena.

Of course statistical analysis can only be applied to measurable ones. It should not be forgotten, however, once we are to analyse the explanation of the fluctuations of some measurable phenomenon, that all influences exerted on such a phenomenon are, in principle, measurable. The influence, e.g. of "psychological factors" or of expectations

on the fluctuations of a price or of consumption are in principle measurable.

This does not mean that it is always easy to measure these influences. In practice they may sometimes be approximated by functions of lagged variables: the influence, e.g. of expectations may be represented by a function of past price increases.

(2) Variability of coefficients.

It is often maintained that the method of multiple correlation is too rigid since it assumes the regression coefficients to be constant. It is very easy, however, to replace those coefficients by functions of other variables which means that they will be variable.

Of course the variability of the coefficients should be systematic. Otherwise one may prove anything. Assuming variability of coefficients to be unsystematic would mean to renounce scientific analysis.

On the other hand it may be observed that experience supplies us with cases of a remarkable constancy of regression coefficients.

(3) Curvilinearity.

A further generalisation of our method consists of replacing the linear formulae by more general mathematical types. This means that instead of the linear relations curvilinear ones are introduced.

Mathematically this is not at all impossible. Experience shows, however, that in a majority of cases it is very hard to find systematic deviations from linearity.

(4) "Results too uncertain to be used."

Often results of statistical analysis are considered to be of little value since their uncertainty is rather large according to what standard deviations or bunch maps show us.

It should be kept in mind, however, that even then the most likely values of the regression coefficients are those found by the calculations. An explanation based upon such calculations still has the advantage of not being contradictory to observations, which is more than can be said of quite a number of theories.

(5) Multicollinearity with cyclical analysis.

As to the application of multiple correlation analysis to the explanation of cyclic movements the general objection is made that in this case the chance for multicollinearity is very high.

This would be true if the time shape of the cycles in various economic phenomena would be the same. During the period 1919–1932 in the U. S., however (to quote an example), the time shape of cycles in

(i) commodity prices,
(ii) stock prices,
(iii) production, and in
(iv) interest rates

were quite different and hence it was not so difficult to distinguish between the influences exerted by each of these variables on some other phenomenon.

(6) Problems solvable even in the case of multicollinearity.

Even if multicollinearity exists in the relation we are interested in, we may sometimes solve the problems put before us.

As a simple example the following may be quoted. The demand for butter may depend on income, price of butter, and price of margarine. The two latter variables are intercorrelated, meaning that we are not able to separate their influences. As long as the relation between butter prices and margarine prices is not changed, it will be possible,

however, to find what the influence of a government price policy on the demand for butter will be.

Another example: by extrapolating the theoretical values x' for some variable x into a period where a new additional cause for fluctuations in x has been added, it is sometimes possible to find what the influence of this additional cause has been.

This procedure too may be applied in cases of multicollinearity between the other factors determining x'. An example will be given later (see § 61).

(7) Importance of a correct economic analysis.

It cannot be emphasized enough that the first thing to be done in any particular application is to give a correct economic analysis of the relation to be investigated. The emphasis is the more necessary since many young investigators seem to overlook this.

Two things should be carefully kept in mind: first, the necessity to know exactly what relation one really is interested in and second to know what factors enter into this relation.

Sometimes investigators are themselves not very clear about the type of relation they investigate: whether a demand or a supply relation or perhaps a price or sales equation.

In many cases it pays better to have exact figures on one of the factors hitherto neglected than to use very elaborate statistical methods.

The number of papers in *Econometrica* devoted to the description of practical results is still too small in comparison to the number of papers dealing with theoretical subjects!

(8) Difference between short-run and long-run influences.

One of the important practical topics of the day is the question of the difference between reactions during the

business cycle and reactions likely to take a longer period.

During the business cycle the demand for durable goods, e.g. will show an income elasticity which is probably larger than the income elasticity in the long run which presumably may be the income elasticity to be found from family budget statistics. The demand for labour may depend on wage rates in at least two different ways. There will be a more or less immediate reaction in which the influence of wage rate on the degree of capitalisation does not play a very large role; there will also be a long-run influence of wage rates on the degree of capitalisation.

Important parts of our practical programs must be devoted to the analysis and distinction of these two types of dependency.

(9) Independency of consecutive observations.

Once it was believed that more exact results for the regression coefficients could be obtained by taking monthly figures instead of annual ones because of the larger number of observations.

What matters, however, is the number of independent observations. If the series considered are "inert," monthly data will not be new independent data.

For each statistical series there seems to be one minimum "natural" time unit, for which the independency of consecutive residuals is a fact. For crop figures, e.g. this time unit might be one year. Since random series show apparent cycles of three time units, one may even wonder whether or not the short American cycle or Kitchin may be explained by crop variations simply.

(10) Importance of having data for all relevant variables.

Multiple correlation analysis is impossible if the data are not available for all relevant variables to be included in the relations investigated. It is an illusion that one could obtain results without this condition being fulfilled. At-

tempts of earlier investigators to do without this necessary information have not as a rule succeeded.

A typical example may in my opinion be found in the statistical investigations quoted in Prof. Iversen's well-known book on international capital movements. The fluctuations in prices, incomes, the items of the balance of payments, etc. of the countries investigated will be the consequences not only of the capital movements that took place but also of other factors. One cannot hope to find the influence of capital movements on these phenomena without studying simultaneously the effects of these other factors.

This seems to explain why most of the authors quoted hardly succeeded in reaching conclusions, as is rightly pointed out by Prof. Iversen. It would be highly interesting if investigations along the lines of multiple correlation analysis could be substituted for the earlier attempts.

§ 61. Some Successful Examples

In recent decades a number of successful multiple correlation analyses have been published.

It is sufficient to quote the important analyses made of the demand relations for agricultural products in the United States and recently in other countries (in Sweden by Prof. Wold). Other important attempts deal with the demand for services and industrial products as well as the demand for groups of commodities such as food as a whole, consumption as a whole, and investments as a whole. The various recent investigations on the propensity to consume are examples in point. These results deserve particular attention if they form groups of consistent results.

A simple but interesting example seems to be the following. Various authors investigated the demand for tramway transport; for Dutch and Danish towns price elasticities of about 3/4 were found, whereas for Stockholm and Marseille an elasticity of about 1/4 resulted. The difference seems plausible because of

the severe competition of bicycles in the Dutch and Danish towns, where they are much more numerous than in the two other towns mentioned.

The results are not always in the values found for some regression coefficient. They may also be in the extrapolations made for a period in which one new factor acts. The difference between observed and extrapolated theoretical values may then be considered to be a measure for the influence of the new factors. In Figs. 18 to 21 this procedure is exemplified. An attempt is made to explain the fluctuations in imports into the Netherlands. Imports are subdivided into raw materials and finished products. Consumers' goods and investment goods are considered separately. Imports of raw materials as well as those of finished products are assumed to depend on:

(i) the volume of production,
(ii) the price level of finished goods in the Netherlands and
(iii) the price level of imported finished goods.

A fairly good fit is obtained for the period 1923–1933.

To start with 1934 a quota system was introduced. The extrapolated values show very important deviations from the observed ones. The deviations are different for raw materials and for finished goods: the imports of finished products fell short of expectations according to the formula, whereas the imports of raw materials surpassed the extrapolated values. This is quite natural: a quota system has the intention to stimulate home production at the expense of imports of finished products. The four figures therefore show consistent deviations and these deviations may be taken as a measure for the influence of the quota system.

§ 62. Conclusions of Political and Scientific Importance

In rare cases only will it be possible for scientific analysis—and hence also for multiple correlation analysis—to contribute directly to political decisions. In most cases the formulation of

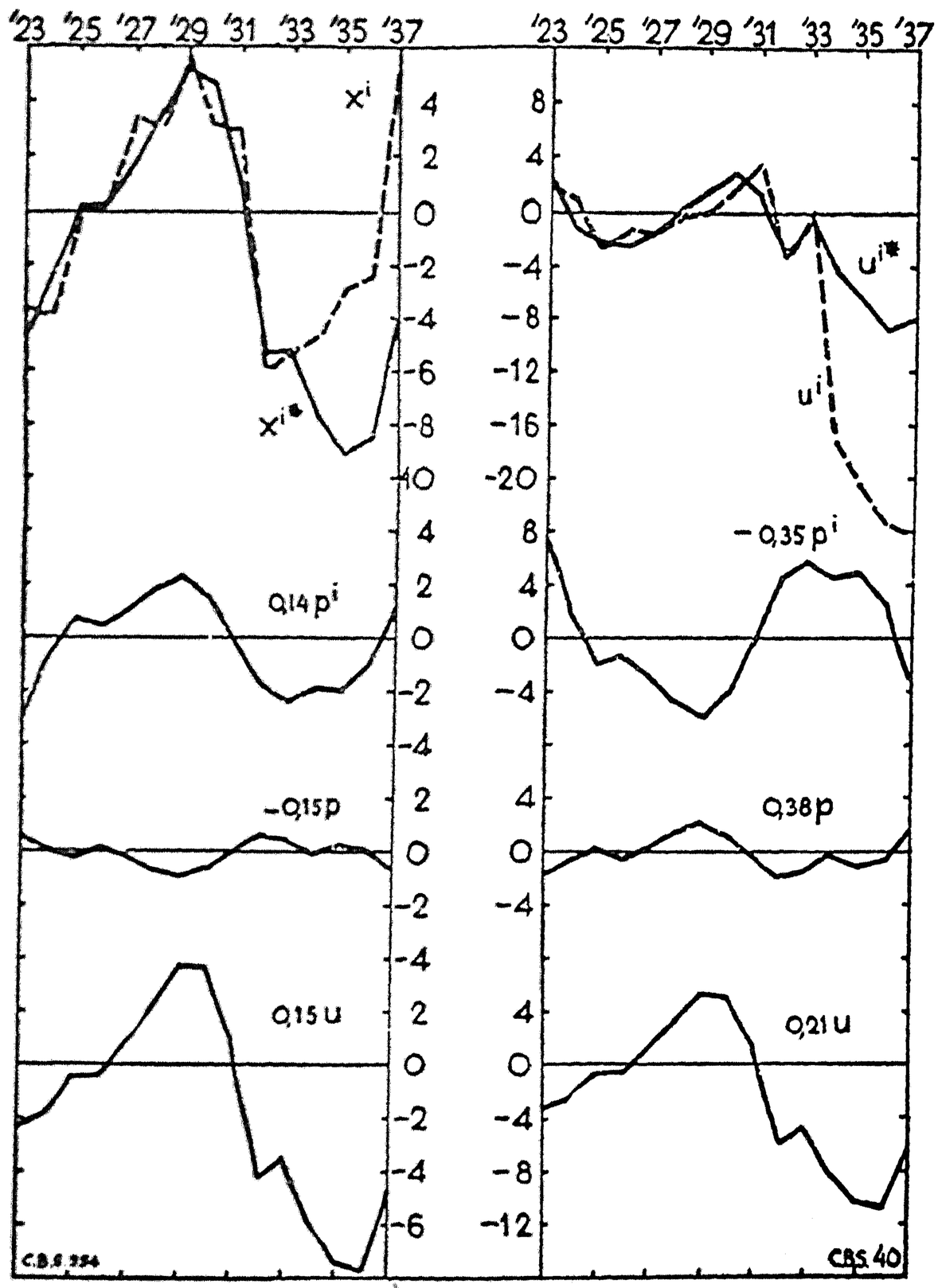

Fig. 18.

x^i	actual imports of raw materials for consumer goods
x^{i*}	calculated imports of raw materials for consumer goods $= 0.15\,u + 0.14\,p^i - 0.15\,p$
$0.15\,u$	influence of the volume of Dutch production of finished consumer goods
$0.14\,p^i$	influence of the price level of imported finished consumer goods.
$-0.15\,p$	influence of the price level of Dutch finished consumer goods

Fig. 19.

u^i	actual imports of finished consumer goods
u^{i*}	calculated imports of finished consumer goods $= 0.21\,u - 0.35\,p^i + 0.38\,p$
$0.21\,u$	influence of the volume of Dutch production of finished consumer goods
$-0.35\,p^i$	influence of the price level of imported finished consumer goods
$0.38\,p$	influence of the price level of Dutch finished consumer goods

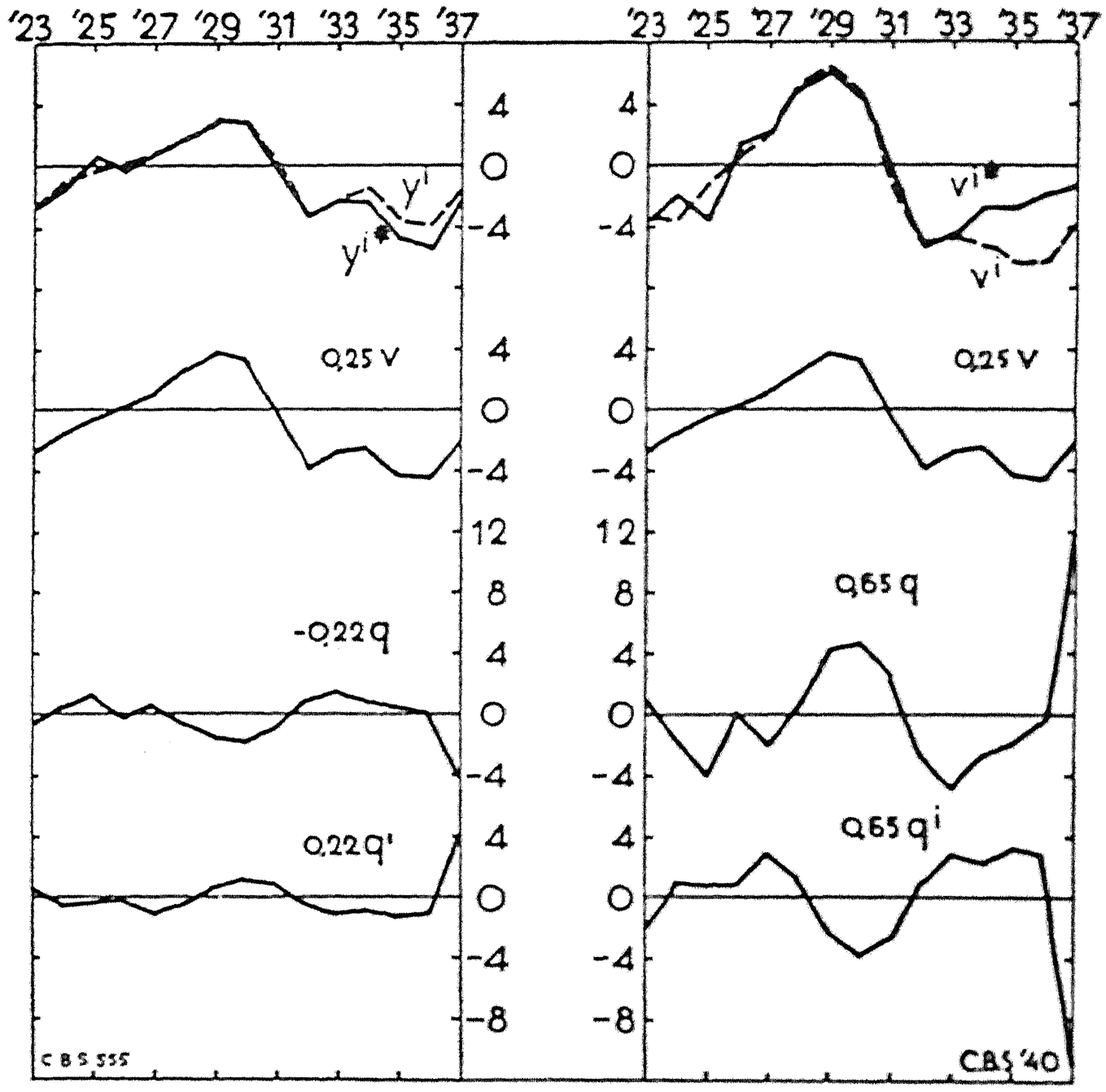

Fig. 20. Fig. 21.

y^i	actual imports of raw materials for investment goods	v^i	actual imports of finished investment goods
y^{i*}	calculated imports of raw materials for investment goods $= 0.25v - 0.22q + 0.22q^i$	v^{i*}	calculated imports of finished investment goods $= 0.25v + 0.65q - 0.65q^i$
$0.25v$	influence of the volume of Dutch production of investment goods	$0.25v$	influence of the volume of Dutch production of investment goods
$-0.22q$	influence of the price level of Dutch investment goods	$0.65q$	influence of the price level of Dutch investment goods
$+0.22q^i$	influence of the prices of imported finished investment goods	$-0.65q^i$	influence of the prices of imported finished investment goods

qualifications and the further consideration of practical complications will be necessary.

The indirect importance seems, however, to be clear. Examples of an almost direct bearing of the results of correlation analysis on practical policy are to be found in the field of agricultural policy. Price forecasts, the calculation of subsidies, etc. have often been based on correlation calculations. An important attempt outside the sphere of agricultural policy was the one to predict the employment level in the United States for the last quarter of 1945 which attempt has been so well analysed by Lawrence R. Klein.*

Cases of a more indirect practical importance may easily be found. The example represented in figures 19 to 21 was part of an attempt to give an appraisal of Dutch commercial policy.

Another example was an analysis made of the tin market† showing that the restriction of production had a considerable influence on the price of that product.

Many investigations upon the effect of devaluations after 1930 and the revalorisation of the Danish and Norwegian crowns in 1925 contributed very much to our knowledge and will be useful to the work of the International Monetary Fund.

Recent investigations of the influence of wage rates on the demand for labour will contribute to future business cycle policy.

Finally some recent results of correlation analysis may be quoted which for the time being are of scientific interest only but which may be of practical importance as well at some later stage. The rather low figure found for elasticities of substitution in international trade may lead to a better knowledge of the nature of the equilibrium in the balance of payments.

A study by Dr. Verdoorn‡ seems to show that the shift from variable to fixed costs was not very important during the last

* L. R. Klein, A Post-Mortem on Transition Predictions of National Product, Journal of Political Economy LIV (1946), p. 289.

† M. J. Schut, Tinrestrictie en Tinprijs, Haarlem 1940.

‡ P. J. Verdoorn, De Verstarring der Productiekosten, Haarlem 1943.

half century contrary to what was generally believed but in agreement with Prof. L. von Mises' point of view.

A closer consideration by Dr. Witteveen* of the price equations obtained so far seems to show that prices are not fixed by producers on the basis of marginal costs: possibly a support of the findings of the Oxford Institute of Statistics.†

* H. J. Witteveen, Loonshoogte en Werkgelegenheid, Haarlem 1947.

† R. L. Hall and C. J. Hitch, Price Policy and Business Behaviour, Oxford Economic Papers No. 2, May 1939, p. 12.

B.

Statistical Evidence on the Acceleration Principle*

§ 63. Theoretical Introduction

1. Much attention has recently been given to the so-called "acceleration principle" or "the Relation" as Mr. Harrod† calls it. Developed by Professors Aftalion, Bickerdike, Bouniatian, Carver, Fanno, Kuznets, and Pigou, it has been given special attention by Professor J. M. Clark in his splendid survey *Strategic Factors in Business Cycles;*‡ and has obtained a place in many theories of the business cycle. Professor Haberler,§ from whom part of the above references are taken, also uses it in his own synthesis of theories; Mr. Harrod regards the acceleration principle and the multiplier principle acting in combination as the chief forces in trade cycles. Quite recently Dr. Chait¶ has developed a theory implying as special cases both the acceleration principle and the multiplier principle.

2. The principle in its simplest form can be best seen in a simplified example.** Let the value of the yearly output of (say) shoes be 100. The cost of the fixed capital equipment required for this output may be 500, 10 per cent of which must be re-

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* Economica, V (May 1938), 164-176. Reprinted by the courtesy of the publisher.

† R. F. Harrod, The Trade Cycle. An Essay. Oxford, 1936.

‡ Cf. also J. M. Clark, "Business Acceleration and the Law of Demand," Journal of Political Economy, XXV, pp. 217-235. (1917.)

§ G. Haberler, "Prosperity and Depression," A Theoretical Analysis of Cyclical Movements. Geneva, 1937, League of Nations.

¶ B. Chait, Les fluctuations économiques et l'interdépendance des marchés. Brussels, 1938.

** Taken from Haberler, loc. cit., p. 84.

placed each year. Then, new machines at the cost of 50 must be constructed each year for replacement. Now, suppose the demand for shoes rises, so that production must be increased by 10 per cent to 110 a year. If there is no excess capacity and if methods of production are not changed, this increase necessitates an increase of 10 per cent in the stock of fixed capital; that is, an additional production of machinery of 50, being an increase of 100 per cent. In this simplest form the principle states that percentage changes in the *production* of consumers' goods are equal to percentage changes in the *stock* of capital goods. As the latter is usually considerably larger than the annual *production* of capital goods, the corresponding percentage changes in the latter are much larger than the percentage changes in the production of consumers' goods. The principle has two aspects between which it is useful to distinguish:

(a) the correlation aspect: there must be correlation between new investment in durable capital goods and the *rate of increase* in consumers' goods production;

(b) the regression aspect: the percentage fluctuations in consumers' goods production are *equal* to the percentage fluctuations in the stock of capital goods. The first aspect states something about the correlation coefficient between two variables; the second aspect about the numerical value of the regression coefficient.

3. There is some doubt about the causal direction of the relation mentioned. Sometimes causality is not considered at all; the principle is only meant to explain the difference in intensity between consumers' goods production and producers' goods production. In other cases, however, the principle is meant to explain investment fluctuations. Changes in consumption are then taken as given; fluctuations in investment activity are deduced. In any case this seems to me the more interesting aspect of the matter and, at the same time, the kind of relationship we need for business cycle theory.

4. In its more rigorous form, the acceleration principle can only be true if the following conditions are fulfilled.

(a) Very strong decreases in consumers' goods production must not occur. If the principle were right, they would lead to a corresponding disinvestment and this can only take place to the extent of replacement. If annual replacement amounts to 10 per cent of the stock of capital goods, then a larger decrease in this stock than 10 per cent per annum is impossible. A decrease in consumers' goods production of 15 per cent could not lead to a 15 per cent decrease in physical capital as the acceleration principle would require. It is interesting that this limit is the sharper the greater the duration of life of the capital goods considered.

(b) There should be no abrupt changes in technique leading to a sudden increase in the amount of capital goods necessary to the production of one unit of consumers' goods.

In its more rigorous form, the principle is equivalent to saying that a constant part of productive capacity is idle and that entrepreneurs never increase production of consumers' goods before having increased correspondingly their capacity. In the case of this constant part being zero—i.e. full occupation of capacity at any moment—this is, at least for increases, a necessity; in all other cases this policy would have to be followed deliberately, and there are hardly enough reasons to suppose this occurs in reality.

In particular there are reasons to expect the principle to break down in times of heavy decreases in production [see what was said under (a)] as well as in the periods immediately following such decreases, until production has reached capacity again. It is only at that moment that the *necessity* of the principle's action recurs.

5. The acceleration principle may, however, be given a less rigorous form. Instead of equality of percentage changes in consumers' goods production and capital goods' stock there may

be assumed to be only proportionality or even only a linear relationship. This means that there would still be correlation but that the regression coefficient would be less than one. Two reasons exist for giving the principle its less rigorous form:

(a) During a period of increasing production, not all firms and not all branches attain at the same moment the point of full capacity. Suppose that for the individual firms the principle acts only—and then of course in its rigorous form—when full capacity is reached; then to a given increase in total production for all firms a smaller percentage increase in total stock of capital goods may correspond. Something similar may happen with heavy decreases in production: some firms may show decreases going beyond the critical one mentioned in 4 (a), others may remain within the critical boundary. Supposing that in such times all firms simply do not replace worn-out capacity, there will again be a percentage decrease in physical capital stock which is smaller than the percentage decrease in consumers' goods production. (This statement holds, of course, equally well for any individual firm going beyond the critical decrease.)

Of course, it is not at all certain in these circumstances that the regression coefficient will have to be the same for every moment. Much depends on the distribution of decreases over firms and on the distribution of idle capacity over firms. But there will be a tendency to a regression coefficient below one. It may be added already here that in fact the material used by Professor Clark in his first exposition of the principle showed this very clearly; the regression coefficient being about one-half instead of one. It may also be added that the magnifying effect may, nevertheless, still exist. If the rigorous acceleration principle would lead to percentage fluctuations in capital goods production ten times as large as those in consumers' goods production, then in the less rigorous form these percentage fluctuations would still be five times as great.

(b) A second reason for the less rigorous form of the acceleration principle might be that even with idle capacity a firm would expand its plant proportionally to the rate of increase

in consumers' goods production, but not by an equal percentage. This would mean that there would not be an immediate necessity for investment but that the willingness to invest would depend chiefly on the rate of increase in consumers' goods production.

It seems important to remind the reader that we have been considering three situations in which a firm may find itself:

(i) full capacity used and increasing production;
(ii) heavily decreasing production, as defined under 4 (a);
(iii) all other situations.

If the less rigorous acceleration principle were caused only in the way indicated under 5 (a), at least part of the firms should be in the situations (i) and (ii) during most of the period studied. There is reason to doubt whether this is so for the post-war period or even for many pre-war years. If this doubt is justified. the only way in which the principle could still be fulfilled would be the one indicated under 5 (b). The behaviour of entrepreneurs assumed there seems, however, less natural than the behaviour assumed under 5 (a). Therefore I think there must be already for theoretical reasons some doubt as to the validity of the acceleration principle. In my opinion an explanation of investment fluctuations by profit fluctuations* is more natural, and for this reason a comparison between these two competing explanations has been made in some of the cases considered. It would, however, lead us too far to go into a detailed consideration of the relationship between profits and investment.

6. One further amendment of the acceleration principle may be mentioned for reasons of completeness. Dr. Staehle, in a private discussion, suggested that the correlation would exist between new investment activity and the *shift in the demand curve* for consumers' goods instead of the increase in the actual quantity demanded or produced. This seems indeed more natural especially in the following situation: suppose that, with

* To be called shortly "the profit principle."

full capacity used, demand increases, but as productive capacity cannot be expanded immediately, the quantities produced cannot rise correspondingly. Prices will rise instead. Nevertheless it is natural to assume that there will be new investment. Dr. Staehle's hypothesis can be given an especially convenient form if the elasticity of demand is unity (which, approximately, will be the case for "all consumption"): then the shift in demand curve is simply equal to the change in the money value of consumption or consumption outlay. Therefore his device would simply be equivalent to saying that new investment is correlated with the rate of increase in consumption outlay.

§ 64. Statistical Verification

7. Depending on the statistical material available, in some cases both aspects—the correlation and the regression aspect—have been tested, whereas in other cases only the correlation aspect could be considered. The regression aspect can only be investigated if figures on the stock of physical capital goods are available and this is only the case for individual branches. The following industries have been taken:

(a) Railways: United Kingdom, pre-war, Fig. 22. France and Germany, pre-war, Fig. 23.
(b) Cotton spinning: United Kingdom, pre-war, Fig. 24.
(c) Shipping: United Kingdom, pre-war, Fig. 25.

As, however, for business cycle theories the behaviour of industry as a whole is of considerable interest, general indices have also been considered. Here the only possibility seems to be comparing the *production* index of investment goods or durable goods with the rate of increase in the index of consumers' goods production. In many respects this is a less reliable comparison, as production of investment goods does not necessarily—although probably—fluctuate parallel to new investment. Replacement, which is included also, may show different fluctuations. It will, however, usually show less pronounced changes.

8. The railway charts have been drawn according to the following principle. The dotted line always represents the deviations from trend (nine-year moving averages) of the percentage rate of increase in rolling stock. For each of the countries it has been drawn twice; it has been compared with a full curve which has everywhere been drawn at such a scale as to give the best fit. The upper full curve represents the fluctuations in the rate of increase of traffic, the lower full curve the profit rate (both as deviations from their nine-year moving averages). The upper graph therefore gives "the explanation of new investment by the acceleration principle," the lower one "the explanation of new investment by the profit principle." The lag of new investment has been taken 1½ year, except for the profit principle for Germany, where it is 1 year. These lags seem to correspond fairly well to the lags in the curves, and, what is more important, they are about equal to the lag between orders and deliveries found for American figures.

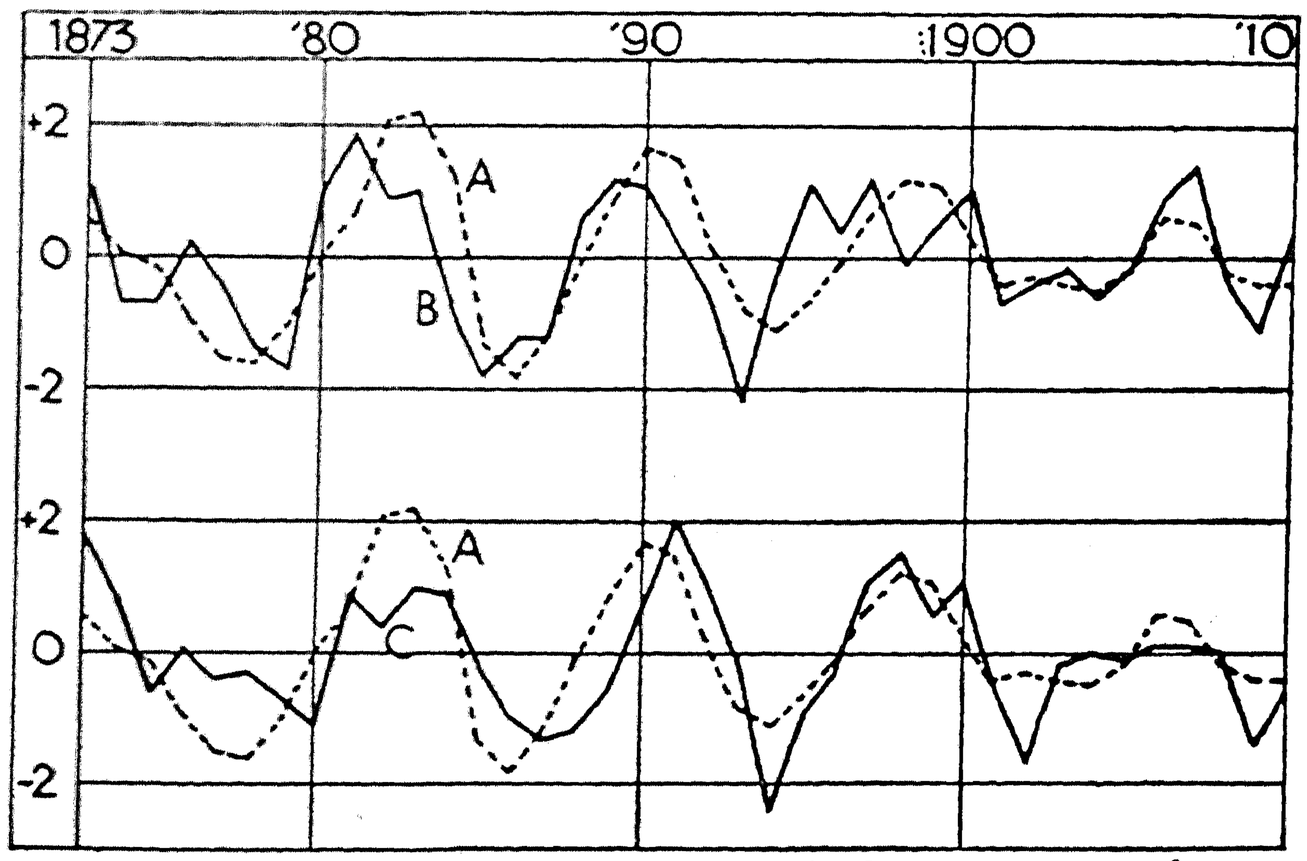

Fig. 22. RAILWAYS, UNITED KINGDOM. A, Percentage rate of increase in rolling stock (average of locomotives and cars weighted roughly according to their average prices in a base period). B, Percentage rate of increase in traffic, 1½ year before, drawn on such a scale as to yield the best fit with A. C, Profit rate, 1½ year before, drawn as B.

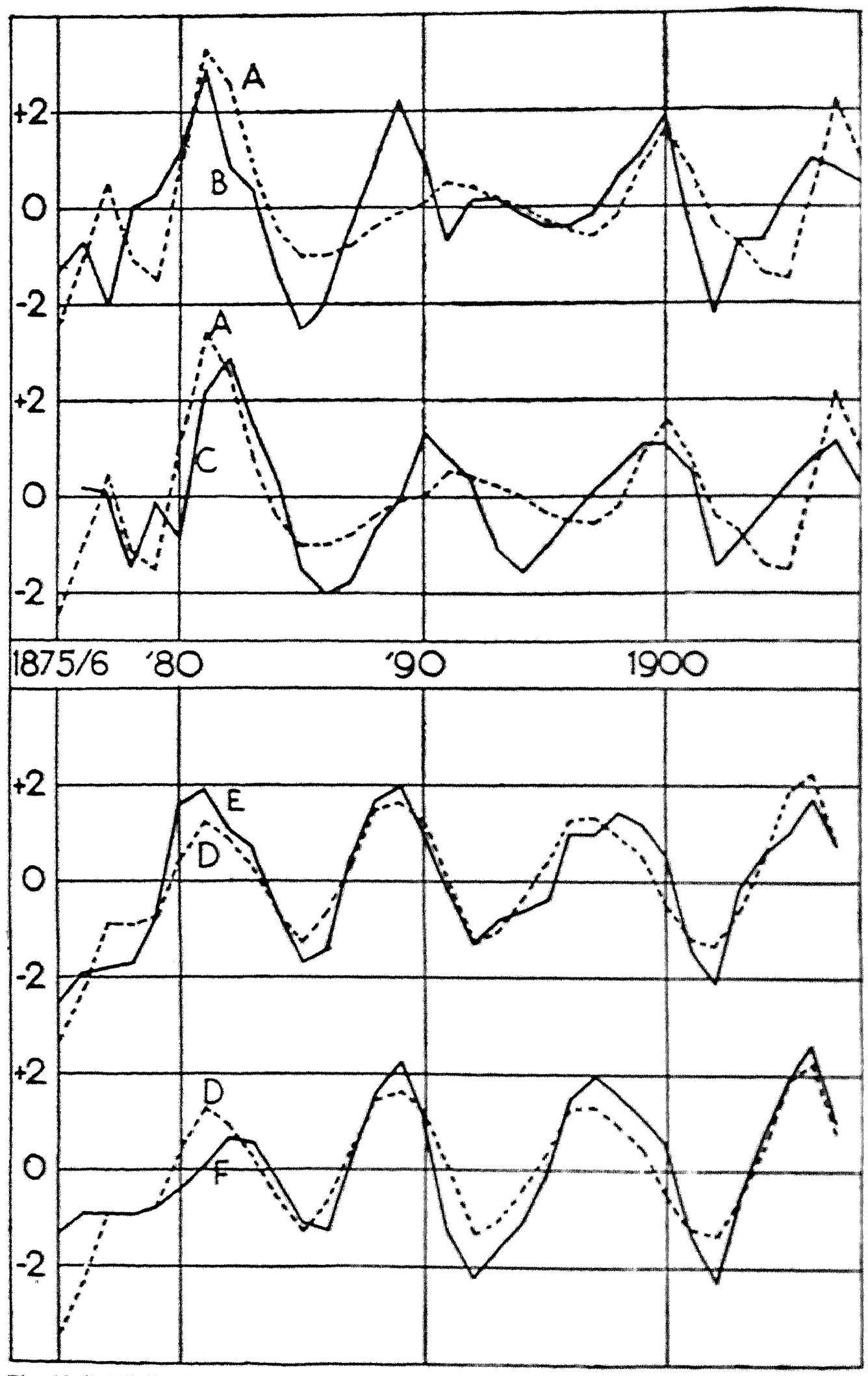

Fig. 23. RAILWAYS, FRANCE (A, B, C) AND GERMANY (D, E, F). A, D, Percentage rate of increase in rolling stock. B, E, Percentage rate of increase in traffic (B, 1½ year before; E, 2½ year before), drawn as B in chart 1. C, F, Profit rate (C, 1½ year before; F, 1 year before), drawn as B.

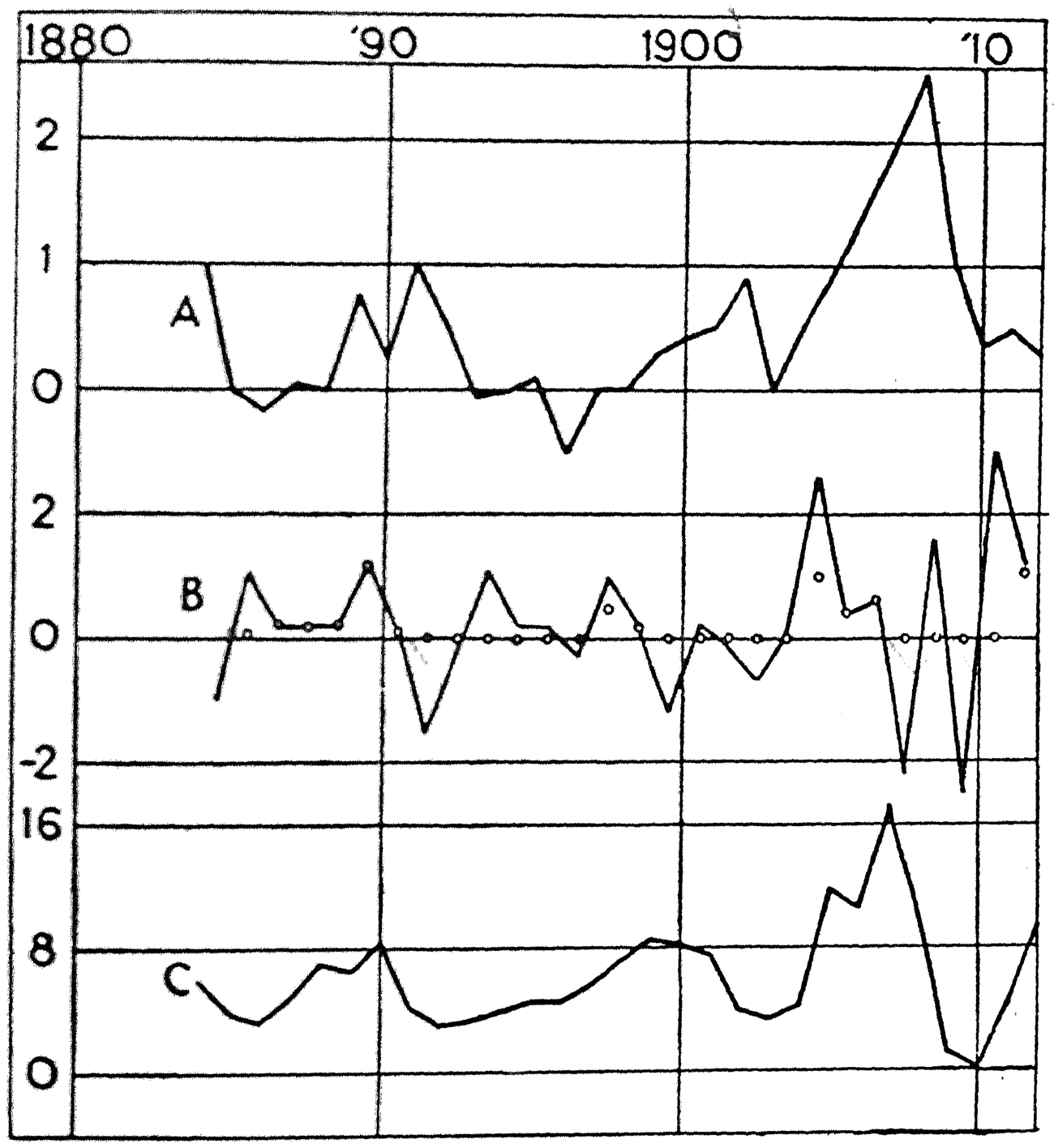

Fig. 24. SPINNING, UNITED KINGDOM. A, Net increase in number of spindles (millions). B, Rate of increase in cotton consumption (million cwt). O, "Corrected curve," i.e. (i) every negative figure replaced by zero, (ii) the subsequent positive figures replaced by zero as long as former maximum in consumption not yet reached again, (iii) the first figure after the former maximum has been surpassed replaced by difference between new maximum and former maximum. C, Average spinning profits (£1,000.–). Figures taken from D. H. Robertson, A Study of Industrial Fluctuations.

The scale chosen for the full line in the upper figures depends on the regression coefficient found for the percentage rate of increase in traffic. According to the acceleration principle in its rigorous form it should be equal to 1; it was, however, found to be for:

United Kingdom	0.53
France	0.44
Germany	0.48

These figures illustrate at the same time what has been called the regression aspect of the principle. Evidently it only agrees with the less rigorous form given.

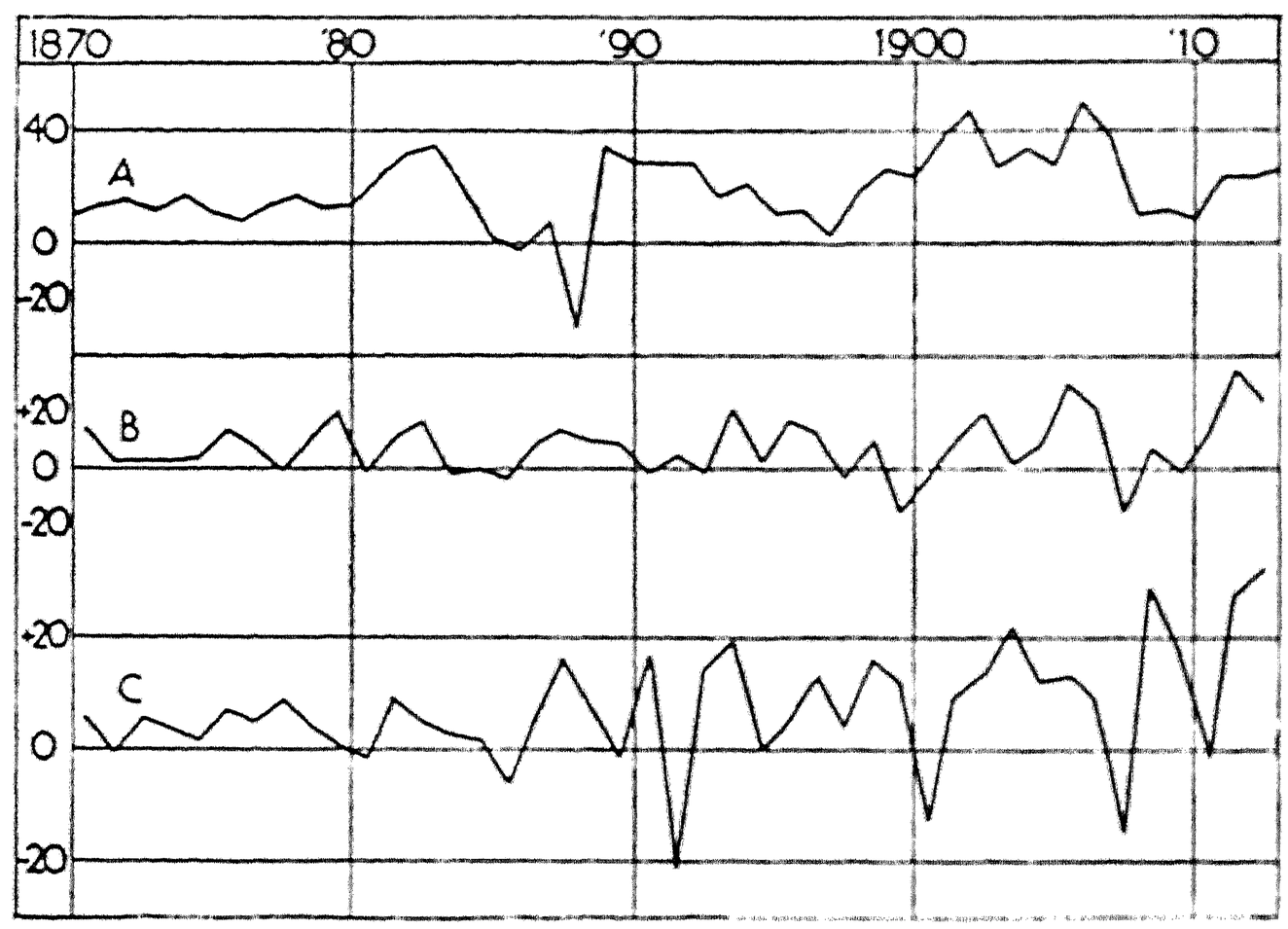

Fig. 25. Shipping, United Kingdom. A, Net rate of increase in total tonnage, 10,000 tons (sailing tons counted for one-third). B, Net rate of increase in total British tonnage entered, million tons. C, Rate of increase in "transport index," indicating ton-miles of transport for chief sea transport commodities (cereals, coal, wood, oil, nitrate). Taken from De Nederlandsche Conjunctuur, March, 1934.

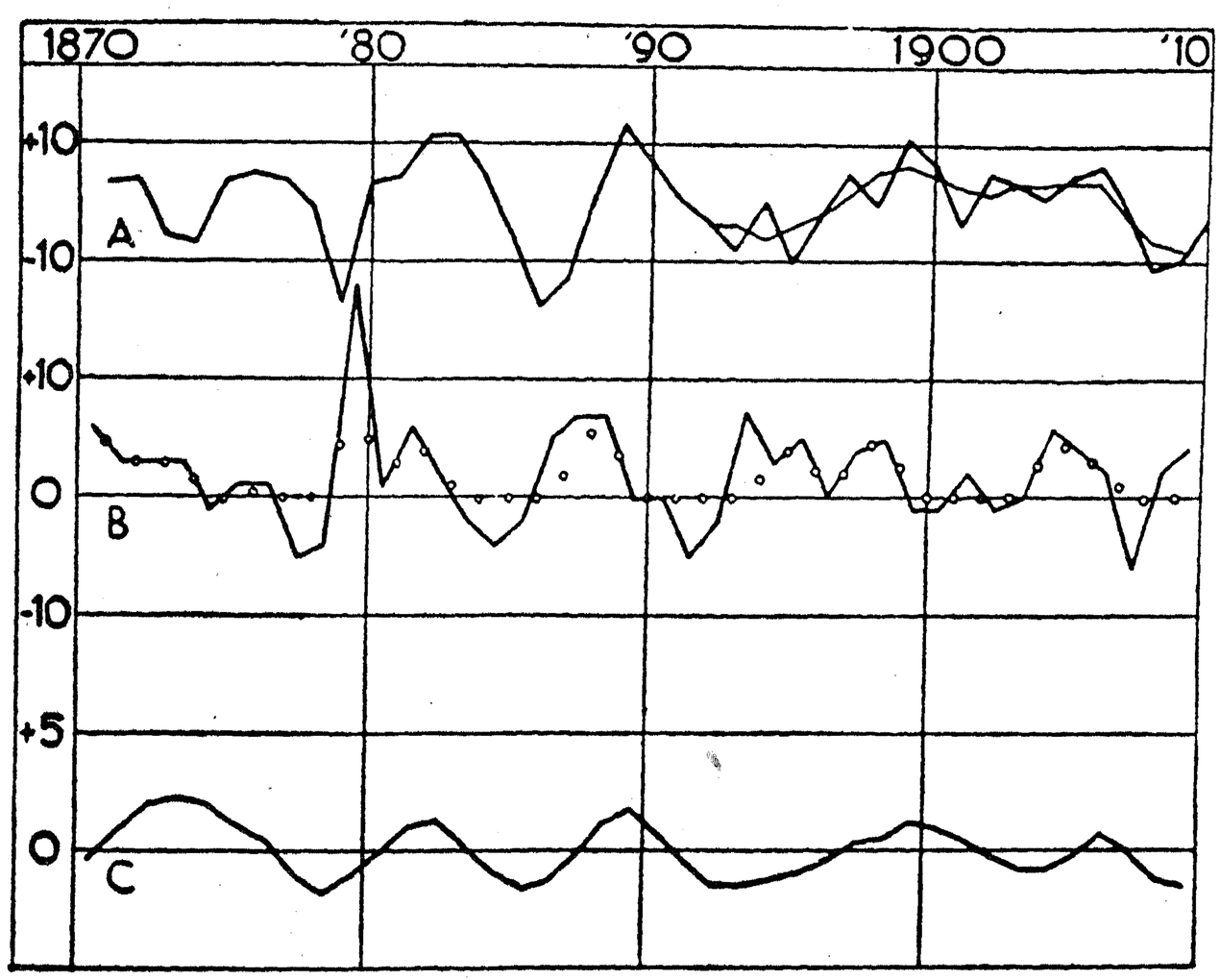

Fig. 26. General Activity, United Kingdom. A, Consumption of pig iron, percentage deviations from trend (nine-year moving average); taken account of pig iron in steel, of changes in stocks and after deduction of iron in exports. B, Rate of increase in industrial production (O, corrected figures, see Fig. 24, two-year moving averages); Hoffmann's index. C, Non-labor income, deviations from trend (Stamp's figures for taxed income).

As to the correlation aspect, it is immediately seen from the charts that the correlation is only high for the case of Germany (as it was for the U.S.A. during the period studied by Professor Clark*); for the U.K. and France, however, it is rather low. But the correlations obtained for the profit principle are by no means higher.

* It was not in the period studied by S. Kuznets: "Relation between Capital Goods and Finished Products in the Business Cycle," Economic Essays in Honor of W. C. Mitchell, New York 1935, p. 209.

It has been tried to improve the correlation by a correction, consisting in two sorts of changes in the figures for the rate of increase in traffic:

(i) putting equal to zero any negative number and the positive numbers following it, as long as the last maximum in traffic has not yet been reached;

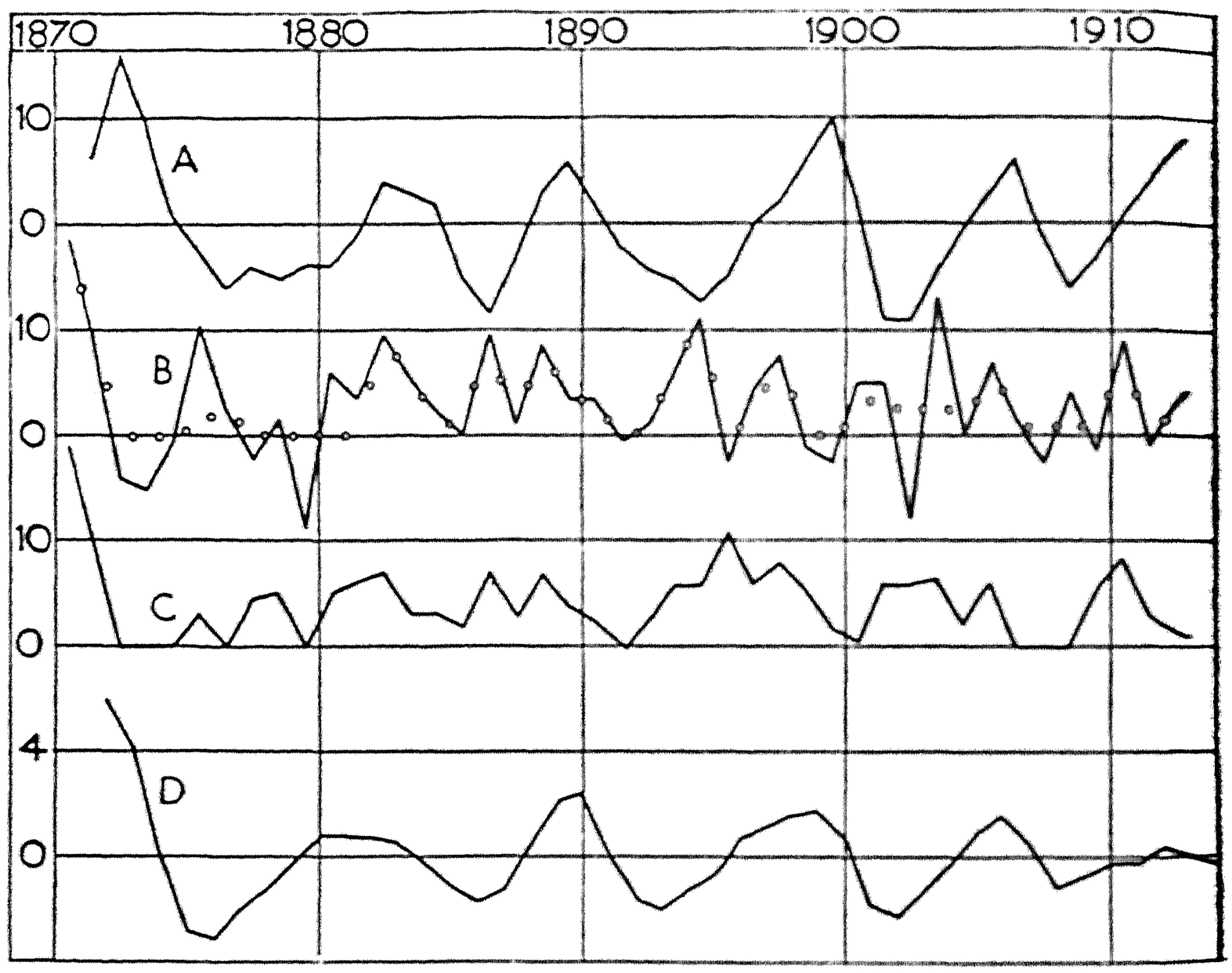

Fig. 27. General Activity, Germany. A, Consumption of pig iron, percentage deviations from nine-year moving average, two-year moving average; taken account of iron in steel and after deduction of iron in exports. B, Percentage rate of increase in industrial consumers' goods production (O, corrected figures, see Fig. 24, two-year moving averages). C, Percentage rate of increase in total industrial production, corrected figures, see Fig. 24. D, Dividends of all societies, percentage of capital, deviations from nine-year moving averages.

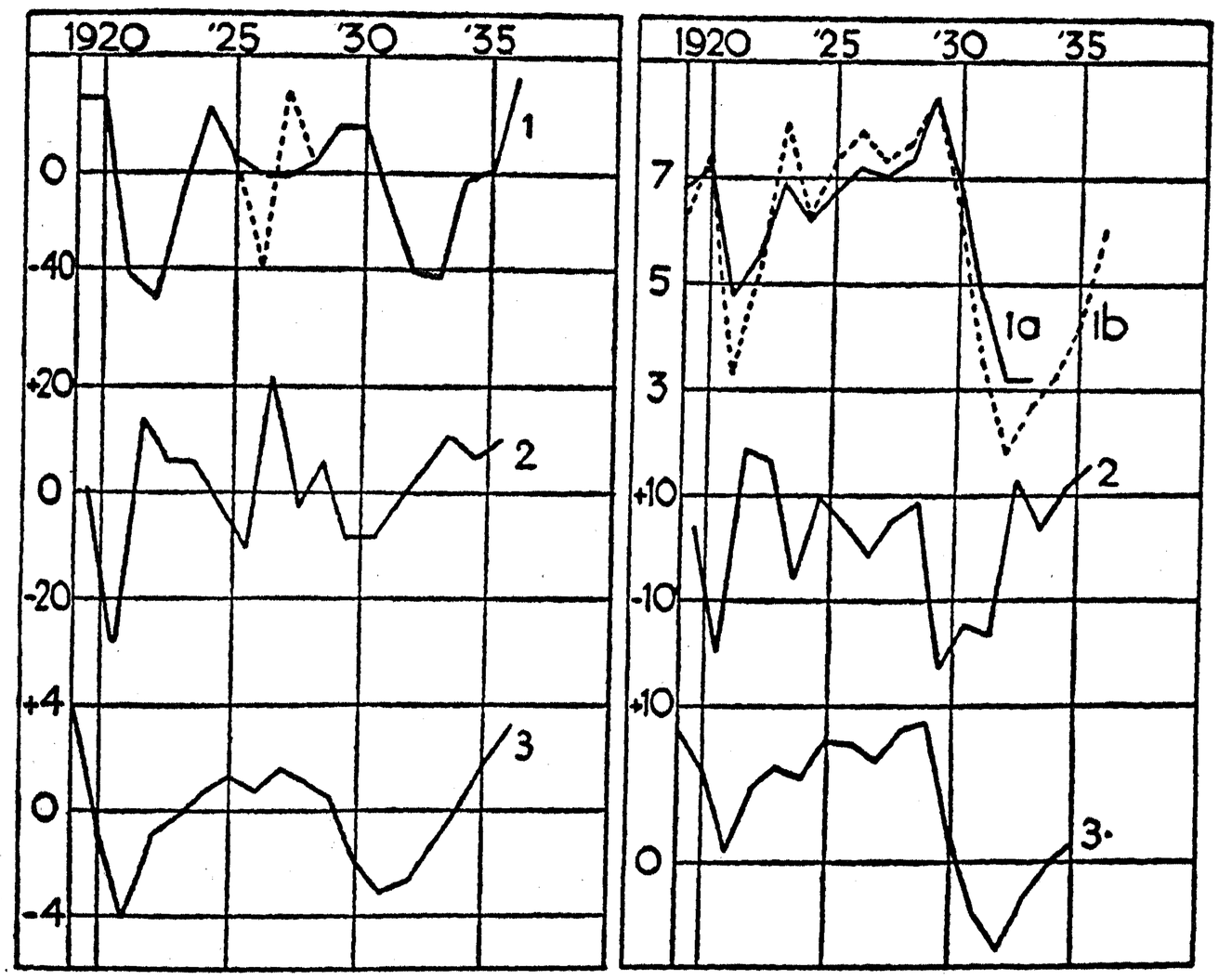

Fig. 28. General Activity, U.K. and U.S.A. U.K.: 1. Consumption of iron, percentage deviation from rectilinear trend; taken account of iron in steel, iron in exports deducted. . . . . actual figures 1926 and 1927; – – – "corrected" figures (corrected for coal miners' strike). 2. Rate of increase in industrial production, 1929 = 100; before 1928 taken from Hoffmann, since from B.o.T. 3. Profits, percentage of capital, deviations from rectilinear trend; Economist. U.S.A.: 1a. Deliveries of producers' durable commodities, Kuznets, 10^9\$ of 1929. 1b. Pig iron production in five million long tons. 2. Rate of increase in industrial production, F.R.B, 1923–5 = 100. 3. Profits of all corporations, 10^9\$, Stat. of Income.

(ii) in those years where a previous maximum in traffic is passed again the rate of increase in traffic is reduced to the extent the new maximum surpasses the previous one.

This correction evidently corresponds to the difficulties mentioned under 4 (a).

In the case of railways, however, it works out very little for pre-war times, as years of decrease were seldom. In some of the other cases considered the influence of this correction is far larger.

9. In all further cases the curves have been given separately, the first always representing the investment series, the next curve (or, for Figs. 25 and 27, two curves) the rate of increase in production (of the goods produced with the help of the investment goods considered) and the lower curve the profits in the industry considered. An exact description has been given with the figures. Sometimes the curve in the middle has been corrected in the way already described; the result is indicated by small circles representing two-year moving averages of the corrected curve, in order to eliminate small and probably meaningless fluctuations.

For cotton spinning and shipping the acceleration principle breaks down completely. No correlation of any importance exists. For cotton spinning, the profit principle works. For shipping no satisfactory profit figures could be found.

Considering the four general cases, where, however, the figures for net investment are not available in any form, I think they are all rather bad for the acceleration principle. One could doubt in the case of the United Kingdom (pre-war), Fig. 26, but even there the explanation by the profit principle certainly gives better results; and in all cases the fit obtained with the acceleration principle would certainly be poor. One has, of course, to be careful with the interpretation of the correlation between profits and investment, which could as well be such that investment determines profits, instead of the other way round. Then the mere fact that their mutual correlation is better than the one required by the acceleration principle would not prove much. There are, however, reasons for believing in the interpretation given* and then it seems that a better ex-

* These will be given in the report on the League of Nations investigations into the subject.

planation of investment is possible than the poor one yielded by the acceleration principle.

10. Summarising, it may be said that the acceleration principle cannot help very much in the explanation of the details in real investment fluctuations, with the possible exception of railway rolling stock.* As a rather rough principle with the chief object of explaining the tendency to more intense fluctuations in durable goods production it remains of value; but that tendency seems to be half as large as would be expected.

* Mr. D. H. Robertson draws my attention to the fact that railways have to carry any freight offered and for that reason might react more exactly in the sense of the acceleration principle.

C.

Long-Term Foreign Trade Elasticities*

§ 65. Importance of Long-Term Elasticities of Imports and Exports

The elasticity of the volume of imports or exports of any one country with respect to its price level may be either conceived of as a short-run elasticity or as a long-run elasticity. In the first case we think of the reaction of that volume which materialises within some short period, say one year; in the second case we think of the total reaction during a long period. From this it follows, however, that these concepts can only be exact if we indicate the periods. And this aspect of the phenomenon is only fully dealt with if we conceive of the volume x_t in question (imports or exports) during time unit t as a function of price levels during a whole range of preceding time units t, $t-1$, $t-2$, $t-3$, etc. In principle this range should be infinitely long:

$$x_t = \epsilon_0 p_t + \epsilon_1 p_{t-1} + \epsilon_2 p_{t-2} + \cdots\cdots = \sum_0^\infty {}_i\, \epsilon_i p_{t-i} \qquad (65.1)$$

In this case, the short-run elasticity could be defined as:

$$\epsilon_S = \epsilon_0 \frac{\bar{p}}{\bar{x}}$$

if $\bar{x}$ and $\bar{p}$ are the particular values of x and p for which we want to know the elasticity; and the long-run elasticity as:

$$\epsilon_L = \frac{\bar{p}}{\bar{x}} \sum_0^\infty {}_i\, \epsilon_i$$

* Metroeconomica, I (December 1949), 174-185. Reprinted by the courtesy of the publisher.

The reasons for the lagged reactions included in our equation (65.1) may be, e.g. that a certain change in price level induces a change in some production process, but only after certain machines now in operation are worn out. Or it may take some time before a switch in imports or exports is effectuated since every switch involves certain costs, which one is only prepared to incur, if the change in price level is considered to be a lasting one, not only a temporary one.

It will be remembered that our equation (65.1) represents a case of what Irving Fisher introduced as a distributed lag.

It seems useful to distinguish between short-term (however defined) and long-term elasticities of foreign trade. It may be useful in many other instances also; our special interest for foreign trade elasticities may be made clear by reminding the reader of some present-day problems for which they are of especial importance. The main problem for Western European countries nowadays is how to balance their foreign accounts. One of the methods used recently has been devaluation; other ones may be wage reductions or increases in productivity. In any of these cases the degree of success will depend on the reaction of imports and exports of the country concerned on the price change provoked. It is a well-known fact that success may even be completely absent if the sum of the elasticities of imports and exports is about one only. Recent statistical investigations* have made it plausible that these elasticities are in fact in many cases as low as that. The consequence would be that not very much could be expected from these methods to

* Hinshaw, R., American Prosperity and the British Balance of Payments Problem, The Review of Economic Statistics, XXVII, 1945; J. Tinbergen: Some measurements of elasticities of substitution, The Review of Economic Statistics, XXVIII, 1946; Tse Chun Chang, International Comparison of Demand for Imports, The Review of Economic Studies, XIII, 1945-46; The British Demand for Imports in the Inter-War Period, The Economic Journal, LVI, 1946; The British Balance of Payments 1924-1938, The Economic Journal, LVII, 1947; A Statistical Note on World Demand for Exports, The Review of Economics and Statistics, XXX, 1948.

equilibrate the balance of payments. The implications of this consequence are far-reaching. They might mean that only by permanent quantitative import restrictions equilibrium may be maintained. Before drawing this conclusion we must, however, ask whether the elasticities as found by the statisticians just quoted are not only short-run elasticities and whether long-run elasticities are not higher. To this question this paper is devoted.

§ 66. Long-Term vs. Short-Term Elasticities

The measurement of elasticities as performed by the authors just mentioned is based on multiple correlation analysis of the year-to-year fluctuations in imports or exports as dependent variables. The independent variables used in the explanation of these fluctuations are, in most cases, some index of home prices (or export prices), some index of foreign prices, and some index of national income of the importing country. Sometimes more variables are included, one of them being an indicator of quantitative restrictions applied. No attempt has so far been made, however, to include lagged values of the price series and hence the measurements must essentially be seen as attempts to measure the short-run value of elasticities. In some cases, where the correlation is very strong—as for a number of separate, well-defined commodities such as wheat, cotton or pig-iron—it may be seen from the figures, however, that the short-term influence explains the fluctuations to a great extent and that influences with small lags, say up to three years, cannot be very large. If they had been so, the correlation would have been a lagged one, easy to detect from the graphs. Much longer lags should evidently be introduced here, in order to get total elasticities much in excess of the figures reported by the authors.

Nevertheless it is probable that the long-run elasticities are higher, and perhaps much higher, than the short-run elasticities. There are various reasons for believing this. A number of the factors making for a relatively low elasticity work only tempo-

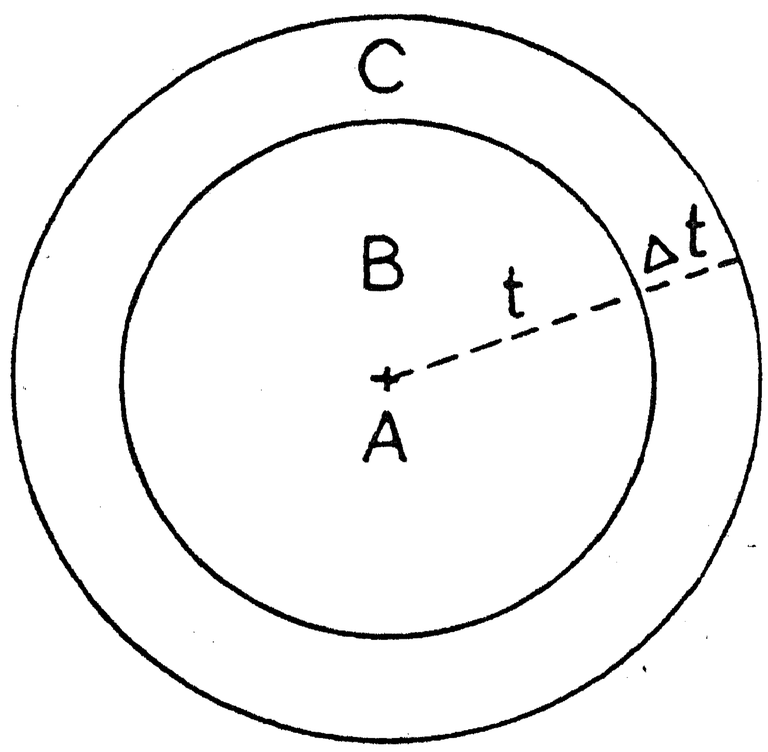

Fig. 29.

rarily. Existing contracts may prevent an immediate shift of demand to the cheapest supplier; after the contracts have expired this factor no longer works. Ties of a psychological or political character may hinder the shift. But after some psychological inertia has been overcome, this may change also. The shift may require the introduction of new machinery—as we already stated—this will therefore last until the old machinery has sufficiently been written off. A switch always causes costs of information, etc. It will not be undertaken therefore, as has been already observed, if the price change is considered a temporary one. But as time proceeds and the price change happens to be a lasting one, there will come a moment where also this cost of switching will no longer work as an obstacle.

Hence there are many reasons for expecting a lagged reaction, surpassing the immediate reaction. There remain, however, factors which are working also in the long run to prevent a complete shift to the cheapest supplier. And it may sometimes be possible to make a priori estimates of the long-term elasticity based on the nature of these factors. The simplest example is that of *distances,* causing differences in *transport costs.*

The existence of distances puts a maximum to the increase in demand caused by a given decrease in prices. Only the customers for whom under the new circumstances extra transport costs will be lower than the price reduction effected will be attracted. This may be illustrated by Fig. 29. Assume that the customers of a small country A are evenly spread over the circle

B. Potential new customers are supposed to be spread with the same density outside that circle. If now a price reduction Δp is applied, the customers in a ring C, of width Δt, will be attracted if transport costs over that distance are equal to Δp. What is the resulting elasticity of demand? Evidently it is equal to $\epsilon = \frac{\Delta x}{\Delta p} \frac{p}{x}$, where

$$\frac{\Delta x}{x} = \frac{\text{surface of ring}}{\text{surface of circle}} = \frac{2\pi t \Delta t}{\pi t^2} = \frac{2\Delta t}{t} = \frac{2\Delta p}{t}$$

Hence $\epsilon = 2 \frac{p}{t}$.

Now if we know something about $\frac{t}{p}$, i.e. the ratio of transport costs to total price, we are able to estimate the elasticity under the simplified conditions assumed. For such bulk goods as wheat and pig iron, transport costs between, e.g. South America and the European coast are of the order of magnitude of 10 per cent of the price. In such a case we would have $\epsilon = 20$. For piece goods the ratio of transport costs to price will probably be lower, leading to even higher elasticities.

There are, however, other factors making for inelasticity even in the long run. One may be *differences in qualities* due to inherent differences in soil, climate or technical skill of the population. These differences are not only important for separate commodities but even much more so for the whole of a country's "export basket." Its composition can only be changed gradually and within certain limits and this is probably the most powerful reason for long-term elasticities to be lower than the figure just quoted as an example. The differences in quality may take the special form of technical risks, i.e. the risk of quality divergencies or the risk of interruption of supply.

Another major class of factors is that of *political factors*. Regardless of distances or other purely technical and economic reasons for cheapness a country's exports may be in demand or not in demand if that country is or is not a part of a political

group. The most outstanding examples one meets when studying the structure of international trade are those of the ties between a country and its overseas territories; the most important case being that of the British Empire. Whereas Germany's and America's customers are mainly the countries in their immediate neighbourhood, Britain's customers are often the Empire members.

§ 67. Measurement from Long-Time Series

From these a priori considerations it appears to be of much interest to measure long-term elasticities of imports or exports. What ways are open to us? There seem to be several, although each of them is beset with special difficulties. The first possibility would seem to use time series over a long period. A formula of the type of (65.1), when the necessary other explanatory variables are added, could be tested by the usual methods of multiple correlation only if we have long series of figures at our disposal. Even then it may prove to be a very risky procedure. There may exist, on closer theoretical examination, a number of factors influencing the observed movements which are hardly accessible to measurement. In the long run factors may be variable which are not so in the short run, especially such factors as quality, technical uses made of the commodity in question, etc. Not all of them will be relevant to our problem, i.e. to the relative competitive position of the countries considered. This can only be judged by insiders. As an example with, at first sight, a good chance of success we will take the demand for British pig iron by the United States during the period 1879–1914. As variables we take:

Q_t quantity of pig iron imported into the United States as a percentage of total American consumption;

P_t import price (including duties) as a percentage of home price in the United States;

v_t an index representing the cyclic situation, viz. production of pig iron as a percentage of its trend value.

It was attempted to explain the fluctuations in log Q_t in two ways, viz. once by the fluctuations in log P_t and log v_t and once by those in log P_t with a distributed lag and in log v_t. In order to account for the distributed lag the following variables were added to log P_t and log v_t: log P_{t-1}, log P_{t-2}, log $\overline{P}_{t-3/6}$ and log $\overline{P}_{t-7/10}$, where $\overline{P}_{t-3/6}$ indicates the average price for the years t-3 to t-6 inclusive and $P_{t-7/10}$ the average price for the years t-7 to t-10 inclusive.

The results of the calculations are shown in Figs. 30 and 31 and in table 1, where R represents the multiple correlation coefficient.

TABLE 1. REGRESSION COEFFICIENTS AND $1 - R^2$ IN THE EXPLANATIONS OF LOG Q_t

Explanatory Variables	*log P_t*	*log P_{t-1}*	*log P_{t-2}*	*log $\overline{P}_{t-3/6}$*	*log $\overline{P}_{t-7/10}$*	*log v_t*	*$1-R^2$*
	Regression Coefficients						
Case I	− 3.6	*	*	*	*	− 1.6	0.106
Case II	− 3.1	− 0.2	− 0.1	− 0.4	− 0.7	− 1.1	0.044

* *Not included.*

It appears that the correlation, although already high in case I, is considerably increased by the introduction of the lagged terms, all of which appear with the correct sign. The long-term elasticity as estimated by case II would amount to $-(3.1 + 0.2 + 0.1 + 0.4 + 0.7) = -4.5$ as against 3.1 when measured by the first member in the same equation or 3.6 when measured by case I. The difference between short-term and long-term elasticity (assuming that no longer lags than a ten-year one would exist) would be only moderate.

The same procedure was used in an analysis of the imports of raw cotton into the United Kingdom comparing imports of American and Indian cotton. Here it appeared impossible to obtain a satisfactory explanation of the fluctuations in the ratio between these two imports.

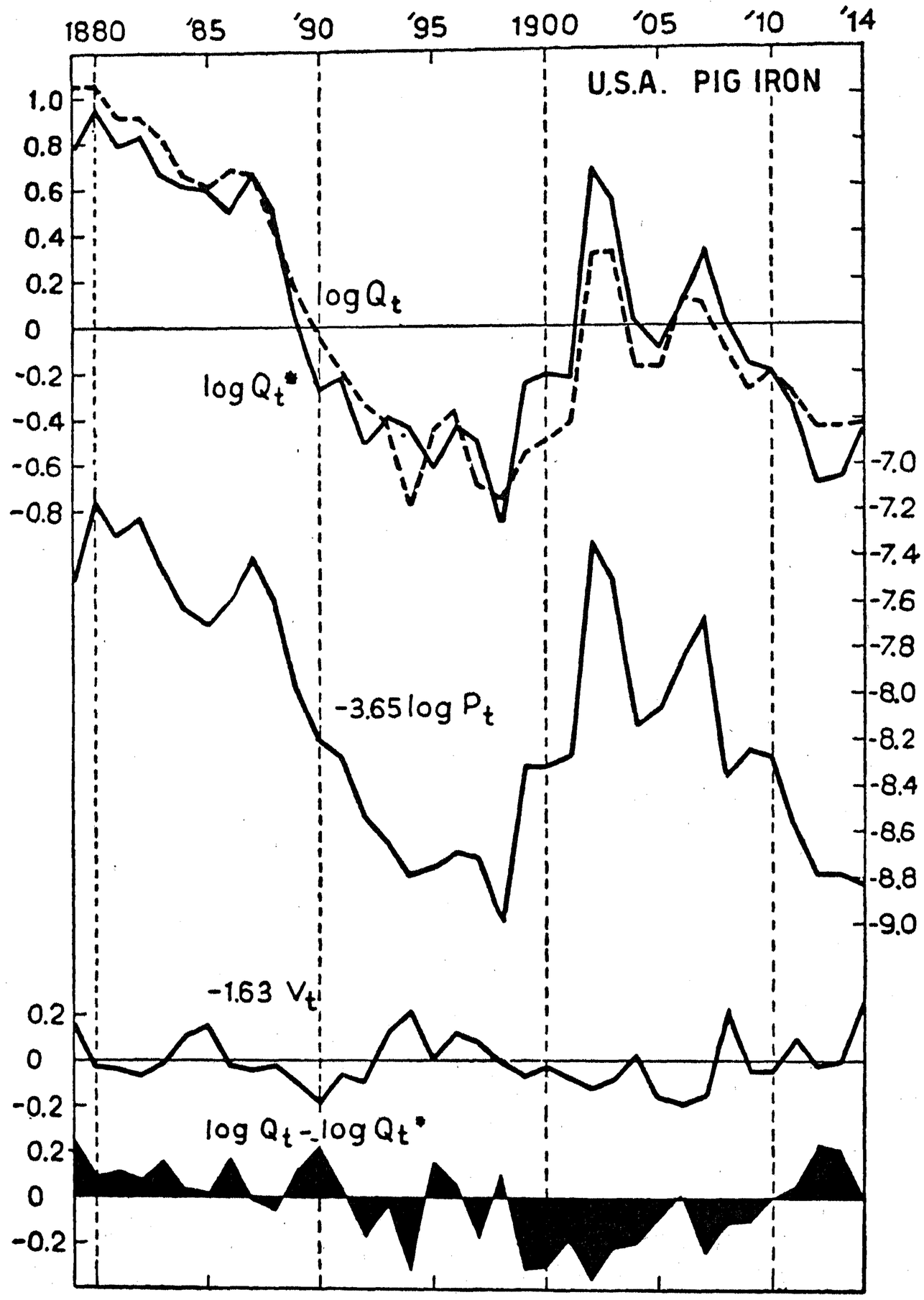

Fig. 30.

An even cruder procedure would consist of a simple comparison of two rather distant time units. This was done for total exports of manufactured articles by a number of coun-

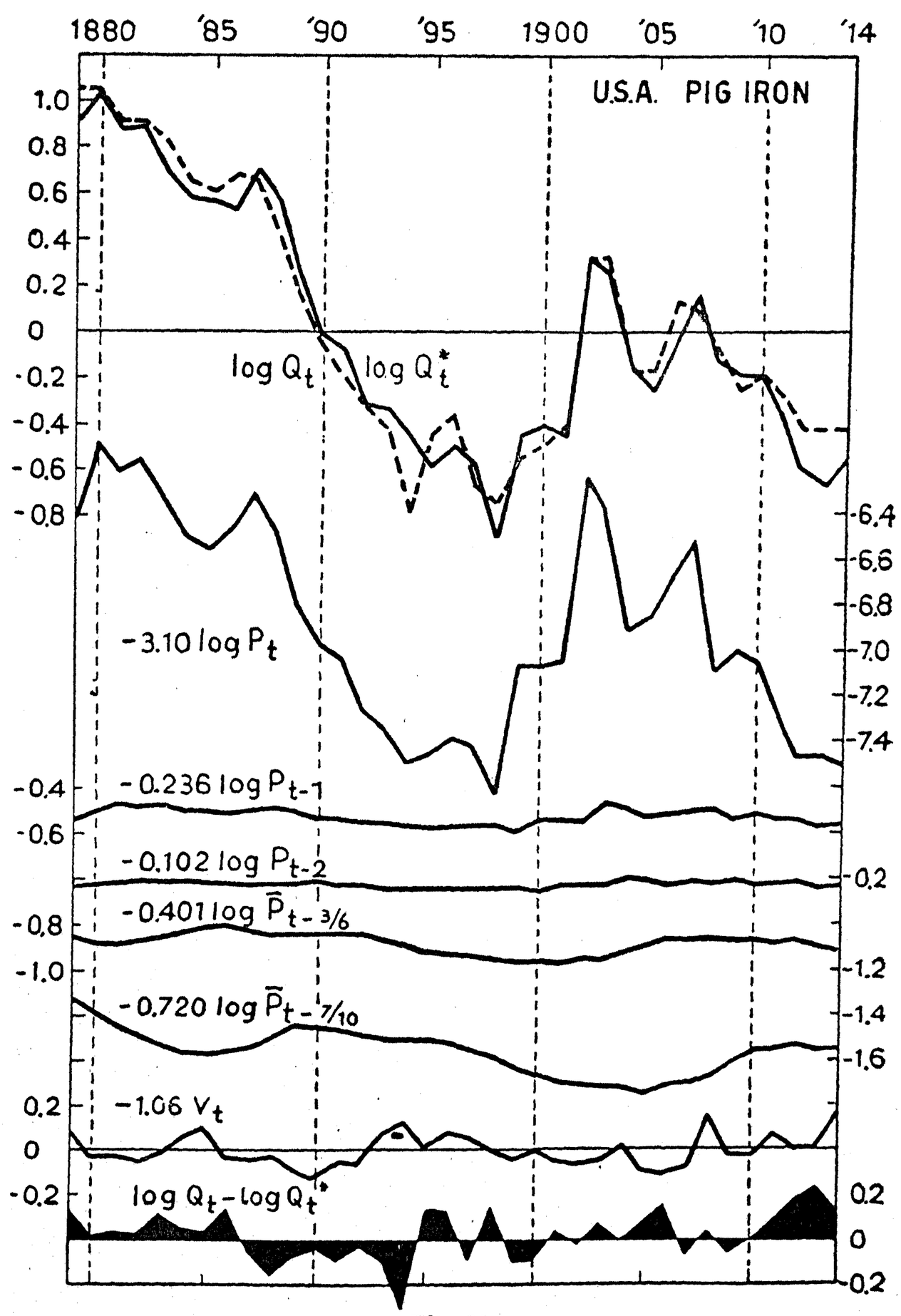

Fig. 31.

tries. These exports, expressed as a ratio to world exports of manufactures (volume figures) were compared with relative prices of these products (price index for country considered

divided by world price index) and a scatter diagram constructed (see Fig. 32). Three periods only were compared, viz. 1881-1890, indicated by (1), 1901-1913 (2) and 1926-1935 (3). The scatter is not very well organized; for such a country as the U.S.A. the slope of the regression line is positive. Of the negative slopes, the one with the lowest elasticity shows an elasticity of about 4; the next one of about 12, and many are considerably

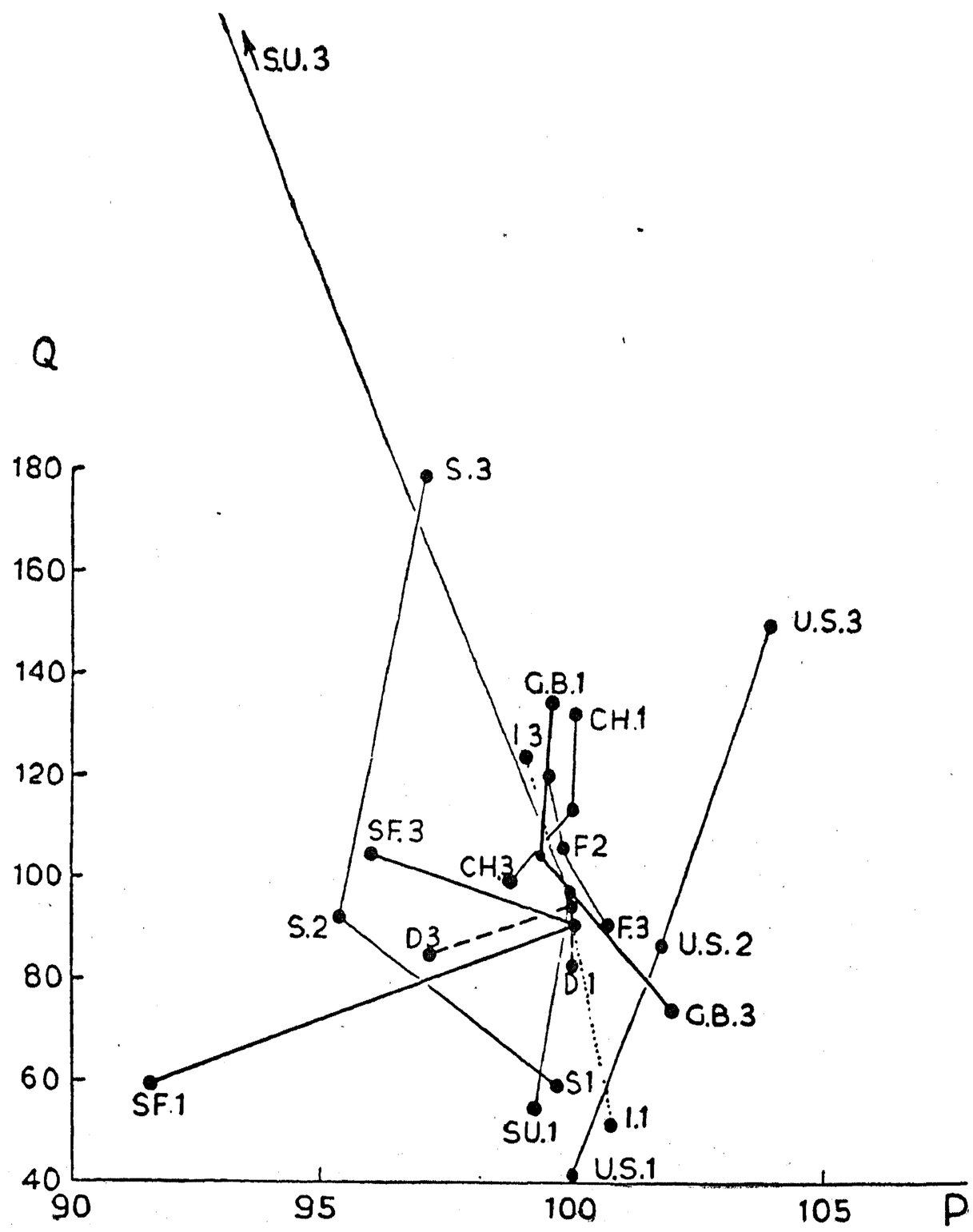

Fig. 32.

higher. These measurements are of little value, however, as long as it is impossible to explain the deviations in the scatter. It seems probable that the price index-numbers are none too good and can hardly take into account the changes in quality that the products under review must have shown.*

§ 68. Measurements from Cross-Section Studies

The other way to measure long-term elasticities is by using cross-section figures, i.e. by comparing different cases observed at the same moment, just as we do, e.g. when deriving long-term income elasticities of consumer demand from family budget statistics. In our case it comes to comparing different countries whose products for some reason or another are different in price. This we may still do in various ways. First we may compare products which for the supplying countries have a different price ratio. This was done by MacDougall who compared products of the United Kingdom and the United States, the relative costs of which diverge widely because of differences in relative efficiency.† The figures suggest a fairly high elasticity, although exact price figures are lacking.

Another possibility consists in comparing relative prices and quantities of the same commodity sold in different markets, price differences varying from one market to another because of distances and perhaps for other reasons as well. This has been attempted by the present writer for a number of commodities, one of which is pig iron. The relative shares in the imports into various countries were calculated (i.e. ratio of imports from the United Kingdom to total imports from U.K., U.S.A., and Germany) and the resulting percentages classified and represented by the intensity of the shade in three maps of which Fig. 33 is an example. From these maps it can be seen that the market shares vary from almost O to almost 100 per

* Most of the figures have been taken from Industrialization and Foreign Trade, League of Nations, 1945.

† Private communication by Mr. MacDougall.

cent and are greatly dependent on the relative distances from the three supplying countries considered.* For our purpose it is of particular interest to note that starting from a market supplied mainly by one of the big three, we have to go only a relatively small distance in order to arrive at a market supplied mainly by one of the others. Small distance meaning that the freight difference corresponding with this distance is only about 5 to 10 per cent of the price. This means that the market share may move by about 80 per cent because of a price change by say about 8 per cent, corresponding with an elasticity of the order of magnitude of 10.

A more direct and more accurate attempt was finally made by collecting figures on imports and import prices of pig iron into a number of importing countries and comparing, for the pairs Germany/U.K. and U.S.A./U.K. the relative quantities imported to and relative import prices charged to the buying countries. The results are shown in Figs. 34 and 35 and in table 2.† In order to eliminate the possible political ties existing between the supplying and the buying countries, two calculations of elasticities were made in each case, the second excluding British territories among the customers.

TABLE 2. CROSS-SECTION ELASTICITIES FOR RELATIVE IMPORTS OF PIG IRON IN A NUMBER OF COUNTRIES
(FIRST ELEMENTARY REGRESSION; DIAGONAL REGRESSION COEFFICIENTS IN BRACKETS)

		Supplying Countries Compared	
		Germany/U.K.	*U.S.A./U.K.*
Including British territories	1913	−3.9 (− 8.3)	−2.0 (−14.6)
	1929	−9.5 (−15.3)	−6.2 (−11.2)
	1932	−1.6 (− 5.0)	−3.4 (− 5.3)
Excluding British territories[a]	1913	−1.9 (− 7.5)	−1.3 (−15.9)
	1929	−6.5 (−14.4)	−0.1 (−16.0)
	1932	−1.6 (− 5.0)	−3.1 (− 5.2)

a Australia, Canada, and Union of South Africa.

* With the political exceptions already mentioned, in particular for Britain.
† The full statistical material will be supplied by the author on request.

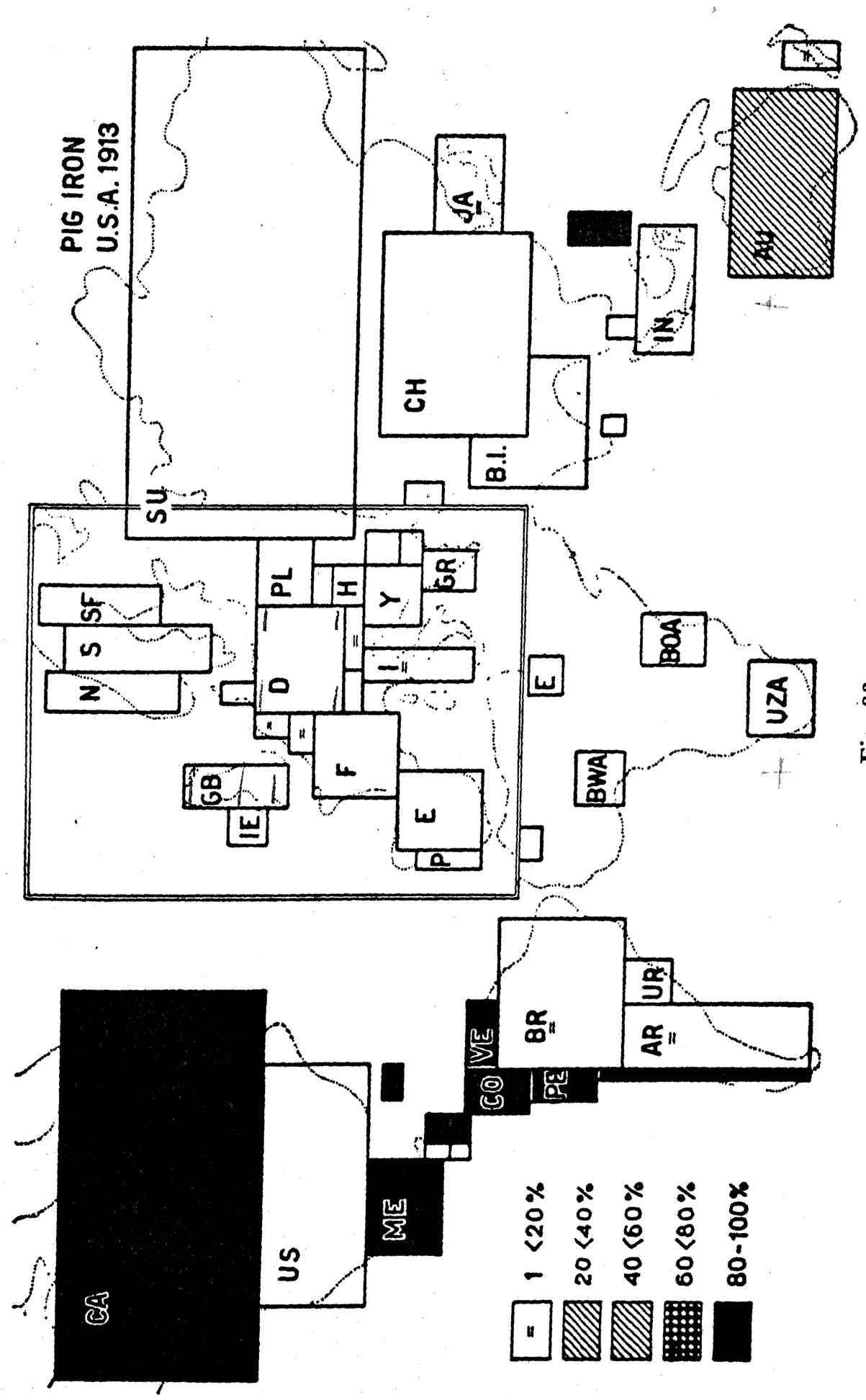
PIG IRON
U.S.A. 1913
SU
CH
JA
B.I.
IN
AU
N
S
SF
PL
D
H
Y
GR
I
F
E
P
GB
IE
BWA
BOA
UZA
CA
US
ME
CO
VE
PE
BR
AR
UR
1 <20%
20 <40%
40 <60%
60 <80%
80-100%

Fig. 33.

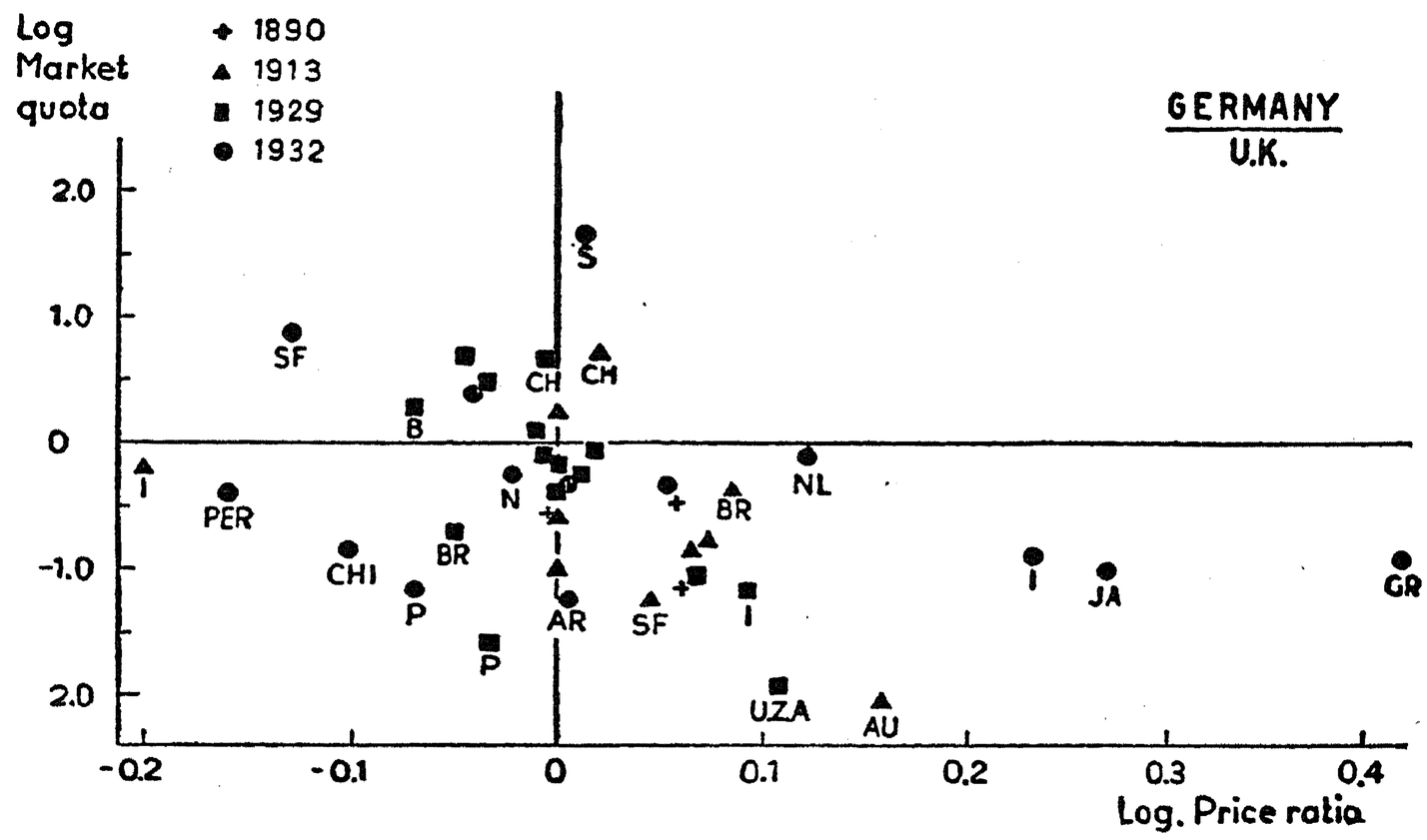
Log
Market
quota
1890
1913
1929
1932
GERMANY
U.K.
2.0
1.0
0
-1.0
2.0
-0.2
-0.1
0
0.1
0.2
0.3
0.4
Log. Price ratio
S
SF
CH
CH
B
I
PER
N
NL
BR
CHI
BR
P
AR
SF
I
I
JA
GR
P
UZA
AU

Fig. 34.

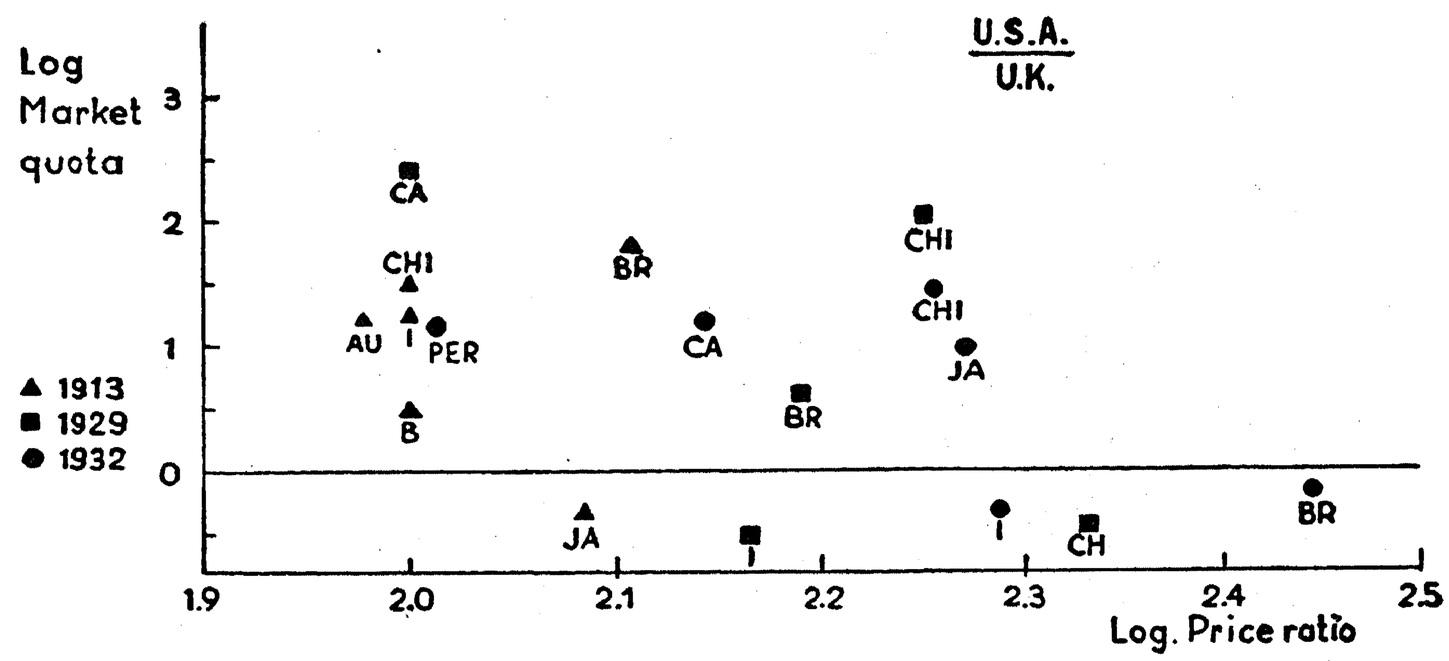
Log
Market
quota
U.S.A.
U.K.
3
2
1
0
1913
1929
1932
CA
CHI
BR
CHI
CHI
AU
I
PER
CA
JA
B
BR
JA
I
I
CH
BR
1.9
2.0
2.1
2.2
2.3
2.4
2.5
Log. Price ratio

Fig. 35.

It is interesting to note that the exclusion of British territories among the buyers does not, as one might have expected, increase elasticity. More interesting still it is, however, that the elasticities found here are not as high as our a priori considerations (based on distances alone) would suggest; the average value being -3.4 for the first elementary regressions and -10 for the diagonal regressions; and their median values -2.6 and -9.8 respectively. When considering these figures it should not be forgotten that they refer to a very well-defined commodity, for which the short-run elasticity also surpasses, necessarily, the figures for export baskets as a whole.

§ 69. Concluding Remarks

In this paper in honour of Constantino Bresciani-Turroni whose work on the measurement of elasticities has been generally admired, an attempt has been made to measure long-term substitution elasticities of imports as distinguished from short-term elasticities. Two methods are tried, viz. one using long time series and "distributed lags," and one using "cross-sections." Their application to the case of pig iron leads to figures of -4.5 and of -3.6 to -10, respectively. From the first method as well as from previous measurements of short-term elasticities it seems probable that these higher elasticities refer to reactions that require at least a couple of years.