3

Statistical Testing

§ 14. Statistical Testing; Measuring Phenomena

We have already stated that econometric investigations should be started by setting up one or more relations between the magnitudes to be studied, with the help of mathematical-economic methods. When this has been done—we have called it the first stage of the investigation—the statistician has the floor. He will try to ascertain whether the supposed relations agree with the facts and if so, whether the measuring of certain magnitudes will make it possible to obtain more concrete information concerning those relations.

The first question the statistician has to answer is whether all phenomena the economist deals with are *measurable*. It is often stated that so many imponderables play a role in economic life that it is fundamentally incorrect to expect that reality can be laid bare by measurement.

This question is not new. It was asked in connection with the natural sciences before it was asked for economics. Physicists have been reproached because they considered heat as something measurable. Practically everyone is now convinced the day has been won by the "measurers," who by their delight in measuring have contributed to enormous progress in the theory of heat. Nor do the contemporary psychologists shrink from measuring many imponderables. However, we shall not reason by analogy, but prefer to go into the question directly. The main problems of economics are concerned with measurable phenomena. We speak of production, of unemployment, of real wages, etc. As far as the influence of imponderables on these measurable phenomena is concerned, we must distinguish between these imponderables themselves and their influence on the measurable phenomena. Even if expectations themselves are perhaps not measurable, their influence on investments can only

be in the form of a certain increase or decrease of the investments, a measurable phenomenon therefore. This latter fact alone interests us. It must moreover be remarked that the expectations themselves are usually based on very concrete and measurable phenomena. It is obvious, therefore, that we may try to investigate the influence of these phenomena on investments, disregarding the fact that expectations themselves might not be measurable.

Another example refers to the "degree of satisfaction." This magnitude is generally considered nonmeasurable. But after examination of the problem it becomes clear that it is not necessary to know the degree of satisfaction if we want to explain the behavior of economic subjects. We only need know the combinations of goods which bring about the same degree of satisfaction—all lying on "indifference-surfaces"—and these are measurable in principle.

We therefore believe that it is not necessary to worry unduly about this matter.

A similar question is: Do psychological relations take place *according to certain laws* in such a way that they can be expressed in mathematical equations? Often doubts are expressed on this question; it is pointed out that free will can, under the same circumstances, lead at one time to a certain reaction and at another time to a different one. The answer to this objection is that if another reaction follows the second time, there must have been a reason for it. Therefore we should speak of different situations when completely describing the circumstances. Moreover—and this is of particular importance to macroeconomic investigations—there is a great likelihood that the joint behavior of large groups of persons shows a more regular pattern than the behavior of separate persons, because individual deviations greatly compensate each other. This is confirmed in many cases by observation; as will become clear from a few of our examples, there is often a surprisingly great regularity in mass reactions.

The simplest form of measurement with which we are concerned in statistics is the measurement of a separate phenom-

enon: the determining of a price, of the quantity consumed, of national income, or of the part of the latter accruing to the working class. From the point of view of econometrics, little can be added to the achievements which were made in this field by government statistics, by statistical evaluation methods, and by theoretical statistics long ago. It is of primary importance that we obtain an impression of the margin of uncertainty in the statistical results. This can often be attained by comparing various alternative observations or calculations; if an average price is concerned, we have to study the variance of the component price data and to apply the formula for the standard deviation of an average. Similar but rougher methods can be applied in other cases where fewer observations are available.

The *standard deviation* σ of a set of figures is computed by ascertaining for each of them the deviation from the average, by squaring these deviations, adding the squares, dividing this total by the number, and finally by taking the square root from the average of the squares of the deviations which has thus been obtained. Expressing this in a formula:

$$\sigma = \sqrt{\frac{1}{n}(x_1^2 + x_2^2 + \ldots + x_n^2)}$$

where $x_1 \ldots x_n$ are the deviations from the average.

The real econometric problems begin when we study whole series of observations, in particular series indicating the course of a certain magnitude in time, the so-called *time series.*

§ 15. Determining the Components of Time Series

Only rarely are all aspects of the behavior of a time series made the subject of econometric study at the same time. Usually various components in this behavior are distinguished; the traditional classification made by business-cycle research workers is in four categories, namely:

1. the trend or general direction of the series, or better, of the curve representing the latter;
2. cyclical fluctuations;

3. seasonal fluctuations, i.e., the fluctuations with a period of usually one year, which are the consequence of the natural change of seasons or of conventions with respect to the calendar and
4. random and incidental changes.

The underlying principle of classification here is the length of the waves. It could be stated that waves of 15 years and more and the one-sided movements (to be considered as waves with an infinite period) belong to group 1, those of from three to 15 years to group 2, those of one year to group 3, and the still shorter ones to group 4. Something, however, will be said about this presently.

As a consequence of the irregular form of most waves, their length often cannot be stated with certainty, or at any rate it is liable to changes. Some waves are not genuine waves, at least according to some writers. The so-called "long business-cycle waves" (Kondratiefs), which have a period of about 40 years, are, according to some authors, only more or less accidental alternations of rising and falling movements. Group 4 also shows rising and falling movements, but we certainly cannot speak here of a real periodicity, or a tendency to it. Changes which occur *only once (structural breaks,* as they are sometimes called) are difficult to classify. Finally, waves of different lengths do not always have different causes, which would make it illogical to separate them.

A few things can be said against the classification itself, i.e., against the choice of the boundaries. There are fluctuations which in origin are perfectly analogous to seasonal fluctuations, but which have a much shorter period, namely, a month, a week, or a day. Examples are the fluctuations in retail sales, in production of electric power, or in traffic. They therefore belong to group 3. On the other hand, there are random fluctuations with a much longer period than one year. The variations in crop yields are a very important example of a random influence. Should we want to ascribe a period to these, a time interval of three years could best be used. Nevertheless, they must be con-

sidered as belonging to category 4. Wagemann only partly meets these objections by his distinction between "free" and "tied" fluctuations.

Much confusion has arisen in the use of the words *long and short* waves in economic life. By "long business-cycle waves," sometimes the waves of 40-years length which have just been mentioned, are meant, sometimes also the waves of approximately nine years; by "short waves," the latter are sometimes meant and in other instances the reference is to the waves of approximately three years which appear in the United States. Schumpeter has proposed to prevent this difficulty by calling the three sorts of waves after the authors who have discovered them, and consequently to identify them as *Kondratiefs, Juglars,* and *Kitchins.* As regards business-cycle fluctuations, there is the additional difficulty that the various writers do not yet quite agree on the causes and therefore a causally founded definition cannot be given easily. Here we usually think of movements generated by endogenous causes. For certain special markets we can also distinguish cycles differing in length from the other ones, for instance, a cycle in house-building activity in the United States (15 years), a coffee-market cycle (also about 15 years), a hog-market cycle (three years), etc.

As regards *seasonal fluctuations,* we can distinguish between the normal or *average* seasonal movement, which is considered to be determined by strict laws—purely periodic, for instance—and the fluctuations which can be ascribed to the seasonal changes actually taking place in a certain year. During a very cold winter, for instance, water traffic will show a deeper trough than it does normally. This trough belongs to the individual seasonal influence of the year concerned, but not to the normal seasonal movement.

These remarks reflect a number of facts which can be useful to the econometrician. At the same time, they can protect him against mechanical application of some of the calculation schemes which have been set up in order to determine the components. Every case should be considered separately. Tak-

ing all these reservations into account, we think that our classification of components is fundamentally sound; in particular the isolation of seasonal fluctuations, of the short random fluctuations, and of a theoretically justified trend, because separate complexes of causes can be made responsible for their presence. We also think that we have given a foundation—albeit conditional—of the method in § 12, because we have shown there that the total movement of an economic variable can—under certain conditions—be split up into a number of additive components.

In the following we shall go a little further into the computation of the separate components; we do not intend, however, to discuss the actual calculation rules in detail, nor to illustrate them by examples. For this, we refer to the literature on the subject.

So far as the distinction based on the length of the periods makes sense—which is often the case—a general method can be given to determine all the components, namely that of *moving averages*. The seasonal fluctuations and all shorter fluctuations can be eliminated, for instance, by taking, instead of a series of monthly figures, the series of the averages of the terms Nos. 1 to 12, Nos. 2 to 13, etc. This series is called a 12-months moving average. Each of its terms has been built up from figures of all seasons, and therefore no seasonal changes are present in this series: the seasonal fluctuations have been eliminated. At the same time, all shorter fluctuations have practically been eliminated. It is true, complete elimination can only be spoken of if the period of the fluctuations goes a certain number of times into 12 months. But all other fluctuations appearing in reality can be assumed to be approximately eliminated. If we want to become acquainted with the shortest of the fluctuations eliminated—let it be assumed that these have a period of three months—we have to determine the three-months moving averages and to subtract these from the original series: we then are left with the shortest fluctuations. If the next period is a 12-months one, we can find the corresponding component by subtracting from the original series both the shortest fluctuations which have just been determined and the 12-months moving

average. In similar ways each of the components can be computed. This method, by the way, is rather rough. Moreover, for seasonal fluctuations it does not show the average or normal seasonal movement. To this end, we can take the averages of all January figures, of all February figures, etc., of the seasonal component determined in the way described previously. More refined methods, however, exist. With these we shall now deal in some more detail.

§ 16. More Refined Methods; the Trend

We shall first consider the determination of the *trend* more closely.

If the wavelength of the longest fluctuations which we do not want to consider as part of the trend amounts to nine years, these fluctuations and the shorter ones will be eliminated if the nine-years moving averages are determined; we then are left with the trend series. By doing this, we make use of the property of shorter fluctuations; that the sum total of their deviations is equal to zero, if taken for one or more periods. This property, strictly speaking, is only valid for nondisturbed, purely periodic movements. It does not exactly apply to damped or antidamped movements or to disturbed movements. In order to avoid this inaccuracy, we can also try to base the computations on a property of the trend itself. In that case we speak of a *mathematical trend* in contradistinction to the method just described.

When calculating a *mathematical trend* we assume *a priori* that the trend is a mathematical graph, a curve of a certain form. The simplest form is the *straight line;* also simple is the *exponential line*, the ordinates of which form a geometric series at mutually equal distances. If we change from the given series to the series of the logarithms of the figures, the trend, if originally an exponential line, will become a straight line. This property is often made use of to reproduce the original series on a logarithmic scale: the exponential trend then is a straight line. Whereas the straight line is characterized by a constant absolute increase per year, the exponential line shows a con-

stant percentage increase. More intricate lines have been used as trends, namely, so-called *parabolas* (of arbitrary order) and also the *logistic curve*. The latter deserves more explanation on account of its fundamental and practical importance. The simplest numerical example is obtained if the trend x_t for each moment t is calculated according to the formula:*

$$x_t = \frac{120}{1+2^{-t}} \tag{16.1}$$

where t goes from $-\infty$ to $+\infty$. The following values are obtained:

$t = -\infty$	...	-3	-2	-1	0	$+1$	$+2$	$+3$	...	$+\infty$
0		$13\frac{1}{3}$	24	40	60	80	96	$106\frac{2}{3}$		120

Representing this graphically (see Fig. 2, curve B), we obtain a curve rising slowly at first, then quicker, and finally approaching a certain saturation value, 120 in this case. In the beginning, the increase is slow if measured in absolute figures, but quickest proportionately.

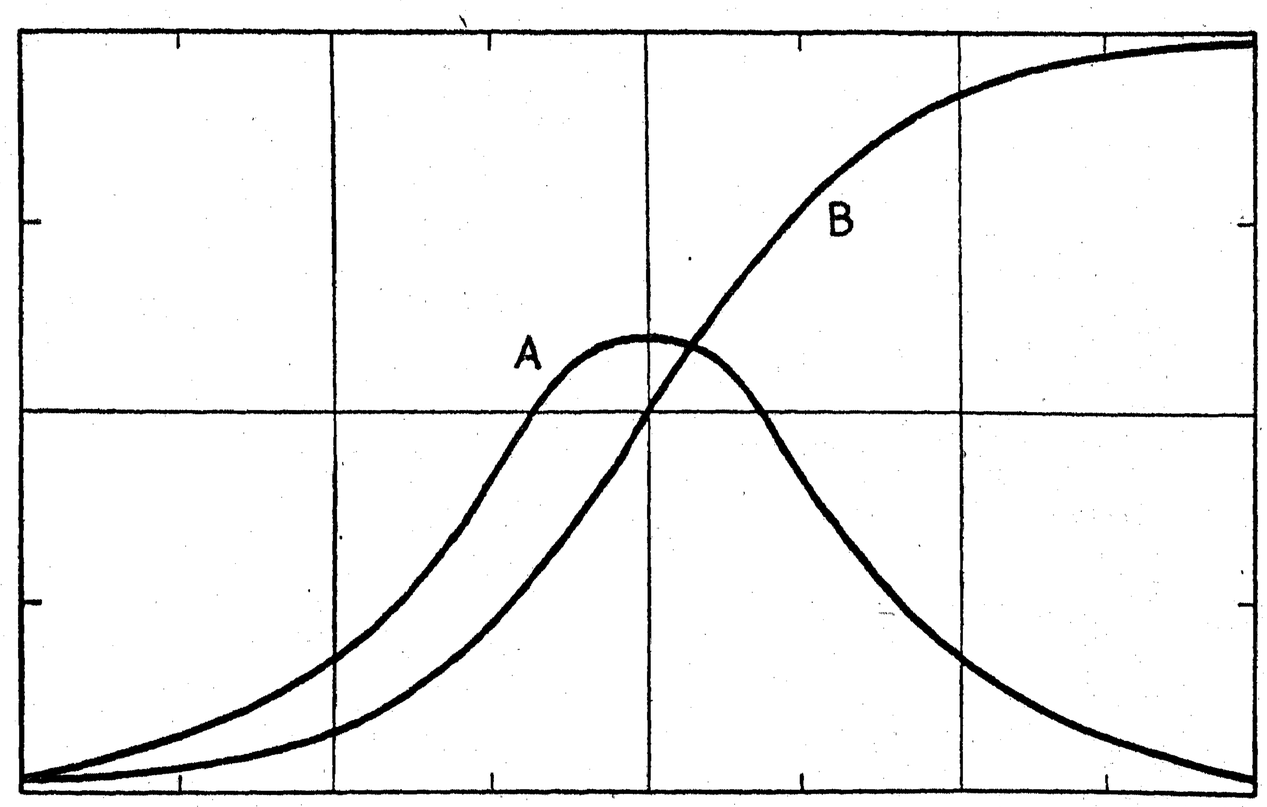

Fig. 2. Course of the Gauss curve (A) and of the logistic curve (B).

* The figures 120 and 2 are only examples; other figures could be used.

Which of these various lines should be used in a concrete case—what is the criterion? In principle, this should be decided by the law determining the generation of the trend. This means that we must have an explanation, a theory, about that movement. In certain simple cases this is actually so. In the case of population growth, for instance, under certain simplifying circumstances, both mortality and birth will be proportional to the size of the population and so will, therefore, the yearly increase of the latter. If this is true, the population line will be exponential. A similar case will be found if the growth of the stock of capital goods under simplified circumstances is studied: the increase of that stock is determined by the savings and these are determined mainly by nonlabor income, which in its turn is often proportionate to the existing stock of capital goods. Also in this case we obtain an exponential growth.

The logistic curve originates if a somewhat more complicated process of growth takes place; namely, if the increase of a population of size x is proportionate both to x itself and to the distance of x to a higher constant figure, a. This law of growth has been found in colonies of lower animals having a fixed stock of food or food source at their disposal. The figure a then corresponds to the population which can be supported with that source of food. The growth slows down if x approaches a and a is not surpassed, but only approximated. The figure 120 in our example corresponds to the figure a. This law has also been observed for human populations, although deviations appear. Moreover, the source of food is usually not constant with human populations. In general a growth approximately following this logistic curve can be expected if after some time a certain slackening, a certain saturation, takes place, and this can actually be observed with many economic phenomena. In particular, the development of the quantity of a new article coming into use often shows approximately this curve, for instance, the number of automobiles or radio sets, the length of the network of railways or of the telephone network, and the size of the production of artificial silk. This development can also be considered somewhat differently. Some people immedi-

ately want to have a commodity that is new on the market. Others prefer first to see which way the cat jumps; others, again, have to overcome a great inertia before deciding to buy. The speed of reaction, therefore, is widely divergent for various people and it can be assumed that like other human qualities, it is approximately distributed according to the "normal law" or Gauss-Laplace law (see also § 17). By accumulating the values of the new purchases, which are distributed in time according to the Gaussian law, we obtain the total number of objects in use, and the course of the cumulated Gaussian curve is very much like that of the logistic curve.

The exponential curve can be considered as an approximation of part of the first stage of the logistic curve; namely, where the saturation tendency practically cannot be noticed yet, because x is so small in comparison to a that from a to x may be considered constant. The increase in this case is only proportionate to x. Besides, the straight line can be considered as an approximation over still smaller intervals of each of these lines, also of the higher parts of the logistic curve; every curve can be approximated by a straight line. Parabolas usually cannot be founded theoretically, and therefore are not recommended strongly; they are usually chosen on the basis of a more external resemblance with the general course of the series to be dealt with.

The presence of an exponential trend in a series x has as a consequence that in the series x′ of percentage increases of x, no trend is present; the figures x′ are all grouped around a figure not equal to zero. Instead of eliminating the trend and working with deviations from trend we sometimes use percentage increases, known in American literature as "link relatives."

§ 17. Determining and Eliminating the Random Component

By a "random component" we mean in the broad sense a variable which can assume more values than one, and where it is not certain beforehand which of these values will be assumed in certain circumstances—for instance, at a certain moment of time. A clear example which tells us more than abstract

definitions is the number of times we can throw heads with one or more coins. If we throw with one coin, that number can be 0 or 1 on each throw; if we throw with three coins, it can be 0, 1, 2, or 3. The number of times each of these values appears is its frequency; the table of these frequencies is the frequency distribution. If the latter is expressed relatively, i.e., in figures the sum total of which is 1 and which are proportionate to the frequencies, we speak of a *probability distribution*. The probability or relative frequency of the appearance of one certain result indicates which part of the observations leads to that outcome.

The probability distribution of the outcomes of a head or tail game with one coin, repeated a great number of times, can be given by the following table:

Number of "heads"	Probability
0	½
1	½

Quite a number of other probability distributions of random variables is possible. One very well-known one is the Gaussian distribution. It refers to a continuous random variable, i.e., a random variable which can assume all values in a certain interval and not only integers. The *Gauss law* can be represented graphically by the well-known bell-shaped curve (Fig. 2, curve A); the probability that the observation lies in a very small unit-interval near the value x is

$$y = \frac{1}{\sigma\sqrt{2\pi}} \epsilon^{-\frac{\alpha^2}{2\sigma^2}}$$

If a random variable shows this distribution law, it is called "random" in the narrower sense. Such a distribution can originate from various sorts of mechanisms. It is found approximately, for instance, if the value of the variable at each moment is the result of a very great number of mutually independent forces, each causing a small deviation which itself is random in the wider sense.

A series of figures satisfying some frequency distribution, for instance, the Gauss law, may show various properties as regards the *sequence* of the figures. It is conceivable that the figures are arranged according to size. It is also conceivable that no regularity can be detected in the sequence. In particular, it can be investigated whether any dependence exists between two successive terms. This can be done by considering the frequency distribution of the second item of a number of arbitrary pairs, if the first items of these pairs are all equal to a fixed number a, and checking whether this distribution would be different from the one which is found if the first items are equal to another fixed figure, b. If the frequency distribution of the second items is always the same, no matter which value the first figure has, the subsequent terms are said to be *independent* of each other. If the series of figures we are dealing with has originated in the way described previously, namely, as the result of a very great number of independently working forces, each in itself causing a small deviation which is random in the wider sense, the subsequent terms will also show this independence. In general, the subsequent terms of a series of figures will be mutually independent, if the laws governing the origin of each figure do not depend on the preceding figure.

The determination of the random component in a series of observed figures usually takes place as follows. First, a hypothesis is drawn up concerning the behavior of all other components. On this basis the random component is ascertained and it is subsequently checked whether the latter satisfies the requirements which can be put to a random series. The hypothesis made regarding the sum total of the other components usually is that these have a "smooth course." This can be specified mathematically by assuming that the curve representing this sum total is of the first degree (straight line), or of the second, third, fourth degree, etc. These curves are "smooth" to a decreasing extent; they become more and more complicated and are liable to more and greater oscillations. We can distinguish here between an integral and a differential method. With the former, it is as-

sumed that the sum total of the other components, taken over the whole period under consideration, is a straight line or one of the higher-degree curves. Actually this total then is equated to a trend, namely, to a mathematical trend. With the differential method it is only assumed that these requirements are satisfied for small parts of the period, in other words, that in the vicinity of each point the systematic components together form a straight line or one of the higher-degree curves. Calculation shows that this is equivalent to the assumption that the sum total of these components can be approximated by a moving average. If a straight line is taken as an approximation, it is an ordinary moving average; if, however, higher-degree curves are assumed, it turns out to be a weighted moving average, where higher weights are given to the middle terms than to the extreme terms. The number of terms of these moving averages—nonweighted or weighted—again depends on the length of the shorter periods for which the assumption is made. If it is assumed, for instance, that for every five points the total of the systematic components forms a second-degree curve (an ordinary parabola), we obtain a weighted moving average of the following shape:

$$\frac{1}{35}(-3x_{-2}+12x_{-1}+17x_0+12x_{+1}-3x_{+2})$$

The total of the other components is determined, therefore, either as a mathematical or as a nonmathematical trend. The remaining part which pretends to be the random component is found by subtraction of these trend figures from the given series. Finally, we check whether this remaining part satisfies the requirements.

These can be twofold, namely, independence of the subsequent terms and a Gaussian frequency distribution. The independence is checked by determining the *autocorrelation,* i.e., the correlation (see § 19) existing between the subsequent terms of the series. The checking of the frequency distribution can take place, either rather roughly by drawing up a graph of the distribution and comparing graphically, or more accurately, by mak-

ing use of the mathematical-statistical methods devised for this purpose (the chisquare test).

§ 18. Determining the Seasonal Fluctuations and the Business-Cycle Component

As seasonal fluctuations as a rule have a fixed period, more refined computational procedures are sometimes possible than with the other components. In accordance with the aim of this book we shall only clarify the fundamental significance of some of the most important methods of calculation.

We shall first confine ourselves to determining the normal seasonal movement (see § 15). The first question which is often asked is whether the seasonal influence is *additive* or *multiplicative*. By this, the following is meant: If the general level at which a variable is situated changes, will the addition in the top season remain the same or will it be proportionate to the height of that general level? Further investigation shows that it depends on the mechanism of the seasonal influences whether the first or the second possibility is realized. Moreover, there are many other possibilities. Mostly, however, the statistical material does not allow us to make an exact decision in this case. The ideal scientific research would require an accurate investigation; this however, in general, is still in the realm of unfulfilled wishes.

Further, we can imagine a *fixed* and a *systematically changing* seasonal pattern. With a fixed pattern the seasonal influence is assumed equal every year—be it additive or multiplicative. It is conceivable, however, that the peaks become gradually less high and the troughs less deep. An example is egg prices: it has been possible to stimulate winter laying by night illumination in the henhouses, which has led to a decrease in seasonal differences. This lowering of the peaks can be considered to be regular in various ways: usually it will be assumed that they diminish by an equal amount every year and it can be determined statistically what the size of that amount must be in order to obtain the best possible adaptation to the statistical material.

The systematic change can be conceived to be even more intricate; with Wald's method, for instance, the height of the peaks

and the depth of the troughs are considered to be proportionate to the standard deviation of the seasonal fluctuations, the latter being determined from the deviations of 12-months moving averages.

Shifting seasons offer a particular difficulty. If the shifts take place systematically, they can sometimes be regarded as a special case of the example of changing the seasonal pattern mentioned first: offset against a decline in one of the months there can be a rise in another month. If, however, the shift does not continue in the same direction, this method cannot be applied. One example is the shifting of seasons as a consequence of the change of

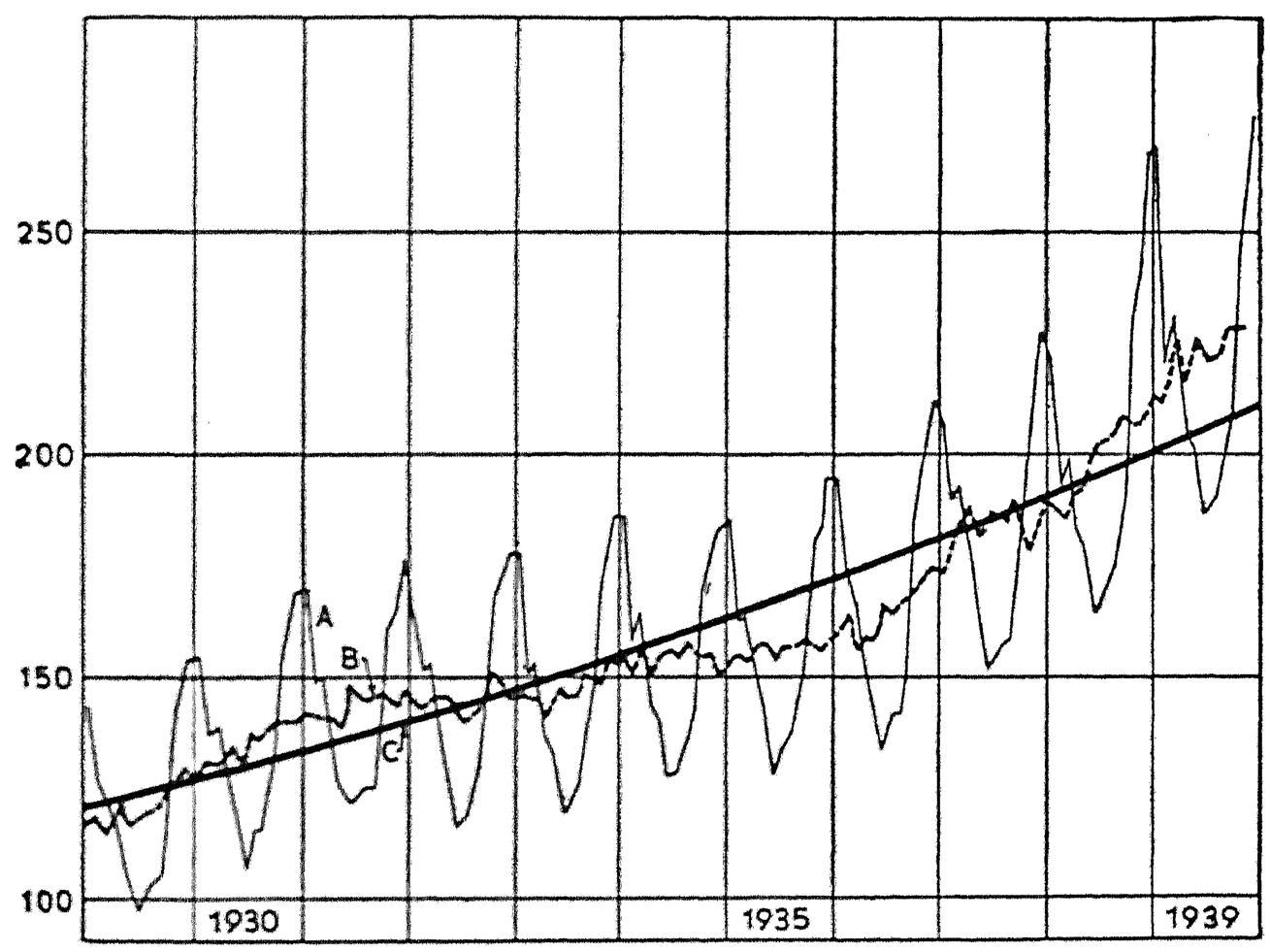

Fig. 3. Example of the computation of the components of a series (A) (electricity consumption in the Netherlands in millions of kwh, thin line). The heavy drawn line (C) indicates the trend, computed according to the exponential method. The dotted line (B) is the series after elimination of the seasonal fluctuations according to the multiplicative method, which is the method to use here because the seasonal fluctuations are proportionate to the yearly averages. The dotted line therefore gives the total of trend, business-cycle component and random component.

the dates of Easter and Whitsunday. Of course, we can try to make the necessary corrections as we go along. Another instance is a changing season brought about by weather conditions. This gradually brings us to the subject of the individual seasonal movement, i.e., the seasonal movement which differs from year to year as a result of a known cause.

The ideal of scientific treatment here is the causal explanation, for instance, the explanation of seasonal movements based on the temperature, the rainfall, etc. They then become correlation problems, to which we shall now turn.

In Figure 3, finally, we give an example of a series (A), a series corrected for seasonal influences (B), and the exponential trend (C).

§ 19. Simple Correlation

We have now come to a further step in econometric research, namely, the comparison of two series of observations bearing upon two phenomena, of which it is assumed on the basis of reasoning that they show a causal relation. This case evidently occurs if, on the basis of economic analysis, it is assumed that the fluctuations of a phenomenon, x, are mainly determined by the fluctuations of another phenomenon, y.

As we shall explain in detail, statistics here has a double task, namely that of *checking* and that of *measuring*.

To begin with, it is always recommended to make use of *graphical methods* in two ways: by drawing up historical graphs and so-called scatter diagrams. A historical graph indicates the course of the series x and y during a certain time period, a scatter diagram gives a series of dots, each of which has y and x as its two coordinates. As a rule the independent variable (the explanatory series, in this case y) will be measured along the horizontal axis and the dependent variable (the series to be explained, in our case x) along the vertical axis. If there is a distinct relation between the two series, the scatter diagram is a finer tool than the historical graph. But the latter shows more clearly whether there is a difference in *trend*, or a *lag*. In those cases scatter diagrams are sometimes so poor that it cannot be

seen what the relation really is. This last remark, by the way, refers to attempts to give suggestions to the econometrician to change the economic hypothesis to be tested, in case the first statistical check has taken place and has given a negative result.

If the two phenomena actually show a close relation, this will manifest itself in the dots obtained, which will lie nearly on one line (see Fig. 4A). If this is the case, the *correlation* is said to be high, possibly complete. If the dots are more scattered, there is incomplete or low correlation. According to the form of the line around which the dots group themselves, we have *linear* or *non-linear* correlation. In the case of linear correlation we also speak

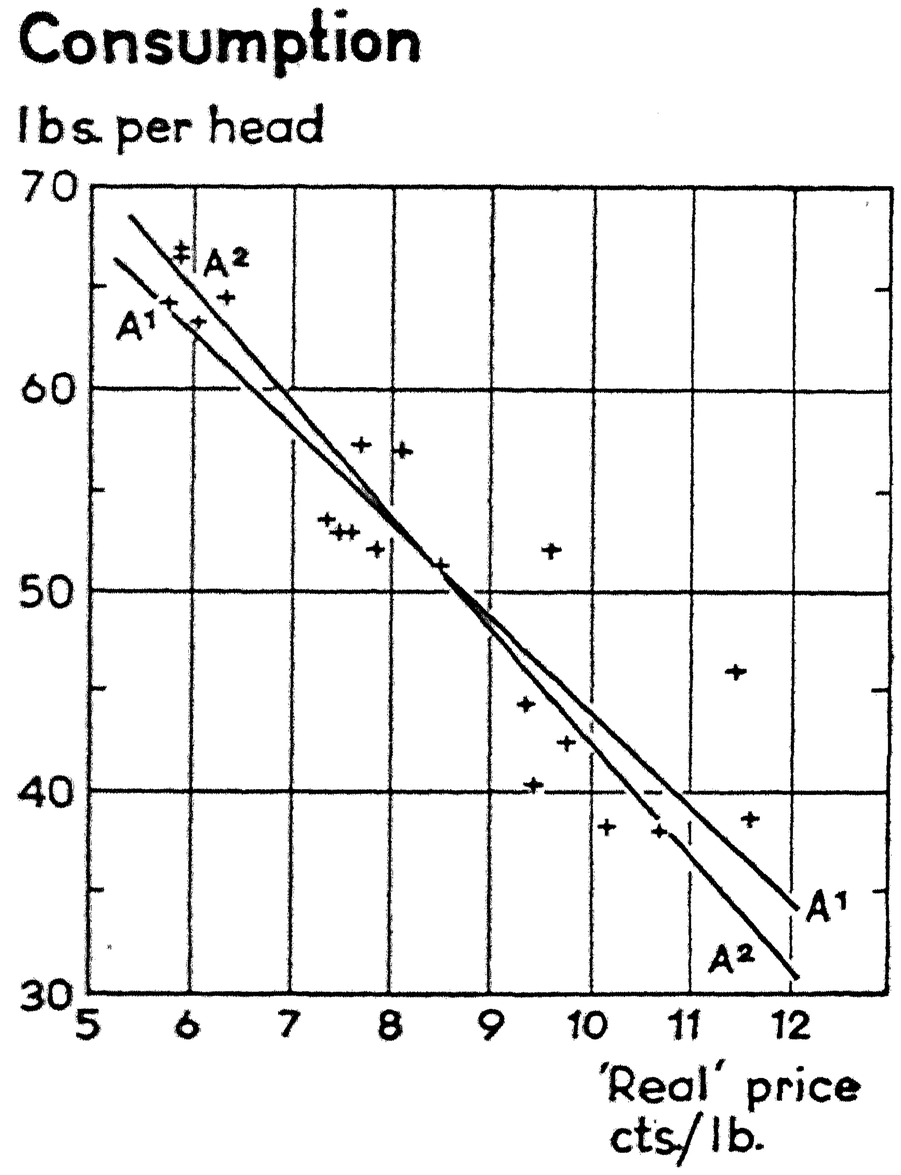

Fig. 4A. Example of simple correlation (the demand line for sugar, taken from Schultz, The Theory and Measurement of Demand). The two straight lines indicate the first (A^1) and second (A^2) regression lines.

of *normal correlation* if for every value of y the deviations of the x values follow the Gaussian frequency distribution with a standard deviation independent of y, and similarly as regards the deviations of y for every value of x.

If the graphical method has shown that there is a certain relation, and therefore, a chance of success, a further refinement of the analysis can take place by algebraic methods.

The *checking* function of statistics must give a reply to the question: Is there any relation between y and x? A reply in the affirmative or in the negative is not adequate here; we need a quantitative reply, because there are various sorts of possible intermediate forms. The relation, therefore, is close to a greater or less extent, and this can be expressed by the *correlation coefficient,* by the *correlation index,* or the *correlation ratio.* For all three, the following equation is valid:

$$r^2 = 1 - \frac{\sigma^2_{x,\,y}}{\sigma^2_x}$$

Here r is one of these three correlation criteria, σ_x is the standard deviation of the series to be explained, x (see § 14), whereas $\sigma_{x,y}$ is the standard deviation of some criterion of the deviations not explained by y, the so-called residuals. If all these residuals are zero, every x can be exactly explained by the assumed relation between y and x; and there is complete correlation. If the residuals, on the average, have the same size as the deviations of x from its own average, the explanation by y has achieved a minimum of success and there is no correlation. The differences between the three criteria of correlation mentioned lie in the way in which the residuals are calculated. To obtain the correlation coefficient, we calculate the deviations of the dots in the scatter diagram from a straight line which is fitted to those dots in the best possible way. For the correlation index the deviations are calculated with respect to a curved line which has been fitted as well as possible. Finally, in the case of the correlation ratio we take the deviations with respect to the averages of all x values belonging to each value of y. The deviations in all these cases

are measured vertically, i.e., in x direction. The correlation coefficient is the severest judge; it is 1 only if the dots lie on a straight line. With the correlation index, the statistical material has, so to speak, been given some more chance, as the curve has been drawn according to requirement; the correlation ratio has even more been fitted to the material by taking the line of the averages, which may jump rather arbitrarily. It is even possible—namely, by measuring the y values finely enough—to see to it that every y value only appears once, so that the corresponding average x value is equal to the (only) x value which appears in that group. Then there is complete correlation in every case in which the y values all differ. This, however, is of course unacceptable and in general such a fine subdivision will not be made. Strictly speaking, no use may be made of the correlation ratio, if the relation to be tested has actually been described beforehand by the economist.

Whether a correlation will be considered satisfactory depends on the tolerance (room for free play) which is allowed, and some difference of tastes exists in this respect. With a great number of observations, moreover, we can be satisfied with a lower correlation than with a small number. We must set a standard for ourselves here.

If this standard has been met, we come to the *measuring* function of statistics. It is now important to determine what relation there is between y and x. In the case of a straight line this question can be expressed as follows: By how much will x increase, if y increases by unity? This amount is called the *regression coefficient;* also in case we have to deal with other than straight lines, the curve indicating the relation is called the *regression line* (or regression *curve*); the equation indicating the relation is called the *regression equation.* The regression coefficient and, in general, the constants appearing in the regression equation can of course not be determined exactly, but with a margin of uncertainty; mathematical statistics offer several criteria for this. The higher the correlation and the greater the number of observations, the smaller this uncertainty will be.

In Fig. 4B a second example of simple correlation is given, about the result of which some remarks have already been made in § 7.

The line or curve indicating the relation can be determined unambiguously if there is complete correlation. As soon as the correlation is not complete, several lines are possible, varying according to the principles applied to make the line fit to the scatter diagram in the best possible way. The line is usually determined by minimizing the sum of the squares of the residuals in the x direction, i.e., by applying the principle of least squares to the residuals in the direction of the variable to be explained. The line obtained in this way, particularly if a straight line has been chosen, is called the *first* regression line.

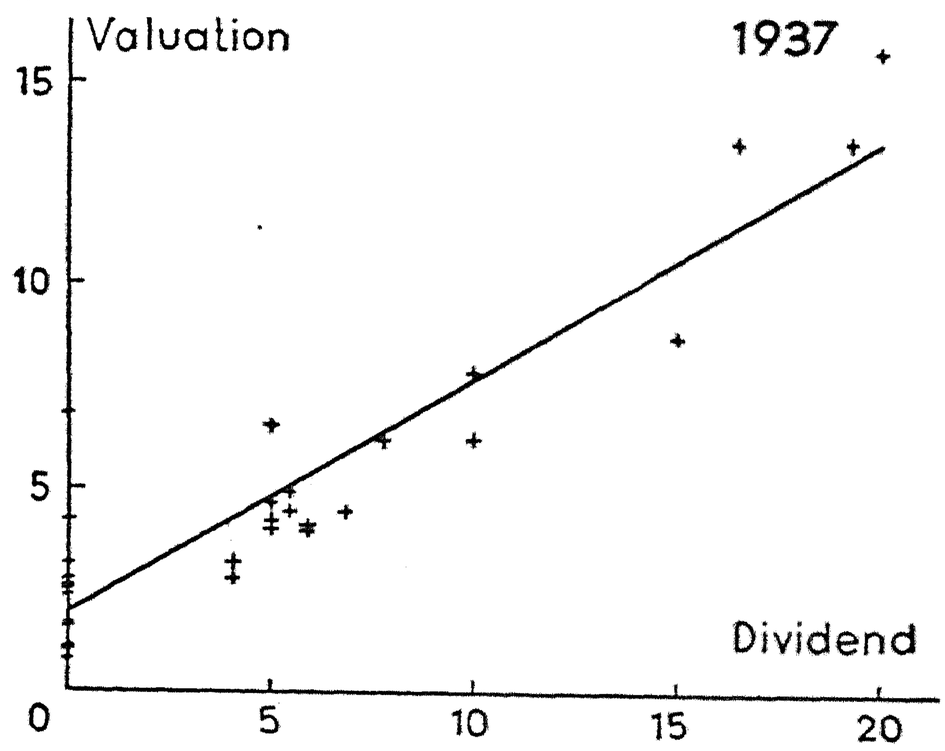

Fig. 4B. Example of simple correlation: the relation between dividends and "valuation" of shares. See § 7.

If the principle is applied in the y direction (that of the explanatory variable), we have the *second* regression line. In case of complete correlation the two lines coincide. If the correlation coefficient is zero, they are perpendicular to each other and coincide with the axes. The deviation between the two lines (the angle, or another criterion) therefore, can also be adopted for measuring the correlation.

If the first regression line is taken, it is assumed that the deviations between the actual and the calculated values are exclusively a consequence of (1) measuring errors in the series to be explained, and (2) incompleteness of the theory. As a rule the greatest importance will have to be attached to (2). If there is reason also to suppose errors of measurement in the explanatory variable, still other regression lines can be used.

§ 20. Possibilities of Application in the Economic Field

As has been mentioned previously, the method of testing by simple correlation can only be used if a magnitude x can be explained by another magnitude, y. We know that this is only rarely the case in economic science. As a rule a variable x is influenced by many other variables. Of old, this has been the difficulty in testing economic laws; the theoretical laws actually do relate two magnitudes at a time, but under the "ceteris paribus" condition: that other factors remain the same, which usually is not the case. Simple correlation, therefore, is important for econometric work only if the theory can show that there are cases in which the ceteris paribus clause is nearly fulfilled. We shall consider to what extent this actually turns out to be the case, in particular as regards demand and supply functions. We therefore interrupt for the moment our consideration of the technical problems of statistical testing. We allow ourselves this encroachment on our systematic treatment of the problem because also when actually carrying out an investigation, the ball has to go to and fro between statistician and economist again and again.

We now come to a classic problem: the statistic determination of demand or supply curves, i.e., the relation between price and quantity sold. This is only possible if the other (demand or supply) factors remain constant; for the demand curve, it is sometimes referred to as the Cournot-case. These other factors are: for the demand function, particularly income and possibly the price of competing articles; and for the supply function, production costs and capacity. Before enumerating a few cases which, in our opinion, satisfy this requirement, we have to correct some-

what the conception which we have just given. It is not necessary, namely, that the other factors remain constant, even approximately; it is sufficient that their influence is small. This happens when they appear in the reaction relation with a small coefficient.

We think that in the following cases there are indeed reasons to expect that the remaining factors will have a small influence.

1. The demand for *primary necessaries of life* will depend little on income; at least as long as they are not highly substitutable or complementary goods their demand will mainly depend on their own price.

2. The demand for goods whose prices *fluctuate very greatly* through circumstances on the supply side will depend relatively little on the other factors. Examples are goods with very strong fluctuations in crop yields, in particular if they are perishable, such as fruits and vegetables; and articles with very strong production fluctuations as a consequence of small adaptation possibilities of supply to demand through the long duration of the production process, such as pork and coffee. An article with very heavy supply fluctuations, moreover, is anchovy, where the catch in one year is sometimes a hundredfold that of another year (see § 40).

3. The demand for goods with very *rapid* supply fluctuations. As changes of income—for the economy as a whole—usually take place relatively gradually, we can reduce the influence of this factor by considering short periods. Examples are various goods with a strong seasonal movement in supply, and which cannot be stored. Fruit and vegetables have already been mentioned in another connection; eggs are another instance, especially in those times when less of them were stored in cold-storage warehouses.

4. The *supply* function of goods with relatively stable costs but great fluctuations in demand, such as mineral raw materials, may also be mentioned.

In a number of other cases the ceteris paribus clause is not satisfied, so that x has to be explained by several variables y, z, etc. Nevertheless, it is often possible to reduce the multiple relation appearing now to a simple one, which subsequently can be

tested by a simple correlation calculation. Usually certain *corrections* are carried out beforehand with the material, or other variables are substituted. We shall enumerate a number of the best-known ones.

1. If in a demand function the most important other factor which has changed is the *size of the population,* the quantity demanded *per capita* is considered. The correction here consists in dividing the quantities by the number of the population.

2. If the most important other factor which has changed is the *price level of all other goods*—where all other goods are assumed to be competitive with the good considered—the price of the good under investigation is divided by the general price level; we work with *deflated prices.* It is tacitly assumed here, by the way, that the influence of the general price level is such that in the demand function exactly the quotient of the price and the general price level appears. It is conceivable that this function is different.

3. If, in a *supply function,* the other factors to be considered are the costs of production per unit, these are subtracted from the price, and the *profit margin* is introduced as a new variable. If these costs, in turn, are supposed to be built up from raw material and wage costs, an index of the raw material prices and a wage index should be merged into an average in such a way that their weights correspond to the composition of the production costs.

4. If, likewise in a supply function, changes in production *capacity* should be taken into account, the quantities supplied are expressed as percentages of the capacity.

5. If, in a demand function, account should be taken of the influence of *prices of competing goods,* it may be assumed that the ratio of the price to that of the competing good appears in the demand function. Moreover, the same remark made under statement 2 applies here. We can even go a step further and also replace the quantity by the ratio of the quantities of the two competing goods. If, therefore, a quantity x_1 of the good considered and a quantity x_2 of the competing good are demanded at prices p_1 and p_2, respectively, it is assumed that

$$\frac{x_1}{x_2} = f\left(\frac{p_1}{p_2}\right)$$

A similar line of reasoning underlies a demand function for investment goods, which has been put forward by some authors. This demand is supposed to be dependent on the quotient

$$\frac{z}{mq}$$

where z is the expected profit,
q is the price of the investment good, and
m is the rate of interest. This quotient can be regarded as a ratio of "profit percentages," namely, $\frac{z}{q}$ for the good to be invested, and m for the amount of money sacrificed.

6. If, in a demand function, *income* is a factor which has to be taken into account besides the price, its influence can be estimated *a priori* on the basis of family-budget statistics and the observed quantities can be replaced by "quantities corrected for changes in income." This may not, however, always be correct.

7. If the remaining factors can all be assumed to change only gradually—for instance, size of the population, changes of tastes and habits, possibly also income changes in very quiet times—it can be assumed that these only influence the trend of consumption. By taking deviations from trend, both for the quantities and for the prices, we then obtain two variables the coherence of which is not disturbed. It would be incorrect to eliminate only the trend in consumption and not in the price, because the trend of the price will also exert its influence on the trend of consumption. The fluctuations in consumption, however, can only be explained by the price fluctuations, if by fluctuations we mean deviations from trend.

If one of these *a priori* reductions cannot be changed to a single relation, we have to make use of multiple-correlation calculations.

§ 21. Multiple Correlation

The problem here is to explain the changes in a series x by the changes in a few other series y, z . . . . Also in this case, *graphical reconnaissance* is very desirable. Time graphs can be used without any difficulty for this purpose. The question now is not whether the fluctuations of x and y are the same—possibly after corrections for trend or lags—but whether the fluctuations of x can be built up from a combination of the fluctuations in y, z . . . etc. Although more general methods are possible, we shall confine ourselves mainly to *additive* relations and we shall ask whether a function

$$x' = ay + bz + \ldots + a'y^2 \ldots \qquad (21.1)$$

with constants a, b, a′ . . . can be found, in such a way that the deviations between x and x′ are as small as possible. In the previous formula, higher-degree terms have been included. Usually even these are omitted in the beginning, and we see whether a *linear* function is sufficient:

$$x' = ay + bz + \ldots \qquad (21.2)$$

With the help of the method of time graphs it is possible to determine by observation whether there is any likelihood of such a relation. This means (see Fig. 5) that it should be possible to approximate the series x satisfactorily by a sum total of y, z, etc., where these last variables may first be enlarged or reduced in a certain proportion, which can be determined freely.

The particulars in the course of x should be found, therefore, either in y or in z, or in both. (See Fig. 5.)

The method of the scatter diagram should be used in this case. If there is a close relation between the variables, the dots are situated in a surface, possibly in a plane. Operating with a spatial diagram, which by the way could only be used if there are only two explanatory variables, however, offers some practical difficulties, and therefore we have to manage to get along in some other way, as, for instance, with Ezekiel's method. We therefore refer to Ezekiel's book *(Methods of Correlation Analy-*

sis) and we shall not go into the methods of finding the best relation along these lines, but only into the way of representing this relation, after it has been determined by calculation.

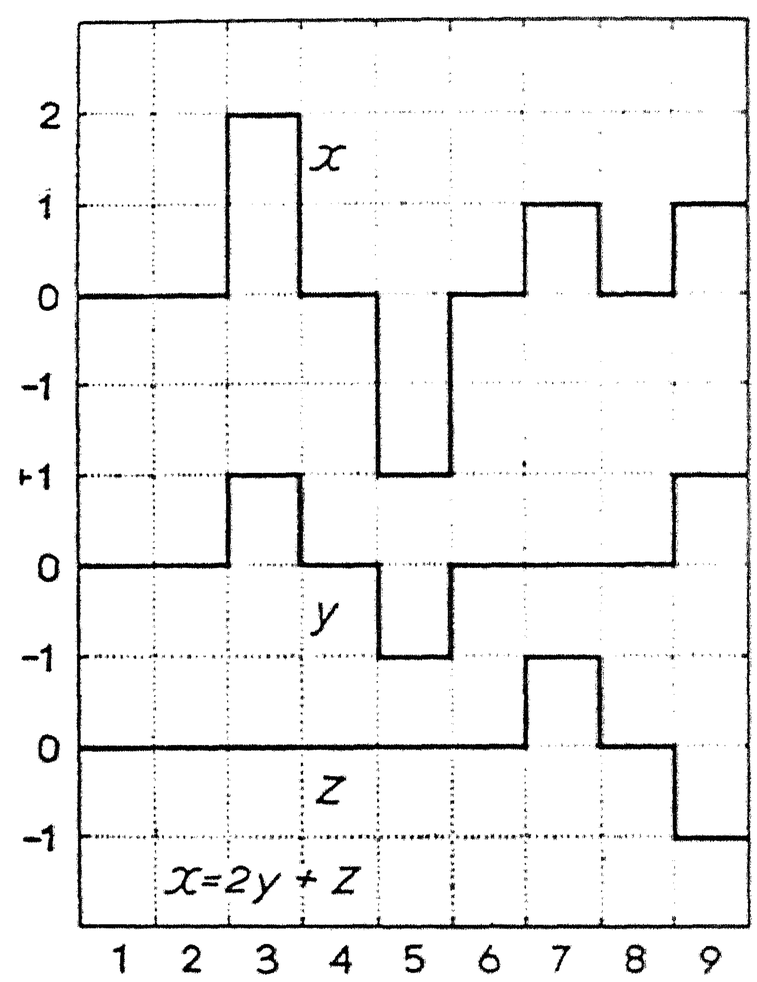

Fig. 5. Theoretical example of multiple correlation. It can be seen that the line x can be built up from the lines y and z, if we take $x = 2y + z$.

The calculation is again carried out by the method of least squares. We determine a, b . . . (and, as the case may be, a′ . . .) in such a way that the sum of the squares of the residuals

$$x - x'$$

is a minimum. We now have obtained the equation of the *first regression line*. Also in this case, other regression lines can be distinguished, namely, the second, third, etc., to the nth, if there are n variables in total and consequently n-1 explanatory variables. The second regression line is obtained by assuming a relation

$$y' = cx + ez + \ldots \qquad (21.3)$$

and requiring that the sum of the squares of $y - y'$ is a minimum. (In the mathematical literature about this subject, a systematic method of indicating all the coefficients appearing in these equations is in use. For the sake of simplicity and in accordance with the purpose of this book we shall not go into this here.) If the correlation is complete—i.e., in the case of a linear regression equation, if the dots lie in a plane—the relation between x and the remaining variables found from the second regression equation will be identical with the one derived from the first.

In equation (21.2), which as a rule is used, because it can usually be assumed that the deviations are consequences of incompleteness of the theory, we call the term ay "the influence of y," the term bz, the influence of z, etc. This term is independent of the unit in which y is measured, because if y is indicated by a smaller number (as a consequence of a greater unit), the term a will automatically be enlarged in the same proportion. If we assume, by way of example, that the investigation refers to a demand curve where

x is the quantity demanded and
y is the price; and where also appear
z and u, two demand factors,

then we shall call

$$x - bz - cu \tag{21.4}$$

"the value of x corrected for the influence of demand factors." This magnitude must show a simple correlation with the price, y. The character of this relation can be represented graphically by drawing a scatter diagram with these two magnitudes as its coördinates; we then speak of a *partial scatter diagram*. Similar scatter diagrams can be made for each of the explanatory variables, for instance, between z and $x - ay - cu$, the values of x corrected for y and u.

The equation

$$x' = ay + bz + cu \tag{21.5}$$

can also be given an interpretation showing its connection with the theory of demand curves. The term $bz + cu$ can be called the "shift." This name fits in with the usual graphic representation of the demand curve, where constant values are allotted to the "remaining factors," in this case z and u, and where we are left with a relation between x and y which is represented graphically. If, now, z and u assume other values, this means that the demand curve is shifted parallel to itself. This method of representation can also be related to the method of representation in space, which we have discussed earlier. For the sake of simplicity we confine ourselves to one demand factor, z. The relation $x = ay + bz$ is represented by an inclined plane, as we have already seen. If a fixed value is given to z, only the points on the plane lying in one straight line are considered. If another value is taken for z, the points of another straight line in the plane are obtained. The projections of these straight lines on the xy plane, then, are the demand lines for the various values of z, "the shifting demand curves."

The determination of the regression coefficients is impossible in a fundamentally important case, namely, if another linear relation exists between two or more of the explanatory variables. The simplest case occurs when the fluctuations in two of those variables, for instance y and z, are proportionate. In that case y can be substituted a certain number of times for z. If a regression equation is to be found, an arbitrary number of other ones could immediately be derived from it, leading to equally satisfactory results. The result, therefore, is not unambiguous. In other words, the influences of y and z cannot be separated.

More complicated cases are also possible. It may happen that y is not proportionate to z but that

$$y = pz + qu \tag{21.6}$$

where p and q are constants. In this case, y can be substituted wholly or partly by the right-hand side of this expression and so the number of possible results is increased.

It should not be inferred that in such cases where *multicollinearity* is said to exist, nothing can be learned from the calculation. If y and z are proportionate, their influences certainly cannot be separated, but u's influence can be determined. If (21.6) is valid, the influence of none of the three explanatory variables can be ascertained, but certain functions of that influence can. Whether the result can be used or not depends on the ultimate purpose of the calculations.

The appearance of multicollinearity may be the consequence of the presence of other structural relations between the magnitudes included in the equation investigated. As an example, we shall take a demand equation

$$u = v_1 Y - v_2 p \tag{21.7}$$

where u is the quantity demanded,
Y is income, and
p is the price.
If, besides, a price-determination equation

$$p = \pi Y \tag{21.8}$$

is valid, the latter is the cause of collinearity of p and Y. It is also said in this case that equation (21.7) is not *identifiable*. This means in practice that if a number of numerical values of u, Y, and p are given, the coefficients v_1 and v_2 cannot be determined. In order to see this, suppose that certain numerical values v_1' and v_2' would have been found, for which the numerical values of u, Y, and p satisfy (21.7). It would now be possible to increase v_2' by any arbitrary figure α, if only v_1' is increased by $\pi\alpha$. By this procedure we would obtain:

$$u = (v_1' + \pi\alpha)\, Y - (v_2' + \alpha)p$$

This equation, which is of the same type as (21.7), would now also be satisfied, as a consequence of (21.8).

This difficulty would not have appeared if in (21.7) the term with p were missing. It would not have appeared either, if in

(21.8) another variable were included besides p and Y. It depends on how the various variables appear in a number of structural equations, whether a certain equation is identifiable or not.

§ 22. Evaluating the Uncertainty of the Results

There are two main reasons why the results of a multiple-correlation calculation are not completely accurate. In the first place, there are random deviations, manifesting themselves in the spreading of the dots in the scatter diagram and caused either by errors of measurement or by neglecting part of the explanatory factors in the underlying theory. In the second place, there may be multicollinearity or a situation approximating it. The two causes may appear independently of each other, but often both will be present.

The uncertainty in the results, in particular in the regression coefficients and also in the correlation coefficient—a consequence of the presence of random deviations—is usually expressed by the size of the *standard deviation* of these magnitudes. The latter indicates the standard deviation which—calculated according to the ordinary definition of this expression—would be obtained if not one complex of observations would be present, but a great number of similar complexes, in which the random deviations would have been different in every case. These complexes, like the one which has appeared in reality, are considered as representing *samples* from a greater imaginary complex indicating "what could have happened just as well." This greater complex, of course, is of a more or less hypothetical character. Fisher, who has given the theory on the basis of which these standard deviations are usually calculated, has assumed that the explanatory variables are exactly known, but that the variable to be explained possesses a random component, which is distributed according to the Gauss law. The usual formulas have been derived from these premises.

These formulas lead to results which themselves can be uncertain to a high degree, if the complex of observations is approximately multicollinear, as has been pointed out by Frisch. The latter, therefore, has tried to reach an estimation of the

uncertainty of the results in another way. In order to understand a few main points of his method, consisting in the determination of so-called *bunch maps,* we start from the theorem that if perfect correlation is present without multicollinearity, all regression equations lead to the same result. This means that the second regression equation (21.3), when solved for x:

$$x = \frac{y}{c} - \frac{e}{c} z \ldots \tag{22.1}$$

leads to coefficients which are the same as in the first regression equation:

$$x = ay + bz + \ldots$$

Therefore: $a = \frac{1}{c}, b = -\frac{e}{c}$, etc.

If, on the contrary, a situation of multicollinearity is approximated, great deviations will be found between the regression coefficients determined in the two ways indicated. Bunch maps (see Fig. 6) give a survey of the various values which are obtained for the same coefficient.

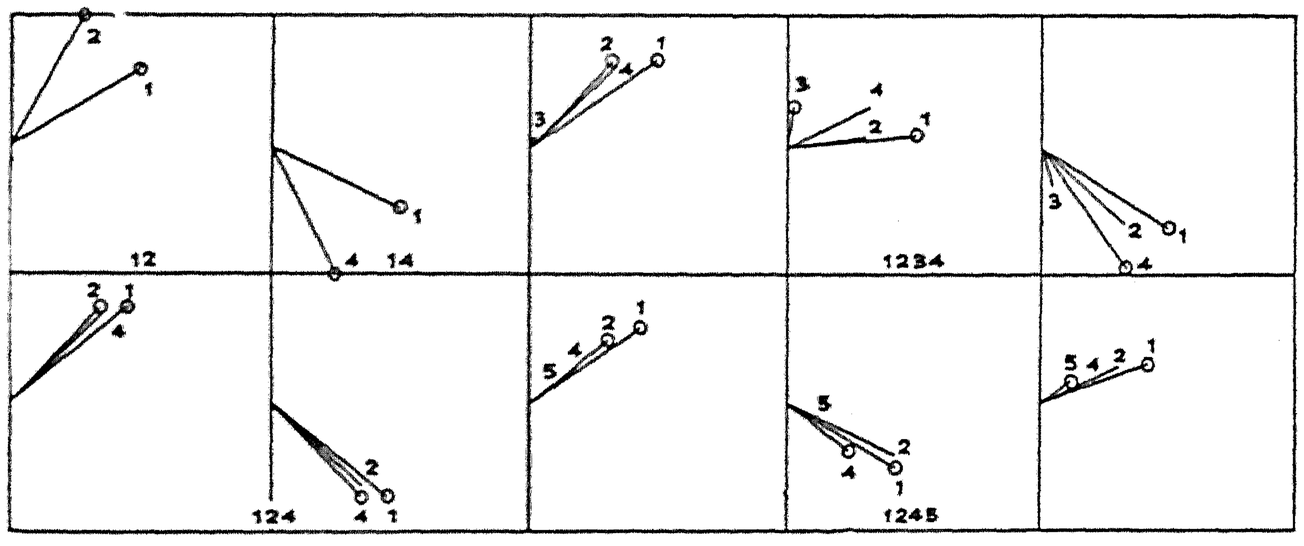

Fig. 6. Example of a bunch map.

These values are indicated by the slope of the lines drawn from the origin, or "beams." These beams carry a number, each indicating a variable. Two of the beams are always indicated in

a particular way; they are the main beams. Their numbers indicate the variables between which the regression coefficient has been determined; the undermost number is the one considered as the dependent variable, the uppermost number indicates the variable considered as the independent one. The numbers further indicate the number of the regression equation from which the value of the coefficient belonging to each beam has been computed, or the direction in which the residuals have been minimized. The number of beams is equal to the number of variables included in the investigation.

Frisch's method consists in particular in a gradual extension of the number of variables. If a new variable is added, we can observe in the regression diagrams what influence this fact has on the bunches of beams. If a variable is added which is essential for the explanation from a statistical point of view, i.e., which approximately explains the residuals not explained so far, it will be seen that the bunches of beams close up. If, however, the added variable is nearly a linear combination of variables already present, the bunches will "explode," whether this new variable is essential from an economic point of view, or not. A typical indeterminateness in the regression coefficients appears. It can be observed, therefore, which newly added variables are "useful" and which are "detrimental." There is a medium class of "superfluous" variables, which have little influence. The statistician will accept only those regression coefficients in which the bunch is closed. Frisch shows that under certain assumptions the margin of uncertainty can be measured by the distance of the extreme values computed in this way, i.e., by the tangents of the extreme beams of the bunch.

The regression diagrams can teach us even more, but that would lead us too far into mathematical details.

A special warning must be given against one incorrect way of using bunch maps. It is sometimes thought that the explosion of a bunch of beams after the inclusion of a new variable is an indication that this variable should not be included. This, however, can only be decided by economic theory, not by regression analysis. It may happen that according to economic theory the

variable in question is essential. The conclusion then must be that we cannot determine all coefficients because—accidentally or not—a few of the most important variables are mutually correlated. As has been remarked, it depends on the problem under investigation whether useful conclusions still can be drawn in such a case.

T. Koopmans has combined the elements of Fisher's and Frisch's theories into one theory which, however, has a complicated mathematical character and therefore cannot easily be paraphrased. We therefore refer to the literature.

§ 23. Simultaneous Equations

The preceding discussion referred to the testing of only one relationship at a time between the economic magnitudes considered. Strictly speaking, it can only be applied if there is but one relation between the magnitudes considered as variables to be explained. As soon as more than one equation exists at the same time, its validity is limited. This is nearly always the case if more than one variable to be explained appears in the problem. T. Haavelmo* has pointed out that the application of the method of least squares to each of the relations separately would mean that mutually contradictory assumptions were taken as starting points. The logical setup in this case should be to consider together the random deviations appearing in each relation as a sample from a higher-dimension complex of such deviations. If each of the relations is considered separately, it is assumed that each deviation is "drawn" from a one-dimension frequency distribution. If, for instance, the deviations in two simultaneous relations are u and v, their joint frequency distribution could be

$$F(uv) = \frac{1}{2\pi\sigma_u\sigma_v}\epsilon^{-\frac{1}{2}\left(\frac{u^2}{\sigma_u^2}+\frac{v^2}{\sigma_v^2}\right)}$$

* T. Haavelmo, The probability approach in econometrics, Econometrica, 12 (1944) Supplement. For a more simplified explanation, see T. Koopmans: Statistical estimation of simultaneous economic relations, Cowles Commission Papers, New Series, No. 11.

If, now, in analogy to § 21, u and v in this equation are replaced by the residuals of the two relations, for instance

$$u = x - x' = x - ay - bz \ldots$$

and an attempt is made to determine the values of a, b, etc., at which a maximum is reached for the probability of the appearance of the observed values of x, y, etc., other values will be found than with the method of least squares (multiple correlation).

The deviations from the regression coefficients found by this last method are only small if high correlations are found for each of the relations. The intricate method can be avoided, however, if the relations are brought into a special shape, namely, the *reduced form* discussed in § 11. The coefficients of the relations in their reduced form may, under certain circumstances, be determined by the method of least squares. We may continue to apply the method of least squares, for instance, if in each equation on the right-hand side only lagged values of the explanatory variables appear.*

§ 24. The Use of Regression Analysis in Econometrics †

The way in which the econometrician makes use of the methods described above has been described systematically by Koopmans along the following lines.‡ The statistician, when testing a relation set up by economic theory, may draw unconditional and conditional conclusions. The *unconditional* conclusions, however, are only *negative*. These conclusions may play a role when the theory is checked. If, namely, the correlation coefficient turns out to be too low (in comparison to the standard set before), the conclusion must be that the relation is incompatible with the facts. In other words, the theory is incorrect, or at least incomplete. The positive conclusions which

* For a simple explanation see Appendix A.

† See Appendix A for a further elucidation.

‡ Koopmans, T.: The logic of econometric business cycle research, Journal of Political Economy, 1940.

can be drawn do not appear when the theoretical relation is checked: it cannot be *proved* statistically that a theory is correct. The correlation may be high: other theories, however, may also lead to a high correlation. Positive conclusions do appear with the measuring function, but then they are *conditional,* for instance; if the assumed relation is correct as regards the choice of the variables and the lags, then the most likely value of the influencing coefficients is such and such.

If we want to draw positive conclusions, additional information is necessary to satisfy the conditions mentioned; this information must be given by economic theory. It is reasonable to submit this additional information to the following principles:

a. the principle of *statistical censorship:* it must not be contradictory to the data;
b. the principle of *scientific efficiency:* it must not be a consequence of theses already assumed, together with the data;
c. the principle of the *solid basis:* it should be as plausible as possible; and
d. the principle of the *sufficient basis:* it should be such that positive conclusions can be drawn. Between these last two principles a certain tension may often exist; we should try to find an optimum.

In most econometric investigations carried out with the help of multiple correlation, the additional information is approximately as follows:

1. The factors not included in the relation only lead to small random deviations or to
2. a trend influence or to
3. deviations in individual years which can be left out of consideration.
4. The factors included in the relation (explanatory variables) are active in a given mathematical way (often linearly, or only additively), whereas
5. the lags are given, or at least should be positive, and
6. certain regression coefficients also have a prescribed sign: sometimes the value of those coefficients is bound within certain limits.

The last three points require further explanation. As we know, the restriction embodied in statement 4 is not serious if the deviations of the variables from their averages are not large. The lags need not be given beforehand; they can be left undecided and be determined by the requirement of the best possible fit between x and x′. This can be done by trial and error, or alternatively, by including in any relation with the explanatory variables y and z, not only these variables for the same moment of time, but also for other moments, for instance

$$x' = ay + a'y_{-1} + bz + b'z_{-1}$$

The relative size of the regression coefficients a and a′ shows approximately how large the lag between x and y should be taken. If, for instance, $a = 2$ and $a' = 1$, then, by rough approximation (at least, if the time unit is small with respect to the period of the fluctuations in y):

$$x' = (a + a')y_{-\frac{1}{3}} + \ldots$$

Point 6 is the most important one. Certain regression coefficients must have a certain sign or a certain value, on account of their economic significance. For instance, the coefficient of the price in a demand equation should be negative, and in a supply equation, positive. A regression coefficient representing wage-price ratio, or the marginal propensity to consume will have to be smaller than 1.

Usually such requirements will not be made at the start, but the regression equation will be computed first.

As has been pointed out earlier, the following results are possible: (a) the correlation is lower than the standard established; the theory is rejected; (b) the correlation is good and the regression coefficients have a small margin of uncertainty; measurement is possible with relatively few limiting conditions; or (c) the correlation is good, but the regression coefficients or lags are relatively uncertain. In this case it is often possible to limit the area of uncertainty with the help of further conditions of the kind mentioned under statements 5 and 6. The value of

one or more of the lags or regression coefficients can be wholly or partly fixed.

The greatest success, of course, is obtained if such additional information is not necessary. Even then, relatively unexpected conclusions are sometimes reached: the influence of one of the factors may be small. From investigations made by the author, for instance, it has become clear that under certain assumptions to be mentioned later (see § 37), the influence of the rate of interest on investment activity is small; similarly as regards the influence of the so-called acceleration principle.

If we meet with success, we can try to mitigate one of the conditions; for instance, the mathematical form may be generalized or more factors may be included in the relation. We can divide the premises originally taken as a starting point into *acceptable* and *doubtful* premises, and from now on we can include only the acceptable ones in the calculation. As regards the doubtful ones, the following conclusions may be reached:

1. they are wrong;
2. they are correct, starting from the acceptable premises;
3. it cannot be decided whether they are correct or not, but if they are omitted, measurement is impossible, or
4. same as 3, but they are irrelevant for measurement.

Conclusions of the kind mentioned under 2 form a type which was not present in the first framework.

As an example of doubtful premises, we mention the assumption that one of the regression coefficients must lie within a given interval:

1. now occurs, if there is not a good correlation in which the coefficient lies within the interval;
2. occurs if there are only good correlations where this is the case;
3. and 4 occur if good correlations exist with values both within and outside the interval.

Another example is the following: assume that y exerts some influence on x, but that it is doubtful whether z also has any influence:

1. applies if the regression coefficient of z is approximately zero;
2. applies if the correlation is good with z and bad without z;
3. and 4 appear if the margins of uncertainty in z's coefficient are large.

Other situations, more favorable to the attainment of positive conclusions, occur if the economic theory leads to the formulation of hypotheses alternative to the doubtful ones; in other words, if the economist says, it is not certain whether z has any influence, but if it does not, then u has some influence.