CHAPTER III

FLUCTUATIONS IN INVESTMENT

§ 7. The Relation tested

Problem chosen for testing.

In this and the two following chapters, a number of the results obtained in applying the method described above to one of the central relations in business-cycle theory will be discussed. The relation in question may be defined as that indicating the " proximate " objective causes of changes in investment activity, looked at from the demand side—*i.e.*, from the side of investing entrepreneurs and public authorities.

Calculations have been made for investment in general, as well as for residential building and railways as important special cases.

As emphasised in Chapter I, the principles underlying the procedure are that economic theory has to suggest the factors to be considered, while the statistical testing process shows the maximum degree of accordance obtainable and the relative strength of each factor required to obtain that degree of accordance.

For the investigation of investment in general, the choice of the relevant factors has been based on the following considerations. Total investment activity is the sum of the investment activity of those individual entrepreneurs who decide to invest at all. The larger this *number*, the greater in general will the volume of investment be. Whether or not an entrepreneur decides to invest depends first of all on whether he expects to make profits or not. Therefore, the *number* of entrepreneurs planning investment will depend on profit expectations.[1]

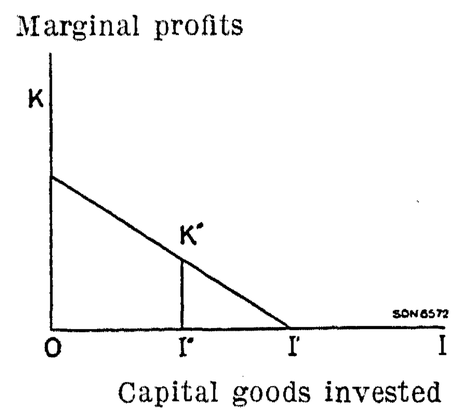
[1] It is almost a tautology to say that investment is governed by profit expectations. It is not quite a tautology, however, since with the same profit expectations there may be different volumes of investment in different conditions, some of which will be mentioned later.

Next, the *extent* of the plans of those entrepreneurs who are planning investment will depend on a number of items entering into their calculations. These calculations, of course, partly reflect the profit expectations of the entrepreneur. There are, however, in them other elements of a more technical order. How much will be invested will also depend, *e.g.*, on the existing unused capacity to produce and on new technical possibilities.

Marginal profits and total profits.

One aspect of these calculations may be considered in somewhat greater detail, with the help of Graph III. 1, which represents marginal profits (K) as a function of the number (I) of capital goods units in existence at a given moment. When this number is I″ at the moment of planning, the entrepreneur will *tend* to invest so many units that the point I′ with zero marginal profits [1] is reached. (If profits are understood to be taken before deducting interest payments, marginal profits minus interest payments are zero at point I′.) Provided that the curve K″I′, when it shifts upward and downward, does not change its slope—as is ordinarily assumed to be the case—I″K″, marginal profits, and the area OI″K″K, total profits on the existing plant, will show proportional changes. This parallelism will cease to be exact as soon as the curve K″I′ changes its slope too. Moreover, each addition to capital will, for the next time-unit, slightly change the base OI″ of the area, and so invalidate somewhat the proportionality between changes in total and changes in marginal profits in successive (instead of alternative) positions. Since, however, the effects of changes in marginal profits seem to be much greater than those due to this correction, in the statement that investment is determined by profit expectations the latter may with a fair degree of approximation be taken as expected total profits on existing plant.

Graph III. 1.



[1] Evidently the argument of this paragraph also holds, *mutatis mutandis*, for *expected* marginal profits and *expected* total profits.

Finally, for many entrepreneurs—especially for public authorities planning to make investments—the *possibility of financing* investment activity will exert a considerable influence. From the theoretical point of view, it is perhaps superfluous to mention this aspect separately, since in profit calculations, in their widest sense, this financial possibility has somehow to be included. From the practical point of view it seems, however, useful to make the distinction, even if it cannot be maintained everywhere in the considerations which follow.

We shall now consider more closely the elements mentioned. Profit expectations themselves are, of course, hardly accessible to statistical measurement; but they will largely be determined by some objective criteria in the minds of most entrepreneurs. These objective criteria will in the first place be included as factors in our analysis.

Factors influencing profit expectations.

The factors which as a rule exert the greatest influence on *profit expectations* are by most authors assumed to be

(1) the magnitude of currently earned profits;
(2) the price of capital goods; and
(3) the rate of interest.

It might seem as if the rate of interest and the price of capital goods ought not to be included separately, since they enter through interest payments and depreciation into currently earned profits; but the interest rates and prices entering into these calculations are some sort of average over a long period and will therefore show almost no connection with the latest prices in both markets. Present investment will be governed by the rate of interest and the price of capital goods now prevailing, or at most will exhibit a fairly small and definite lag. For this reason, these factors are included separately; for most other cost items, this is not necessary.

Profit margin or total profit ?

(4) Other authors have preferred to include as chief explanatory factor profit margins instead of total profits, profit margins representing the margin between average selling price and average prime cost. The chief reason for taking this course must lie in the hypothesis that the entrepreneur who plans to invest takes for granted the amount of additional output which he will

be able to sell as the result of his act of investment; in this case his expectation of profits will depend entirely on the margin between prices and prime cost. The possibility that his sales expectations will depend on the general business situation seems to have been neglected in this hypothesis. For entrepreneurs far from the margin, this may be justified to some extent; profit margins, as well as profits, have therefore been included as an explanatory variate. It may be asked whether it would not be preferable to include profit margin and volume of production separately as explanatory variates instead of total profits (which is about the same thing as the product of margin and volume of production) and profit margin. Since, however, statistics of total profits are, in general, more reliable than those of profit margins, and it may be expected that total profits as such have an important influence, it seems better to take total profits as one of the variates.

Profits and profit rates.

No distinction has so far been made between amount of profits and profit rate—*i.e.*, profit as a percentage of capital. Actually, this rate is commonly considered as the factor which has to be taken for our purpose. As will be seen below, in a number of cases figures are available and have been used. In other cases, however, only the amount of profits is available. From the statistical point of view the difference is very small. Generally the two series are very highly intercorrelated, as a consequence of the smooth movements of total capital stock. The results will therefore be very nearly the same—*i.e.*, the regression coefficient found for profits may be assumed to equal that which would be found for profit rate if the latter was used as explanatory series, provided that due corrections for changes in units were made.

(5) Another factor which has been mentioned is the rate of increase in prices. The underlying idea is that rising prices stimulate and falling prices curtail investment activity. Objects bought or constructed in times of rising prices show rising values and, therefore, rising possibilities of yielding a profit when sold; the reverse is true in periods of falling prices. The argument is of course more especially true for goods that are easily marketable, but applies also to some extent to capital goods, which may, *e.g.*, be constructed in advance when prices rise.

Financial possibilities.

The factors determining the financial possibi of investment are already included in the list given. They are, first of all, currently earned pr which are not only important for a great numb smaller enterprises, but also for public authorities. Public buc will, with some lag, reflect private economic conditions and d mine, in a high degree, the possibilities for public authoritie make large investments.

Indirectly, high current profits are also important for big er prises, in so far as the raising of money by issuing new shares be facilitated. The " easiness " of getting money in this way also be inversely indicated by the share yield, which in some s indicates the movements in the " interest rate " which the pu expects to receive on new shares.

In addition, ordinary interest rates will be another indicat Long-term interest rates may play a rôle for those enterprises wl usually finance their investments by bonds or mortgages (railwa building), or which base their calculations on those rates; short-t rates may influence financing by bank credits.

Technical circumstances.

Apart from profit expectations, some techni circumstances also influence the volume of inve ment, as has already been observed. Apart fr " autonomous " technical changes, to be discus later, two factors seem outstanding.

Investment activity will be lower, the lower actual producti is in proportion to existing productive capacity. As in gener and especially in the pre-war period, total capacity is a ve smoothly moving series, the influence of that series may be n lected, since all our investigations deal only with the short fluctuations. As for the actual volume of production, it is highly correlated with the volume of investment that it must n be included in our analysis as a separate series. Any calculati aiming at explaining investment activity will automatically ta account of the circumstance discussed here.[1]

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[1] Indicating by v the volume of investment, by $u + v$ the total volu of production, and summarising in R all other factors in the " explanation of investment activity, the relation to be tested will be: $v = a\ (u + v) +$ Since $u + v$ is very near to bv (b constant), it follows that the equation m

(6) The second factor will be the rate of increase in the volume of production.[1] The larger this rate of increase, the larger the need for new capital goods. This is the tendency upon which the well-known " acceleration principle " is based. This connection is only a close one if no excess capacity is available; and this will be the case for only a small proportion of enterprises. In consequence, the strength of the force will be less than the pure theory of the simplest case would suggest.[2] Nevertheless it may have an influence.

Here, again, a mathematical difficulty presents itself under certain conditions—namely, when, as is the case in various periods and countries,[3] investment activity and consumers' goods production (and therefore also general production) are highly correlated. The rates of increase in the general production level and in investment activity are then also highly correlated. To include the series now proposed would therefore be almost the same as to include the rate of increase of the variate that is to be explained. It may be proved mathematically that this means introducing a small change in lag.[4] The new variate may therefore be given any importance (within some limits) one likes, if only the lag in the relation explaining investment activity be accordingly changed. Only if that lag may be known *a priori* will the problem be absolutely solvable. A further condition is, of course, that the lag, as well as the coefficient found for the rate of increase in total production, must be positive.

Series rejected.

The list of explanatory series omits two other series which have been emphasised by some authors and might possibly be considered as influencing investment fluctuations. These series have been rejected partly for *a priori* reasons, partly as a consequence of some provisional correlation calculations.

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also be written: $(1 - ab)\ v = R$ or $v = cR$, meaning that $u + v$ could be left out as an explanatory variate.

[1] Not only consumers' goods production, since for the production of other goods machines, etc., are equally necessary.

[2] *Cf.* G. HABERLER, *Prosperity and Depression*, page 84.

[3] *Cf.* discussion of results in § 10.

[4] *Cf.* end of § 8, p. 48.

(1) *"Weighted average of previous production."*

Some theorists have emphasised a new "explanatory" series by paying attention to what has been called the "echo effect". Suppose the duration of life of all capital goods were strictly equal to a given period: then production required for replacement of worn-out capital goods would be an exact repetition of total production of investment goods some time before. If, *e.g.*, that period were strictly seven years, then production in 1929 would be partly destined to replace capital goods produced in 1922, production in 1930 to replace those produced in 1923, etc. Now it is clear that (i) the life of various capital goods shows an enormous spread and (ii) even the life of, *e.g.*, one particular machine depends on business conditions in the year in which it is replaced. There will be a tendency to replace more in good than in bad years, even if the technical duration of life be the same. The first circumstance leads to the necessity of taking, for the explanation of 1929 production, not the year 1922 only, but a weighted average of a number of years which perhaps have their centre in 1922. Some experiments give the impression that the weighting flattens the curves so radically that practically no movements are left.[1] For this reason, as also on account of the uncertainty of the exact distribution of the duration of life, the variate "weighted average of previous production of capital goods" has been omitted. The second circumstance mentioned above—viz., the influence of the business position on replacement—is already taken into account by the inclusion of profits as one of the determining factors.

(2) *Autonomous changes in investment.*

Apart from the factors discussed, a number of extra-economic or autonomous factors will influence investment activity. Important inventions may do so; or political events which suddenly change expectations. These influences are considered, in this analysis, as non-systematic disturbances which act largely accidentally, in an irregular way, like lottery drawings. In general, such influences will exist whenever many mutually independent and small forces are acting, which will be the case in

[1] See also: P. DE WOLFF, "The Demand for Passenger Cars in the United States", in *Econometrica*, 6 (1938), page 113.

normal times. This is the approach to business-cycle problems which is known as the " shock theory of cycles ".[1] Some very exceptional events which do not obey these " laws " will be generally known, so that they may easily be eliminated before the analysis. This has been done, *e.g.*, with the English coalminers' strike in 1926, while for the American calculations the period since 1933 has been treated separately. With the exception of such events, the other autonomous influences are assumed to be included in the statistical residuals.

Two stages of investigation.

To sum up, there would be reason to include at least six explanatory series (indicated by the numbers 1-6), namely:

(1) current profits;
(2) price level of capital goods;
(3) interest rates (long, short or both);
(4) profit margin;
(5) the rate of increase of prices;
(6) the rate of increase in the volume of production.

To include this large number of series in all calculations would have meant such an amount of work that it seemed advisable to make the investigation in two consecutive stages. In the first stage, where a general orientation about the importance of each variate is the object, all series are included, but only three cases are considered (Germany and the United Kingdom before the war;

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[1] *Cf.* R. Frisch: " Propagation Problems and Impulse Problems in Dynamic Economics " in *Economic Essays in Honour of Gustav Cassel*, London, 1933. The difference between this type of business-cycle theory and the other theories may be shortly characterised as follows: whereas most theories do not pay very much attention to external disturbances, and in most cases only start their argument with the assumption of an initial disturbance, the shock theory supposes that such disturbances work at very short intervals and are each of them of only little importance. From this simple fact, this theory concludes—by mathematical deductions—that the resulting movement may show, apart from the " endogenous " periods, apparent periods which are only the consequence of the cumulative effect of disturbances. The word " shock " has been chosen in analogy with physics, where such problems were already known.

the United States after the war), and the calculations are limited to regression formulæ and correlation coefficients. In the second stage, the calculations include the more important series only, but they are more detailed and cover six cases (adding to the above France and the United States before the war and the United Kingdom after the war). In addition, the significance of the latter calculations has been tested in various ways.

A rough and fairly easy first test of the significance of results consists in dividing up the period studied and repeating the calculations for the shorter periods. Similar results, with coefficients of the same order of magnitude, should be found: otherwise the significance of the results must be doubted. A number of calculations for the pre-war period have accordingly been repeated for two sub-periods—up to 1895 and from 1895 onwards. The year 1895 has been chosen as it is the turning-point of one of the so-called long waves.

In addition, another experiment has been made. As it has often been suggested that the "laws" governing upward phases and those governing downward phases of the cycle are different, the calculations have been repeated for (1) all years in upward phases and (2) all years in downward phases.

In addition, the more exact significance calculations mentioned in Chapter II have been made for some characteristic cases. They have not been repeated for all cases, as they are very laborious.

§ 8. The Statistical Material

The following details of the statistical series used in the calculations may now be given.

The countries and periods studied are:

Countries and periods.	Pre-war:	Germany 1871-1912 (42 years);*
		United Kingdom 1871-1910 (40 years);*
		United States 1877-1913 (37 years);
		France 1871-1908 (38 years).
	Post-war:	United Kingdom 1920-1936 (17 years);
		United States 1919-1933 (15 years).*

Only the cases indicated with an asterisk have been included in the first stage.

The post-war figures for Germany are too much vitiated by autonomous events to afford a good basis for research. The period after 1933 for the United States has been left out, as the policy of the Government may have changed the relations investigated; the calculations have, however, been extrapolated (*cf.* Section 11).

Series used.

(a) *Description of series.*

I. *Total investment* has as a rule been represented by estimates of the consumption of iron and steel (v_i). Alternatively, pig-iron production has been used.

The main difference between the two series consists in exports of iron and steel (not included in consumption) and in scrap used in steel production (not included in production of pig-iron). The inclusion of exports may be interpreted as an attempt to take into consideration not only the home market, but also foreign markets. The exclusion of the second item is in most cases not serious, as there is a good parallelism between cycles in pig-iron and steel production. Only for the United Kingdom 1900-1910, where there was a marked divergency, must the consumption figures be preferred; it is interesting, however, to compare the results of the two attempts.

In the case of the United States, a more accurate estimate of the volume of investment (v) is available for the post-war years in the figures calculated by KUZNETS.[1] These figures, which distinguish between producers' durable goods, consumers' durable goods, and building, have also been used. Building has been excluded in all cases except one, where residential building (as estimated by WICKENS and FOSTER)[2] has been subtracted, and non-residential building retained.

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[1] S. KUZNETS, *Gross Capital Formation*, Bulletin 52, National Bureau of Economic Research, New York, 1934.

[2] D. L. WICKENS and R. R. FOSTER, *Non-farm Residential Construction* 1920-1936, Bulletin 65, National Bureau of Economic Research, New York, 1937.

The exact series used is indicated in each table. In general, the results obtained for the various series are not very different.

II. The explanatory series are taken from the following sources.

(1) *Profits earned* have been represented by the series indicated in the following table:

Country and period	Description of series	Source
United States, pre-war	(*n*) Share price index.	*Review of Economic Statistics.*
United Kingdom, pre-war	(E) Non-labour income.	BOWLEY, *Economic Journal*, 1904, completed with the help of data by STAMP, *British Incomes and Property* (based on Income Statistics).[a]
Germany, pre-war	(*d*) Dividends earned in % of capital.[b]	DONNER, *Die Kursbildung am Aktienmarkt, Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36.
	(*n*) Share price index.	Same source.
France, pre-war	(*n*) Share price index.	Statistique generale de la France.
United States, post-war	(Z^c) Net income of corporations.	*Statistics of Income.*
	(*n*) Share price index.	Standard Statistics Co.
United Kingdom, post-war	(Z^c) Net profits earned in % of capital.[b]	*Economist.*

[a] Some experiments with statistical methods devised to find the annual figures out of the three-year-moving averages given by Income Statistics seemed to show that the cyclical movements of the latter and those of the former do not differ very much.
[b] Year of earning is taken to precede year of distribution.

(2) *Price of capital goods.* — The price of pig-iron (q_1) has been taken throughout, as it represents the most fluctuating item in the cost of capital goods. The sources are indicated in the following table:

Country and period	Description of series	Source
United States, pre-war	Price of No. 1 foundry pig-iron at Philadelphia.	*U.S.A. Statistical Abstract.*

Country and period	Description of series	Source
United Kingdom, pre-war	Sauerbeck's index of price of pig-iron.	S. KUZNETS: *Secular movements in Production and Prices.*
Germany, pre-war	Average price of pig-iron produced.	*Statistisches Handbuch (1907); Statistische Jahrbücher.*
France, pre-war	Average price of pig-iron produced.	*Annuaire statistique de la France.*
United States, post-war	Price of Bessemer pig-iron at Pittsburgh.	*U.S.A. Statistical Abstract.*
United Kingdom, post-war	Price of Cleveland-Middlesbrough pig-iron.	Statist's index number of wholesale prices, *Journal of the Royal Statistical Society*, Part II.

(3) *Interest rates.* — In most cases, calculations have been made with (i) (m_s) market rate of discount or some other short-term interest rate and (ii) (m_{Lb}) bond yields.

In addition, some calculations have been made with (m_{Ls}) share yield, as representing a special category of "interest rates", viz., the rate attributed to funds raised by share issues. The sources are summarised in the following table:

Country and period	Description of series	Source
United States, pre-war	(m_{Lb}) Long-term: 1890-1899: Yield on ten American railroad bonds.	*Review of Economic Statistics*, 1919.
	1900-1913: Yield on sixty bond issues combined.	Standard Statistics Co.
	(m_s) Short-term: market rate on 60-90 days paper.	I. FISHER, *The Theory of Interest.*
United Kingdom, pre-war	(m_{Lb}) Long-term: Yield on 2½% Consols.	,,
	(m_s) Short-term: market rate of discount.	,,
Germany, pre-war	(m_{Lb}) Long-term: Yield on fixed interest bearing securities.	DONNER, see under (1).
	(m_s) Short-term: market rate of discount.	
France, pre-war	($^1/m_{Lb}$ Long-term: Index of price of 3% "rente".	*Bulletin de la Statistique générale de la France*, 1919/20.

Country and period	Description of series	Source
United States, post-war	(m_{Lb}) Long-term: Bond yield, 60 issues combined.	Standard Statistics Co.
	(m_s) Short-term: market rate on 4-6 months commercial paper.	I. Fisher: *The Theory of Interest*; *League of Nations Monthly Bulletin of Statistics*.
	(m_{Ls}) Share yield: (i) Cash dividends of corporations in % of (ii) total capital stock.	(i) Statistics of Income. (ii) Statistics of Income; prior to 1925, estimates based on new security issues and index of share prices.
United Kingdom, post-war	(m_{Lb}) Long-term: Yield of 2½% Consols.	Statistical Abstract of the United Kingdom: Statistical Summary of the Bank of England.

(4) The calculation and sources of the figures for *profit margins* ($p-\frac{1}{2}\,l_w$) are given in this table:

Country and period	Description of series	Source
United States, pre-war	Index of cost of living — ½ index of hourly earnings.	National Industrial Conference Board.
United Kingdom, pre-war	(i) Index of prices of exported finished products — ½ (ii) index of wage rates.	(i) Calculation L.o.N. based on trade statistics. (ii) Index of Bowley and Wood, reproduced from Layton: *Introduction to the Study of Prices*.
Germany, pre-war	(i) General index of wholesale prices — ½ (ii) index of wages (both in % deviations from trend).	(i) Jacobs & Richter: Grosshandelspreise. *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft No. 37. (ii) J. Kuczynski: *Löhne und Ernährungskosten in Deutschland*.

(5) The sources of the figures for (u) *production of consumers' goods* are:

Country and period	Description of series	Source
United Kingdom, pre-war	Index of production of consumers' goods.	Hoffmann: *Weltwirtschaftliches Archiv*, vol. 40.
Germany, pre-war	Index of the Institut für Konjunkturforschung.	Wagenführ, Die Industriewirtschaft. *Vierteljahreshefte zur Konjunkturforschung*. Sonderheft No. 31.

Country and period	Description of series	Source
United States, post-war	League of Nations estimates based on study by Warburton in:	*Journal of the American Statistical Association*, vol. 30.

(6) The figures for the *rate of increase in general price level* and their sources are:

Country and period	Description of series	Source
United Kingdom, pre-war	(Δq_i) Rate of increase in price of pig-iron. [a]	*Vide supra.*
Germany, pre-war	(Δq_i) *Idem.* [a]	*Idem.*
United States, post-war	(Δp) Rate of increase in cost-of-living index.[a]	National Industrial Conference Board.

[a] For pre-war times the general price level seemed to be well represented by that of investment goods. For post-war times iron prices are no more representative.

(b) *Trends.* — As the relation studied claims to represent only the causation of short-run movements in the volume of investment, deviations from trend have been taken throughout, except when otherwise stated. In general, trends have been calculated as nine-year moving averages for pre-war periods—which are long enough to allow of the first and last four years being omitted—and as rectilinear trends for post-war periods—which are too short to allow of omitting eight years.[1]

(c) *Other series included.* As has already been stated (Chapter III, § 7), an attempt has been made throughout to explain production of pig-iron or consumption of iron and steel (v_i) or total physical investment (v) by some of the six explanatory series mentioned above.

[1] The trend chosen for the American figures (post-war period) may be somewhat biased by the fact that the period starts with a boom year and ends with a slump year.

In order to judge the importance of the trend, the first calculation of Table III. 4 was made without correction for trend. The correlation coefficient was hardly affected, nor was the regression coefficient of profits (0.33 instead of 0.29). The combined influence of price and interest must therefore have been nearly the same; of the respective influence of these two factors, nothing can be said owing to a very high intercorrelation between the two variates (*cf.* Chapter II, § 6, page 29).

In some cases it has seemed useful to include still further series, sometimes in substitution for one of those mentioned already. In the case of the United States (pre-war), in order to obtain a fairly good explanation, it seemed necessary to include building volume (v_B) as a separate variate. This may be justified by the fact that the factors affecting the building volume are rather poorly represented in the four explanatory series: *e.g.*, building profits, which might play a rôle, are not reflected immediately in share-price indices. The same applies even more to other factors. The inclusion of building volume means, of course, only a postponement of the problem; the factors affecting building itself are to be studied afterwards (Chapter IV).

Lags considered.

(i) As a general starting-point, a lag of *half-a-year* was assumed to exist between the explaining series and investment activity.[1] In most cases, this seemed to be not far from reality.[2]

(ii) In the second stage, various experiments were made with *other lags*. They consisted either in assuming a lag of one year for all variates, or in introducing the same variate both with a lag of one year and without any lag and comparing the regression coefficients. For example, if the best explanation turns out to be: $0.8\ z + 0.4\ z_{-1}$, where z_{-1} stands for z one year earlier, this result represents a case of so-called " distributed lag " [3] with an average lag of 1/3 year. Similarly, lags of half-a-year may be introduced by simply taking as a variate $\frac{1}{2}\ (z + z_{-1})$ instead of z.

(iii) Here it should be remarked that an infinity of different interpretations can be given to the above formula, which, mathematically speaking, all come to the same. Instead, *e.g.*, of giving the above interpretation, one could read the formula:

$$1.2z - 0.4\,(z - z_{-1})$$

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[1] Except for the United Kingdom, where preliminary calculations showed a lag of one year for non-labour income to be preferable.

[2] A number of estimates available for the length of the production period would seem to indicate a few months for machinery, six months for house building, and one year for shipbuilding. *Cf.* " The Length of Certain Production Processes " (Dutch), *De Nederlandsche Conjunctuur*, August 1934, page 32.

[3] This term is due to Professor Irving Fisher. Distributed lags seem to be even more probable than simple lags.

saying that there are two influences, viz.:

(*a*) an influence of z, without lag and with strength 1.2;

(*b*) an influence of the rate of increase in z, with an (average) lag of half-a-year and a strength — 0.4.

§ 9. Chief Results

Before giving a detailed account of all the cases considered and results obtained, the conclusions drawn may be shortly summarised as follows.

(*a*) On the assumption that our estimate of iron and steel consumption (or the alternatives used) is a just index of investment activity, there is fairly good evidence that the fluctuations in investment activity are in the main determined by the fluctuations in profits earned in industry as a whole some months earlier.

(*b*) The influence of the other factors included is not considerable and is therefore, in many cases, numerically uncertain. This fact is reflected in

(i) the significance calculations in the ordinary sense, and

(ii) the fact that the "influence" of these other factors is sometimes positive and sometimes negative, and almost always small (*cf.* Chapter II, § 5).[1]

Nevertheless, for particular countries, fairly certain results are obtained (*cf.* next few sections).

(*c*) No systematic differences of a general character have been found to exist between upward and downward phases;

(*d*) As was to be expected, the difficulties arising from "multicollinearity"[2] increase with the number of variates and prevent the complete solution of a number of problems.

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[1] The chief significance of this result is that in past cycles the rôle of interest rates and of prices of iron has been far less important than that played by profit changes. It does not follow for all cases—though it does for most of them—that interest rates might not have exerted a great influence if they had fluctuated more violently than in fact they have done.

[2] See Chapter II, § 6.

§ 10. Details of Results, First Stage.

The results of the calculations of the first stage are summarised in Tables III. 1, III. 2 and III. 3, giving the correlation and regression coefficients for the cases and variates indicated. The tables have been constructed in the following way. The chief explanatory variate, namely profits, has always been included: in the case of the United States two such variates were taken, namely profits and share yield. The selection of these variates has been based on their coefficient of simple correlation with investment activity σ' and on the "influence" (*cf.* Chapter II) which they show in the more complete "explanations".

The chief variate or the couple of chief variates has then been combined with each of the other variates, and with certain combinations of larger numbers of the latter. Variates showing regression coefficients with wrong signs have sometimes been excluded in order to reduce the number of possible combinations; *e.g.*, the variate profit margin, $(p_A - \frac{1}{2} l_w)_{-1\,2}$ in the case of the United Kingdom in the four- and five-sets, and the variate rate of increase in price level (Δq) in the three-set. The same applies to variates showing a very small influence. This influence may be found by multiplying the standard deviation, given at the bottom of the tables, by the regression coefficients. In order to give an *a posteriori* test, this has been done for the highest regression coefficient in each column.

The tables have been used for three purposes. First, the increase in the correlation coefficient as the consequence of the inclusion of a new variate may be studied, and secondly, the stability of the regression obtained for one variate in various cases. Thirdly, conclusions as to the relevance of the variates may be drawn. These latter are based on:

(i) whether or not the variate in question increases the correlation coefficient to any considerable extent,

(ii) whether or not the sign of a fairly stable regression coefficient is right, and

(iii) whether or not the influence of that variate is perceptible.

Carrying out this programme for each of the variates, the following conclusions seem legitimate.

Table III. 1. "Explanation" of Investment Activity, First Stage.

United Kingdom, 1871-1910: Iron and Steel Consumption (v_i).

Case	Regression coefficients of: [1]							Correlation coefficients
	E_{-1}	$(q_i)_{-1/2}$	Δq_i	$(m_{Lb})_{-1/2}$	$(m_s)_{-1/2}$	Δu	$(p_A - \frac{1}{2} l_w)_{-1/2}$	R
11	1.85							0.552
21	3.20	— 0.31						0.664
22	1.92		0.17					0.649
23	1.92			— 0.366				0.643
24	2.85				— 0.051			0.611
25	1.79					0.43		0.606
26	2.65						— 0.44	0.655
31	3.29	— 0.32	0.13					0.729
32	3.16	— 0.28		— 0.330				0.726
33	3.41	— 0.26			— 0.021			0.670
34	3.16	— 0.31				0.45		0.714
35	3.16	— 0.20					— 0.24	0.680
41	3.28	— 0.30	0.15	— 0.239				0.783
51	3.45	— 0.27	0.15	— 0.229	— 0.016			0.786
52	3.25	— 0.31	0.13	— 0.223		0.27		0.797

Standard deviations of variates: [1]

v_i								
7.48	2.23	13.25	14.90	6.74	57.87	4.38	7.26	
Maximum influence [2]	7.69	4.24	2.53	2.47	2.95	1.97	[3]	

[1] Units used: all series in percentage deviations from trend, except series m_{Lb} and m_s, which are in absolute deviations from trend, in units of 0.01 % and Δq_i and Δu, which are in percentages of the trend values of q_i and u respectively.

Meaning of symbols (for fuller explanation, see § 8).

v_i: consumption of iron and steel.
E: non-labour income.
q_i: price of iron.
Δq_i: rate of increase in $q_i = q_i - (q_i)_{-1}$.
m_{Lb}: bond yield
m_s: short-term interest rate.
Δu: rate of increase in consumers' goods production.
$p_A - \frac{1}{2} l_w$ profit margin.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.
[3] Wrong signs.

Table III. 2. "Explanation" of Investment Activity, First Stage.

Germany, 1871-1912: Iron and Steel Consumption v_i.

Case	Regression coefficients of: [1]							Correlation coefficients
	$d_{-1/2}$	Δu	$(p-\frac{1}{2}l_w)_{-1/2}$	$(m_{1.8})_{-1/2}$	$(q_i)_{-1/2}$	Δq_i	$(m_g)_{-1/2}$	R
11	5.37							0.853
21	5.37	0.23						0.857
22	5.18		0.10					0.854
23	5.63			— 0.013				0.854
24	5.39				— 0.006			0.853
25	5.51					— 0.05		0.854
26	4.96						0.027	0.865
31	4.98	0.32	0.21					0.859
32	5.70	0.25		— 0.017				0.857
33	5.19	0.28			0.038			0.857
34	5.44		0.09	— 0.013				0.854
35	5.32		0.28		— 0.101			0.856
36	5.78			— 0.015	— 0.020			0.854
41	5.80	0.36	— 0.13	— 0.024		— 0.07	0.054	0.875
Standard deviations of variates: [1]								
v_i 11.09	1.76	3.52	5.25	44.80	12.85	9.42	65.88	
Maximum influence [2]	10.21	1.27	1.47	1.08	1.30	3	3	

[1] Units used: all series in percentage deviations from trend except series d, $m_{1.8}$ and m_g, which are in absolute deviations from trend, in units of 1%, 0.01% and 0.01% respectively. For Δu and Δq_i, see table III. 1.

Meaning of symbols (for fuller explanation, see § 8).

v_i: consumption of iron and steel.
d: dividends in % of capital.
Δu: rate of increase in consumers' goods production.
$p-\frac{1}{2}l_w$: profit margin.
$m_{1.8}$: share yield.
q_i: price of pig-iron.
Δq_i: rate of increase in price of pig-iron.
m_g: short-term interest rate.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.

[3] Wrong signs.

Table III. 3. " Explanation " of Investment Activity, First Stage. United States, 1919-1933: Deliveries of Producers' Durable Commodities + Non-residential Building (v').

Case	Regression coefficients of:[1]								Correlation coefficients
	$Z^c_{-1/2}$	$(m_{LS})_{-1/2}$	$(q_i)_{-1/2}$	$(p-\frac{1}{2}l_w)_{-1/2}$	t	Δp	Δu	$(m_S)_{-1/2}$	R
11	0.19	— 0.076							0.986
21	0.19	— 0.074	— 0.02						0.987
22	0 17	— 0.083		0.10					0 987
23	0.18	— 0.079			— 0.13				0 986
24	0.20	— 0.074				— 0.29			0 990
25	0.19	— 0.076					0.01		0.986
26	0.18	— 0.078						0.004	0.986
31	0.16	— 0.077	— 0.01	0.27					0.989
32	0.18	— 0.081	— 0.07		— 0.63				0.987
33	0.20	— 0.071	— 0.02			— 0.29			0.991
34	0.17	— 0.076		0.19	0.57				0.987
35	0.19	— 0.076		0.02		— 0.29			0.990
36	0.20	— 0.072			0.10	— 0.29			0.990
41	0.16	— 0.075	— 0.14	0.31	0.13				0.989
51	0.18	— 0.071	— 0.07	0.14	0.10	— 0.23			0.991
52	0.16	— 0.077	— 0.17	0.31	— 0.13		—0.06		0.989
53	0.17	— 0.067	— 0.18	0.24	0.27			0.012	0.990
Standard deviations of variates:[2]									
v' 19.85	96.50	154.20	25.49	17.81	4.32	7.44	7.48	130.23	
Maximum influence[3]	19.30	12.80	4.54	5.52	2.72	3	0.07	3	

[1] Units used: All series in percentage deviations from the average, except series m_{LS} and m_S, which are in absolute deviations from average, in units of 0.01 %. For Δp and Δu, see table III. 1 (Δq_i and Δu).

Meaning of symbols (for fuller explanation see § 8).

v': deliveries of producers' durable goods + non-residential building.
Z^c: profits of corporations.
m_{LS}: share yield.
q_i: price of pig-iron.
$p-\frac{1}{2}l_w$: profit margin.
t: trend.
Δp: rate of increase in cost of living.
Δu: rate of increase in consumers' goods production.
m_S: short-term interest rate.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.
[3] Wrong signs.

The importance of profits (Z), and, in the case of the United States, of share yield (m_{Ls}), is confirmed: the increase in correlation and the influence of the variate are considerable; the signs are right.

Increase in correlation. The increase in correlation obtained by the inclusion of each variate may be taken from the following table:

Average Increase in Correlation Coefficient obtained by adding Each of the Following Variates.

Country	Profits	Share yield	Short-term interest rate	Price of iron	Rate of increase in production	Rate of increase in prices	Profit margin
United Kingdom 1871-1910	0.552	0.069 [a]	0.023	0.078	0.039	0.073	0.060 *
Germany 1871-1912	0.853	0.001	0.012 *	0.001	0.004	0.001 *	0.002
United States 1919-1933	0.795	0.191	0.001 *	0.001	0.000	0.003 *	0.001

[a] Bond yield.
* In this case the sign of the regression coefficient is wrong.

Signs. For the variates: short-term interest rate (m_s), rate of increase in price (Δq or Δp) and profit margin ($p - \frac{1}{2} l_w$), the results vary in different countries. All these variates show wrong signs in all cases for at least one country.

For the other variates and countries, the signs are right, but share-yield data for the United Kingdom are not available. The variate Δu, rate of increase in consumers' goods production, shows positive signs in at least one case for each country, but its influence is found to be small. In addition, as has been observed in the theoretical part of this chapter (§ 7), this influence may always be replaced by a shift in the lag assumed, especially in case of a high correlation between investment activity and consumers' goods production. Price of iron (q_i) shows negative signs in most cases for each country.

Influence. Taking the standard deviation of v_i equal to 100 in each country, the maximum influence found is the following:

Country	Profits	Share yield	Short-term interest rate	Price of iron	Rate of increase in:		Profit margin
					production	prices	
United Kingdom	103	33 [a]	39	57	26	34	*
Germany	92	10	*	12	11	*	13
United States	97	64	*	23	0	*	28

[a] Bond yield. * Wrong signs in all cases considered.

Conclusions. From the foregoing we conclude that, as stated already, the factors short-term interest rate, price of iron, rate of increase in production and in prices, and profit margin are, in the mean, far less important than profits and share yields. In particular cases some of them seem to be important, but a general indication is lacking. The most important variates that are to be considered in the second stage are therefore profits (and for the United States, post-war, also share yield), and in addition the price of iron and interest rates, since the theoretical considerations given in § 7 require that these should be considered at the same time. A second reason for including these two factors is the great importance which many economists attach to them as causal factors of investment.

§ 11. Details of Results, Second Stage.

A. *Examination of Regression Equations found.*

Correlation found. The correlation coefficients found (*cf.* Tables III. 4 to III. 9) are, in general, fairly high, their median value being about 0.80; and it is satisfactory that —in view of the better statistics available for post-war years—the post-war correlations are considerably higher than the pre-war ones for the same country. Especially interesting is the improvement obtained by taking such a careful estimate

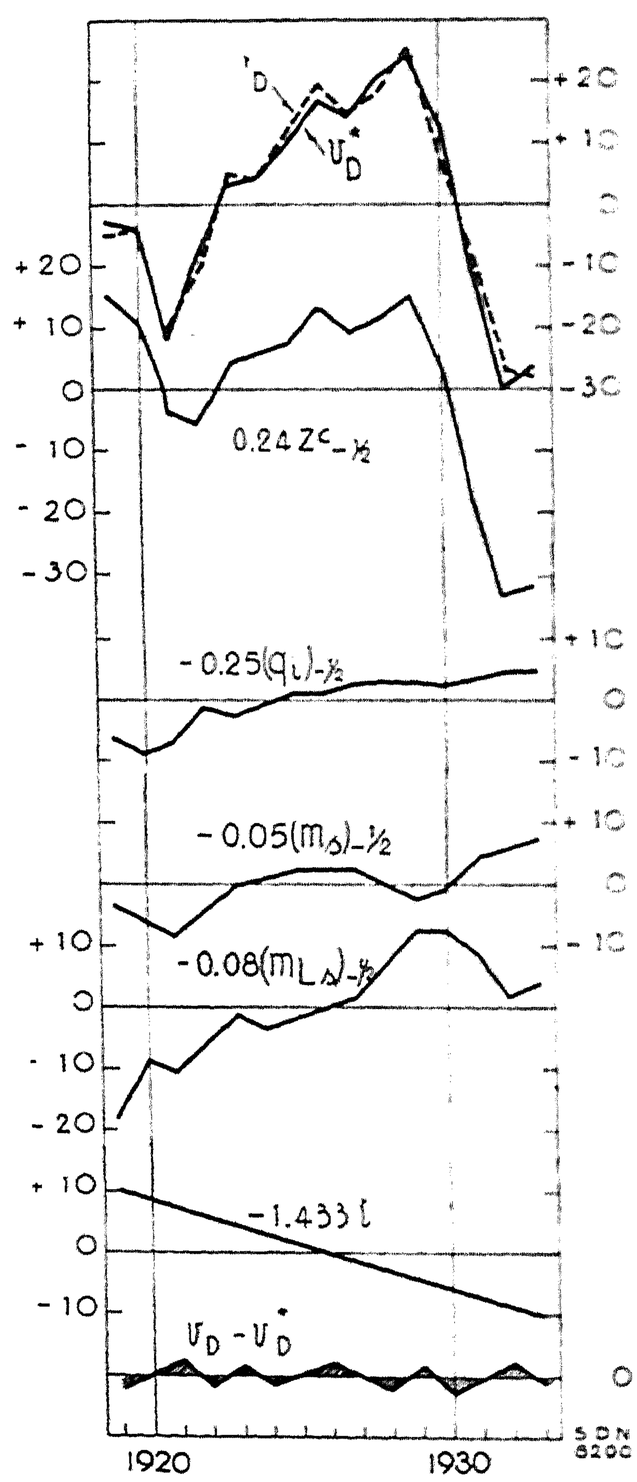
as Kuznets' index of investment activity instead of pig-iron production.

"Influence" of profits.

A feature common to all results is, as has already been observed, the important part which profits (or one of the series reflecting them) play in the "explanation". The other factors play only a secondary part, as is easily seen from Graphs III. 2—III. 5, constructed in accordance with the rules laid down in Chapter II. § 5.

Graph III. 2.

"Explanation" of Investment Activity.
United States of America, 1919-1933.



Investment = Flow of producers' durable goods to enterprises, plus flow of consumers' durable goods to enterprises and households.

v_D: investment activity, actual.

v_D^*: investment activity, as explained by:

$Z^c_{-1/2}$: profits

$(q_i)_{-1/2}$: price of iron

$(m_s)_{-1/2}$: short-term interest rate

$(m_{L\Delta})_{-1/2}$: share yield

} aged ½ year.

t: trend.

Graph III. 3.
"Explanation" of Iron and Steel Consumption.
United Kingdom, 1920-1936.

v_i: iron and steel consumption, actual.

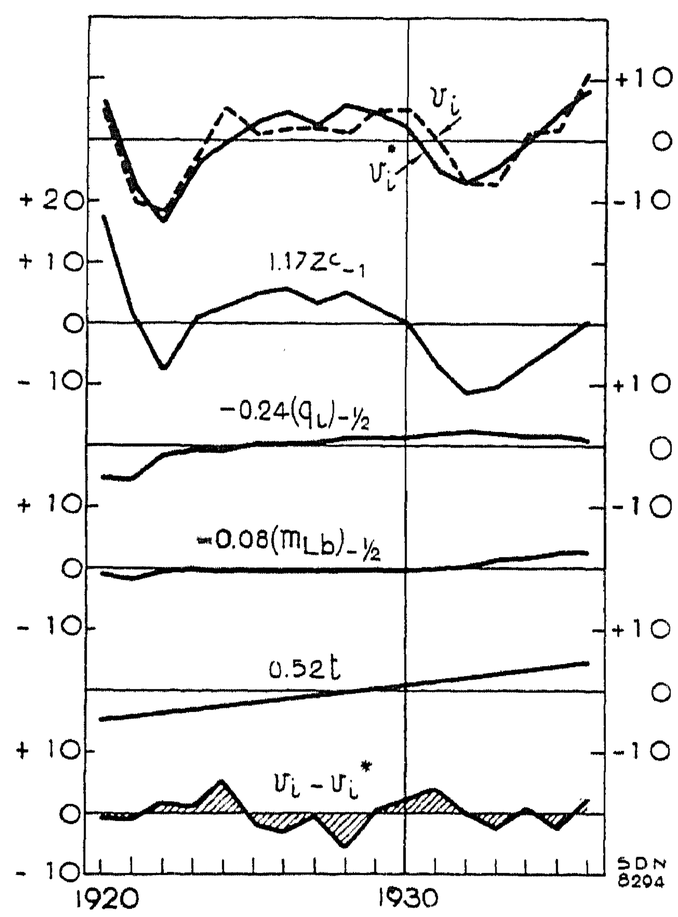
v_i^*: iron and steel consumption, as explained by:

Z^c_{-1}: profits, lagged 1 year;

$(q_i)_{-1/2}$: price of iron } lagged ½-year;

$(m_{Lb})_{-1/2}$: bond yield }

t: trend.



Regression coefficients. The regression coefficients of profits vary according to the series taken to represent them. It is found in using such a comprehensive figure as corporation profits, for the United States, that when there is an increase in these profits of 1% of their average level, there is a corresponding increase in investment activity of 0.3% of its average level. The figure found for Germany (pre-war) is about twice as great, whereas the English figure (post-war) is nearly four times as great; but the latter is based on the *Economist* sample of profits, which seems to show relatively small percentage fluctuations.

It is natural that share prices,[1] if used as representative of profits, obtain a larger coefficient, lying between 0.6 and 1.0.[2] This means that the change in investment caused by a 1% change in share prices is also of the order of 1%.

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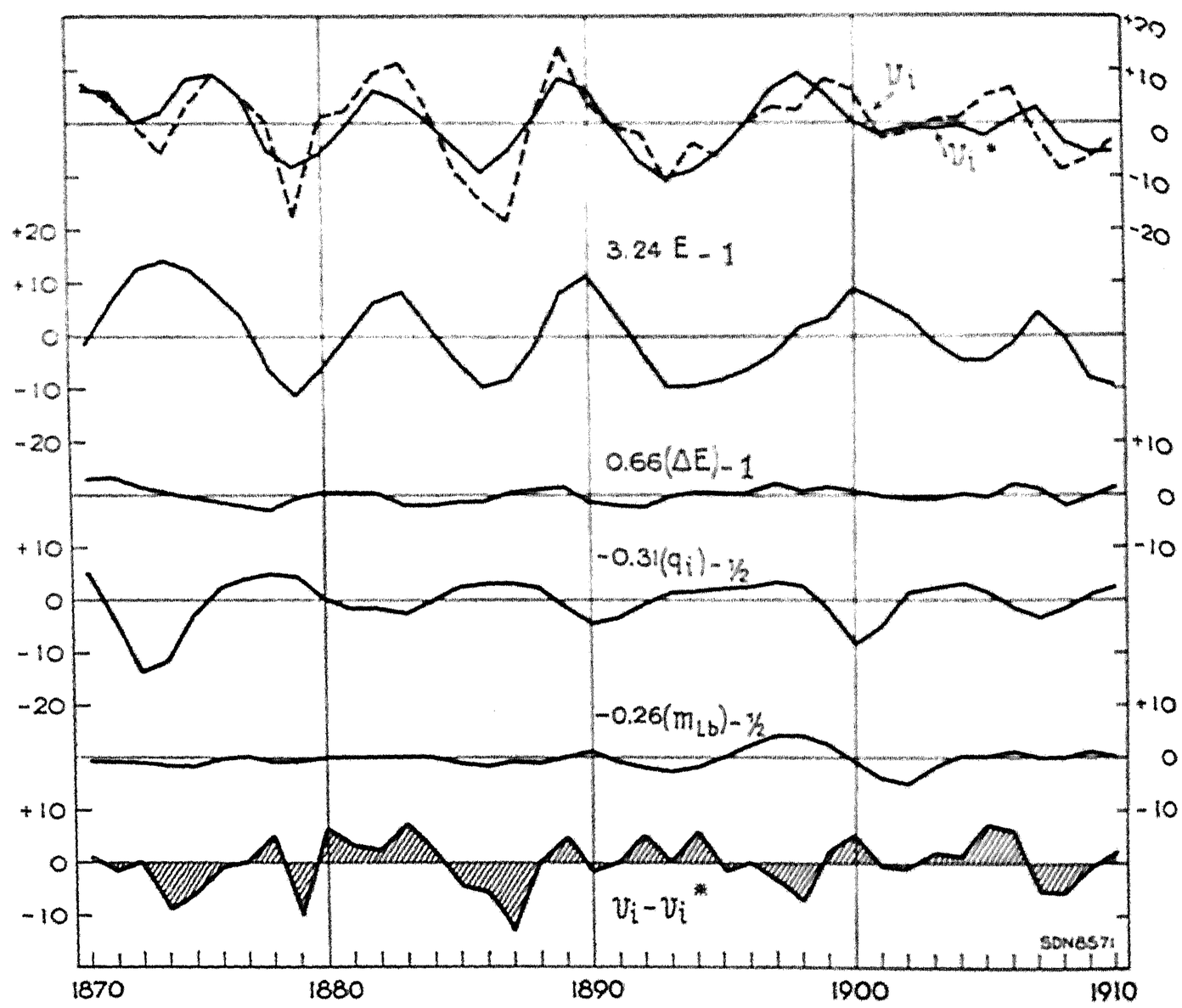
[1] For in pre-war times share prices in general showed smaller percentage fluctuations than dividends or profits. In the United States after the war the situation was different; the share yield was at a minimum in 1929 when prices were at a maximum.

[2] As far as the figures in Table III. 4 are concerned. Some of the exceptional figures in the other tables are explained below.

Graph III. 4.

"Explanation" of Iron and Steel Consumption.
United Kingdom, 1871-1910.



v_i: iron and steel consumption, actual.

v_i^*: iron and steel consumption, as explained by:

E_{-1}:	non-labour income	lagged 1 year.
$(\Delta E)_{-1}$:	rate of increase in non-labour income	lagged 1 year.
$(q_i)_{-\frac{1}{2}}$:	price of iron	lagged ½-year.
$(m_{Lb})_{-\frac{1}{2}}$:	bond yield	lagged ½-year.

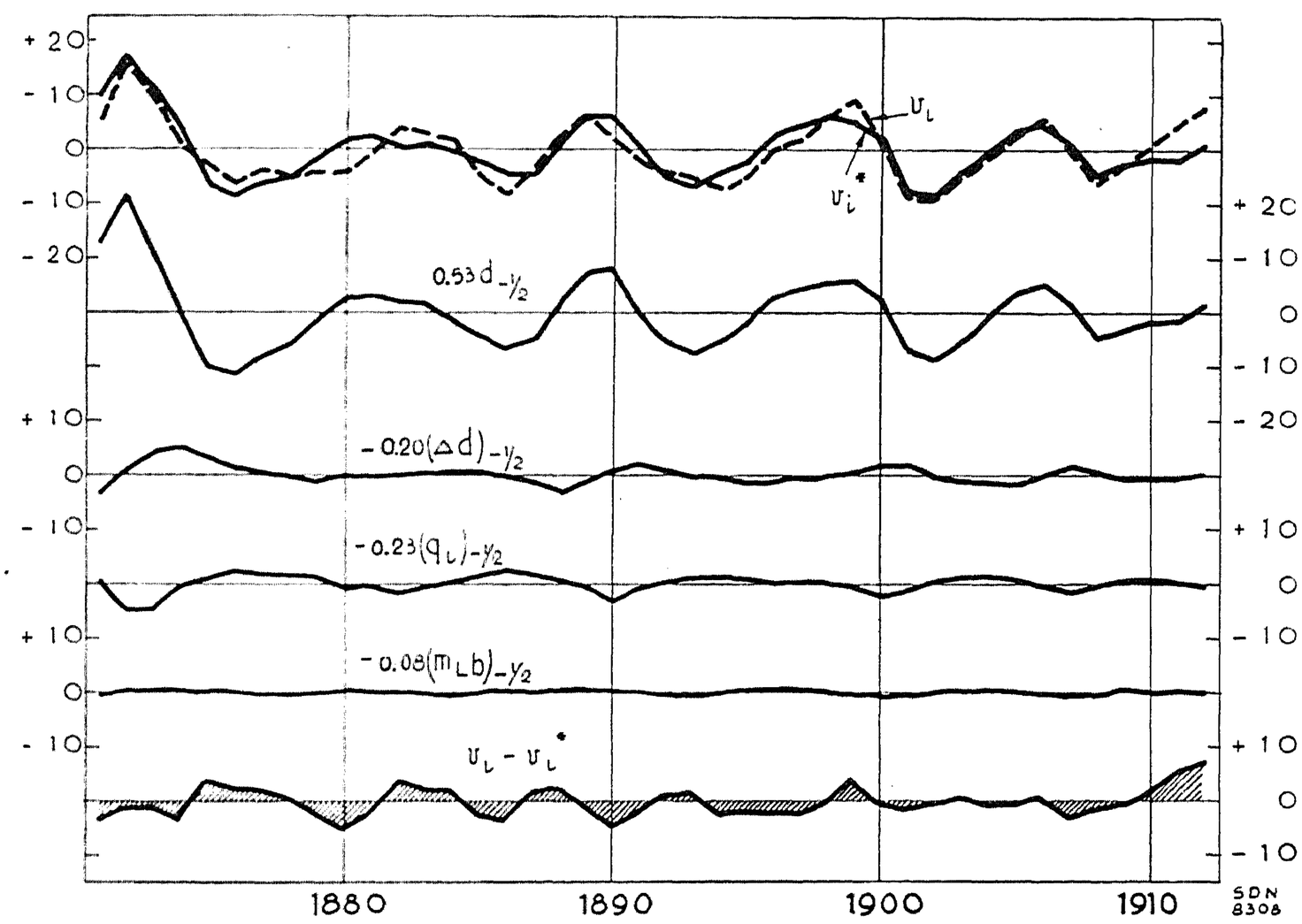
Lag. In order to obtain an impression of the empirical evidence regarding the lag between profits and investment activity, Graph III. 6 has been constructed from all cases where a " free " lag was used in the calculations, *i.e.*, where both profits without lag and with one-year lag[1] were intro-

[1] Or, which comes to the same thing, profits and the rate of increase in profits. In Graphs III. 4 and III. 5, the latter's influence has been shown separately, whereas in the tables (except Table III. 10) it has been combined with the influence of profits.

Graph III. 5.

"EXPLANATION" OF IRON AND STEEL CONSUMPTION.

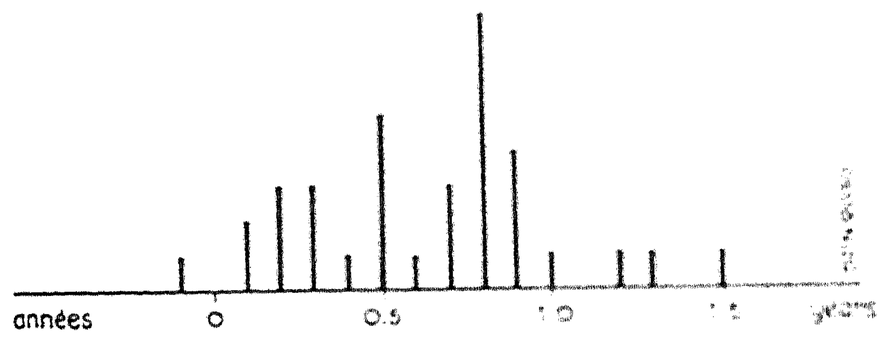
GERMANY, 1871-1912.



v_i:	iron and steel consumption, actual.	
v_i^*:	iron and steel consumption, as explained by:	
$d_{-1/2}$:	dividend in % of capital	lagged ½-year.
$(\Delta d)_{-1/2}$:	rate of increase in dividend in % of capital	
$(q_i)_{-1/2}$:	price of iron	
$(m_L b)_{-1/2}$:	bond yield	

duced and the real lag was assumed to be the average of these two, weighted by the regression coefficients corresponding to each. Graph III. 6 gives the frequency distribution of the lags found, the average length of which is about eight months. This figure supports fairly well the *a priori* reasoning in § 8.

Graph III. 5.
FREQUENCY DISTRIBUTION OF LAGS.



The two relations between profits and investment activity.

At this point, the validity of our general result, that profits have a large influence on investment activity, may be discussed in a more general framework. It is clear that, between these two variates, profits ($\bar{\bar{Z}}$) and investment activity ($\bar{\bar{v}}$), there exists another relation—viz., the definition (or, if one prefers it, the calculation) of profits. Writing

$\bar{\bar{u}}$ for consumption goods production

and

$\bar{\bar{z}}$ for the general profit margin,

profits will be defined (or calculated) by the equation [1]

$$\bar{\bar{Z}} = (\bar{\bar{u}} - \bar{\bar{v}})\,\bar{\bar{z}} \ .$$

Hence we would have two relations between profits and investment activity.

This fact might give rise to doubts as to the value of the preceding results in two respects:

(1) Is it not possible that only the second relation between profits and investment exists and that hitherto this has been wrongly taken as an *influence* of profits on investment ?

(2) Granted that there are two relations, is there not reason to fear that the coefficient found for the influence of profits is partly attributable to the other relation ?

It seems reasonable to answer both questions in the negative.

(1) As has been stated in the Introduction,[2] an essential element of our method is that the variates playing a rôle in each equation

[1] For the purpose of our argument, it is immaterial whether or not slightly different definitions (or methods of calculation) are adopted.

[2] *Cf.* p. 12.

must be known on *a priori* grounds. This also applies here. Even without any statistical evidence, few economists would deny that there is a causal influence of profits on investment.

But let us suppose, nevertheless, that evidence on this point is sought for in the statistics. A test should then be devised to prove that the apparent parallelism between profits and investment activity is not only due to the composition of profits but also to the causal connection between the two variates.

The obvious high correlation between Z and a combination of u, v and z does not imply that each of these three variates is closely correlated with Z. If the other relation between v and Z also exists, it is to be expected that the correlation between v and Z will be closer than that between u or z and Z. In most cases considered, however, u is closely correlated with v—as a consequence of what has been called the "multiplier" effect—which results in the fact that a high correlation between v and Z also causes a high correlation between u and Z and, hence, to some extent between the third variate z and Z. In these cases, therefore, the proposition cannot be tested along these lines. Only where u and v are not closely correlated will the test be possible.

Such a case is that of Germany before the war (*cf.* Graph III. 7, left-hand part). The upper pair of curves compares profits—represented here by dividends, d—with the *a priori* combination $u + v$, and z (for lack of better data, z has been approximately represented by the expression $(p - \frac{1}{2} l_w)$ where p is the general price level, l_w the wage rate, $\frac{1}{2}$ being approximately the wage quota in prices). The three other pairs of curves compare d with each of the variates u, v and z separately.

The correlation between d and the combination is close. Of the three other correlations, that between d and v is the nearest. This is demonstrated by the following correlation coefficients:

Between	d	and	combination	: 0.80
,,	d	,,	$v_{+\frac{1}{2}}$	: 0.70
,,	d	,,	u	: 0.44
,,	d	,,	$p - \frac{1}{2} l_w$	: 0.64
,,	u	,,	v	: 0.26.

The result is therefore favourable to our thesis, though not strikingly so.

Another clear case is that of Germany in the years around 1929. The fluctuations in profits (Z) may be accounted for by the fluctuations in the factors $u + v$, volume of production, and p/l_w, the proportion between prices and wage-rates (following Donner,[1] who prefers the proportion to the margin). It is especially noticeable that the volume of total production is not correlated with home investment (v) so closely as profits are (cf. Graph III. 7, right-hand part). In this particular case, this must be ascribed to the fact that 1929 already showed a decline in the internal business-cycle position, whereas exports were even higher than in 1928. The satisfactory correlation between profits and investment activity —including, as a further variate, interest rates (m_2) with a negative sign—cannot therefore be a consequence of profits' depending chiefly on the volume of production and the latter's depending chiefly on the volume of investment; for the volume of production shows a lower correlation both with profits and with investment than these two series do with each other.

In most other cases for which data are available, the correlation between u and v is much closer. The only test possible in such cases is to ascertain whether the correlation between profits (Z) and v is higher than that between Z and z. This is found to be the case for the United Kingdom before the war—where, however, the figures for z are very unreliable—and for the United States after the war.

But, it must be repeated, the greatest importance must be attributed to the *a priori* argument to include profits in the explanation of investment.

(2) The problem of the reliability of the coefficients when two equations exist between two variates arises also in the statistical determination of supply and demand curves: the variates "price" and "quantity exchanged" occur both in the demand and in the supply function. Often a doubt is expressed as to whether in

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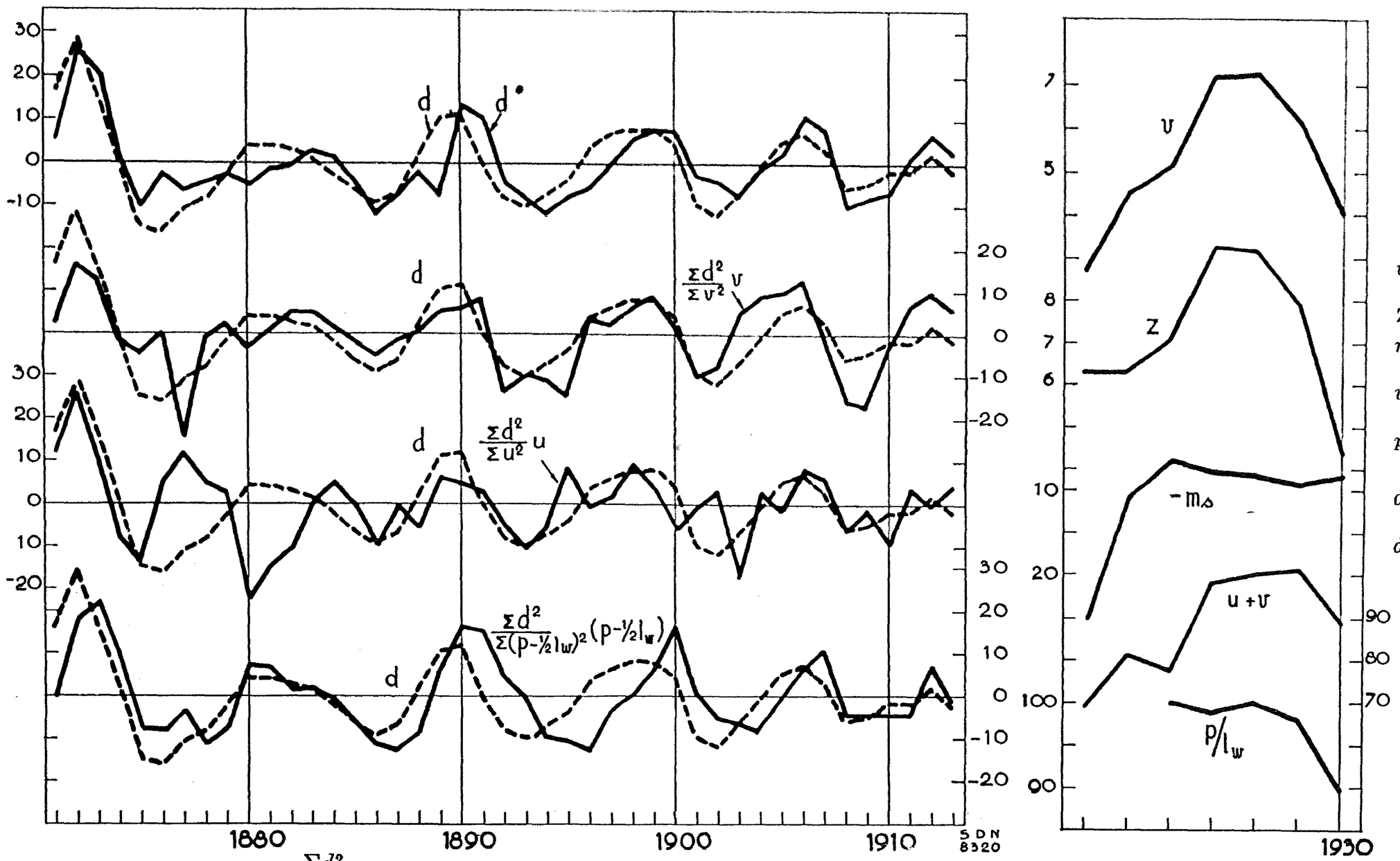
[1] DONNER: "Die Kursbildung am Aktienmarkt", *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36.

Graph III. 7.

THE TWO RELATIONS BETWEEN PROFITS AND INVESTMENT.
GERMANY 1924-1930 AND 1871-1911.



v: production of capital goods.
Z: profits.
m_s: short-term rate of interest.
u: production of consumption goods.
p/l_w, $(p - \frac{1}{2} l_w)$: estimated profit margin.
d: dividend in % of capital, actual.
d^*: dividend in % of capital, as explained by v, u, and $(p - \frac{1}{2} l_w)$.

Multiplication by $\frac{\Sigma d^2}{\Sigma v^2}$ ..., etc., in order to equalise the standard deviations (amplitudes) of explained and explaining series.

such a situation both relations are statistically determinable. This doubt seems to be based on what happens in a special case—viz., when both relations are supposed to contain no, or no important, other variates, or when, for lack of statistical data, these other variates are left outside the calculations. In fact, in such a case, only one statistical equation will in general be found, and it would be difficult to maintain that this represents both economic relations.

In principle, this difficulty disappears, however, when, in at least one of the relations, other important variates play a part, provided, of course, that not exactly the same set of supplementary variates occurs in both equations. As soon as different variates occur in the two relations, statistical calculations will yield different results. A special case is the one where the lags are different in both relations. In the case of demand and supply relations for one market, consumers' income, or the price of a competitive commodity will, e.g., occur as complementary variate in the demand relation, whereas, in the supply relation, cost of production or productive capacity may come in. In some special cases, these variates may be of only minor importance, but the lag between price and quantity exchanged may be different for the two relations, as was assumed to be the case for sugar in Professor Schultz's investigations.[1]

The doubt referred to above is the less justified the more important the complementary variates are. And it is interesting to note that if both relations actually contain only the two variates, without differences in lag, these variates in general will no longer be variates but constants, since, in general, two equations are sufficient to determine the value of two unknowns.

In the case of investment activity and profits, it is clear that the relation representing the calculation of profits does contain other variates—viz., those indicated above: production of consumption goods and profit margin. There is therefore no particular reason for doubt as to the significance of our coefficients.

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[1] H. Schultz: *Statistical Laws of Demand and Supply*, Chicago, 1928.

Coefficient of price of iron.

The regression coefficients found for iron prices sometimes show positive signs, which contradict theoretical expectation. But these results are probably not significant, and simply illustrate the degree of indeterminateness of the figures. This makes it rather difficult to give figures for the elasticity of demand. The coefficients given in the tables are, owing to the method of measurement employed, estimates of that elasticity corresponding to trend values for the variates. None of the negative values is larger than unity; those for the three European countries are all between 0 and — 0.5, and in the case of France they are more often small positive than small negative numbers. The elasticity in those countries must have been small. The British figures are all between + 0.16 and — 0.31, and here it would appear that the elasticity is below one-half. The significance calculations (see page 80 and Table III. 10) indicate, however, that there is a wide range of uncertainty, though, even in the worst case, there is about a 95% chance that the elasticity is below 0.90. The American post-war figures are all between — 0.03 and — 0.36, which also suggests an elasticity of less than one-half; here also a rather large range of uncertainty exists.

The clearest historical example of the influence of iron prices on demand seems to be the upturn in demand in England in 1875, which took place in full depression, and followed the heaviest drop in prices among our observations. (See Graph III. 4 above.)

Coefficient of interest rates.

Even greater uncertainty seems to exist concerning the coefficient of interest rates. Here again, French and German figures are centred around zero, and American figures are not very far from it (*cf.* corresponding columns in Tables III. 4—III. 9). Only the English figures (relating to the influence of long-term interest rates) show a decidedly negative tendency, again with a large spread. Their median value is — 0.31 for the pre-war calculations; the post-war figure is considerably smaller: about — 0.1. The figure — 0.31 is in itself not a small figure: it means that a reduction of 1% in the long-term interest rate would cause a 31% increase in investment activity. It should not be forgotten, however, that the largest fall per annum in the long-term interest rate

in any cycle before the war was 0.18 and that in most cases it was far less.

Not very different results are obtained in cases where share yield has been introduced as representing the interest rate, or where both share yield and the short-term interest rate have been used, except for the United States in the post-war period. Here share yields are found to have a considerable influence, possibly because they showed large fluctuations in a direction opposite to the usual fluctuations showed in pre-war times in Germany. This must be attributed chiefly to the exaggerated stock-exchange boom in 1928-29, which forced the yields down to very low levels, and to the confidence crisis in 1932, which resulted in very high share yields.

In addition, it must be noted that the fluctuations in share yield did not show any correlation with short-term interest rates; therefore the importance of share yield as an explanatory factor does not involve any proof of the importance of interest rates in the narrower sense.

Finally, the question may be put as to whether the influence found for a change of 1% in interest rates is the same as for a change of 1% in profit rate, (*i.e.*, profits as a percentage of capital). If it is true that the difference between these two rates is the guide for investment activity, these two influences must be about equal. Unfortunately, the question can only be answered for the cases where profit rates are known, *i.e.*, for the United Kingdom after the war and Germany before the war. Here the regression coefficients calculated on this basis compare as follows:

Influence of 1% change in	Profit rate	Long-term interest rate
United Kingdom post-war	12	— 8
Germany pre-war	6	— 8

The result is not bad: the order of magnitude is the same, a deviation in one direction being counteracted by a deviation in the opposite direction. This would, in a sense, support the results obtained. For the United Kingdom before the war the coefficient

of interest rates is about 3 times as great as that found for the post-war period, but roughly the same proportion seems to apply to the coefficient for profits.

Comparison between countries.

Table III. 4 (pages 68 and 69) enables us to compare the results obtained for the four countries studied. Most of the differences between these results are hardly significant, the most important being probably the difference in lags in the influence of profits. In particular, the lag of one year found for the United Kingdom (post-war) is decidedly longer than in most other cases. In addition, the influence of the price of iron and interest rates seems to be smaller in Germany and France than in England.

Comparison between production and consumption of pig-iron.

Table III. 5 (pages 70 and 71) shows an interesting difference in the lag with which profits enter into the English explanations of pig-iron production and consumption. Production reacts to profits much more quickly than does consumption. It follows that consumption probably lags behind production. One has the impression that the fluctuations in foreign demand for British iron must have had a leading influence on the business position in the United Kingdom. In the case of Germany, on the other hand, the difference in lag is much smaller. Apart from this difference, no essential feature emerges from Table III. 5. As was to be expected, the influence of profits is greater on consumption than on production, the difference being greater in Germany than in England.

Comparison between periods.

Table III. 6 (pages 70 and 71) shows comparable figures for three periods: (i) before 1895, the turning-point of the " long cycle ", (ii) between 1895 and the war, and (iii) after the war. If structural changes have occurred in the relation investigated, they must be reflected in changed coefficients. If, *e.g.*, it be true that investment activity at present reacts more violently to profit changes than it did before the war, this would be translated into a larger regression coefficient for profits. Unfortunately, it is almost impossible to obtain comparable figures for the pre-war and the post-war periods.

"EXPLANATION" OF INVEST

Table 1

Note.— All series are in % deviations from trend, *except* series (9) and
indicated in brackets after each coefficient: when the lag has been deter

Country	Period	Series explained	Correlati coefficie R
(1)	(2)	(3)	(4)
United States	1919-1933	Production of pig-iron	0.94
,,	,,	Investment activity [a]	0.98
United Kingdom	1871-1910	Consumption of pig-iron	0.75
,,	1920-1936	,, ,, ,,	0.90
Germany	1871-1912	,, ,, ,,	0.87
United States	1895-1913	Production of pig-iron	0.77
Germany	1871-1912	Consumption of pig-iron	0.79
France	1871-1908	,, ,, ,,	0.81
United States	1895-1913	Production of pig-iron	0.76
Germany	1871-1912	Consumption of pig-iron	0.83

a Flow of producers' and consumers' durable commodities, building excluded.

ACTIVITY, SECOND STAGE.

Comparison of Countries.

which are in absolute deviations from trend in units of 0.01%. The lag in years is "freely", the coefficient is in italics.

Regression coefficients and lags of:					
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
† 0.29 (½)			— 0.03 (½)		† — 0.32 (½)
† 0.28 (½)			— 0.36 (½)		† — 0.05 (½)
	3.24 (0.8)		— 0.31 (½)		— 0.26 (½)
1.17 (1)			— 0.24 (½)		— 0.08 (½)
0.53 (0.9)			— 0.23 (½)		— 0.08 (½)
		0.85 (½)	— 0.51 (½)		— 0.27 (½)
		0.72 (½)	— 0.13 (½)		0.21 (½)
		0.94 (0.9)	0.10 (½)		— 0.08 (½)
		1.05 (0.4)	— 0.54 (½)	0.02 (½)	
		0.61 (0.3)	— 0.17 (½)	0.06 (½)	

† Intercorrelation between 0.75 and 0.80.

" EXPLANATION " OF INVESTMEN

Table III. 5. Comparison

Note. — See note to Table III. 4.

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United Kingdom	1871-1910	Production of pig-iron	0.79
,,	,,	Consumption ,, ,,	0.75
Germany	1871-1912	Production ,, ,,	0.75
,,	,,	Consumption ,, ,,	0.88
France	1871-1908	Production ,, ,,	0.69
,,	,,	Consumption ,, ,,	0.81

Table III. 6. Compariso

Note. — See note to Table III. 4.

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United States	1877-1895	Production of pig-iron	0.75
,, ,,	1895-1913	,, ,, ,,	0.68
,, ,,	1919-1924	,, ,, ,,	0.83
,, ,,	1919-1933	,, ,, ,,	0.61
United Kingdom	1871-1895	Consumption of pig-iron	0.77
,, ,	1896-1910	,, ,, ,,	0.68
,, ,,	1920-1936	,, ,, ,,	0.90
Germany	1871-1895	Consumption of pig-iron	0.90
,,	1895-1912	,, ,, ,,	0.88

† Intercorrelation between 0.75 and 0.80.
a Because of the lack of correlation, the calculation of a lag has no point in this case.

TIVITY, SECOND STAGE.

iron Production and Consumption.

	Regression coefficients and lags of:				
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
	1.96 (0.1)		— 0.17 (½)		— 0.39 (½)
	3.24 (0.8)		— 0.31 (½)		— 0.26 (½)
23 (½)			0.08 (½)	— 0.01 (½)	
53 (0.8)			— 0.26 (½)	0.02 (½)	
		0.49 (0.1)	0.17 (½)		— 0.03 (½)
		0.94 (0.9)	0.10 (½)		— 0.08 (½)

Various Periods.

	Regression coefficients and lag of:				
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
		0.94 (0.2)			
		0.74 (0.2)			
		2.71 (0.3)			
		0.18 [a]			
	3.29 (0.8)		— 0.33 (½)		— 0.56 (½)
	† *1.43* (lead 0.1)		† — 0.11 (½)		— 0.07 (½)
1.17 (1)			— 0.24 (½)		— 0.08 (½)
0.31 (½)			† 0.10 (½)	0.02 (½)	
0.90 (½)			0.13 (½)	— 0.03 (½)	

Profit figures for the United Kingdom and for the United States as used in post-war calculations are not available for pre-war years. Share prices, which in pre-war United States could probably be used as an indicator of profits, are no longer representative of profits in the entire post-war period, as they are quite out of line at the top of the 1929 boom. As a consequence, (i) the correlation obtained with share prices is no longer good and (ii) the regression coefficient obtained for the period 1919-1933 is quite different from that obtained for the period 1919-1924. The intensity with which investment activity reacts to share prices would seem to have decreased as compared with pre-war times according to the figure for the whole period, whereas it would seem to have increased very much according to the 1919-1924 figures. For the United Kingdom a post-war estimate for "non-labour income" by C. CLARK,[1] which is about comparable to pre-war figures, leads to a regression coefficient considerably lower (1.36). There is therefore some evidence of a more intensive reaction after the war in the United States and a less intensive in the United Kingdom.

Comparison of the figures for the two pre-war periods shows that the coefficient for profits after 1895 is, in Germany, considerably higher than in the period before 1895, slightly lower in the United States and considerably lower in the United Kingdom. The American coefficients for the two sub-periods, as well as the 1871-1895 English coefficient, are in line with those for the entire pre-war period; the 1896-1910 English is quite out of line and, moreover, a lead instead of a lag has been found here between profits and investment. This lead may, however, be interpreted as an influence of the rate of increase in profits. In addition, the British business-cycle pattern was rather weak, partly perhaps because the Boer war counteracted the 1901 depression;[2] with less pronounced fluctuations, disturbing elements become more important and the results less reliable. Anyhow, the result for the period 1896-1910 is not satisfactory.

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[1] *National Income and Outlay*, London, 1937, page 60.

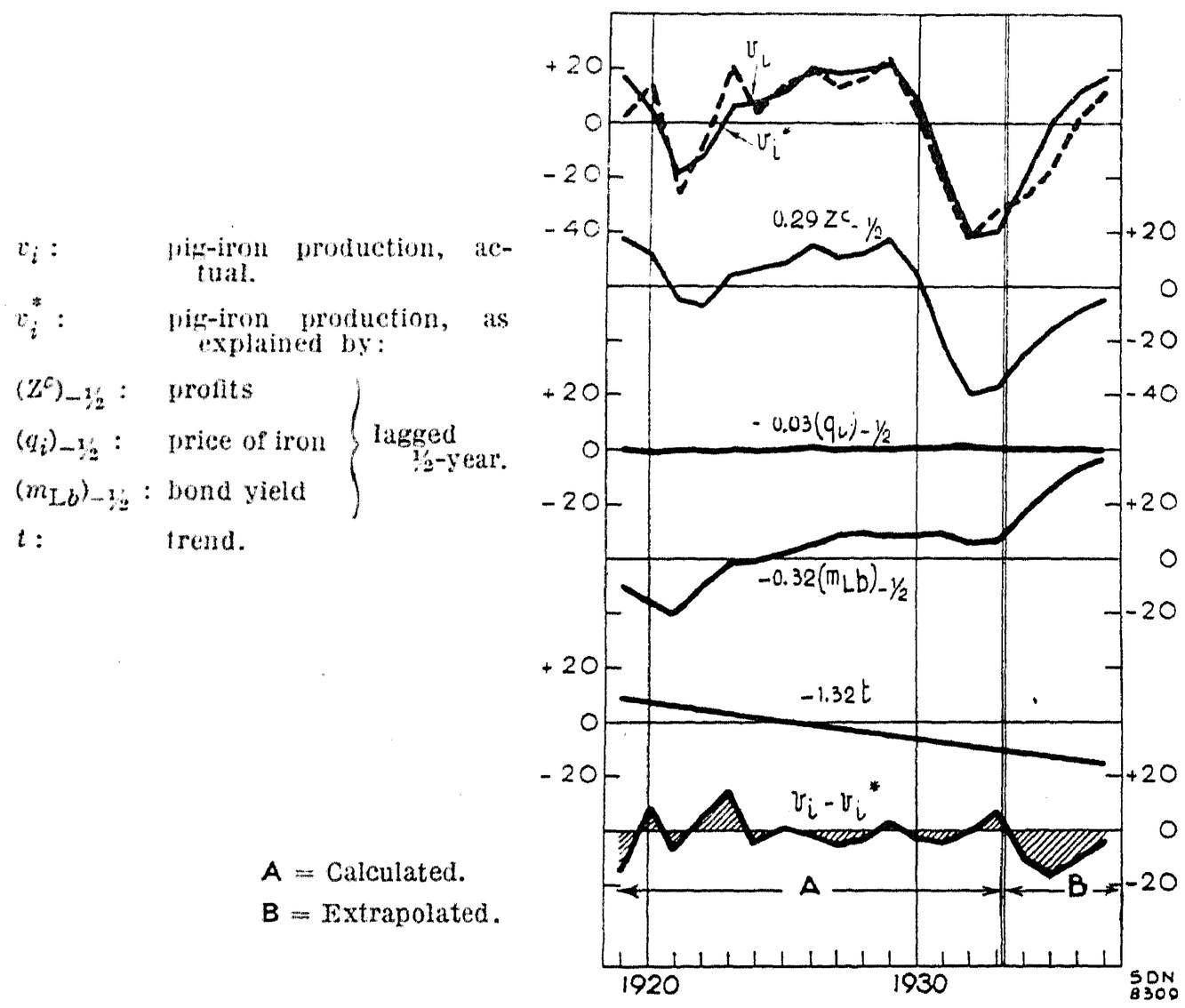
[2] *Cf.* Graph III. 4. It shows that pig-iron consumption fell only by about 10% from 1899 to 1901, whereas it fell by some 20 to 30% in the crises preceding 1901 and by 40% in Germany, *cf.* Graph III. 5, in 1901. The employment figures also showed the smallest fall ever seen in a crisis.

Extrapolation for the United States, 1934-37.

It has already been mentioned that one of the calculations for the United States has been extrapolated in order to cover the years 1934-37. In other words, the regression coefficients for the period 1919-33 have been applied to subsequent years (*cf.* Graph III. 8).

Graph III. 8.

"EXPLANATION" OF PIG-IRON PRODUCTION.
UNITED STATES, 1919-1937.



As Kuznets' estimates of investment activity are not available after 1933,[1] it was only possible to extrapolate the calculation for pig-iron production. One of the determining factors in the calculation being total profits, these had to be estimated for 1936 and 1937.

[1] It has not been possible to make use of the figures that have recently been published by KUZNETS in *National Income and Capital Formation in the United States, 1919-1935.*

" EXPLANATION " OF INVESTMENT

Table III. 7. Comparison of

Note. — See note to Table III. 4. — Series (1) is in absolute deviations from

Country	Period	Series explained	Correlation coefficient
(1)	(2)	(3)	(4)
United States	1877-1913 Upward	Production of pig-iron	0.86
	Downward	.. .. ..	0.78
United Kingdom	1871-1910 Upward	Consumption of pig-iron	0.79
	Upward	.. .. ..	0.78
	Downward	.. .. ..	0.59
Germany	1871-1912 Upward	Consumption of pig-iron	0.90
	Downward	.. .. ..	0.89
France	1871-1908 Upward	Consumption of pig-iron	0.85
	Downward	,, ,, ,,	0.79

† Intercorrelation between 0.75 and 0.80. * Multiple correlation over 0.80.

Table III. 8. Comparison of

Country	Period	Series explained	Correlation coefficient
(1)	(2)	(3)	(4)
United States	1877-1913 Upward	Production of pig-iron	0.78
	Downward	,, ,, ,,	0.60
United Kingdom	1871-1910 Upward	Consumption of pig-iron	0.68
	Downward	,, ,, ,,	0.53
Germany	1871-1912 Upward	Consumption of pig-iron	0.84
	Downward	,, ,, ,,	0.84
,,	1871-1912 Upward	,, ,, ,,	0.72
	Downward	,, ,, ,,	0.85
France	1871-1908 Upward	Consumption of pig-iron	0.81
	Downward	,, ,, ,,	0.75

IVITY, SECOND STAGE.

ard and Downward Phases I.

d, in units of 0.01%; series (12) is in % deviations from trend.

Regression coefficients and lags of:						
rofits[a]	share prices	price of pig-iron	short-term interest rate	long-term interest rate	share yield	building volume
(5)	(7)	(8)	(9)	(10)	(11)	(12)
	* *0.26* (0.3)	* 0.42	—0.03 (½)			*0.27 (0)
	0.69 (0.9)	0.23	0.01 (½)			0.55 (0)
46 (0.8)		0.16 (½)		—0.27 (½)		
79 (0.8)				—0.29 (½)		
38 (0.8)				—0.31 (½)		
	0.36 (½)		0.04 (½)		0.13 (½)	
	† 0.54 (½)		0.04 (½)		†— 0.03 (½)	
	0.88 (1.3)	0.27 (½)		—0.29 (½)		
	† *3.15* (0.7)	—0.36† (½)		0.25 (½)		

[a] United Kingdom: non-labour income.

ward and Downward Phases II.

Regression coefficients and lags of:		
profits	non-labour income	share prices
(5)	(6)	(7)
		0.73 (0.2)
		0.58 (1.2)
	2.24 (1.5)	
	1.41 (0.7)	
.46 (½)		
.41 (½)		
		0.57 (½)
		0.58 (½)
		1.92 (0.8)
		2.24 (0.7)

The estimate was based on two very high correlations found for the period 1919-1933, which held also for 1934 and 1935. The first " explains " profits by (1) total receipts of all corporations during the same year and (2) total receipts during the preceding year. The inclusion of lagged receipts is justified by the fact that " total deductions " (*i.e.*, the amounts to be deducted from total receipts in order to obtain profits of all corporations) are very closely correlated with receipts lagged over a few months, since they represent an " adaptation of costs to receipts " which takes some time. The second correlation is between " total receipts of all corporations " and

(1) Index of industrial production (Federal Reserve Board), multiplied by index of wholesale prices (Bureau of Labor Statistics); and
(2) Department-stores sales.

This latter correlation served to estimate total receipts for 1936 and 1937, which in turn determined the estimate of profits for 1936 and 1937. The result of the extrapolation is shown in Graph III. 8. The general direction of actual and calculated production of pig-iron is the same, but there seems to be a more or less systematic difference in level which might reflect the result of the changed structure.

Comparison between phases.

Tables III. 7 and III. 8 (pages 74 and 75) give comparable results for two sets of years; those showing a lower, and those showing a higher, investment figure (measured in deviations from trend) than the preceding year. The former set of years is called " downward phases ", the latter " upward phases " of the cycle. As the calculations for upward and downward phases are chiefly intended to be an illustration of the degree of uncertainty in the general results, it did not seem worth while to construct bunch maps, etc. As in Tables III. 4 to III. 6, only cases in which the correlation coefficients between the " explanatory " series do not exceed 0.80 have been included, in order to exclude—although admittedly only in a very rough way—cases of multicollinearity. Some of the cases given in Table III. 7 are probably still unreliable, however, in that respect, and Table III. 8 has therefore been added; in this table, only the chief variate—profits—or a representative series is employed, and multicollinearity is therefore impossible.

No systematic differences can be discovered. The regression coefficients of profits in Table III. 7, although rather divergent, do not differ from those for the whole period by more than three times the standard errors of the latter (*cf.* Table III. 10) in the case of the United Kingdom and Germany. For the United States and France, the results of either the upward or the downward phases are very uncertain because of a high intercorrelation between some of the explanatory variates. In Table III. 7, downward phases show a lower coefficient for the United Kingdom and a higher for Germany and the United States. These differences disappear, however, almost entirely in Table III. 8, except for the United Kingdom. The coefficients of the secondary factors sometimes show larger relative differences, but even these are not significant, as the coefficients are rather uncertain (*cf.* standard errors, Table III. 10), the only possible exception being the influence of building in the United States. It is therefore difficult to obtain evidence regarding the necessity of explaining downward phases by other relations than upward phases.

Inclusion of more variates.

Table III. 9 (pages 78 and 79) gives, for a few cases, the successive results obtained if more and more variates are included in the explanation. It is chiefly intended to show the great importance of profits as against the other variates in the " explanation ", and the relatively small improvements in correlation and the small change in regression consequent upon their inclusion. Nevertheless, these improvements will prove to be significant.

Partial scatter diagrams.

Graphs III. 9 to III. 11 represent partial scatter diagrams (*cf.* Chapter II, § 5) for three cases. As has already been observed, they enable us to test whether the hypothesis of rectilinear relationship is fulfilled or not. It may be seen from the graphs that there is no wide departure from rectilinearity, but that nevertheless a tendency to curvilinearity is present in a number of cases. The graphs containing the price of iron as the independent variate represent the demand curve for iron. (This is not quite correct in the case of the United States as the investment index includes other investment goods, but, owing to the rather large parallelism between the production of the various kinds of investment goods, the error cannot be important.)

EXPLANATION OF INVEST

Table III. 9. Influe

Note. — See note to Table III. 4 and III. 7. Δ stands for "rate of increas

Country	Period	Series excluded	Correla coeffic R
(1)	(2)		(4)
United States	1919-33	Investment activity [a]	0.9
,, ,,	,,	,, ,,	0.9
,, ,,	,,	,, ,,	0.9
United Kingdom	1871-1910	Consumption of pig-iron	0.5
,, ,,	,,	,, ,, ,,	0.6
,, ,,	,,	,, ,, ,,	0.7
Germany	1871-1912	Consumption of pig-iron	0.7
,,	,,	,, ,, ,,	0.7
,,	,,	,, ,, ,,	0.8
,,	,,	,, ,, ,,	0.8

a For United Kingdom, non-labour income.
b Flow of producers' and consumers' durable commodities, building excluded.

Table III. 10. Results of Significance Calculations

Case	Correlation coefficient	Serial correlation of residuals	Regression coefficien: Profits	Δ profits	Non-l inc
(1)	(2)	(3)	(4)	(5)	(
I	0.99	—0.46 ± 0.27	0.24 ± 0.02 (½)		
II	0.77	0.01 ± 0.20			3.29 ±
III	0.75	0.16 ± 0.16			3.24 ±
IV	0.90	0.00 ± 0.25	1.17 ± 0.17 (1)		
V	0.87	0.43 ± 0.16	0.53 ± 0.09 (½)	—0.20 ± 0.11 (½)	

I United States, Investment activity *a*, 1919-1933.
II, III, IV } United Kingdom. Consumption of pig-iron. { 1871-1895. 1871-1910. 1920-1936.
V Germany. Consumption of pig-iron, 1871-1912.

a Flow of producers' and consumers' durable commodities, building excluded.

ACTIVITY, SECOND STAGE.

Number of Variates included.

Series (13) is expressed in % of average level of price of pig-iron during period.

Regression coefficients and lags of:						
profits α	share prices	price of pig-iron	short-term interest rate	long-term interest rate	share yield	Δ price of pig-iron
(5)	(7)	(8)	(9)	(10)	(11)	(13)
0.31 (½)						
0.28 (½)					— 0.04 (½)	
0.24 (½)		— 0.25 (½)	— 0.05 (½)		— 0.08 (½)	
1.82 (0.6)						
1.90 (0.8)				— 0.33 (½)		
3.24 (0.8)		— 0.31 (½)		— 0.26 (½)		
	0.59 (½)					
	0.59 (½)	0.00 (½)				
	0.61 (0.3)	— 0.17 (½)	0.06 (½)			
	0.56 (0.9)	— 0.15 (½)	0.03 (½)			0.35 (½)

Correlation of Residuals and Standard Errors of Regression Coefficients.

their standard errors, and lags of:				
Δ Non-labour income	Price of pig-iron	Short-term interest rate	Long-term interest rate	Share yield
(7)	(8)	(9)	(10)	(11)
	—0.25 ±0.12 (½)	—0.05 ±0.02 (½)		—0.08 ±0.02 (½)
0.54 ±0.54 (1)	—0.33 ±0.13 (½)		—0.56 ±0.49 (½)	
0.66 ±0.42 (1)	—0.31 ±0.10 (½)		—0.26 ±0.14 (½)	
	—0.24 ±0.14 (½)		—0.08 ±0.10 (½)	
	—0.23 ±0.17 (½)		—0.08 ±0.14 (½)	

B. *Significance calculations.*

Table III. 10 (pages 78 and 79) gives details regarding the significance calculations made. These have been restricted to five cases which seem representative and call for the following comments.

Serial correlation for residuals.

First, the serial correlation for the series of the residuals has been calculated: *i.e.*, the correlation of that series with itself if lagged one year. This calculation serves to test the hypothesis at the basis of Fisher's theory—viz., that the residuals are to be considered as sample drawings from a " normally distributed universe ". At the same time, it gives information as to whether the regression chosen satisfies the scheme of the shock theory (*cf.* § 7). In order to see whether the serial correlation differs from zero to any significant extent, the serial correlation coefficient has to be compared with its standard error, which equals $\frac{1}{\sqrt{N-1}}$.[1] It then appears that the greatest deviation from zero is found for Germany before the war, where the result is, however, still within a distance of three times the standard error.

Standard errors.

The standard errors, calculated with Fisher's formula (*cf.* Appendix A, § 4) are such that all the regression coefficients for profits or the series replacing them are with a very high probability significantly positive: in all five cases, the regression coefficients are more than three times their standard error.

Only five out of ten coefficients tested for iron prices and interest rates are rigorously significant, *i.e.*, with a probability of over 95 %. This fact is illustrated by Graph III. 12, where for each case the ranges within once the standard error on either side of the regression coefficient (*i.e.*, $b \pm \sigma_b$) have been indicated in black, those within twice the standard error (*i.e.*, $b \pm 2\sigma_b$) by shading. It will be seen that five of the latter areas go beyond the zero point. Nevertheless, there is more probability of the regressions, being negative. Moreover, it is satisfactory to the statistician that the range of error is generally smaller for post-war than for pre-war figures.

[1] *Cf.* M. S. Bartlett: " Some Aspects of the Time Correlation Problem ", *Journal of the Royal Statistical Society*, 1935 (98), page 537, quoted in T. Koopmans, *loc. cit.*, page 129.

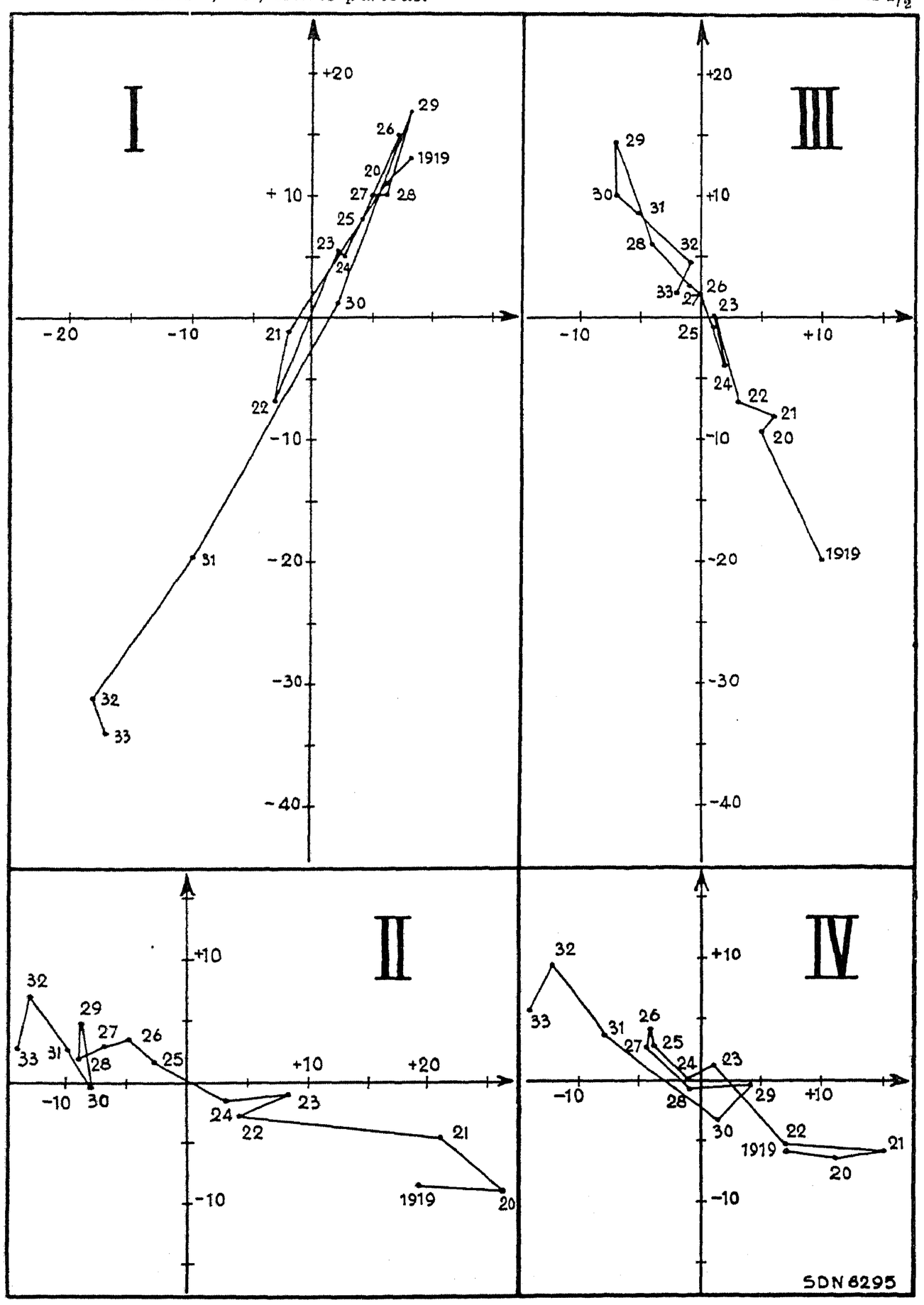
Graph III. 9.
Partial scatter diagrams.
INVESTMENT ACTIVITY: UNITED STATES, 1919-1933.

Ordinates

I, II, III, IV — Investment activity (flow of durable producers' goods to enterprises, plus flow of durable consumers' goods to enterprises and households) corrected for influence of other "explanatory variates", *i.e., ceteris paribus.*

Abscissæ

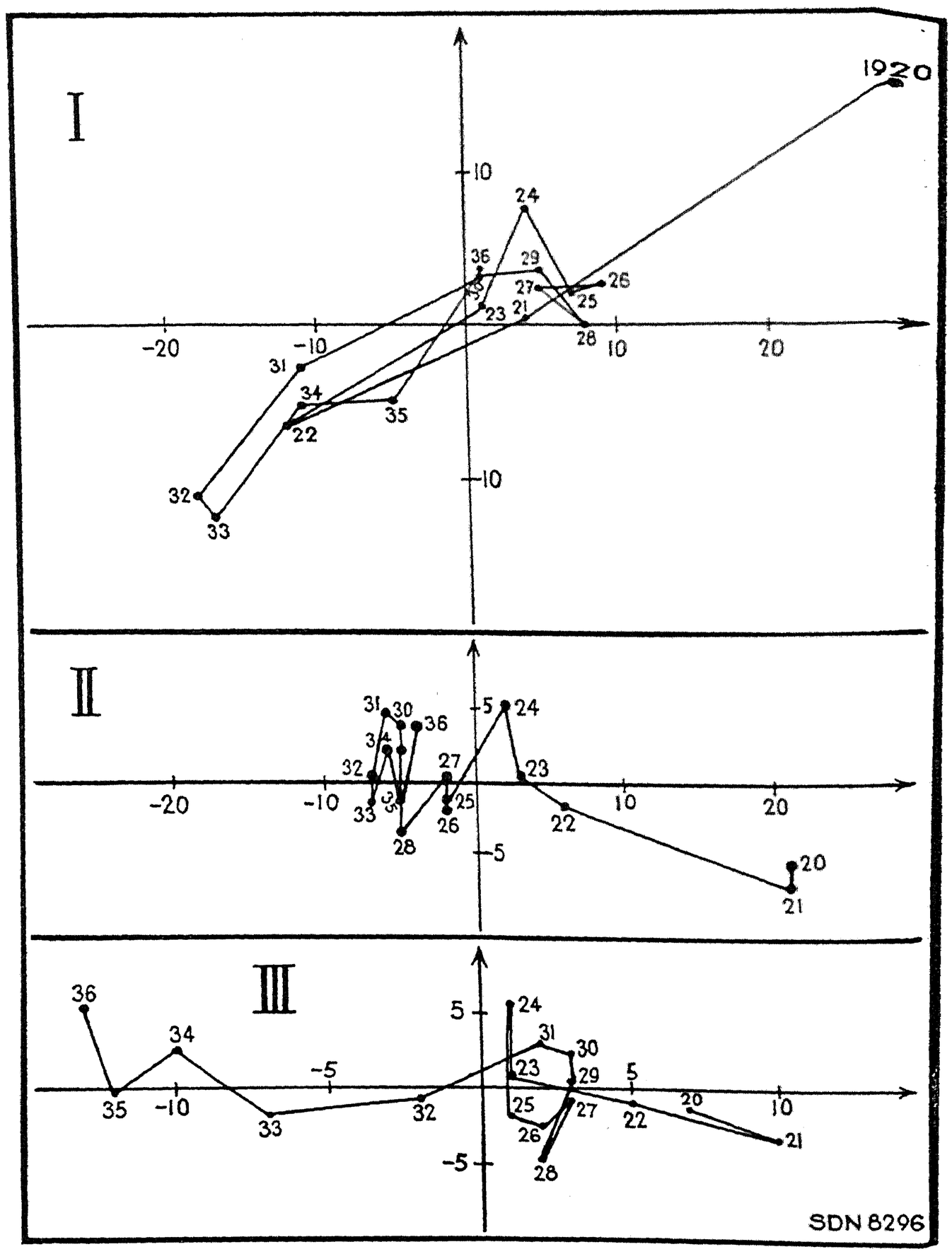
I Profits $_{-1/2}$
II Price of pig-iron $_{-1/2}$
III Share yield $_{-1/2}$
IV Short-term interest rate $_{-1/2}$



Graph III. 10.
Partial scatter diagrams.

PRODUCTION OF PIG-IRON: UNITED KINGDOM, 1920-1936.

	Ordinates		*Abscissæ*
I	Production of pig-iron, corrected for influence of other "explanatory variates", *i.e., ceteris paribus.*	I	Profits – 1
II		II	Price of pig-iron – 1½
III		III	Long-term interest rate – 1½



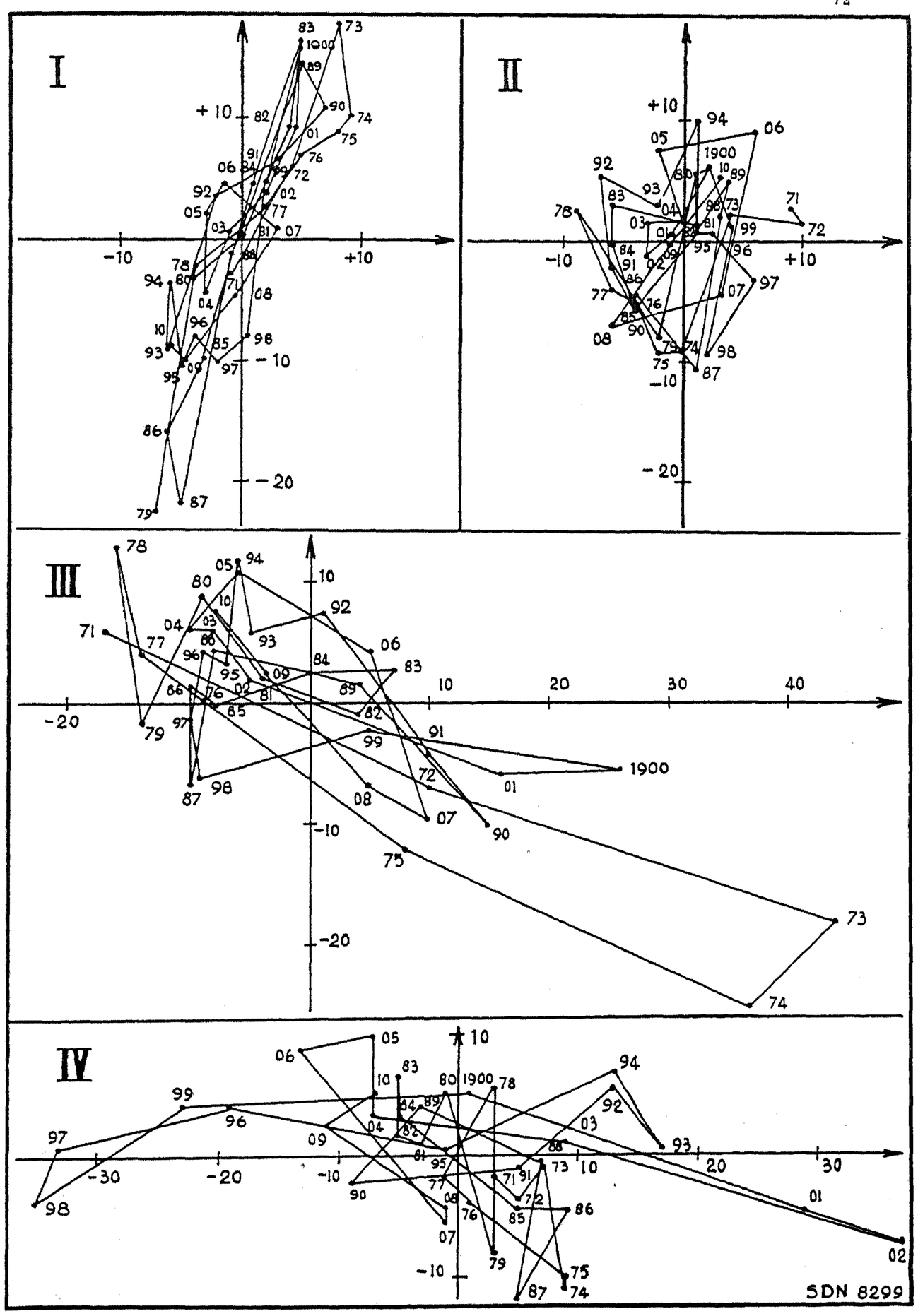
Graph III. 11.
Partial scatter diagrams.
CONSUMPTION OF IRON AND STEEL: UNITED KINGDOM, 1871-1910.

Ordinates

I, II, III, IV — Consumption of pig-iron, corrected for influence of other "explanatory variates", *i. e.. ceteris paribus.*

Abscissæ

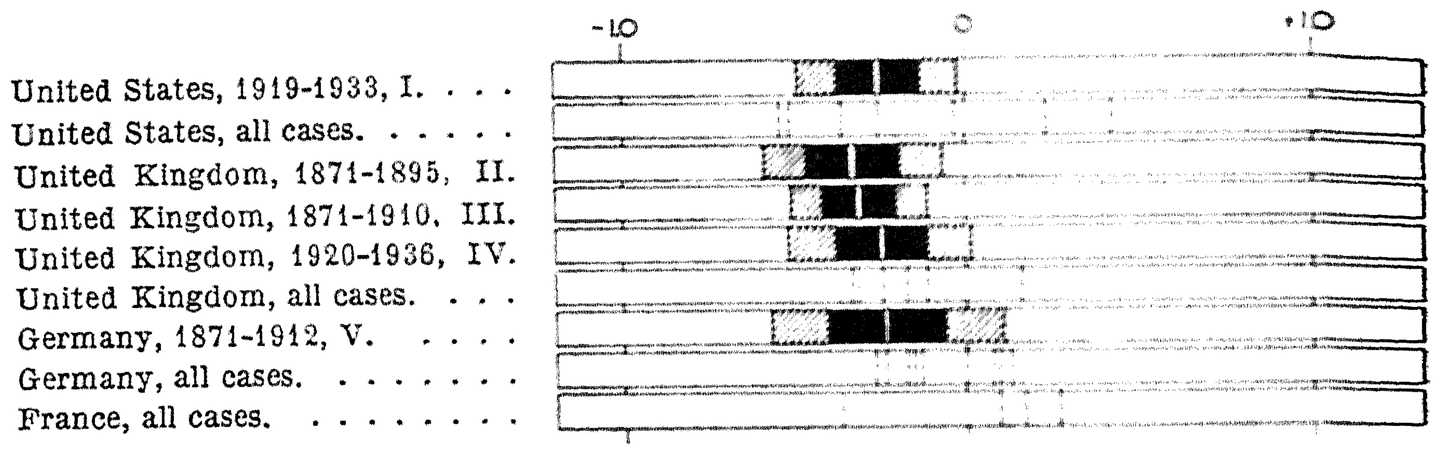
I Non-labour income $_{-1}$
II Δ non-labour income $_{-1}$
III Price of pig-iron $_{-1/2}$
IV Long-term interest rate $_{-1/2}$



Graph. III. 12.

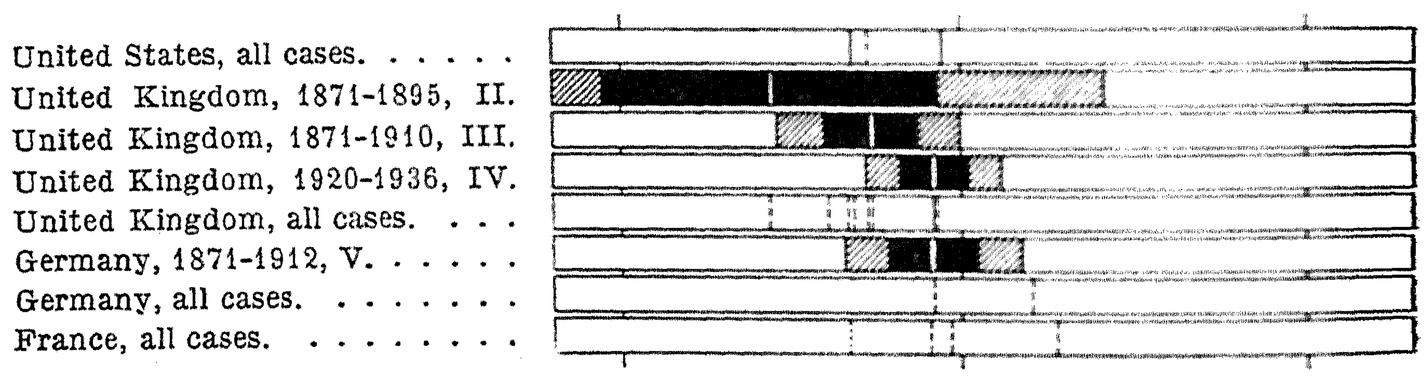
REGRESSION COEFFICIENTS AND RANGES OF INCERTITUDE.

Regression Coefficients of Iron Price.



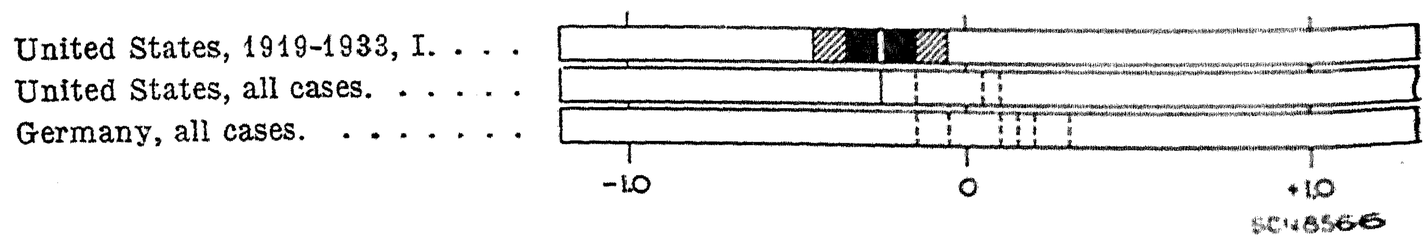
Regression Coefficients of Interest Rate.

A. Long-term, in 0.01 %.



Regression Coefficients of Interest Rate.

B. Short-term, in 0.05 %.



Explanation: Black range: range between b and $b \pm \sigma_b$ . . . . . ; Shaded range: range between b and $b \pm 2\sigma_b$ . . . . . — only for cases where σ has been calculated (cases I-V, table III. 10).

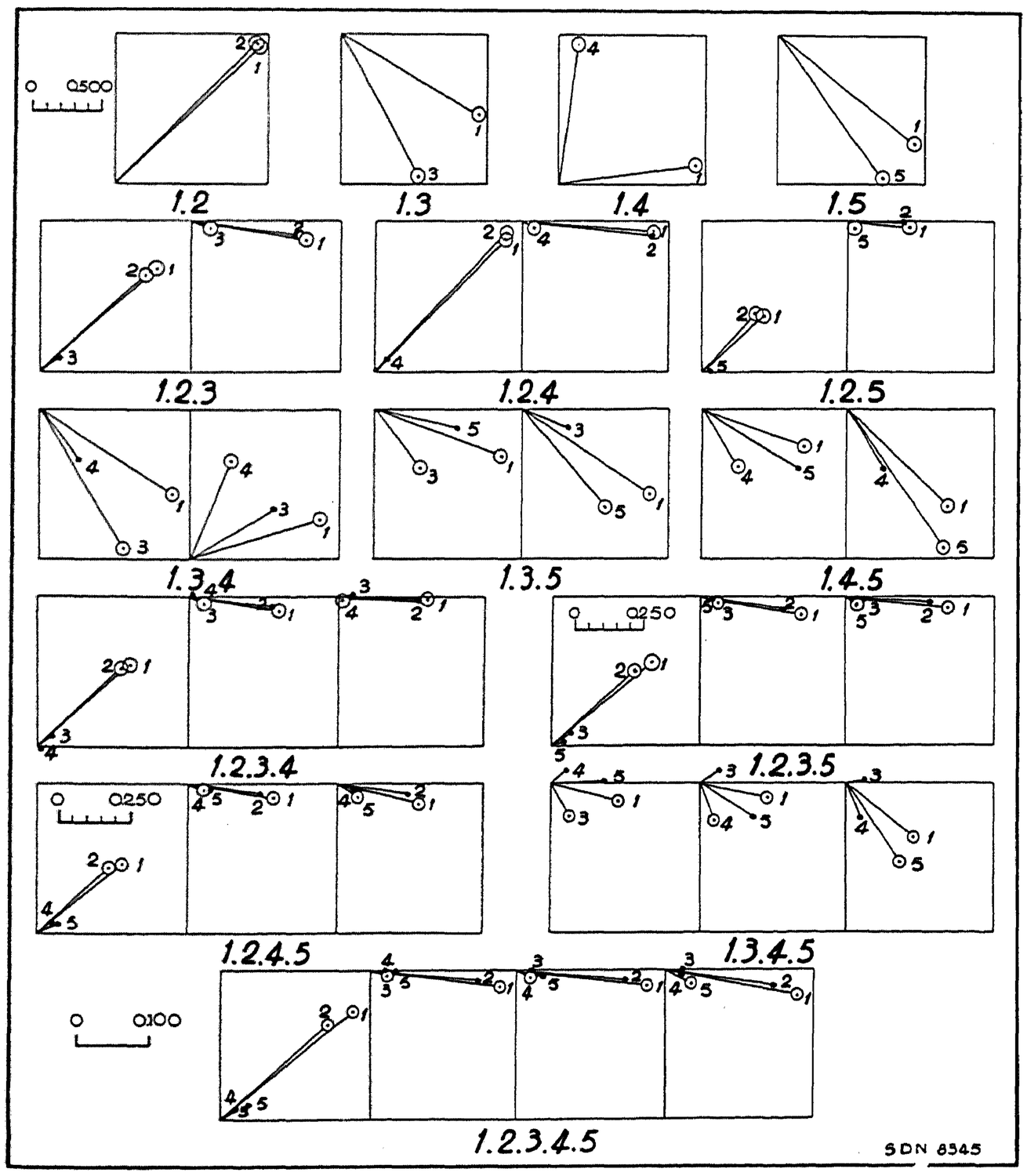
Solid line : post-war result; Broken line : pre-war result — for all cases.

Graph III. 13.

Bunch map.

INVESTMENT ACTIVITY: UNITED STATES 1919-1933

1 = Investment activity (flow of durable producers' goods to enterprises, plus flow of durable consumers' goods to enterprises and households).
2 = Profits$_{-1/2}$. 3 = Price of iron$_{-1/2}$. 4 = Short-term interest rate$_{-1/2}$.
5 = Share yield$_{-1/2}$.



Bunch maps.

In Graphs III. 13 to III. 16 a number of bunch maps are reproduced.

Graph III. 13 (page 85), relating to one of the United States post-war calculations, is very satisfactory, and even more so if—following Koopmans' argument—only the beams with numbers 1 and 2 are considered. In fact, in the 5-set (the case including all five variates) they coincide almost entirely, indicating a high degree of determinateness of all regression coefficients found. These findings are in close agreement with the findings on standard errors.

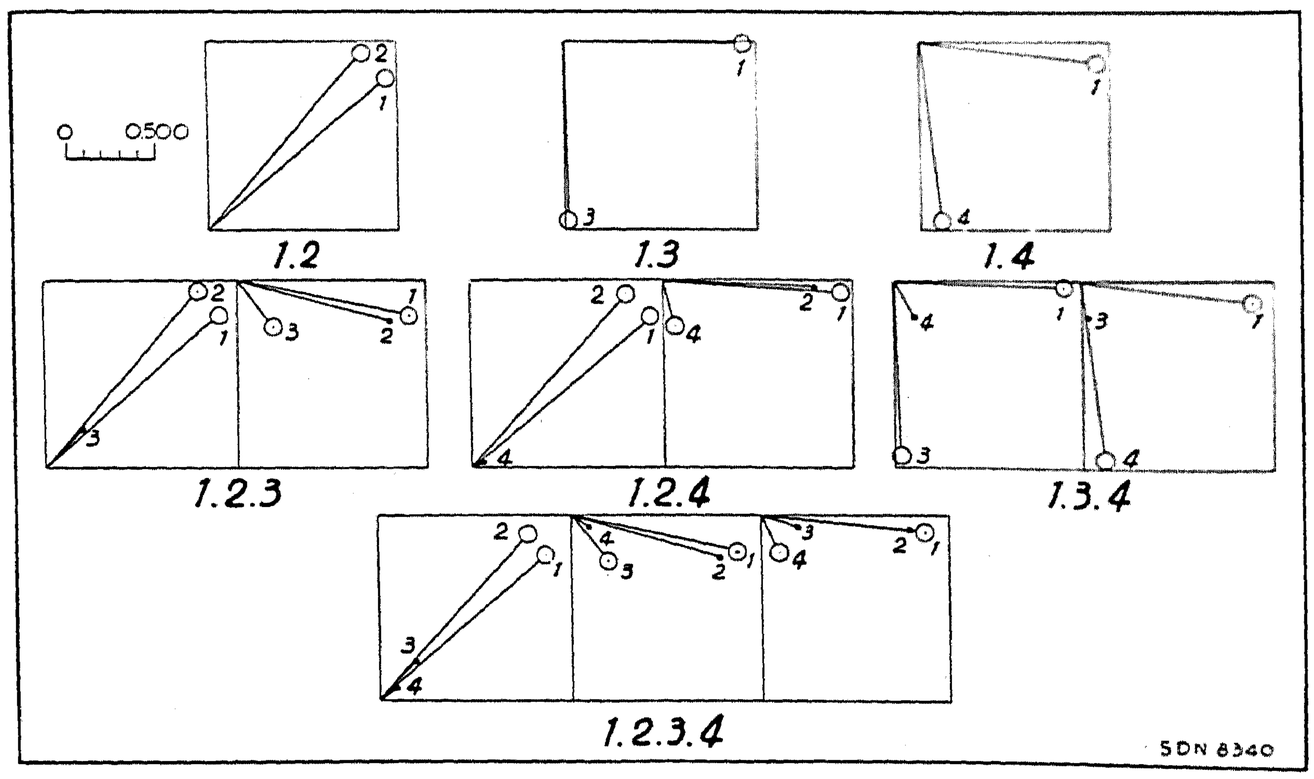
Graph III. 14, relating to the post-war calculation for the United Kingdom, is also very satisfactory, and the above observations apply. This case is particularly suitable for illustrating Koopmans' result regarding the minor importance of beams corresponding to variates which exercise only a secondary influence. Table III. 11 shows the coefficients of the four elementary regression equations, together with the probable limits to the true regression coefficients that take into account both the error of weighting

Graph III. 14.

Bunch map.

CONSUMPTION OF IRON AND STEEL: UNITED KINGDOM, 1920-1936.

1 = Consumption of iron and steel. 2 = Profits$_{-1}$. 3 = Price of iron$_{-1\frac{1}{2}}$
4 = Long-term interest rate$_{-1\frac{1}{2}}$.



and the error of sampling in these coefficients. For the calculation of these limits, see Appendix A. 5. The assumption underlying the calculation of the limits to the error of weighting is that the standard deviation of the disturbances in each of the three explaining variates is not greater than a third of the standard deviation of the corresponding variate itself. Allowance for sampling errors of regression coefficients has been made by extending the limits to both sides by twice the standard error of sampling.

Table III. 11. Probable Limits to the True Regression Coefficients. Consumption of Iron and Steel: United Kingdom, 1920-1936. [a]

Explaining variate		2 Profits$_{-1}$	3 Price of iron$_{-\frac{1}{2}}$	4 Long-term interest rate$_{-\frac{1}{2}}$
1st	elementary regression coefficient	1.17	—.24	—.07
2nd		1.49	—.29	—.07
3rd		1.44	—1.31	—.21
4th		.11	—.07	—1.74
Limits to the error of weighting given by ultimate beams in bunch map	upper	1.49	—.07	—.07
	lower	.11	—1.31	—1.74
Maximum fraction of standard deviation admitted for disturbances		1/3	1/3	1/3
Narrower limits to the error of weighting:				
According to the rule of thumb	upper	1.34	—.07	—.07
	lower	.99	—.28	—.09
According to the strict rule	upper	1.28	—.24	—.07
	lower	1.17	—.28	—.09
Final limits including allowance for sampling errors	upper	1.62	.04	.12
	lower	.84	—.56	—.28
		1.46	-0.26	-0.08

[a] For units, *cf.* Table III. 4.

Graph III. 15 concerns one of the calculations for the United Kingdom before the war. It is certainly less good than the post-war cases: the beams spread much more widely. Nevertheless the result indicates the significance of all coefficients found. Here,

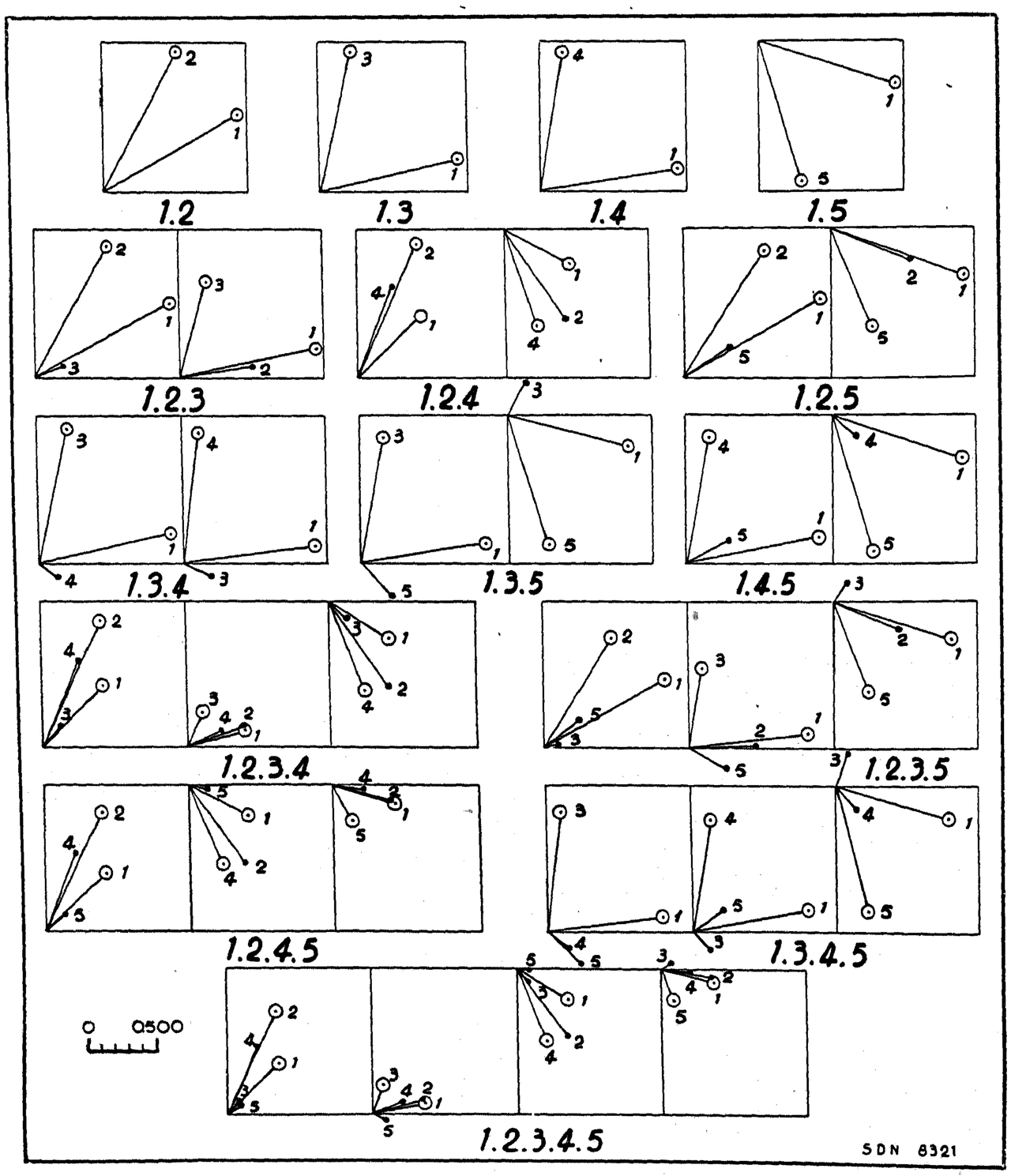
it is interesting to see how the direction of the relation between variates 1 and 4 (consumption and price of iron) is changed by the inclusion of the other variates; even if only 2 is added (*cf.* set 124). The significance of variate 3 might be doubted, but if only the beams 1 and 2 are considered, the regression coefficient for 3 is again very well determined.

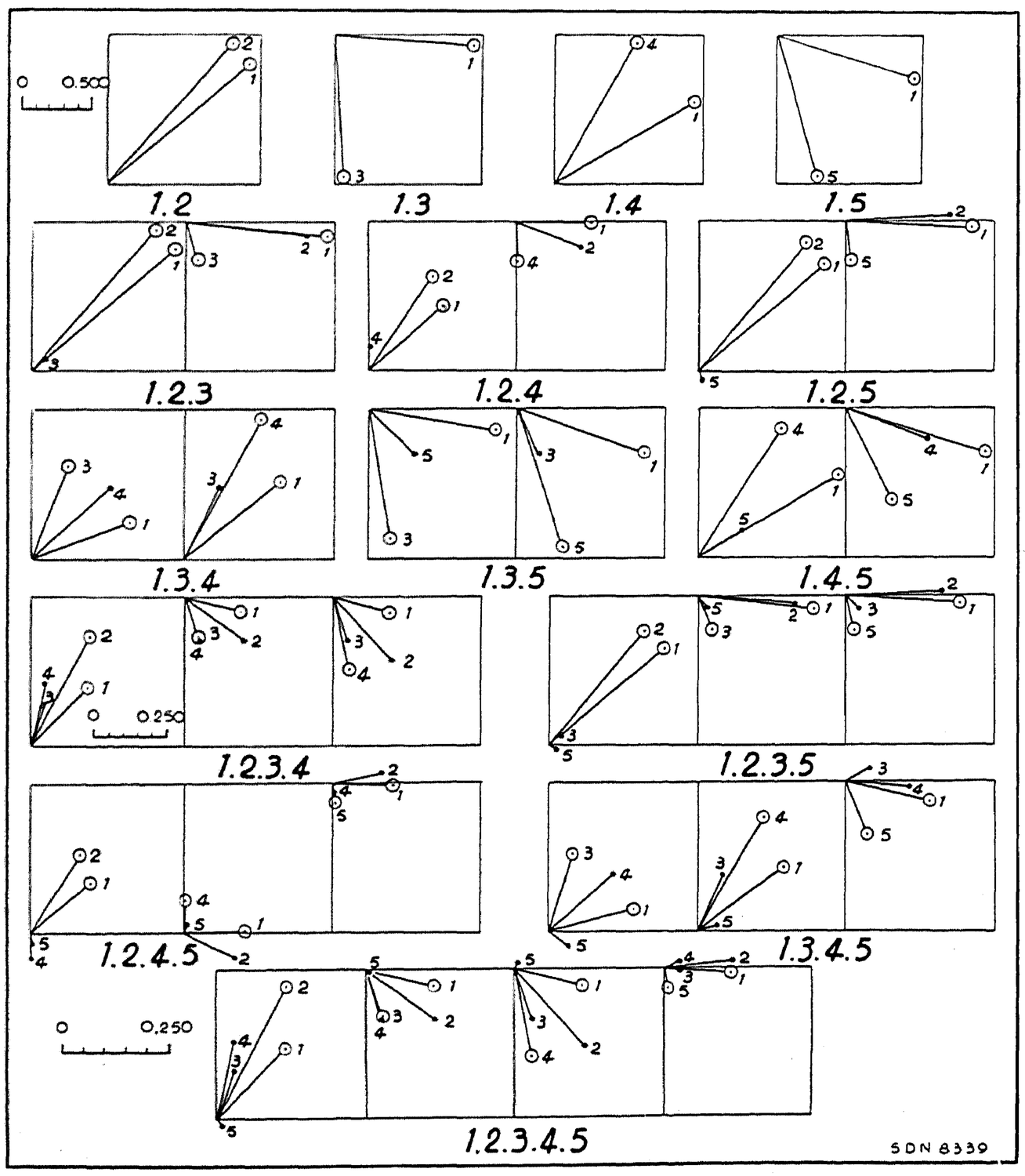
Graph III. 15.

Bunch map.

CONSUMPTION OF IRON AND STEEL: UNITED KINGDOM, 1871-1910.

1 = Consumption of iron and steel. 2 = Non-labour income$_{-1}$.
3 = Δ non-labour income$_{-1/2}$. 4 = Price of iron$_{-1}$. 5 = Bond yield$_{-1/2}$.



Graph III. 16, concerning one of the German (pre-war) calculations, is very similar to Graph III. 15. The spread of the beams is about the same; the relation between variates 1 and 4 is also reversed by the introduction of other variates and the regression coefficients of variates 3 and 5 are well determined if only beams 1 and 2 are considered. But the bunch map for Germany is less satisfactory in that the relation between variate 1 and variate 2 (profits) is not improved by the addition of any or all of the secondary variates.

Graph III. 16. Bunch map.
CONSUMPTION OF IRON AND STEEL: GERMANY, 1871-1912.
1 = Consumption of iron and steel. 2 = Dividends$_{-1/2}$. 3 = Δ dividends$_{-1/2}$.
4 = Price of iron$_{-1/2}$. 5 = Long-term interest rate$_{-1/2}$.