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The Use of Econometric Research for the Purpose of Economic Policy (An Example)

§ 44. Object and Summary of This Chapter

The object of this chapter is:

1. to estimate, for the Netherlands, the influence of wage, tax, and price policy (including devaluation) and an increase in productivity on the balance of payments and employment;
2. to give some examples of "inefficient regulators" in economic policy; and
3. to illustrate a number of questions of methodology in economic analysis, viz.:
   a. the question when the results of economic analysis can be proved verbally;
   b. what simplifications in the analysis are possible by the introduction of new ("composed" or "derived") variables;
   c. to indicate the role of "directives" and "instruments" of economic policy in the analysis, and
   d. to illustrate the role of "boundary conditions" in modern economic analysis.

Using a new expression of Frisch's, these points will be dealt with by the introduction of a "decision model"* of a very simple nature; indeed on purpose as simple as seems possible without violating the nature of the subject (§ 45).

The coefficients used in the model are as much as possible based on statistical regression analysis for the interwar period. For this model, in order to close the balance of payments gap,

* Ragnar Frisch: "A Memorandum on Price-Wage-Tax-Subsidy Policies as Instruments in Maintaining Optimal Employment"; The University Institute of Economics, Oslo; publ. as an UN Document, April 1949.

the wage policy appears to be inefficient in that wage reductions of an acceptable size won't do the job (§ 48). The reason is that real incomes of nonworkers will rise and hence imports for their expenditures, which renders the total influence of wage reductions on the balance of payments only weak. The same appears to be true for reductions in nonlabor margins, since these raise real wages (§ 49). A combined policy, most easily obtained by a devaluation of the currency, gives somewhat better results, but still appears to be inefficient in one respect, viz. in that it increases incomes as a consequence of increased exports and hence still tends to charge imports "unduly" (§ 50). It is better, therefore, simultaneously to keep home consumption down by indirect taxation or government budget economies; by combining all these measures it appears to be possible to reach the goal by relatively small changes, viz. a 15 per cent devaluation (vis-à-vis the outside world as a whole) and an increase in indirect taxation by some 3 per cent of total national expenditure (§ 51). Some alternative tax measures are indicated (§ 52).

The same model enables us to study the effect of wage increases and changes in labor productivity on employment (§ 53). It appears necessary and possible to distinguish sharply between problems with different "side conditions," e.g. wage increases without or with automatic demands for a rise in real farm incomes or automatic financial measures to maintain balance of payments equilibrium.

It further appears that not all these results can be "proved verbally" in the usual sense of this phrase. As soon as simultaneous equations are involved this is essentially impossible (§ 54).

Finally it appears to be efficient to use, instead of the variables "wage rate" and "employment," two others, viz. "wage cost per unit of product" and "volume of production" (§ 46).

§ 45. Description of the Model and Its Alternatives

The model to be presented is a model in which all coefficients have been measured; in order to demonstrate the margins of uncertainty in the results, one central case and five alternative cases, showing different numerical values of the coefficients, have been considered. From the theoretical point of view the models represent a logical combination of two lines of thought in present-day literature with a refinement for the labor market. The two lines of thought may be indicated as the income approach, for which the Keynes-Machlup-Meade models are typical and the price approach, as given by Bickerdike,* Brown,† Joan Robinson,‡ Polak,§ Metzler,|| Faxén and Savosnick,¶ Neisser,** and others. Some of the latter authors have combined the two approaches, others have not, i.e. have assumed full employment throughout.

In our model, two products are distinguished, viz. a *national product,* sold abroad (exports) as well as at home (total expenditures) and an *import product,* used as raw material in the national product. The population is divided into *wage earners* and *entrepreneurs.* The former spend the whole of any additional income, and the latter only a certain part of it (mainly because of taxes which are supposed not automatically to be spent wholly at an increase in receipts). It appears to be possible, and attractive, to boil down the system of relations describing the model to three equations only (after elimination of a number of definition equations), which might conveniently be called

1. the demand equation for the national product by the home market;
2. the demand equation for that product by the world market; and
3. the supply or price-fixation equation for that product.

* C. F. Bickerdike, The instability of foreign exchange, The Economic Journal, 30 (1920), p. 118.

† A. J. Brown, Trade balances and exchange stability, Oxford Economic Papers, 6 (1942), p. 57.

‡ Joan Robinson, "Essays in The Theory of Employment," Oxford, 1947, p. 142.

§ J. J. Polak, Various unpublished reports of the International Monetary Fund.

|| L. A. Metzler, The theory of international trade, in: "A Survey of Contemporary Economics," Philadelphia, 1948, p. 210.

¶ K. O. Faxén and K. Savosnick, Växelkurs och handelsbalans, Ekon. Tidskr., 1949, p. 168.

** H. Neisser, Unpublished report of the Institute of World Affairs.

Since the purpose of the model is to discuss questions of economic policy, a number of phenomena will be considered as constants which could not be so in a model meant to explain economic history, such as world market prices, world market demand at constant supply prices, crops, etc.

Equation (1) is obtained from the demand equation proper (1, 1) and the definition of income (1, 2) by eliminating, as is usual in Keynesian analysis, total expenditure: the result is a kind of "multiplier equation."

The demand equation proper expresses that the increase in real expenditure $\bar{\bar{x}} - \bar{x}$, where $\bar{x}$ represents the mid-1949 figure of x, depends on the increases in real incomes of nonworkers $\bar{\bar{Z}}' - \bar{Z}'$, and of workers $\bar{\bar{L}}' - \bar{L}'$ in the following way:

$$\bar{\bar{x}} - \bar{x} = (1 - \sigma)(\bar{\bar{Z}}' - \bar{Z}') + (\bar{\bar{L}}' - \bar{L}') \qquad (1, 1)$$

This means that the marginal propensity to spend is $1 - \sigma$ for nonworkers and 1 for workers.

It will be clear that

$$\bar{\bar{Z}}' = \frac{\bar{\bar{Z}}}{\bar{\bar{p}}} \quad \text{and} \quad \bar{\bar{L}}' = \frac{\bar{\bar{L}}}{\bar{\bar{p}}}$$

where $\bar{\bar{Z}}$ is the nominal nonworkers' income

$\bar{\bar{L}}$ is the nominal wage bill, and

$\bar{\bar{p}}$ the price level.

Further, if Y is national income,

$\bar{\bar{Z}} = \bar{\bar{Y}} - \bar{\bar{L}}$

Indicating by $\bar{\bar{X}}$ the total expenditure and by $\bar{\bar{D}}$ the deficit on the balance of payments, we have in addition:

$$\bar{\bar{X}} = \bar{\bar{x}}\,\bar{\bar{p}}, \text{ and}$$
$$\bar{\bar{Y}} = \bar{\bar{X}} - \bar{\bar{D}} \qquad (1, 2)$$

which is the definition of the national income.

It follows from (1, 1) and the subsequent identities that

$$\bar{\bar{X}} = \bar{\bar{x}}\bar{\bar{p}} = (1 - \sigma)\bar{\bar{Z}} + \bar{\bar{L}} + \{\bar{x} - (1 - \sigma)\bar{Z}' - \bar{L}'\}\bar{\bar{p}} \qquad (1, 1')$$

The elimination of $\overline{\overline{X}}$ between (1, 1′) and (1, 2) and the substitution of $\overline{\overline{Z}}$ by $\overline{\overline{Y}} - \overline{\overline{L}}$ yield:

$$\overline{\overline{Y}} = (1 - \sigma)(\overline{\overline{Y}} - \overline{\overline{L}}) + \overline{\overline{L}} + c\overline{\overline{p}} - \overline{\overline{D}} \qquad (1, 3)$$

where $c = \bar{x} - (1 - \sigma)\, \bar{Z}' - \bar{L}'$; (1, 3) may be written in the form

$$\overline{\overline{Y}} = \overline{\overline{L}} - \frac{\overline{\overline{D}}}{\sigma} + \frac{c\overline{\overline{p}}}{\sigma} \qquad (1')$$

which reminds us of the multiplier equation; it is somewhat more complicated because of the different behavior of workers and nonworkers.

It will appear useful to rewrite this equation after we have discussed equations (2) and (3); hence we indicate it provisionally as (1′).

Equation (2) is meant to express the balance of payments deficit D as a function of the relevant variables. In our "decision model" only such phenomena have to appear as variables which may vary during any change in economic policy of the Netherlands, i.e. only Dutch phenomena. According to present-day views there are two main variables influencing the balance of payments, viz. the production level y of the country and its price level p. Imports m will depend on both; in a linear approximation* we have then:

$$m = \mu y + \bar{m}\epsilon^{m} p \qquad (2, 1)$$

where $m, y,$ and p are measured as deviations from initial values; for y and p these initial values are taken equal to 1. Under these circumstances, μ is the marginal import quota and ϵ^{m} the elasticity of imports with respect to the Dutch price level (foreign prices being constant). Value of imports is indicated by the same figures as volume, since foreign prices are equal to 1 throughout.

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* The restrictions implied in the use of linear approximations will be discussed in §§ 48, 49 and 50.

The volume of exports e will only depend on p:

$$e = -\bar{e}\epsilon^e p$$

if also e is measured as a deviation from initial value, and ϵ^e represents (the absolute value of) the elasticity of exports with respect to the Dutch price level. The money value of exports E (deviation from initial value) will be:

$$E = -\bar{e}(\epsilon^e - 1)\,p \tag{2, 2}$$

From equations (2, 1) and (2, 2) we deduce

$$D = \mu y + \{\bar{m}\epsilon^m + \bar{e}(\epsilon^e - 1)\}\,p$$

or

$$D = \mu y + \delta p \tag{2}$$

Equation (3) will be written as a "price-fixation equation"; the price level will depend on labor costs per unit of product l' (deviations from initial value which will be taken equal to 1), volume of production y, and possible autonomous changes π_0 of the profit margin as may be the consequences of either a government price policy or, e.g., the liberalization of international trade. Writing p for the price level (deviation) we have*

* This formula may be deduced in the following way. To begin with, assuming free competition, we have: price = marginal cost. The changes in marginal cost will depend on at least three factors, viz.:

1. the development in labor cost per unit of product in a fixed enterprise, viz. the marginal enterprise in 1949;
2. differences in costs due to the expansion or contraction of production, causing a shift of the marginal unit of production;
3. a number of autonomous factors such as indirect taxes, price regulations, or changes in the remuneration of nonworkers.

We shall assume that the development meant under 1 may be represented by the movements of our variable $\bar{l}'$, i.e. we assume a parallelism between average labor cost of unit of product and such costs in a fixed enterprise. The difference meant under 2 will be a function of the general level of production $\bar{y}$; the autonomous factors will provisionally be indicated by $\bar{f}$. Hence, marginal cost may be written as $\phi\,(\bar{l}', \bar{y}, \bar{f})$ and $\bar{p} = \phi\,(\bar{l}', \bar{y}, \bar{f})$. This function we develop into a series, taking only the members of zero and first degree in the deviations l', y, etc.:

$$\bar{p} + p = \phi\,(\bar{l}', \bar{y}, \bar{f}) + l'\,\frac{\overline{\delta\phi}}{\delta l'} + y\,\frac{\overline{\delta\phi}}{\delta y} + f\,\frac{\overline{\delta\phi}}{\delta f}$$

Leaving out the 1949 values which cancel, we have

$$p = l'\,\frac{\overline{\delta\phi}}{\delta l'} + y\,\frac{\overline{\delta\phi}}{\delta y} + f\,\frac{\overline{\delta\phi}}{\delta f}$$

The differential coefficients are constants and the first two may be written as π_1 and π_2; the third term as a whole as π_0.

$$p = \pi_1 l' + \pi_2 y + \pi_0 \tag{3}$$

where π_1 represents the marginal wage quota, and
π_2 the "flexibility of prices," being the inverted supply elasticity.

At this moment we may return to equation (1′) and rewrite it in a more convenient form. It will be possible to express both Y and L in terms of y, p, and l'. Using the symbols, as was done in (1′), in their meaning of absolute values, we have:

$$\overline{Y} = (1 + \mu')\, \overline{y}\overline{p} - \overline{m}$$

where μ' is the average import quota; in fact, $(1 + \mu')\, \overline{y}\overline{p}$ represents the gross value of the national product, from which imports have to be deducted in order to obtain the net value. Since for the Netherlands the marginal import quota happens to be equal to the average import quota, we have substituted μ for μ' in what follows.

We have also: $\overline{L} = \Lambda \overline{l'} y$, where Λ represents the initial value $\overline{L}$ of L; since $\overline{Y} = 1$, Λ represents the wage quota in the national product.

We shall now replace all the variables by their deviations from initial values and simplify the resulting equations by omitting initial values at both sides of each one (since they are equal) and products of deviations (since they are supposed to be small). This leads to:

$$Y = (1 + \mu)\, p + y - \mu\epsilon^{m} p$$
$$L = \Lambda\, (l' + y)$$

Equation (1′) may, also using deviations, then be written:

$$\sigma(1 - \Lambda)\, y + D + \{1 - \overline{x} + \sigma\,(\mu + \Lambda - \mu\epsilon^{m})\}\, p = \sigma\Lambda l'$$

or

$$\xi_1 y + D + \xi_2 p = \xi_3 l'$$

In this equation we shall add a political parameter ξ_0 representing an autonomous increase in national expenditure, e.g. by an increase in investment outlay or government expenditures, or by certain measures of taxation or subsidizing.* This gives to this equation the form:

$$\xi_1 y + D + \xi_2 p = \xi_3 l' + \xi_0 \tag{1}$$

The other two equations may now be added using the same order of the variables:

$$-\mu y + D - \delta p = 0 \tag{2}$$

$$-\pi_2 y \qquad + \ p = \pi_1 l' + \pi_0 \tag{3}$$

All of the seven coefficients have more or less exactly been measured statistically; those appearing in equations (2) and (3) may be taken from multiple-correlation studies for the period 1923 to 1938, not only macroeconomic, but also to some extent microeconomic in character.† The coefficients of equation (1) depend, apart from μ and ϵ^m, on the structural parameters λ and $\bar{x}$ and the coefficient σ. For the situation in 1949, λ and $\bar{x}$ are fairly exactly known and equal to 0.55 (portion of national income going to workers) and 1.04 (national expenditures expressed in national income).‡ The coefficient σ is by far the least certain; under present conditions it seems, however, to be definitely positive. This is primarily due to the high marginal rate of taxation and the probability that an increase in tax receipts will only slightly influence the level of public expenditure. It is partly due also to the circumstance that total expenditure of the nation already surpassed its income for years as a consequence of the war and the resulting desire to do away with dissaving.

In view of the uncertainty margin in some of the coefficients and in particular σ, a number of alternative models have been

* A more detailed treatment will be given in § 52.

† For *exports* see J. B. D. Derksen and A. L. G. M. Rombouts, The influence of prices on exports, De Ned. Conjunctuur, Special Memorandum Nr. 1, The Hague 1939. For *imports* see J. Tinbergen, The fluctuations of the Netherlands imports, 1923-1938, Stat. en Econometr. Onderzoekingen 3 (1948), p. 52 (Dutch with an English summary). For *prices* see J. Tinbergen en A. L. G. M. Rombouts, De samenhang tussen de prijzen van ingevoerde grondstoffen en die van afgewerkte producten in Nederland, De Ned. Conjunctuur 10 (1939), p. 150 (Dutch).

‡ See Centraal Economisch Plan 1949, The Hague (Dutch), p. 20.

used, which are characterized by the following values of the coefficients (where the alternative values have been printed in italics):

	Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
$1-\sigma$	marginal prop. to spend	0.7	*1.0*	*0.5*	0.7	0.7	*0.9*
ϵ^m	price elasticity of imports	0.3	0.3	0.3	*0.5*	0.3	0.3
ϵ^e	price elasticity of exports	2.0	2.0	2.0	*3.0*	2.0	2.0
π_1	marginal wage quota	0.3	0.3	0.3	0.3	*0.5*	0.3

The variation in the last coefficient may be interpreted not so much as a variation in the marginal wage quota proper but more as the tendency for certain trade margins to move in sympathy with direct (mostly labor) costs.

Model Nr. 1 is considered to be the most likely representation of the year-to-year reactions of the Dutch economy.

From the above-listed values of the fundamental coefficients the following values for the dependent coefficients in the equations may be found:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
$\xi_1 = 0.45\,\sigma$	0.135	0	0.225	0.135	0.135	0.045
$\xi_2 = -0.04 + \sigma(0.99 - 0.44\epsilon^m)$	0.217	−0.04	0.389	0.191	0.217	0.046
$\xi_3 = 0.55\,\sigma$	0.165	0	0.275	0.165	0.165	0.055
$\delta = \bar{m}\epsilon^m + \bar{e}(\epsilon^e - 1)$	0.53	0.53	0.53	1.02	0.53	0.53

The coefficients μ and π_2 have been taken equal to 0.44 and 0.125 throughout; the first since it seems to be pretty exact and the second since its influence on the results appeared to be only small, within the probable limits of variation.

For the reader's convenience the initial (1949) values of the various variables may be summarized here:

$\bar{y} = \bar{p} = \bar{l}' = \bar{Y} = \bar{1}$; $\bar{x} = 1.04$; $\bar{m} = 0.44$; $\bar{e} = 0.40$; $\bar{D} = 0.04$; $\bar{L} = \bar{L}' = 0.55$; $\bar{Z} = \bar{Z}' = 0.45$.

§ 46. Choice of Variables; Boundary Conditions

Before discussing the use that has been made of the models, a few preliminary remarks may be made about what seems to the author to be a feature of efficiency in the choice of the variables and equations. The complaint is rather general that the older models used in econometrics are often too complicated to be understood. On purpose therefore a very simple setup has been chosen here, where only the most strategic variables are left. As the characteristic parameter for the labor market, whose inclusion was essential for the subjects to be dealt with, the *index of labor cost l' per unit of product* has definite advantages in comparison to the *wage rate l*. Essential for the whole argument is the influence of wages on prices at the one hand and labor income on the other hand. Now for prices l' is decisive indeed rather than l; whereas total labor income may either be written as the product of l' and y or as the product of l and employment a. Introduction of l into the "central system" of the three equations, therefore, would have meant the introduction of a. The choice of l' enables us to avoid this. Of course a may and will be considered later on; but it need not be included into the central simultaneous kernel of relations. This seems to be a good example of an efficient choice of a "composed" or a "derived" variable able to simplify the system. The choice of D instead of imports and exports separately, although more familiar, seems to be another example. Both are examples of what may be called the "art" of scientific analysis and are of some interest for the understanding by general economists of econometrics.

Another remark may be made before using our equations. They have only a restricted "range of validity"; as soon as some of the variables surpass certain limits or boundaries they are no longer valid. Such *boundary conditions* may be of a different nature and there may be several of them, partly overlapping, for the same variable or set of variables. A few examples will be given. One is the condition that y cannot surpass the value y^F corresponding with full employment. If in any problem a solution for y would be found greater than y^F, equation (1) no longer applies and instead we have to replace it by:

$$\mathbf{y} = \mathbf{y}^{\mathbf{F}}$$

At the same time (3) should be changed so as to express that p may rise more than indicated by (3).

Other boundary conditions are of course that neither y nor p or l' be less than -1 (or, in absolute amount, negative). Long before such a limit would be reached, however, certain psychologic or political boundaries would have been attained, the exact place of which is not known and must be left to the judgment of the reader.

The role of these boundary conditions may be very important; in fact they may sometimes far more influence the situation than our equations. They may even be such as to exclude any solution at all if their number, together with the number of valid equations, surpasses the number of unknowns.*

§ 47. Directives and Instruments; the Strategy of Economic Policy

With the traditional setting of the problem, the political parameters (in our example l', ξ_0, and π_0) are considered as given and the economic phenomena proper (in our example y, D, and p) as unknown. We then ask: what are the consequences of a given increase in wage rates l', or of a given increase in government expenditure ξ_0, etc.† and our equations enable us to answer such questions easily. Since, however, an economic policy has a certain purpose, it seems more appropriate to invert the roles of these two classes of variables. In order to clarify the matter entirely, we introduce the concepts of *"directives"* and *"instruments"* of economic policy. If the wage rate is used to obtain a closed balance of payments, we call the wage rate the instrument

* There is an interesting development of economic analysis toward the discussion of such situations, as is, e.g. clear from the recent work on linear programming.

† It is not essential, in this respect, that, as a simplification, we have considered wage rates as autonomous and not as consisting of an autonomous and a dependent component, viz. a component determined by the automatic reactions of workers on prices, etc. Under present social and political conditions this seems to be a feasible approximation to reality.

of this policy and the directive is that particular value of the variable D which corresponds with equilibrium (in our case $D = -0.04$). The directives are, with this new setting of the problem, the data, and the instruments are the unknowns. This is characteristic of the decision models. Also, now our equations may be used to determine the unknowns, provided that their number is not too large or too small. This latter remark reminds us of the necessity, in this case in particular, of a correct statement of our problems and the choice of a correct number of unknowns. Among these, some of the old unknowns may remain, if the number of unknown political parameters is smaller than the number of equations. It may be observed finally that further political parameters may be introduced: in fact one could say, e.g. that the Marshall help represented such a parameter in equation (2). The parameter l', in addition, may be split up into two components, the wage rate l and the index of productivity h (we shall again take $\bar{h} = 1$), in the following way:

$$l' = l - h \tag{4}$$

where under the present conditions each of these components is, to some extent, an autonomous parameter.

Each of the political parameters can be determined by a restricted number of "pressure groups": l' by trade unions (or more precisely by trade unions, employers, and government together), π_0 by employers (more precisely by employers and government), ξ_0 by government (and parliament). Economic policy, therefore, is a strategic game in the sense of Morgenstern and Von Neumann. Recent experience in this field shows a number of interesting examples of this play of chess: devaluation as a pure act of government, followed by a demand for 5 per cent wage increase by the workers. The government may be compelled to decrease subsidies as a further response, etc.

Actual economic policy will therefore differ from some well-advised welfare planning based on an attempt to indicate an "optimum solution" (see § 51).

§ 48. Isolated Wage Policy; Consequences of the Double Function of Wages

As a first example of the various problems one may alternatively formulate let us consider isolated wage policy. The well-known question how wage-rate changes influence employment may at once be answered by our system of equations. We shall assume proportionality between employment and volume of production y; we could have supposed a general linear relationship as well. Solving our equations for y we have:

$$y = \frac{\begin{vmatrix} \xi_3 l' + \xi_0 & 1 & \xi_2 \\ 0 & 1 & -\delta \\ \pi l' + \pi_0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \xi_1 & 1 & \xi_2 \\ -\mu & 1 & -\delta \\ +\pi_2 & 0 & 1 \end{vmatrix}} = \frac{\{\xi_3 - \pi_1(\delta + \xi_2)\} l' + \xi_0 - (\delta + \xi_2)\pi_0}{\xi_1 + \mu + \pi_2(\delta + \xi_2)} = \eta l' + \eta' \xi_0 + \eta'' \pi_0 \tag{5}$$

The numerical values for η, η', and η'' are:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
η	-0.09	-0.29	0.00	-0.27	-0.31	-0.21
η'	1.50	2.00	1.28	1.38	1.50	1.80
η''	-1.12	-0.98	-1.18	-1.67	-1.12	-1.03

It should be kept in mind that this formula is only a correct approximation for small values of the political parameters l', ξ_0, and π_0. Even then η, η', and η'' could have been unreliable, if the determinant in the denominator were close to zero; this, however, does not appear to be the case.

From the results it is clear that the elasticity of the demand for labor (coinciding with η) is very modest according to this model. A closer study of the numerical values obtained for the other variables shows that this is due to the well-known double function of wages, viz. that they are on the one hand an element of consumer demand with a high propensity to spend and, on the other hand, a cost and price element.

A second problem on wage policy arises if, as is advocated in certain quarters, this policy is proposed in order to restore balance of payments equilibrium. This means assuming $\xi_0 = \pi_0 = 0$ and asking for what value of l' the condition $D = -0.04$ is fulfilled. This is an example of the (partially) inverted problem referred to in § 46; D now being given and l' unknown. The other political parameters ξ_0 and π_0 are, however, given and hence the other variables y and p must be considered as unknowns: they cannot be chosen freely.

We might solve the present problem by solving, similarly to (5), our system (1) to (3) for D and putting $\xi_0 = \pi_0 = 0$ and $D = -0.04$. A somewhat shorter way consists, however, of rewriting our equations:

$$\left.\begin{array}{rl} \xi_1 y - \xi_3 l' + \xi_2 p = -D = 0.04 \\ -\mu y \qquad\qquad - \delta\, p = -D = 0.04 \\ -\pi_2 y - \pi_1 l' + \quad p = 0 \qquad\qquad \end{array}\right\} \tag{6}$$

and solving for l'; leading to:

$$l' = \frac{0.04\,(\xi_1 + \mu + \delta\pi_2 + \pi_2\xi_2)}{-\xi_1\delta\pi_1 - \xi_3\mu - \xi_3\delta\pi_2 + \xi_2\mu\pi_1} \tag{7}$$

the numerical values of which are:

Model Nr.	*1*	2	*3*	*4*	*5*	*6*
$l' =$	-0.35	-3.75	-0.25	-0.26	-0.37	-0.77

One general remark should be made here before interpreting these results. Most of them are outside the range of what could be considered small changes; hence the values are only exact solutions to the linearized system of equations. They will be somewhat different from the solutions of the nonlinear equations from which we started; they may even be quite different. We shall not, however, make use of the numerical results as such.

The interpretation of the result for model 2 is that it is meaningless in that it surpasses the possible boundary condition that l' in absolute value shall be positive. It is not meaningless, however, in that it shows that no equilibrium in the balance of payments can, in this model, be obtained by small wage reductions.

For the other models too the results show that small wage reductions cannot do the job. Even the real wage reductions would be quite different from small, as the figures for p, obtainable from (6), show:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
$p =$	-0.10	-1.00	-0.08	-0.07	-0.17	-0.21

Wage rates may therefore be said to be bad regulators of the balance of payments under present conditions in the Netherlands (and probably in many other countries). A closer study of the other variables shows that the reason is that wage reductions tend to increase real incomes of nonworkers and thus to burden the balance of payments which partly compensates the favorable direct effects; hence such a strong reduction is necessary.

It is interesting to note that this failure of wage rates as "single regulators" of the balance of payments is not due to the phenomenon of "critical elasticities" of external trade, frequently discussed.* This we see from the denominator of (7) which does not vanish for positive values of δ; in fact, when we vary δ it only vanishes for:†

$$\delta = \delta_0 = \mu \frac{\xi_2 \pi_1 - \xi_3}{\xi_1 \pi_1 + \xi_3 \pi_2} \tag{8}$$

the numerical value of which is negative for all models. This difference between wage rates and the general price level or the rate of exchange is also due to the dual role of wages just mentioned and expresses itself *inter alia* in the appearance of l' in equation (1). If it did not appear there it would only influence D through p and the case of critical elasticities would apply.

* See L. A. Metzler, The theory of international trade, in: Ellis, "A Survey of Contemporary Economics," Philadelphia, 1948, p. 210.

† This argument is not quite exact, since ϵ^m also influences ξ_2; it is only exact if we consider ϵ^e as subject to variations. Since ϵ^e is the more important of the two, there is no serious objection against doing so.

§ 49. Isolated Price Policy

As a counterpart to an isolated wage policy we may now consider an isolated price policy, i.e. a policy of deliberate lowering of price margins, say by price control or by a liberalization of international trade tending to increase competition. In our formulas this comes to handling π_0 as a political parameter and asking for the consequences. We shall consider only the question, what decrease in π_0 would be necessary in order to fill the gap in the balance of payments? Since the method of answering this question is similar to the one used in the preceding section we need only give the numerical results which are:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
π_0 (necessary for $D = -0.04$)	$+1.11$	-1.14	$+0.60$	-0.54	$+1.11$	$+5.7$

The meaning of these apparently meaningless figures is that in none of the models a price fall within the limits set by reality, or by the range of validity of our equations, will be able to close the gap. Even in model 4, where the decrease is less than 1, it surpasses nevertheless the margin left after deduction of wages (30 per cent) and imports (31 per cent). In model 2 it is more than unity and in the other models only a price rise would lead to a (possibly unstable) equilibrium in the balance of payments. If we restrict ourselves—as we should—to the regions of validity of the equations, the answer must be that the influence of prices on the balance of payments is small and of doubtful algebraic sign. A closer examination leads to the explanation of this result by two major factors: first, there is the counterpart to the influence of wages. A fall in prices lowers nonlabor incomes but raises real wages, leading to extra imports. Secondly, there is the question of critical elasticities, referred to at the end of the preceding section. The analytical expression for π_0 in this problem being

$$\pi_0 = \frac{0.04\ (\xi_1 + \mu + \pi_2\delta + \pi_2\xi_2)}{\mu\xi_2 - \delta\xi_1} \tag{9}$$

we find that its denominator becomes equal to 0 for $\delta = \delta_0 = \frac{\mu\xi_2}{\xi_1}$, being a positive figure not far from the actual value of δ. The introduction of wages into our model has therefore brought the critical elasticity of the balance of payments with respect to prices to a higher level and in fact to the neighborhood of δ.

For practical policy our conclusion is therefore that neither wage policy nor price policy should, as an isolated policy, be applied in order to close the gap in the balance of payments. Each of them would disturb the equilibrium between the two big classes of society too seriously and as a consequence be ineffective.

§ 50. Devaluation

It seems natural therefore to think of a combined policy of lowering prices and wages at the same time, in order to preserve this "social equilibrium" and consequently get an efficient tool to close the balance of payments. The combination may be tried with different "weights"; but by far the most practical form of this type of policy would seem that of a devaluation of the national currency. This in fact comes to lowering, in the same proportion, wage rates and all other national cost items expressed in foreign currency. For our formulas this would mean that the percentage fall in prices would be less than that of wages since 31 per cent of prices in the initial situation is imports; hence we would have the condition

$$\lambda l' = 0.69\, l' = p \tag{10}$$

Substituting this condition into our equations and taking $\xi_0 = 0$, we obtain:

$$\left.\begin{array}{l} \xi_1 y + D + \xi_2 \lambda l' = \xi_3 l' \\ -\mu y + D - \delta\lambda l' = 0 \\ -\pi_2 y \qquad + \lambda l' = \pi_1 l' + \pi_0 \end{array}\right\} \tag{11}$$

Again we may study, in the traditional way, the influence of devaluation on y and D, e.g. by solving these equations, now considering l' as given. The third equation would be superfluous for finding y and D, but would give us π_0. Again it is easier, however, to impose a given value of D and instead consider l' as an unknown. The structure of the equations then becomes a simpler one from the mathematical point of view; the third equation, being the only one which contains π_0, may be disregarded as long as we are not interested in π_0 and the two others yield us y and l'. It is particularly interesting to solve for l', indicating at once the percentage of devaluation which would be necessary if devaluation were the only policy pursued. Its analytical expression is:

$$1' = \frac{0.04\ (\xi_1 + \mu)}{-\xi_1 \delta\lambda - \mu\xi_2\lambda - \mu\xi_3} \tag{12}$$

and its numerical values are:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
l' (devaluation)	−0.41	−1.46	−0.31	−0.21	−0.41	−0.72

Apart from the figure for model 2 these figures are already less unrealistic than those obtained in the case of wage policy or price policy separately. Nevertheless they are still pretty high, even in the central model. A closer study of the values found for the other variables in this case shows that devaluation as the only policy to obtain balance of payments equilibrium is still not very efficient. This may be illustrated by the figures of the following table.

Values for the Other Variables in the Case of Devaluation

(Degree of devaluation* necessary to equilibrate the balance of payments.)

	Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
p	price level	−0.28	−1.00	−0.22	−0.15	−0.28	−0.50
y	volume of production	0.25	1.12	0.17	0.24	0.25	0.51
x	real expenditures	0.13	0.81	0.07	0.18	0.13	0.34
Y	national income	−0.12	−0.19	−0.11	+0.07	−0.12	−0.14
L	wage bill	−0.09	−0.18	−0.08	0.02	−0.09	−0.12
Z	other income	−0.03	0.00	−0.03	0.05	−0.03	−0.03
L'	real wage bill	0.07	0.37	0.04	0.10	0.07	0.16
Z'	real other income	0.09	0.44	0.06	0.11	0.10	0.19
M	imports	0.07	0.36	0.05	0.08	0.07	0.16
E	value of exports	0.11	0.40	0.09	0.12	0.11	0.20

* As indicated above.

Again, these figures should not be taken too literally; the reservations made in § 48 are again valid. They only show two things; first, that a small degree of devaluation will not be sufficient, and second, what the ratios between small changes in the wage-and-price level on one side and the corresponding changes in the other variables will be.

From the table it is clear that devaluation will automatically lead to a considerably higher volume of consumption and production; the most obvious reasons being that larger exports lead to larger incomes and hence larger consumption. Of course, these findings are subject to the qualification that they are not outside the region of validity of our equations. Under the actual conditions of the Netherlands, e.g. with almost full employment, a further rise of production by 25 per cent would hardly be possible at short notice. Apart from this it would seem more efficient to maintain consumption as far as possible until the balance of payments would be in equilibrium and only then to increase it. Evidently this will not happen automatically by devaluation. It should be aimed at on purpose, i.e. by an additional political parameter.

§ 51. Combined Wage, Price, and Tax Policy: the "Optimum" Solution

It is here that we may introduce our third parameter, ξ_0, one interpretation being that by a decrease in subsidies or an increase in indirect taxes, home prices may be raised in comparison to export prices, tending to keep consumption on the predevaluation level. The same might be done by a direct decrease in other government expenditures.

Almost the same problem, and even a somewhat simpler mathematical version of it, arises if by mere technical necessity we have to keep down the volume of production on the predevaluation level and if we wish to make room for the increased exports by pressing consumption somewhat down. In this latter case the problem is: given D and y, given also that $p = \lambda l'$, how shall we choose ξ_0, l', and π_0? The role of data and unknowns in our system of equations has now been completely inverted. And the three equations enable us to find the degree of devaluation l', the reduction ξ_0 in national expenditures, and π_0 and from them all other variables. The interesting thing from the mathematical point of view is that even equation (2) immediately yields l', the only unknown appearing in that equation; equation (1) then enables us to find ξ_0 and equation (3) to find π_0. It is easily found that for models 1, 2, 3, 5, and 6 the degree of devaluation is now:

$$l' = \frac{-0.04}{\lambda\delta} = \frac{-0.04}{0.69 \times 0.53} = -0.11 \tag{13}$$

and for model 4:

$$l' = \frac{-0.04}{0.69 \times 1.02} = -0.06$$

The reduction of national expenditure appears to be very near to -0.04 everywhere, being equivalent to an increase in home prices by about 4 per cent.

The problem we started this section with is a little bit more complicated. Here we may express all the unknowns, i.e. the three political parameters, ξ_0, l', and π_0 and all the variables except D, p, and y in terms of y which now acts as a mathematical parameter.

We find:

$$\left.\begin{aligned} l' &= -\frac{\mu y + 0.04}{\delta\lambda} \\ \pi_0 &= -\pi_2 y - \frac{(\lambda - \pi_1)(\mu y + 0.04)}{\delta\lambda} \\ \xi_0 &= \xi_1 y - \frac{(\xi_2\lambda - \xi_3)(\mu y + 0.04)}{\delta\lambda} - 0.04 \end{aligned}\right\} \tag{14}$$

Using equation (1, 2) and expressing l' and p $(=\lambda l')$ in y with the aid of (14) we find:

$$x = y + (0.04 - \mu + \mu\epsilon^{m})\left(\frac{\mu y + 0.04}{\delta}\right) - 0.04 = 0$$

Putting this expression equal to 0, we find y and hence, by (14), the political parameters. The results are:

Model Nr.	*1*	*2*	*3*	*4*	*5*	*6*
y volume of production	0.08	0.08	0.08	0.05	0.08	0.08
l' degree of devaluation	−0.20	−0.20	−0.20	−0.09	−0.20	−0.20
ξ_0 incr. in national expenditure	−0.027	−0.034	−0.021	−0.030	−0.027	−0.032

It is interesting to note that

1. the necessary reduction of national expenditure lies within the range of real possibilities, whereas
2. the resulting increase in production seems possible at short notice;
3. the degree of devaluation now necessary is of the order of magnitude of the actual relative devaluation that took place in Sepetmber 1949.*

In fact, this is the only case of those so far considered where small values for the unknowns are found and, hence, some reliance can be placed upon the numerical values. It is the only case where, on the other hand, this is necessary; in the other cases only a negative use needed to be made of the results.

There is some reason to speak of an optimum solution now since (a) some sort of equilibrium between workers' and non-workers' incomes is preserved, (b) the increase in production is devoted to the balancing of foreign accounts before admitting an increase in consumption, which (c) makes it possible to devalue only slightly. One could, however, formulate other opti-

* The guilder devalued 30 per cent with respect to the dollar, but only 13 per cent with respect to the average of foreign currencies when weighted according to Dutch exports, and 20 per cent when weighted according to imports.

mum solutions and of course this depends upon the practical choice one makes as to the relative weights of the various interests concerned. Some alternative choices were made and the corresponding solutions calculated. One was to postulate that real incomes of workers and nonworkers should be affected proportionately; it results in almost the same solution as the one presented. Others might be tried, according to whether one judges the labor share in national consumption too small or too large (it is, in the Netherlands, considerably larger than before World War II).

Also in another, and perhaps even more important, respect our solution represents an "optimal" solution; it is an attempt to handle the three political parameters at the same time with one single directive. In this respect it is fundamentally different from the situation which would develop as a result of the strategic game between the various "pressure groups" mentioned before (including the government as the representative of the general interest). Each of these groups acts on different directives. It would be interesting, and in fact constitutes an item on the author's agenda, to find out what type of results emerges.

Dealing with the "inverted" problems typical for "decision models" we may observe at this moment that there is, in our special case, one of the possible problems which is not soluble, viz. the problem to find the values of the three political parameters if the variables y, D, and p are given. This is due to the special circumstance that in equation (2) no political parameter enters and that hence (2) cannot be fulfilled if all three variables are given. This is again an illustration of the many anomalies from general theory that may occur if the equations have special forms: a possibility too often forgotten by the mathematical economists of the Lausanne school and only detectible by a specific choice of the equations. Even if these are only approximations to reality there will be only a very narrow range of possible combinations of y, D, p.

§ 52. A Closer Examination of Tax and Subsidy Policy

We may introduce somewhat more explicitly some elements of tax and subsidy policy by writing:*

τ' for the excise tax rate, to be applied to the value of home sales $\bar{X}$;

θ'_1 for the profit tax rate, to be applied to gross profits $\bar{Y} - \bar{L}(1-\theta_2)$;

θ'_2 for social charges, to be applied to total wages $\bar{L}$, as an additional payment to be made by entrepreneurs (these charges may be also such taxes on wages as are paid by entrepreneurs, even if formally assessed to workers); and

θ' for all benefits received by workers, expressed equally as a multiplier to $\bar{L}$.

We have accordingly to define more exactly some of our variables which are affected by the tax system. Let $\bar{L}$ and $\bar{\bar{Z}}$ represent wages and other income *after tax*, let $\bar{Y}$ be national income *"at factor cost,"* $\bar{X}$ be home sales, and $\bar{p}$ be prices *excluding excise tax*.

The following changes have to be made in our equations:

$$(1, 1)\colon\ \bar{\bar{x}} - \bar{x} = (1 - \sigma)(\bar{\bar{Z}}' - \bar{Z}') + (\bar{\bar{L}}' - \bar{L}') + \bar{\bar{\theta}}'\bar{\bar{L}}' - \bar{\theta}'\bar{L}'$$

$$\text{where } \bar{\bar{Z}}' = \frac{\bar{\bar{Z}}}{\bar{\bar{p}}(1 + \tau')} \text{ and } \bar{\bar{L}}' = \frac{\bar{\bar{L}}}{\bar{\bar{p}}(1 + \tau')}$$

$$\bar{Z} = \{\bar{Y} - \bar{L}(1 + \theta'_2)\}(1 - \theta'_1)$$

(1, 2): remains unchanged.

$$(1, 1')\colon\ \bar{X} = \bar{\bar{x}}\bar{\bar{p}} = (1 - \sigma)\frac{\bar{Z}}{1 + \tau} + \frac{\bar{L}}{1 + \tau} + \frac{\theta'\bar{L}}{1 + \tau} + c'\bar{\bar{p}}$$

$$(1, 3)\colon\ \bar{Y} = (1 - \sigma)\frac{\{\bar{Y} - \bar{L}(1 + \theta'_2)\}(1 - \theta'_1)}{1 + \tau} + \frac{\bar{L}(1 + \theta')}{1 + \tau'} - \bar{D} + c'\bar{\bar{p}}$$

* I have added primes to these symbols in order to avoid confusion with the symbols used by Frisch (Mathematical Appendix to Memorandum on Price-Wage-Tax-Subsidy Policies as Instruments in Maintaining Optimal Employment). These are all conceived of as multipliers to national income at market prices. Our symbols are rates in the practical sense.

where $c' = \bar{x} + (1 - \sigma)\bar{z}' - (1 - \theta'_0)\overline{L}'$ and θ'_0 is the value of θ' prevailing in 1949.

So we get for equation (1):

$$\begin{aligned}
(1 + \tau')\xi_1 &= \tau' + \sigma + \theta'_1 - \sigma\theta'_1 + \Lambda\{-\} \\
(1 + \tau')\xi_2 &= (1 + \mu - \mu\epsilon^{m})(\tau' + \sigma + \theta'_1 - \sigma\theta'_1 - c(1 + \tau') \\
(1 + \tau')\xi_3 &= -\Lambda\{-\} \\
(1 + \tau')\xi_0 &= -\tau' - \sigma - \theta'_1 + \sigma\theta'_1 + (c - 0.04)(1 + \tau') \\
&\quad - \Lambda\{-\}
\end{aligned}$$

$$\text{where } \{-\} = \{-\sigma(1 + \theta'_2)(1 - \theta'_1) + \theta'_2 - \theta'_1 - \theta'_1\theta'_2 - \theta'_2\}$$

For small tax rates we may neglect their products and we find:

$$\begin{aligned}
(1 + \tau')\xi_0 = {} & -\tau'(1.04 - c) - \theta'_1(\sigma - 1 - \Lambda - \Lambda\sigma) \\
& - \theta'_2(\Lambda - \sigma\Lambda) + \theta'\Lambda - \sigma + c + \Lambda\sigma - 0.04
\end{aligned}$$

Assuming $\sigma = 0$, we have, for the case in which no private hoarding takes place,

$$\xi_0 = \frac{1}{1 + \tau'}\{-\tau'(1.04 - c) - \theta'_1(1 - \Lambda) - \Lambda\theta'_2 + \theta'\Lambda\}$$

In that case, $c = 0.04$ and the "weights" with which the various tax rates now appear in this political parameter may be easily understood: an increase in τ' decreases, in the same proportion, national expenditure since it "presses" upon the total of them; increases in θ'_1 and θ'_2, do so for the part of national income going to nonworkers and workers, respectively, whereas an increase in θ', on the other hand, stimulates national expenditure in the same intensity as θ'_2 would press it down. With the degree of approximation used here, all these changes can be substituted in the proportions indicated. As soon as the tendency to hoard is introduced by making σ greater than 0, their substitutability is changed as indicated in the last formula but one. If supply of labor had been relevant (with the situation of supply pressure, we consider, it is not), still other differences would have been found. See Frisch's memorandum just quoted.

§ 53. The Influence of Wages upon Employment under Various Side Conditions

In § 48 we discussed the consequences and possibilities of wage policy as regards the balance of payments. An important further aspect of wage policy is its influence on employment. Our equations enable us to compute the influence of an autonomous change in wage rates on employment, if a relation between the volume of production and employment is added. As long as labor productivity remains unchanged, they are simply proportional and may be represented by the same index y. As soon as changes in labor productivity h (also considered as an index number with $\bar{h} = 1$ and measured as deviations from this value) are introduced we have to distinguish between y and employment a (with $\bar{a} = 1$, say). Their relation is

$$a = y - h \tag{15}$$

At the same time, then, we have to distinguish between the true wage rate l ($\bar{l} = 1$) and the index of labor cost per unit of product l', their relation being

$$l' = l - h \tag{4}$$

as was stated in § 47.

Under the prevailing conditions we may and will, in fact, assume that changes in h are autonomous, depending as they do on (1) the psychologic-political attitude of certain groups of workers, and (2) such entrepreneurial initiatives as the introduction of work classification, better industrial relations, or the organization of many plants.

To begin with, we will assume that labor productivity is constant. The influence of the wage rate on employment, other political parameters remaining constant, may then be found as indicated in § 48, equation (5). Under present conditions there are good reasons, however, to drop the assumption of other constant parameters, and take into account, when considering the consequences of wage policy, the probable reactions of the government and possibly of others. This comes to considering the consequences for employment of wage changes with "side conditions." Of course these consequences are different from those expressed in equation (5). An important special case is the one in which we assume that the government, by tax measures, tries to preserve the balance of payments equilibrium. In this case we have as data: l', $D = -0.04$, and $\pi_0 = 0$. Our unknowns are y, in which we are particularly interested, p, and ξ_0. The equations, written in the relevant order, are:

$$\left.\begin{array}{l} \xi_1 y + \xi_2 p - \xi_0 = \xi_3 l' + 0.04 \\ -\mu y \ \ - \delta p \qquad\quad = +0.04 \\ -\pi^{2} y + \ \ p \qquad\quad = \pi_1 l' \end{array}\right\} \qquad (16)$$

Since ξ_0 does not interest us in particular, we have only to eliminate p from the last two equations, leading to

$$y = -\frac{0.04 + \delta'\pi_1 l}{\mu + \delta\pi_2} \qquad (17)$$

The corresponding conjectural elasticity of the demand for labor appears to be considerably larger than was found in § 48, viz. -0.31 in all models, except in model 4, where it amounts to -0.54.

We may also express y as a function of the real wage rate $l - p$ instead of l. This leads to an elasticity of about -0.4 (-0.7 in model 4; -0.6 in model 5).

As another problem consider the case where the government and perhaps the entrepreneurs together respond to the wage policy by changing both ξ_0 and π_0 so as to maintain (1) balance of payments equilibrium, and (2) "social equilibrium" as we understood it in the case of devaluation. This comes to taking as given $D = -0.04$, and $p = \lambda l'$ and l' and as unknown ξ_0, π_0 (which do not primarily interest us now), and y. We may now use equation (2) and find:

$$- \mu y - \delta\lambda l = 0.04$$

$$y = - \frac{0.04 + \delta\lambda l}{\mu} \quad (18)$$

yielding an elasticity of -0.83 (in model 5: -1.60).

Let us now suppose that labor productivity is no longer constant, but constitutes another policy parameter in the hands of workers and employers. Instead of l' we have now to introduce two other data, viz. l and h, whereas y has to be replaced by $a + h$, according to (15). In the two problems just considered this yields us:

Bal. of paym. equil. by tax measures $a + h =$

$$\frac{0.04 + \delta\pi_1 (l + h)}{\mu + \delta\pi_2} \quad (17')$$

Bal. of paym. equil. and "social equilbrium" by tax and price

$$\text{policy } a + h = - \frac{0.04 + \delta\lambda (l + h)}{\mu} \quad (18')$$

This leads to the following numerical results:

	Models 1 to 3, 5, 6	*Model 4*
(17′)	$a = -0.31l - 0.69h - 0.08$	$a = -0.54l - 0.46h - 0.08$
(18′)	$a = -0.83l - 0.17h - 0.09$	$a = -1.60l + 0.60h - 0.09$

We may also calculate p as a function of the data and hence use the real wage rate $l_R = l - p$, instead of l, yielding:

	Models 1 to 3, 6	
(17″)	$a = -0.42l_R - 0.59h - 0.08$	$a = -0.73l_R - 0.28h - 0.07$
(18″)	$a = -2.7l_R + 1.7h - 0.09$	$a = -5.2l_R + 4.2h - 0.09$

Model 5

$$a = -0.58l_R - 0.43h - 0.07$$
$$a = -2.7l_R + 1.7h - 0.09$$

These results deserve careful consideration. It is interesting to note, first, that generally the autonomous influence of labor productivity on employment is, in the short run, negative, with the exception of the second case in model 4. It is obvious that this is correlated with the elasticity of demand. This negative influence of labor productivity on employment is no difficulty only if the value of a would otherwise (i.e. without the considered change in h) have surpassed full employment and would therefore have been impossible. Another interesting feature of our results is that the elasticity of employment with respect to real wages is always higher than the elasticity with respect to nominal wages, a well-known fact that explains a large part of the discrepancy between Douglas' findings and those of various other authors. It is also noteworthy that the elasticity of employment with respect to real wages is, in absolute value, well above one in all models in the second case, that of balance of payments equilibrium and "social equilibrium."

§ 54. Summary; Is It Possible to Translate Our Analysis into Verbal Deductions?

We may briefly summarize our findings. We constructed a simple model intended to approach short-run reactions (i.e. reactions within the next few years) of the economy of a small country upon autonomous changes in wage, price, and budget policy as well as in labor productivity. We used numerical values (in six alternatives) believed to be valid for the Netherlands. The essence of the model is a combination of the basic ideas of Keynes-Machlup on the influence of real income on the balance of payments, the influence of prices in this respect, and the role of wages for prices and workers' income. An appropriate choice of the main variables led to a very simple system with production volume, price level, and balance of payments deficit as representing the economic phenomena, unit wage cost, autonomous profit margin, and autonomous part of national expenditure as "political parameters." Various problems of economic policy were dealt with. With given "directives" the values of the parameters were the unknowns. It appeared that closing the balance of payments cannot very efficiently be done by wage reductions alone or profit reductions alone. Their combination, in the special form of devaluation, appeared more efficient, although still not very attractive. Double the devaluation applied in September 1949 might have been necessary to close the gap. Budget policy (in either of three forms: decrease of subsidies, increase in indirect taxes, or reduction of expenditure generally) should be added. An "optimum solution" is presented, which seems to be within the range of practical possibilities. That solution may be made even more acceptable by increases in labor productivity.

Finally, the influence of wage rates and labor productivity, considered as autonomous variables, on employment is studied. A distinction should be made between a number of cases according to whether the other political parameters do or do not react. Other parameters being equal, the elasticity of the demand for labor is found to be very weak. It is higher, however, if the other parameters are supposed to change in response to wage increases, e.g. in order to maintain balance of payments equilibrium and "social equilibrium." The elasticity may then approach or even surpass unity. If the real wage rate instead of the nominal rate is considered to be the action parameter, the elasticities are higher. The influence of changes in labor productivity on employment is negative as long as the demand elasticity is below one, otherwise it is positive.

Having summed up our main findings, we now ask whether this analysis, in view of the interest for practical politics it may arouse, and a certain aversion of mathematics, can be given in nonmathematical form. The answer clearly depends on what exactly is meant by nonmathematical analysis. It is of course possible to replace every equation by a series of verbal statements but by doing so we would certainly not get an easy-reading exposition of our analysis. I am inclined to consider as "verbal treatment" a sequence of statements (including calculations) each of which is based on one or more of the preceding statements. An example may be: "A wage rate fall of 1 per cent, production remaining constant, leads to a 0.3 per cent fall in prices. This price fall, production remaining constant, improves the balance of payments by an amount $\delta \times 0.3$. Since we assume production constant and imports increase, a decrease of so and so much in consumption must have occurred. This may have been obtained by a decrease in subsidies of so and so much." Characteristic of this piece of logic is that it is one-dimensional. It does not and cannot involve a simultaneous relation between two or more variables. Hence verbal treatment is only possible for such problems where the system of relations involved is of the "consecutive" type, i.e. where one equation only contains one of the unknowns, a second equation contains one more unknown, a third equation again one more, etc. This is the type of system considered by Bentzel and Wold* for other purposes. Of the problems dealt with in our present paper some are of this type, viz. the first one in § 51 (given D, y, and $p = \lambda l'$, and unknowns ξ_0, l', and π_0) and the second in § 53 (given D, l', and $p = \lambda l'$, and unknowns y, ξ_0, π_0). These may therefore be treated verbally and in fact the above example of verbal treatment refers to the former of these two. The other problems treated are not, however, of this type. Some simultaneous relation comes in.

I see a confirmation of my viewpoint in the fact that literary economists, when dealing with simultaneous problems, have to recur to graphical treatment, which is a substitute for proof: the usual demand-and-supply scheme is the best-known case in point.

There is of course one other escape: dynamization with sequence analysis. It is needless to say that matters are often unduly complicated by this method. As a practical method, the one followed in § 50 may be recommended, where the numerical results are given in a table enabling every reader at least to test that the figures satisfy the equations and further enabling him to understand why (in that particular case) the influence of devaluation on the balance of payments is only restricted.

* R. Bentzel and H. Wold, On Statistical Demand Analysis from the View-Point of Simultaneous Equations, Skandinavisk Aktuarietidskrift, 1946, p. 95.