CHAPTER VI

POSITIVE CONCLUSIONS ABOUT CYCLICAL MOVEMENTS IN THE UNITED STATES, 1919-1932

(6.1) CONCLUSIONS ON "DIRECT" RELATIONS

The system of relations established permits of a considerable number of conclusions about the actual course of events in the United States between 1919 and 1932.

A rather elementary way of reaching conclusions is simply to consider the graphs representing the result of each correlation calculation made. In this way it may be seen, for each year and each variable, in what proportions the various causes of changes — as far as they have been considered — have contributed to these changes. Some examples may be given.

Equation (5.10) shows the relative strength of the various components in the combined profit calculations of all entrepreneurs. Considering movements from 1928 to 1929, it appears that the value of consumers' goods production was still increasing, whereas that of producers' goods production was already decreasing. In the same interval, wages were increasing, tending to decrease profits.

Graph 1.13 shows that the decrease in value of investment goods in those same years is wholly due to residential building and not to other investment. Further, graph 2.5 indicates the causes of the decline in residential building. The number of houses some four years before was very high, and this discouraged building in 1929.

Taking the fall in general investment from 1929 to 1930 — which contributed considerably, according to graph 5.10, to the fall in profits in 1930 — we find from graph 2.4 that profits one half-year before were the chief explanatory series. Here we meet a very important feature. It would seem as if this were a circular reasoning: profits fell because investment fell, and investment fell because profits fell. This is, however, an inexact statement. Profits in period t fell because investment in period t fell, but the latter fell because of a fall in profits in period $t_{-\frac{1}{2}}$; and owing to this time lag there is no danger of circular reasoning.[1] Moreover, this lag is important in that changes in it may change considerably the resulting movements of both series, as will be shown below.

Let us go back to the fall in profits in 1930 and study the influence of consumption U′ as affecting production of consumers' goods U. This fell considerably, and the fall in costs which accompanied it was not able to compensate it. Relation (2.1) tells us that one of the proximate causes of the fall in consumption was a decline in wages and in other consumptive expenditure. The result of the fall in wages is, however, almost entirely counterbalanced in Z by the rôle of wages as costs. Graph 2.1 gives also the proximate causes of the fall in non-workers' consumption. Here we find that a fall in capital gains had already caused a decline in consumption out of capital gains [2] between 1928 and 1929. Consumption out of other income was still rising. Capital gains fell because the *rate of increase* in stock prices, upon which, of course, they depend, falls before stock prices themselves fall. Here, a sort of "acceleration principle", but of an economic significance quite different from the ordinary acceleration principle, has an important influence.

Taking graph 1.3, we find a remarkable divergence between the various income types; it appears that dividends D, especially, remained high in 1930, and interest income K_I remained high all through the depression. Entrepreneurial withdrawals — corresponding to profits in non-corporate enterprises — fell heavily.

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[1] Even without lag it is possible to avoid circular reasoning, but the argument would be somewhat more complicated. Investment activity and profits would then both be determined by other variables.

[2] This is shown by figures for the sales of expensive motor-cars.

The foregoing conclusions are examples of the type of information obtainable for one special turning-point. The graphs may also be used in a slightly different way — viz., in order to obtain a number of statements valid for the period studied as a whole. This may, even more than the analysis of a single turning-point, give an impression of the forces which are most important in the business-cycle mechanism as a whole. The following is an attempt to formulate some of these statements.

The fluctuations in total value of production of both consumers' goods and investment goods have been caused much more by quantity fluctuations than by price fluctuations (*cf.* graphs 1.10, 1.15 and 1.16). The fluctuations in total profits, which are chiefly caused by fluctuations in total sales (*cf.* graph 5.10) have therefore also been chiefly governed by quantity fluctuations; only in a closer approximation are prices important. Clearly, an exception must be made for agricultural raw materials, where the reverse is true; their proportion in total production is, however, restricted to about 10%.

The influence on investment activity of what are usually considered as the most important "brakes" on an expansion — viz., interest rates and other costs — seems to have been very moderate (*cf.* graph 2.4). This is due not so much to the moderate size of fluctuations in interest rates and prices, as to the low elasticities.

Consumption outlay depends on two types of income, which are governed by rather different laws. Wages, salaries, dividends, rent and interest payments lag more or less behind general profits, whereas capital gains, by their very nature, lead (*cf.* E and G, graph 2.1).

The monetary sphere seems to be much less narrowly in contact with the physical sphere than one might expect. A superficial inspection of the graphs shows that the fluctuations in interest rates do not correlate narrowly with those in general production. The shape of the waves is clearly different for both groups. Equation (4.422) and graph 4.92 suggest that fluctuations in gold stock are a very important factor influencing interest rates; graph 4.63 suggests in addition that, the supply of money being much more elastic than the demand for it, the fluctuations in gold stock will hardly be found in total money in circulation. Apart from the influence of gold movements, there is an influence of general activity — productive as well as speculative — on interest rates. As already stated, however, the influence of interest rates on production and speculation seems to have been minimal (*cf.* sections (2.4), (2.5), (4.3) and (4.7). It must not be forgotten, of course, that these conclusions cannot be generalised for any business-cycle period in any country; to some extent they seem, on the contrary, to be very specific.

(6.2) Conclusions on Indirect Relations; the Elimination Process and the "Final Equation"

The rather elementary types of conclusion given above, which deal with one equation at a time, and hence with proximate causes only, are for that very reason somewhat superficial. The method used is not expedient, either for arriving at a picture of the course of business cycles as a whole, or for considering the consequences of economic policy. To attain the first object, starting for instance with the fall in profits after 1929, we should have to pass in endless procession from one equation to another, to find more and more remote causes. On the other hand, when studying, say, the consequence of a sudden lowering, in 1929, of wage rates by 10%, one cannot of course simply deduce that profits would have been increased by 10% of the wage sum L_w, and stop at that. A change in wage rates changes prices (3.5) and production (2.4); it changes consumption (2.1) and thereby production and. . . wage rates (3.1). Here, again, we would have to follow the effects through all equations, but now in the opposite direction.

For both purposes it is therefore necessary to have recourse to another method. The general characteristics of the business cycle may, as it is exposed in the Introduction, be found by the elimination process, which will now be taken up. Problems of policy will be dealt with in section (6.8).

In principle, we shall now try to eliminate all variables but one from our equations, and to obtain one equation, to be called the "final equation", in which only one of the variables — say Z^c — will appear together with a number of data. This elimination process is very laborious, and can in fact only be carried out with the help of further simplifications. According to whether more or fewer of these are adopted, we may obtain a rough first approximation or more refined second, third, etc., approximations. The latter are, of course, more exact, but far more complicated; for reasons of clearness it will therefore often be more helpful to take the less exact formulæ.

In the elimination process, all trend terms will, from the start, be disregarded. This does not involve any special simplification, but simply means that our results are obtained not for the variables as they stand, but for the deviations they show from some straight line in time (a different one for each variable).[1] This straight line will be considered as a structural development, in which we are for the moment not interested.

Further, all terms containing cumulants, like $\int Z$, will be omitted, since some calculations have shown that they have no large influence on the shape of the shorter fluctuations.[2]
found for one variable may afterwards be transformed for another variable.

The exact course of the elimination process is largely dependent on the mathematician's choice. In principle, he may start where he likes and may eliminate variables in what order he likes. He may also freely choose what variable or variables he likes to keep in his final result. This does not matter very much, at least in principle, since any result found for one variable may afterwards be transformed for another variable.

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[1] This straight line need not be the rectilinear trend of each series. It would be so if we had introduced a trend in every equation. For then, owing to a well-known theorem of multiple correlation analysis (proved by Frisch and Waugh), the regression coefficients would have been the same as if beforehand each variable had been replaced by its deviations from trend.

[2] A more exact argumentation can only be given at a further stage. *Cf.* pages 147 *sqq.*

Here, the extremely simple example of the Introduction:

$$V_t = \beta Z_{t-1} \qquad (6.21)$$

$$U_t = L_t + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2}) \qquad (6.22)$$

$$Z_t = U_t + V_t - L_t \qquad (6.23)$$

may be reconsidered.

Z_t may be kept by eliminating V_t and $U_t - L_t$[1] by substituting (6.21) and (6.22) in (6.23):

$$Z_t = \beta Z_{t-1} + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2})$$

or:

$$Z_t - (\beta + \varepsilon_1 + \varepsilon_2) Z_{t-1} + \varepsilon_2 Z_{t-2} = 0 \qquad (6.24).$$

It is also possible to keep V_t by first solving (6.21) for Z_{t-1}:

$$Z_{t-1} = \frac{1}{\beta} V_t, \qquad (6.25)$$

from which it follows that:

$$Z_{t-2} = \frac{1}{\beta} V_{t-1} \text{ and } Z_t = \frac{1}{\beta} V_{t+1} \qquad (6.25').$$

In addition, (6.23) must be solved for $U_t - L_t$:

$$U_t - L_t = Z_t - V_t = \frac{V_{t+1}}{\beta} - V_t \qquad (6.26)$$

and the result substituted in (6.22):

$$U_t - L_t = \frac{V_{t+1}}{\beta} - V_t = \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2})$$

$$= \frac{\varepsilon_1}{\beta} V_t + \frac{\varepsilon_2}{\beta} (V_t - V_{t-1})$$

or:

$$\frac{V_{t+1}}{\beta} - \frac{\beta + \varepsilon_1 + \varepsilon_2}{\beta} V_t - \frac{\varepsilon_2}{\beta} V_{t-1} = 0 \qquad (6.27).$$

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[1] It will be seen that $U_t - L_t$ must and can be considered as a single variable in these cases.

It will readily be seen that this equation for V is the same as (6.24) for Z; the only differences being that (6.27) has been divided by β and relates to one time unit later than (6.24).

Nevertheless, in practice it sometimes makes a good deal of difference where one starts, and the particular structure of the equations may very much facilitate some course. On closer examination of the system (*cf.* Appendix B, table I) one finds that the equations may be ranged in four groups. First there is the group of monetary equations, which may, by elimination, easily be reduced to only two equations, expressing n and m_{Lb} in non-monetary variables. Secondly, there is a group of equations which may immediately be substituted in the others, each reducing thereby the number of equations and of the variables by one. Equations (1.2), (1.3), (1.7), (1.9), (1.13), (1.14), (1.15), (1.16), (2.2), (5.1), (5.2), (5.3), (5.4), (5.5), (5.6), (5.7), (5.9) and (5.11) belong to this group. After the substitution, there remains the set given in table III, which may now be subdivided in two groups:

(i) A "price" group, containing equations (1.8′), (1.10′), (1.11′), (1.12′), (3.1′) to (3.5′) and (5.8′); and

(ii) What for reasons to be mentioned later may be called the "strategic" group, containing the remaining equations (2.1′), (2.4′), (2.5′) and (1.4′), (2.6′), (4.6′), (4.91′) and (5.10′).

The structure is such that the first group consists of a number of relations "explaining" variables that play only a secondary role in the second group. The chief variables in the "strategic" group are: Z^c, corporation profits, n, share prices, v_B, residential building activity, v', other investment activity, and U′, consumption. They may be called the "strategic variables". This grouping suggests the following treatment: the "price" variables may first be found as functions of the "strategic variables" and then be substituted in the "strategic" group of equations [1]. This has been done in tables

[1] The substitution must necessarily be repeated when, after further elimination, some of the strategic variables are expressed in others.

IV and V, where the whole process is given step by step. We are then left with a kernel of relations which can more easily be treated. It is, of course, not by chance that we are left with these equations and these variables. The logical structure of our system of equations, which after all is nothing but a reflection of the structure of the business-cycle mechanism, is such that they play the central rôle. This is why we call them the strategic group for the understanding of the mechanism. Their coefficients will be seen to have the largest influence on the character of business cycles.[1] In order to simplify still further, we may, within the strategic group, eliminate the variable v', which is easily expressed as a function of the other variables. For reasons to be given fully in section (6.7), we consider v_B as an external variable. Greater difficulties arise if we try also to eliminate n and Z^c; for these variables do not occur only once in these equations, but several times, with various lags. This reflects the economic fact that these variables are connected by many causal chains, working in various directions and with various lags.

The expression of n in terms of the other variables is made especially difficult by the presence of the term $2.40\,(n - n_{-1} - 20)''$ and of H, which is equal to $0.3\,(Z^c_m - Z^c - 7)''$. The first term intends to indicate that in a speculative boom n is much affected by its own previous rate of increase. This has the result that n moves much by its own laws, pulling with it the other variables, as we shall be able to show. The H-term means that n is depressed by currency-hoarding in a severe depression. On account of these complications, it is useful to split up our considerations into three parts relating to the three forms of the equation for n:

(i) Case I, the "normal interval", where $n - n_{-1}$ is less than 20 and has therefore no influence on n, and where Z^c either rises or does not fall so deeply that hoarding takes place;

[1] *Cf.* section (6.9).

(ii) Case II, the "boom interval", where $n - n_{-1} > 20$ and consequently influences n, without hoarding;

(iii) Case III, the "depression interval", where $n - n_{-1} < 20$ and n is further kept down by the occurrence of hoarding. A fourth case, where a boom development of n would coincide with the depression phenomenon of hoarding, need not be taken into consideration.

The equation found for Z^c which still contains n-terms next to terms with Z^c and a number of variables that are taken as given for our system of equations, *e. g.*, f, runs:

$$\begin{aligned} 0.770\,Z^c = {} & 0.179\,Z^c_{-1} + 0.006\,Z^c_{-2} - 0.015\,Z^c_{-3} + 0.007\,Z^c_{-4} \\ & - 0.131\,h_{-2} - 0.083\,h_{-3} - 0.290\,h_{-4} - 0.845\,f \\ & + 0.335\,f_{-1} + 0.081\,v_B - 0.017\,(v_B)_{-1} + 0.090\,n \\ & - 0.049\,n_{-1} + 0.001\,n_{-2} + 0.003\,n_{-3} \qquad (6.28) \end{aligned}$$

(*cf.* Appendix B, table V, line 262 + 264).
In the next sections, we shall consider cases I, II and III for n.

Detailed Description of the Elimination Process.

The process starts in table II, where the equations of group 4 in table I are combined. Let us follow somewhat more closely the beginning of this process.

In line 1, equation (4.4) is copied, with the omission of the VC-term (the indication of the "explanatory" series is given at the top of each column to save space; for the same reason, the heading in one column is sometimes changed). In line 2, equation (4.63) is written, but transformed in an equation "explaining" M by applying equation (1.5) and adding M′ on both sides. Subtraction, in line 3, eliminates M; this equation may be written with m_S on the left-hand side (4); the factor -0.42 has immediately been applied since it is with this coefficient that m_S occurs in M (2). (2) + (4) gives again M (5), but now without m_S. If now, in all places where M or m_S occur, we replace them by the expressions (4) and (5) so found, these variables are eliminated from the system of equations (*cf.* lines 7 and 8). The same procedure is applied to other monetary variables until, in (21) and (22), m_{Lb} and n

are expressed in non-monetary or external variables (except H, which is kept for reasons of convenience).

The eliminations in table III are of a simpler type: certain variables in some of the equations, especially of the "strategic group", are simply replaced by the expressions by which they are "explained" or defined in some other equation. Thus, in (2.1), $0.77\mathrm{E}$ is first replaced by $0.77(\mathrm{D} + \mathrm{L}_c + \ldots)$ with the help of (1.3); subsequently $0.77\mathrm{D}$ is replaced by $0.77\,(0.151\,\mathrm{Z}^c + \ldots)$ equation (5.1) and finally the S_{-1}-term so introduced is reduced to 0, according to equation (1.9) in its simplified form: all cumulants and trend terms omitted. The equations used are mentioned in the column "References".

In table IV, the procedure is again of a somewhat different type. The purpose of this table is to express the prices m_{R}, p^f, p, l, q, and the variables $u + v$ and L_w in "strategic" variables F^1, F^2 and $\mathrm{U} + v$, where the F's represent certain expressions in the strategic variable Z^c and the external variables h and f. In line 101, $0.80p'$ in (1.8′) is substituted by $0.80(0.47l_{-0.21} + 0.25p^f_{-0.21})$. In the next line, the l-term in 101 is replaced by (3.1′). This gives an expression with p on the left-hand side and $0.147p_{-0.63}$ on the right-hand side. In line 103, these two terms are combined to $0.853p_{+0.11}$, where 0.11 is the weighted lag (lead) $+\ 0.11 = [(1.000 \times 0) - (0.147 \times -0.63)] \div 0.853$; then all terms of the equation are divided by 0.853 ($0.200 \div 0.853 = 0.234$, etc.) and shifted in time by -0.11 year. This procedure as a whole is indicated by the reference "R". This way of elimination is continued throughout the table. Attention should be drawn to the groups of terms taken from (3.4′) and (3.2′) respectively which are introduced as a whole in (110) and (111). Here, as well as in the subsequent introduction of F^1 and F^2, the procedure was dictated by considerations of simplicity and the avoidance of unnecessary calculations.

In table V, the different "strategic" variables are successively eliminated. In the first place U and U′ are treated. With the help of (2.1) and the results of the preceding table,[1] U′ may be replaced by $\mathrm{U} + v$, F^1, F^2, F^3, where F^3 is a new

[1] *Cf.* also note 1 on page 136.

combination, provisionally to be kept in this form. Equation (2.6′) gives another relation between U′ and U (u' and u being transformable into U and U′ via (1.12) and (1.10)). These two equations suffice to eliminate both variables, and to find U expressed in v and F's (216). It may be remarked that the same expression (206) is applied three times (in 207, 208, 209) but with different lags (0, 1 and 2 years).

In lines (217) to (224), v (or v', *cf.* (1.14)) is eliminated, and hence the result of (216) may be improved by expressing U without using v (225).

In (227) to (230), p^f is treated. The difficulty is here that F^3 contains $0.049\Delta p^f$, which causes the small p^f terms in the last column. But the latter are so small that, if they are replaced (229) by the expression for p^f in (228), the p^f terms they yield are no longer perceptible.

In (231) to (237) certain variables of tables IV and V are expressed in F^1, F^2, F^4, F^5 and v_B. Terms with lags of parts of a year are split into two terms with the same average lag.[1]

Certain combinations of these variables occur in n and Z^c (*cf.* (4.91′) and (5.10′)); their expression in F^1, etc., may now easily be found: S_n in (242) and S_z in (248).

The F's are then decomposed into terms with Z^c, n, h and f (249-252; 258-261). If we add the other terms in (4.91′) (253), we find, after a few transformations, n expressed in Z^c and external variables. We only need to substitute this expression for the n-terms in S_z to find Z^c expressed in values assumed by Z^c at moments lying 1, 2, 3 and 4 years back and in external variables (266).

(6.3) The Character of the Movements in the Absence of a Stock-exchange Boom and of Hoarding

To study case I, we omit both the term with $(n - n_{-1} - 20)''$ and that with H in line 257 (Appendix B) which explains n. We may now replace the n-terms in the Z^c-equation (*cf.* line

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[1] The same procedure is applied to small leads, *e.g.*, $-0.022F^1_{+0.23}$ is replaced by $-0.027F^1 + 0.005F^1_{-1}$, with the same average lag $(-0.022 \times +0.23 = +0.005 \times -1)$.

262) by an expression containing only Z^c at various moments and exogenous variables. In this way we get a "final equation" for Z^c, running:

$$0.445Z^c = 0.177Z^c_{-1} - 0.098Z^c_{-2} + 0.006Z^c_{-3} + 0.012Z^c_{-4} - 0.135h_{-2} - 0.077h_{-3} - 0.305h_{-4} + 0.74(Au+P) - 0.40\,(Au+P)_{-1} - 0.822f + 0.315f_{-1} \qquad (6.30).$$ [1]

In order to facilitate the understanding of this equation and its consequences, it may be written in a somewhat more condensed form:

$$Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2} + e_3 Z^c_{t-3} + e_4 Z^c_{t-4} + (AU + HO + F + R)_t \qquad (6.31).$$ [2]

Here e_1 to e_4 are numbers depending, in principle, on almost all regression coefficients in all elementary equations. They describe in an abbreviated form the structure of the economic mechanism with regard to business cycles; they will be different in other countries, or under another regime, where the economic structure of society is different.

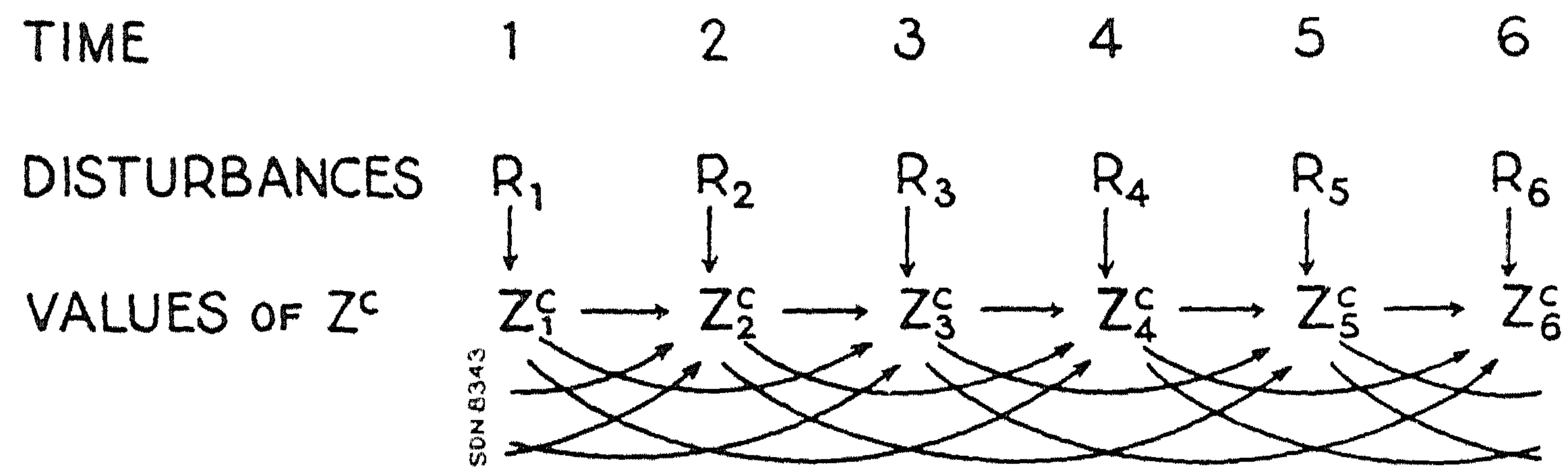
The other four new symbols have this in common, that they may be considered as *largely independent* of the general business-cycle position. Their exact meaning may be discussed later. This co-existence in formula (6.31) of two types of terms — independent terms and terms depending on previous values of Z^c — is of importance. It represents the fact that at any moment profits Z^c (and quite similar propositions hold, as we have already indicated, for the other variables) are the product of two types of forces: forces connected with previous business-cycle situations $(e_1 Z^c_{t-1} + e_2 Z^c_{t-2} + e_3 Z^c_{t-3} + e_4 Z^c_{t-4})$, and independent forces which are often indicated as *disturbances*, since their changes cause Z^c not to follow the regular pattern of cycles. They are also indicated as *external* or *extraneous forces* (which indicates their origin; for some of them this expression is more appropriate than for others, as we shall see), or *shocks* (which of course bears

[1] *Cf.* Appendix B, table V, line 265.
[2] The meaning of the last four symbols will be described in the next pages.

somewhat more upon the possibility of their sudden changing and is not therefore equally applicable to all of them), or *starters* (which reminds of the possibility — especially if they come in a rather quiet period — that they may be the beginning of a new cyclic movement).[1] The causal connections which are described in equation (6.31) may be illustrated by the following diagram, where one symbol R stands instead of the sum AU + HO + F + R:

Graph 6.31.

Causal Connections between
Disturbances and Profits



The arrows indicate causal connections. Each value of Z^c depends immediately on certain disturbances, but it depends also on the earlier ones through its connection with the Z^c-values for one, two, three and four[2] years back.

Equation (6.31) tells more than this very general statement. It tells what is the origin of the first three disturbances AU, HO and F. Going back to the equations that describe these three variables, we will even be able to determine the magnitude of the disturbances in any year.

AU represents influences coming from changes in the gold stock and in the autonomous component P in central banking policy; it is

$$AU_t = 1.66(Au + P)_t - 0.90(Au + P)_{t-1} \qquad (6.32).$$

[1] Dr. Johan Åkerman considers as the "real causes" of the business cycle these external forces; our own preference being to indicate by that term the structure of the economy, represented by the coefficients e_1, e_2, e_3 and e_4. It is, of course, only a question of definition.

[2] In order not to overload the diagram, the arrows from Z^c_1 to Z^c_5, Z^c_2 to Z^c_6, etc., corresponding with the term $e_4\, Z^c_{t-4}$ have not been drawn.

The isolation of these terms seems especially interesting in judging the influence of banking policy and gold movements — past as well as potential.

HO represents influences coming entirely from the housing market and, more exactly, from a development in the housing market that is largely a product of events more than three years back (and therefore, as already observed, in a very high degree independent of Z^c_{t-1}, Z^c_{t-2}, which are most important in the final equation). It is given by the formula:[1]

$$\mathrm{HO}_t = -0.303h_{t-2} - 0.173h_{t-3} - 0.685h_{t-4} \qquad (6.33)$$

and depends on the number of houses in existence two to four years before. Through its large influence on the actual building volume, this number acts also on the present value of Z^c. The usefulness of taking it as a separate item is (i) that in no other part of the economic system were such large lags found to have a considerable influence on the cyclic movements;[2] (ii) that it shows almost autonomous cycles, to be discussed later, and (iii), that, for that reason, we are able to evaluate the influence of housing on the general business situation.

F stands for the influences, chiefly climatic, which change crops; they are generally accepted as important external forces. Again it seemed useful to isolate these terms.

$$\mathrm{F}_t = -1.847f_t + 0.708f_{t-1} \qquad (6.34).$$

R, finally, is an agglomerate of a non-discernible multitude of disturbances which, each in itself, seem far less important than the three types mentioned, but taken together may still be important. Because of their large number and, in all probability, mutual independence, they may, however, be treated as random disturbances.

Although we have succeeded in giving separate terms for at least some of the most important external factors, there are two categories which may also be important and have not been

[1] *Cf.* Appendix B, table V, line 266.

[2] One could have expected that the so-called "echo principle" would also give an example of such forces, and wonder why it has no place in this system of equations. Very probably, however, these forces are of importance only for the explanation of trend movements. *Cf.* Vol. I, Chapter III, and section (2.1) of this volume.

included, viz., inventions and, in the United States especially for the period after 1932, Government policy. The latter, if well devised, will, however, belong rather to the class of regular exogenous factors such as the terms HO in our example.

From the business-cycle point of view the first four terms in equation (6.31) are the more interesting. They represent the systematic cyclical forces. They tell us that, apart from disturbances, the situation of to-day will depend on the situations of one, two, three and four years ago; and — if the problem is studied more accurately — even of a number of more remote years; the influence of the latter is, however, found to be small.

Looking for a moment at this systematic part only, we get the relation

$$Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2} + e_3 Z^c_{t-3} + e_4 Z^c_{t-4}, \qquad (6.35)$$

which is called a "difference equation". It enables us to calculate the future movements (in the absence of new disturbances) if there are given:

(i) four initial values, say Z_{1917}, Z_{1918}, Z_{1919} and Z_{1920}, and

(ii) four coefficients e_1, e_2, e_3 and e_4, which depend on the coefficients in our elementary equations and therefore in the widest sense upon the economic structure.

In our example, the period and the damping degree of the endogenous movements depend only on e_1 to e_4 (*i.e.*, on the structure), whereas the amplitude depends on the initial values, say Z^c_0, Z^c_1, Z^c_2 and Z^c_3. In more complicated cases these influences may be mixed up in various ways. If the endogenous movement is damped, it will vanish after some time and a new movement will develop only if fresh disturbances occur.

The numerical values for the coefficients in our case lead to the following formula:

$$Z^c_t = 0.398 Z^c_{t-1} - 0.220 Z^c_{t-2} + 0.013 Z^c_{t-3} + 0.027 Z^c_{-4} \qquad (6.36).$$

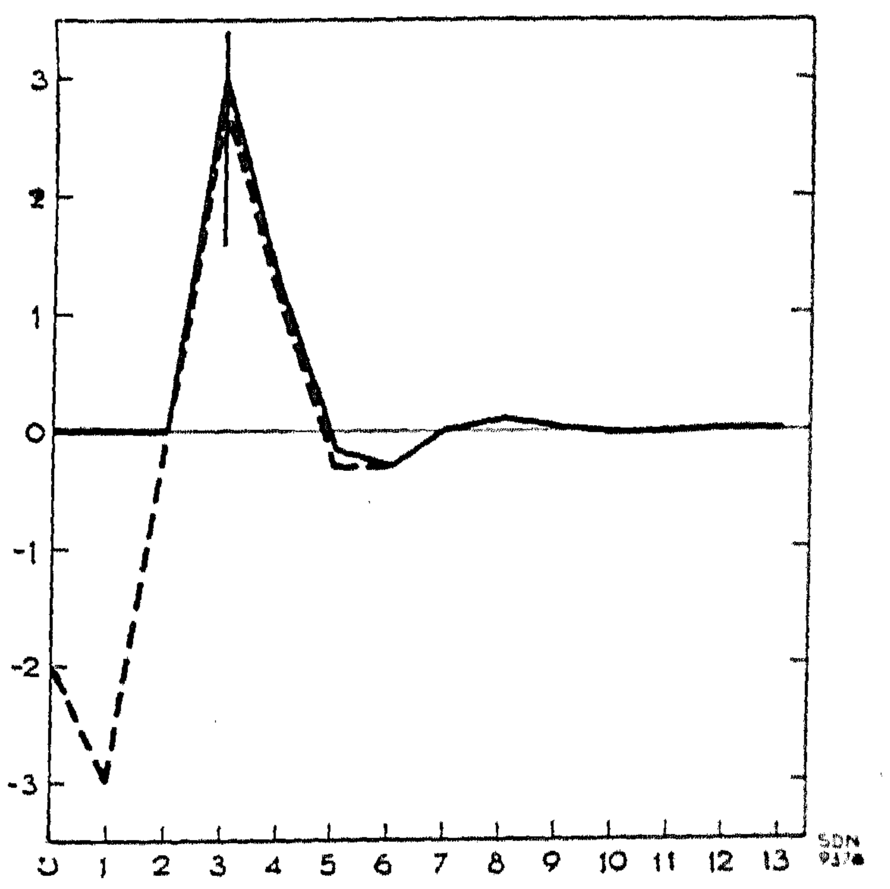
Choosing some arbitrary initial values, *e.g.* $Z^c_0 = 0$, $Z^c_1 = 0$, $Z^c_2 = 0$, $Z^c_3 = 3$, the further development may be calculated. It is given in graph 6.32, and consists chiefly of a damped cycle with a period of 4.8 years. It may be proved mathematically that this is the case independently of the initial values of Z^c chosen;[1] this statement could be tested by some trials with other values. This cycle is somewhat longer than the well-known "short American cycle", of which the average length has been estimated at forty months. Neither of the figures should, however, be taken too literally; for, on the one hand, the average length of cycles just quoted is based upon measurements of actual cycles which are always subject to disturbances, and, on the other hand, our result too is subject to a considerable margin of error.[2] It may be shown, however, that the other conclusions — those regarding the influence of policy and of external factors — are far more certain.[3]

Graph 6.32.

ENDOGENOUS MOVEMENTS OF Z^c (within the "normal interval" for share prices and in the absence of hoarding) FOR TWO SETS OF INITIAL VALUES. Time in Years.

——— Initial values: $Z^c_0=0$, $Z^c_1=0$, $Z^c_2=3$.

- - - - Initial values: $Z^c_0=-3$, $Z^c_1=0$, $Z^c_2=3$.



The mechanism may be made somewhat more understandable by the following analysis: Equation (6.36) shows two large forces acting on Z^c_t: first, a force in the same direction as Z^c_{t-1};

[1] *Cf.* below.
[2] *Cf.* page 179.
[3] *Cf.* J. TINBERGEN: "On the Theory of Business-cycle Control", *Econometrica* VI (1938), page 22.

secondly, one in the direction opposite to Z^c_{t-2}. If now Z^c_{t-1} and Z^c_{t-2} have the same sign—*i.e.*, if the system finds itself either distinctly above normal or distinctly below normal—the positive and negative forces will counteract each other and the new Z^c will be small, *i.e.*, profits will be nearer to normal. This, in a nutshell, is the turning-point situation. If, on the other hand, Z^c_{t-1} has a sign different from that of Z^c_{t-2}, the forces will reinforce each other and the new Z^c_t will be of the same sign as Z^c_{t-1}. If the absolute value of Z^c_{t-1} is small in comparison with Z^c_{t-2} — *i.e.*, if the normal position is only just passed — Z^c_t will be larger than Z^c_{t-1}: this is the cumulative process. For a good understanding of this result, as well as of some of our further statements, it may be observed that even in the simple case with only two coefficients, $Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2}$, the connection between (i) the coefficients e_1 and e_2, and (ii) the character of the endogenous movement is not so simple as one might expect. The following types of movement are possible:

(*a*) Cyclic movements, with any period of at least two years, either
  (i) damped, or
  (ii) undamped, or
  (iii) anti-damped;

(*b*) Damped "one-sided" movements which gradually carry the system back to the "equilibrium position" $Z^c = 0$;

(*c*) "Explosive" movements which carry the system away from that position without ever returning to it;

(*d*) Several combinations of these types.

The rules indicating the connection mentioned above cannot easily be formulated in ordinary language; a mathematical formulation is the only one possible. The problem is to find a function of time,[1] say Z^c_t, which identically satisfies the equation

$$Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2} \qquad (6.371).$$

[1] Throughout the following pages, the variable time is supposed to assume entire values $t = \ldots 0, 1, 2 \ldots$ only, the corresponding values of Z^c_t being conceived as annual averages of profits for the year t.

The mathematician finds this function by trial, and proves afterwards that the solution found is the general one. A function [1]

$$Z_t^c \equiv Kx^t, \tag{6.372}$$

where K and x are constant, is tried. The substitution of this function in (6.371) gives

$$Kx^t = Ke_1x^{t-1} + Ke_2x^{t-2}.$$

From this it is clear that K may be chosen arbitrarily, since each value for K will be correct once x has been chosen so as to satisfy $x^t = e_1x^{t-1} + e_2x^{t-2}$, or

$$x^2 - e_1x - e_2 = 0 \tag{6.373}.$$

This quadratic equation, formed with the coefficients of the equation (6.371), is called the characteristic equation. Its roots x_1 and x_2:

$$x_1 = \frac{1}{2}e_1 + \sqrt{\frac{1}{4}e_1^2 + e_2}, \quad x_2 = \frac{1}{2}e_1 - \sqrt{\frac{1}{4}e_1^2 + e_2}, \tag{6.374}$$

are the only values of x for which the function (6.372) satisfies (6.371). All other values of x would yield no solution at all, whatever values might be chosen for K.

As the coefficients e_1 and e_2 are real numbers, the roots x_1 and x_2 are either both real or conjugate complex. If x_1 is real and positive, but smaller than 1, the curve

$$Z_t^c = Kx_1^t \tag{6.375}$$

represents a gradual approach to an equilibrium situation ($Z^c = 0$). The deviation from equilibrium in the year $t + 1$ is found from that in the year t by dividing it by the factor

$$D = \frac{x_1^t}{x_1^{t+1}} = \frac{1}{x_1} \tag{6.376}$$

which may be called the "damping ratio" of the movement. The larger D, the faster the equilibrium is approached. If x_1

[1] The sign $\equiv$ indicates that the equality of Z_t^c and Kx^t is meant to be true for every value of t.

is larger than 1, however, the "damping ratio" is smaller than 1, and the movement leads farther and farther away from the equilibrium situation (upwards or downwards according to the sign of K).[1]

If both real roots x_1 and x_2 are positive, the movement

$$Z_t^c = K_1 x_1^t + K_2 x_2^t, \qquad (6.377)$$

which is a combination of two solutions, each of one of the above types, also constitutes a solution. Substitution of the expression (6.377) in (6.371) shows this at once. The arbitrariness of the constants K_1 and K_2 makes it possible to choose them in such a way as to give prescribed values to Z_t^c for two points of time — *e.g.*, $t = 0$ and $t = 1$. This only requires that K_1 and K_2 should be determined by the two equations:

$$\begin{aligned} K_1 x_1^0 + K_2 x_2^0 &= Z_0^c \\ K_1 x_1^1 + K_2 x_2^1 &= Z_1^c \end{aligned}$$

which, since $x_1^0 = x_2^0 = 1$, reduces to:

$$\left.\begin{aligned} K_1 + K_2 &= Z_0^c \\ K_1 x_1 + K_2 x_2 &= Z_1^c, \end{aligned}\right\} \qquad (6.378)$$

where Z_0^c and Z_1^c are the prescribed values of Z_t^c for $t = 0$ and $t = 1$.

Thus it will be possible to find one determinate solution for each given pair of values for Z_0^c and Z_1^c.[2]

If the roots x_1 and x_2 are conjugate complex, it may be shown that the expression (6.377) is equivalent to

$$Z_t^c = K a^t \sin \frac{2\pi}{T}(t - \tau) \qquad (6.379).$$

If the factor a^t were absent from this expression, Z^c would move in an undamped harmonic oscillation with a period of T years. The factor a^t brings about a gradual increase or decrease

[1] $y = Kx^t$ also represents the value in the year t of a capital K, invested in the year $t = 0$ at compound interest at the annual rate of $100\,(x - 1)$ per cent.

[2] It is possible to prove that no other solutions exist.

n the amplitude of this oscillation, according as $a > 1$ or < 1. Calling, again,

$$D = 1/a$$

the damping ratio, the oscillation is damped if $D > 1$, and anti-damped if $D < 1$.

The period [1] T and damping ratio D of the solution (6.379) depend as follows on the coefficients of (6.371):

$$\tan \frac{2\pi}{T} = \frac{x_1 - x_2}{x_1 + x_2} = \sqrt{1 + 4\frac{e_2}{e_1}} \qquad (6.380)$$

$$D = \frac{1}{\sqrt{x_1 x_2}} = \frac{1}{\sqrt{e_2}} \qquad (6.381).$$

On the other hand, the initial amplitude K and phase τ of the oscillation are not prescribed by the coefficients in (6.371), but depend only on the initial values Z_0^c and Z_1^c of Z^c. They may be found in the same way as K_1 and K_2 were found in (6.378).

From (6.380) it follows that a value above two years may always be found for the period T. A limiting case is formed by a negative real root x_1, which leads to an oscillation of an exact two-year period, with a damping ratio $D = \frac{1}{|x_1|}$ and an initial amplitude determined by the initial values of Z^c.

In the case of the simple model quoted in the Introduction, we have (*cf.* equation (0.4), page 17) $e_1 = +1.6$, $e_2 = -1.0$, and therefore $x^2 - 1.6x + 1 = 0$; the roots x_1 and x_2 are: $0.8 \pm \sqrt{0.64 - 1}$ or $0.8 \pm 0.6i$; the corresponding movements are cyclic and undamped, and show a period of ten time units of four months. This number is found as the quotient of 360° (or 2π radiants) by arc $\tan \frac{0.6}{0.8}$.

[1] Though equation (6.380) admits of an infinite series of solutions T_k satisfying $\frac{2\pi}{T_k} = \frac{2\pi}{T_o} + k\pi$, $k = 0, \pm 1, \pm 2 \ldots$, with $0 < \frac{2\pi}{T_o} \leqq \pi$, the restriction of t to entire values (see note 1 on page 142) makes the expression (6.379) for each of these solutions mathematically equivalent to that corresponding to the value T_o.

No other types of movement than those described above occur when the final equation, unlike (6.371), contains also terms with Z^c_{t-3}, Z^c_{t-4}, etc. Every additional term, however, increases by 1 the degree of the characteristic equation, and with that the number of its roots. Thus, the general solution of a final equation containing terms up to Z^c_{t-4} is a combination of four specific movements, each of one of the above types, and each corresponding to one root of the characteristic equation. Though in general the initial relative importance of the specific movements is determined by the four initial values of Z^c, the specific movements most important for business-cycle analysis are those which have the smallest damping ratio, as such movements are most likely to persist for a longer time.

In the present case, the final equation (6.30) contains as the most important movement a cycle with a period of 4.8 years and a damping ratio of 1.89.

The period and damping ratio depend, in principle, on each coefficient in each single equation which has been used in deriving the final equation. In section (6.9), the influence of changes in some of the more important coefficients on damping and period are studied, and some indication is given of the uncertainty in the above figures ensuing from given margins of error in those coefficients. One instance, however, is sufficiently interesting to be mentioned here, as it indicates a probable bias in the above figures.

Owing to the large damping ratio, there is a tendency for damping and period to be particularly sensitive to changes in coefficients in the elementary equations that represent causal connections acting with a large time-lag. The only case of a lag above one year in the elementary equations (leaving aside the term with h_{-4} which is considered as an external variable) is in equation (5.7), and it appears that, if (5.7) is replaced by (5.7′) — see page 120 — the period works out at 5.0 years and the damping ratio at 1.59. As the fit in the explanation of L_s is nearly equally good in the two alternatives, it would seem that the choice has to depend on other considerations. Damping and period for any set of coefficients in the L_s-equation intermediate to the extreme cases (5.7) and (5.7′) may be found approximately by linear interpolation between the figures given.

For most problems to be studied in the following sections, the choice between the two possibilities is not important; to save work, only the final equation based on (5.7) has been applied in such cases. Only in the second half of section (6.9) some figures are given on the basis of both equations for L_s.

According as more terms occur in the final equation, the computation of the damping ratios and periods of its specific solutions becomes more laborious. Now that the principles have been explained for the simplified case (6.371), we shall make only two general inferences which are of great importance for business-cycle theory and business-cycle policy.

(i) The damping and period, and even the type, of the possible movements defined by the final equation depend very much on the numerical values of its coefficients.

(ii) The effect of certain measures of business-cycle policy on the stability of the economic system may be studied by means of the effect of such measures on the coefficients in the final equation. Stability will be promoted by measures that increase the damping ratio of the solutions to that equation.[1]

The omission of the cumulants in the elimination process may now be explained in somewhat more detail.

In a preliminary calculation, these terms were retained throughout the elimination process. The resulting occurrence of terms containing cumulants in the final equation influences in two ways the possible movements of the system:

(i) The period and degree of damping of the cyclical movement are to some extent affected by the presence of such terms.

(ii) Besides that, the cumulants introduce an additional root into the characteristic equation, which is real and positive, giving rise to a one-sided movement. This movement is explosive (away from the equilibrium situation) if the algebraic sum of all coefficients of cumulation terms in the final equation is positive; the movement is damped (gradual approach of the equilibrium situation) if that sum is negative.

[1] All this may be formulated independently of where the equilibrium lies. It is, however, quite possible that the desirability of a high equilibrium level might conflict with the requirement of a "very stable" or a "strongly damped" movement. In a private publication this problem has been studied somewhat more closely: *cf.* J. TINBERGEN, *Fondements mathématiques de la stabilisation des affaires* (Hermann, Paris, 1938).

The upshot of the preliminary calculations was that the $\int Z^c$-terms in the final equations, which resulted from the cumulants taken into consideration in the elementary equations, (i) had a small negative influence (about — 0.05) on the damping ratio and a small positive influence on the period of the cyclical solution (about 0.5 year); (ii) gave rise to a solution corresponding to an explosive movement (root $x_5 = 1.14$).

If such a solution really constitutes a possibility inherent to the economic system of the United States, simple calculations — for instance, numerical solutions as given in sections (6.5), (6.6) and (6.7) — show that, granted certain initial conditions, the one-sided type of movement could lead, after a few years, to values for the variables so far from the average, and, consequently, so far beyond the range in which the elementary equations have been determined, that the latter, and hence the final equation, could no longer be assumed to hold. This exponential solution, if it were kept to, would have to be interpreted as the expression of a cumulative process to which a special explanation of the turning-points, by bottle-necks and other non-linearities in the equations, would have to be added.

This last result seems unsatisfactory in two respects. First, though the conception of a cycle as consisting of two alternating cumulative processes, an upswing and a downswing, linked by two turning-points, is not uncommon in general cycle theories, [1] in few of these theories are the cumulative processes attributed to those phenomena that are represented by the $\int Z^c$-terms in the final equation — the surplus of corporations (equation (1.9)), working through its influence on dividends (equation (5.1)), the stock of capital goods through its influence on depreciation (equation (5.9)), and the amount of long-term debt outstanding, working through interest payments (equation (5.4)). An explanation of the cycle that would be largely dependent on the growth of these three assets may therefore be rejected *a priori* on theoretical grounds.

The second objection is that the cumulants in the elementary equations that have led to this explosive exponential

[1] *Cf.* G. Haberler, *Prosperity and Depression.*

root are not all, and perhaps not even the most important of, the cumulants to which the economic mechanism gives rise in reality. At many places, trends have been used which properly represented cumulations of physical quantities or values. Of the cumulant, for example, of physical investment,

$$\int \bar{\bar{v}} = \int \bar{v} + \int v,$$

the first component moves very gradually, and the influence of $\int \bar{\bar{v}}$ on any other variable will be statistically distinguishable, in the sense of the correlation technique, from that of other gradually operating factors only if the oscillations in the much smaller term $\int \bar{v}$ render $\int \bar{\bar{v}}$ different from a smooth curve — which is not markedly the case for most cumulants (see graphs 1.9, 5.4, 5.9). Thus in a number of cases an accurate determination of the coefficients of cumulants was not possible, or not important since it was impossible in so many other cases (equations (1.2), (4.1) and (4.2), and their use in (4.82), (4.63), (5.2), and perhaps also the trends in prices: (3.3), (3.4), (3.5)).

A rough estimate of the possible effects of these "hidden" cumulants in the elementary equations showed (i) that their influence on the cyclical solution could not be very large, and would change the damping factor by a figure for which $\pm$ 0.05 could be set as an extreme limit; (ii) that the sign of the algebraic sum of the coefficients for the cumulants in the final equation, including terms resulting from the "hidden" cumulants, could not be determined without a more precise knowledge of the coefficients of these hidden cumulants than seems attainable at present. On the basis of the information available, not much more could be said than that the positive real root probably lay somewhere between 0.75 and 1.25, which leaves the possibility of either a damped or an explosive one-sided movement. The latter has to be rejected for reasons already given above; the influence of the former on the cyclical movements would be moderate and therefore in accordance with the movements observed in reality. To sum up, on the ground of their small influence under (i) and our ignorance of their effect under (ii), it seemed both advisable and justified to keep all terms containing cumulants out of the elimination process.

(6.4) An Economic Interpretation of the Final Equation in the Absence of a Stock-exchange Boom and of Hoarding

The final equation for Z^c discussed in section (6.3) may be interpreted economically. This interpretation clearly holds only for the non-speculative interval, as outside that interval the coefficient of the n-equation upsets the structure of the Z^c-equation. There are, nevertheless, two reasons for considering the Z^c-equation more closely. First, in the period up to 1927, stock-exchange speculation was in fact not very important; secondly, it seems probable that many pre-war cycles can be explained without giving so much weight to the stock exchange. Much the same factors coming into play in the Z^c-equation may have been important in that period, some of them even more important, since the explanation of undamped waves is impossible with the present coefficients.[1]

The economic explanation may be started by repeating the importance of equation (5.10) in the original system, from which the final equation has been derived. This equation states that total profits are the difference between total receipts and total costs of all enterprises. All items in receipts and costs depend, either directly or through a number of channels, on profits (for convenience, corporation profits Z^c have been taken), either at almost the same moment or some time before. The table on page 151 indicates the expression of each of these items in terms of Z^c, together with the result of adding them up or subtracting as may be necessary.[2]

The relative importance of the various components may now easily be seen. In comparing the expressions with the elementary equations, the direct influence of Z^c (which is found there) may be compared with its indirect influence. The most important terms in the table may now be considered separately. A distinction may be made between positive and negative terms, the former tending — if the totals in the columns with

[1] Except, of course, by the occurrence of fresh shocks (*cf.* section (6.3)).
[2] Exogenous terms are omitted.

Z^c and Z^c_{-1} are larger than 1.44 and 0.27[1] respectively — to reinforce the original deviation in Z, the latter to counteract this tendency.

	Z^c	Z^c_{-1}	Z^c_{-2}	Z^c_{-3}	Z^c_{-4}	No.
Receipts:						
U =	1.420	0.504	0.080	— 0.040	0.011	(6.41)
V =	0.467	0.529	0.109	0.046	0.006	(6.42)
Total receipts	1.887	1.033	0.189	0.006	0.017	(6.43)
Costs:						
L_w =	0.529	0.283	0.026	— 0.015	0.003	(6.44)
$L_s + L_c + K_I + K_R + N$ =	0.355	0.313	0.262	0.014		(6.45)
Total costs	0.884	0.596	0.288	— 0.001	0.003	(6.46)
Difference Z =	1.003	0.437	— 0.099	0.007	0.014	(6.47)
Also Z =	1.450	0.260				(5.11)
Therefore 0 =	— 0.447	0.177	— 0.099	0.007	0.014	(6.48)
App. B, V, 265 * 0 =	— 0.445	0.177	— 0.098	0.006	0.012	(6.49)

* Small differences between (6.48) and (6.49) are attributable to repeated omissions of small terms.

Big positive influences are those acting through U and V. They express the simple fact that high profits lead to high consumption and high investment. Profits work directly as well as indirectly: on consumption outlay as, *e.g.*, farm prices and farm consumption are high if general incomes are high; on investment outlay as high share prices facilitate high investments and are themselves — through dividends — causally correlated with high profits.

Big negative influences are, apart from the quite natural influence of higher wage totals and other incomes which are paid in times of higher employment, the following.

[1] These coefficients are those by which Z^c (corporation profits) and Z^c_{-1} must be multiplied in order to yield Z (total profits).

(i) The negative term in U (6.41) is partly due to the influence of commodity stocks. After a year of peak consumption, production falls somewhat more than consumption, as the readjustment of stocks to the lower level of consumption permits of a certain destocking.

(ii) The other part of the negative term in U is due to the influence of speculative gains (capital gains and gains on commodity speculation) on consumption, and thereby on production. These gains are always proportional to some rate of increase, which introduces a positive and a (necessarily somewhat more lagging) negative term, *e.g.*, $Z^c_{-1} - Z^c_{-2}$.

(iii) No negative term will be found in V (6.42); there is, of course, a negative influence of share yield (which represents a type of interest rate) due to its negative sign in (2.4) and the fact that share yield depends positively on dividends, which, in turn, depend on profits; but this negative influence is more than compensated by the direct positive influence of profits. The influences of interest rates in the narrower sense of the term, as well as of prices of investment goods, would also work negatively. These factors were, according to our calculations, almost negligible in the period studied; it is probable that they were stronger in pre-war times, and that they contributed essentially to the formation of cycles in those times.[1]

(iv) The greater incomes paid out (*cf.* (6.44) and (6.45)) at times of higher profits are the consequence not only of higher employment, but also of a higher rate of payment. This may also be a factor tending to reverse the movement of Z^c, especially as there is a lag in the correlation between profits and these rates. This influence is, however, weak, as 0.95 of an increase in wages and salaries is consumed and therefore reflected in U, and since the influence of a lower profit margin on investment activity is, too, not very large (*cf.* relation (2.4)).

---

[1] *Cf.* the results given for investment in the United Kingdom in Vol. I.

(6.5) Character of Movements introduced by a Stock-exchange Boom

We have now to consider the second possibility mentioned above, where the rate of increase of our index in stock prices exceeds 20 points per annum. In this case the elimination process may most easily be carried out by the following approximative method. Equation (257), table V,[1] is written in the form

$$n = 3.607Z^c + 1.941Z^c_{-1} - 0.142Z^c_{-2} + 0.012Z^c_{-3} + 2.40\,(\dot{n}_{-\frac{1}{2}} - 20)'' \qquad (6.51)$$

in order clearly to indicate that n is supposed to depend on a preceding rate of increase.[2] It may be combined with equation (6.28) for Z^c (omitting external terms):

$$0.770\,Z^c = 0.179\,Z^c_{-1} + 0.006\,Z^c_{-2} - 0.015\,Z^c_{-3} + 0.007\,Z^c_{-4} + 0.090\,n - 0.049\,n_{-1} + 0.001\,n_{-2} + 0.003\,n_{-3} \qquad (6.52).$$

The character of the movement is now considerably changed; the important fact being that the original form of the Z^c-equation matters much less to the result than the coefficients in the n-equation (6.51). The mathematical solution of equations (6.51/2) shows the movements of the system now to be unstable, *i.e.*, an initial movement in the upward direction will be reinforced in an ever-increasing degree. In order better to understand the character of the changes, we may first study the movements generated by the relation

$$n = 2.40(\dot{n}_{-\frac{1}{2}} - 20)'' \qquad (6.51')$$

i.e., relation (6.51) in the assumption of stable profits $Z^c = 0$. The movements may be studied for all values of $\dot{n}$ by assuming, as we did in section (4.8), that the relation between $\dot{n}_{-\frac{1}{2}}$ and n is as follows:

---

[1] External terms omitted.

[2] This, in fact, has been the sense of our equation (4.8). It is only for simplicity that until now $n - n_{-1}$ has been written. Logically, this is less correct, and for that reason it is now dropped.

$$\left.\begin{array}{l} \text{I. For } \dot{n}_{-\frac{1}{2}} < 20: \quad n = 0; \\ \text{II. For } \dot{n}_{-\frac{1}{2}} > 20, \text{ but } < 62: n = 2.40\,(\dot{n}_{-\frac{1}{2}} - 20); \\ \text{III. For } \dot{n}_{-\frac{1}{2}} > 62: \quad n = 100; \end{array}\right\} \quad (6.53).$$

In order to give the problem its simplest shape, we may reduce time units to one-third of their original length, *i.e.*, to four months; $\dot{n}_{-\frac{1}{2}}$ in our previous notation may then be replaced by $3\,(n_{-1} - n_{-2})$, and equation (6.51′) becomes:

$$n = 7.2\,(n_{-1} - n_{-2} - 6.7)'' \qquad (6.51'')$$

which, as before, is only valid for interval II.

Table (6.53) turns into:

$$\left.\begin{array}{l} \text{I. For } n_{-1} - n_{-2} < 6.7: \quad n = 0; \\ \text{II. For } n_{-1} - n_{-2} > 6.7, \text{ but } < 21: \\ \qquad\qquad n = 7.2\,(n_{-1} - n_{-2} - 6.7); \\ \text{III. For } n_{-1} - n_{-2} > 21: \quad n = 100; \end{array}\right\} \quad (6.53').$$

The movements possible under the laws contained in this table are of various types. Starting from an initial level of share prices equal to zero[1] (*i.e.*, some average level), the following possibilities exist.

If no disturbance from outside occurs, the level will remain zero; because $n_1 - n_0$ will be zero, we are in interval I, and $n_2 = 0$; again $n_2 - n_1 = 0$, therefore $n_3 = 0$, etc.

(i) If a small disturbance occurs, viz.,

$$n_1 - n_0 < 6.7,$$

then again $n_2 = 0$; therefore $n_2 - n_1 < 0$, $n_3 = 0$, etc. Share prices will immediately become stable again.

[1] It is of no great importance whether this level is indicated by $n = 0$ or by n equal to any other constant. It is essential, however, that the level indicated by 100 is higher.

(ii) A somewhat larger disturbance,

$$n_1 - n_0 > 6.7 \text{ but } < 8.85,$$

has similar consequences. Although n_2 will now be positive, this will not suffice to make its increase over n_1 larger than 6.7; and for $n_2 - n_1 < 6.7$, n_3 will again be zero.[1]

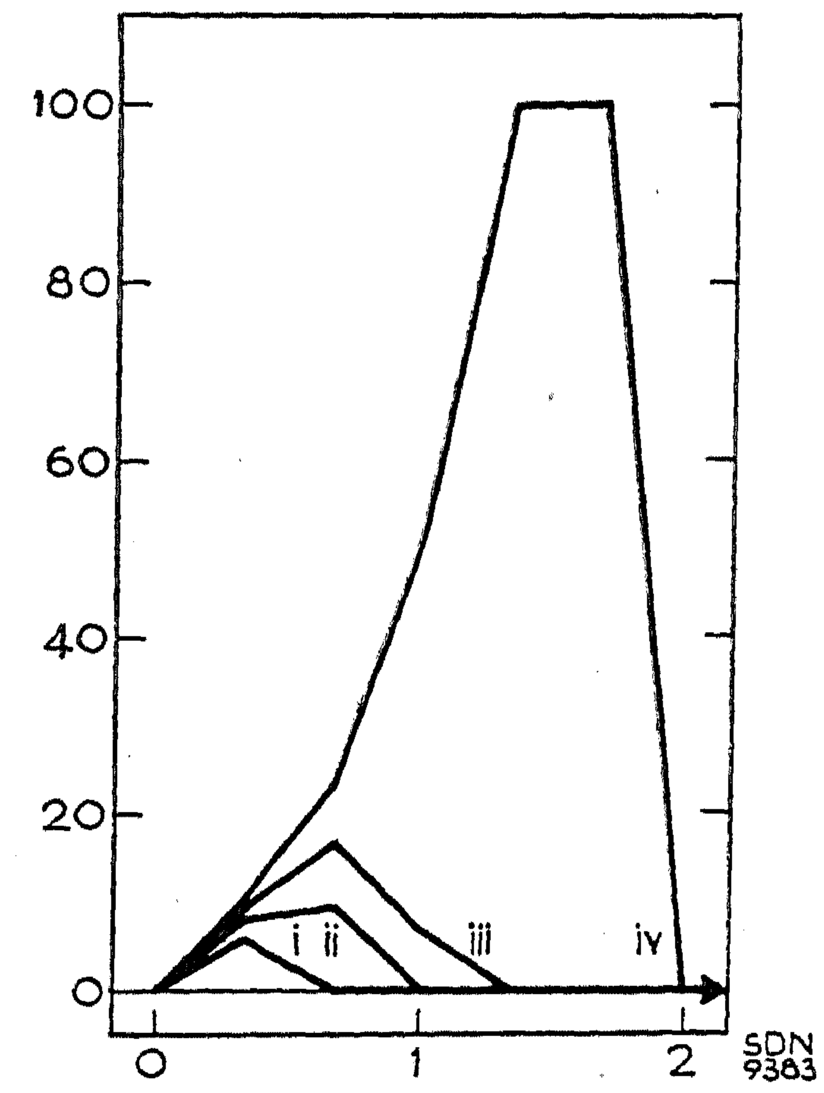
Graph 6.51.

MOVEMENTS OF SHARE PRICES, WITH STABLE PROFITS.

Initial values:

(i) $n_0=0$, $n_1=6$.
(ii) „ $n_1=8$.
(iii) „ $n_1=9$.
(iv) „ $n_1=10$.

Time in years.



(iii) If $n_1 > 8.85$, n_2 will be at least 15.55, or 6.7 more; therefore n_3 will be positive; but it may be less than n_2; then, and even if it is less than $n_2 + 6.7$, n_4 will again be zero.

[1] The limiting value 8.85 for n_1 is found by asking for what value of n_1, $n_2 - n_1$ will be > 6.7; $n_2 - n_1 = 7.2\,(n_1 - n_0 - 6.7) - n_1 = 7.2\,(n_1 - 0 - 6.7) - n_1 = 6.2\,n_1 - 48.2$. In order to make this > 6.7, n_1 has to be larger than $\frac{48.2 + 6.7}{6.2} = 8.85$.

(iv) It can be proved mathematically[1] that if n_1 surpasses 9.5, the movement will be "explosive", *i.e.*, will not return in the way indicated under (i) to (iii). It may be more simple to give an example. Taking $n_1 = 10$, we find:

$$n_2 = 7.2\,(10 - 0 - 6.7) = 23.8;$$
$$n_3 = 7.2\,(23.8 - 10 - 6.7) = 51.1;$$
$$n_4 = 7.2\,(51.1 - 23.8 - 6.7) = 148;$$
$$n_5 = 7.2\,(148 - 51.1 - 6.7) = 649.$$

This development, however, stops as soon as the "third interval" is reached, in this case n_4; here the formula $n = 100$ is used instead of the formula $n = 7.2\ (n_{-1} - n_{-2} - 6.7)$. The calculation would continue: $n_4 - n_3 = 100 - 51.1 = 48.9 > 21$, therefore $n_5 = 100$ again; but now $n_5 - n_4 = 0$ and we are brought back at once into interval I, yielding $n_6 = 0$. Graph 6.51 illustrates our results.

It may easily be found that any upward movement, once it passes into the "explosive" type, suddenly falls back upon the zero level in the same or a similar way.

The mechanism described by (6.53′), however simplified, seems to represent correctly the typical movements on the stock exchange; accelerated rises followed by a sudden fall. It may be changed in some respects — *e.g.*, the sudden change from interval I into II and from II into III may be smoothed out somewhat, or the coefficient 2.40 may even be lowered considerably — without changing this fundamental conclusion. The latter must certainly be completed by the remarks that (i) small external shocks may, especially at the beginning, easily interrupt the explosive development, and (ii) the top level of 100 assumed here is of course in a high degree arbitrary.

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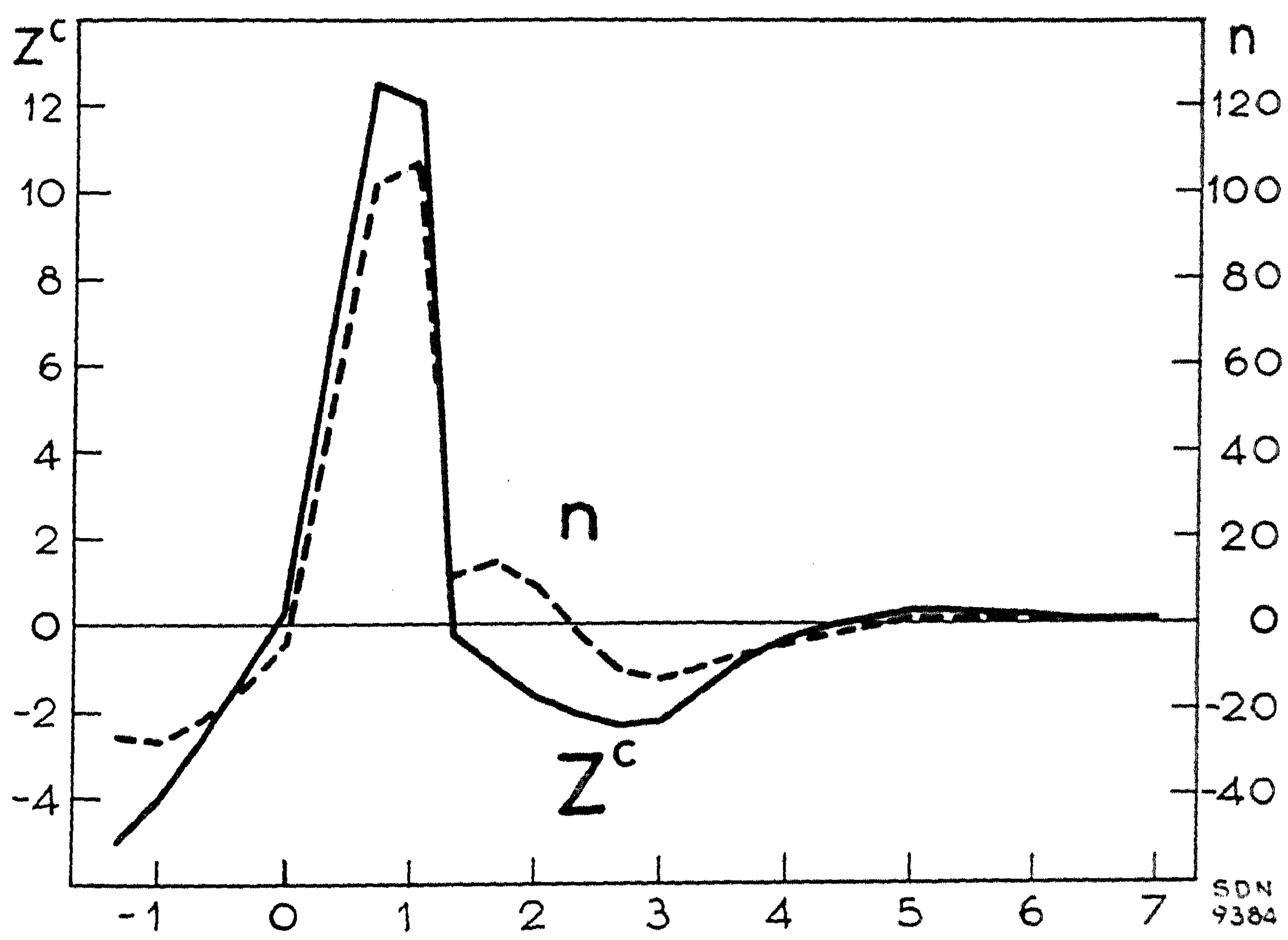
[1] By solving the difference equation $n_t = 7.2\ (n_{t-1} - n_{t-2} - 6.7)$. Introducing as a new variable $n'_t = n_t + 48$, it is homogeneous: $n'_t = 8.7\,n'_{t-1} - 8.7\,n'_{t-2}$, and the roots of the characteristic equation are 6.0 and 1.2. This means that as soon as $\frac{n'_2}{n'_1} > 1.2$ or $n_2 + 48 > 1.2\,(n_1 + 48)$, explosive movements will develop from the start. As $n_2 = 7.2\ (n_1 - 6.7)$, this leads to the condition $n_1 > 9.5$.

But, though the details are incalculable, the main conclusion of the danger of this mechanism to the stability of the system holds. Only if the coefficient 2.40 were less than about 0.33 would the danger of explosive movements be wholly removed.[1]

A combined solution of the equations (6.51) and (6.52) for n and Z^c is very difficult. Only a numerical solution, therefore, has been given in graph 6.52.

Graph 6.52.

Movements of
CORPORATION PROFITS (Z^c) AND SHARE PRICES (n),
taking Account of the Three Intervals of Share Prices
(Time in years).



It shows one speculative boom, followed by a damped cycle like that of graph 6.32. Evidently it is largely a matter of chance whether or not another speculative boom will occur when the system recovers from the depression ensuing upon the first boom. Quite small shocks could easily lead $\dot{n}$

[1] Coefficients of this magnitude and lower were found for all other countries investigated in an other publication of the author: "The Dynamics of Share Price Formation". *The Review of Economic Statistics*, November 1939.

into the speculative interval again. After the boom, the asymmetry of the n-curve is somewhat less than in the case of graph 6.51, owing to the "support" given to share prices by the high profits still prevailing at the start of the crisis. The period of the Z^c equation is approximately maintained in this combined system.

In the numerical solution reproduced in graph 6.52, a reduction of the time-unit to four months is again needed to bring out adequately the short-term movements connected with a speculative boom. The speculative term in the n-equation is again given the form (6.53′), while the other terms in the right-hand members of both equations are adapted to the four-month unit by replacing Z^c_{-1} (lag one year) by

$$\frac{1}{16} Z^c_{-1} + \frac{4}{16} Z^c_{-2} + \frac{6}{16} Z^c_{-3} + \frac{4}{16} Z^c_{-4} + \frac{1}{16} Z^c_{-5}$$

(distributed lag with an average of three four-month units), etc.

(6.6) Hoarding

We shall now consider the rôle of hoarding in the cyclical mechanism. The evidence derived from one instance of hoarding over a few years clearly does not sustain a general conclusion on the cyclical importance of hoarding in the United States economy. It may be of interest, however, to study what would be the consequences if the features of hoarding observed over these few years constituted a regular system of behaviour, recurring when similar conditions recur. As described in section (4.6), we then assume hoarding to be initiated only in a deep depression where Z^c comes more than 7 milliard dollars below its previous peak value. Our form of analysis is not qualified to discover whether or when this situation will occur: the system of equations only describes, as was shown in (6.3), the *propagation* of certain shocks, and it is therefore these shocks that determine, generally, the *amplitude* of the movements, and, in particular, the occurrence of the above situation.

We can only analyse what happens to the cyclical mechanism when the situation is there, *i.e.*, when

$$Z^c_m - Z^c > 7 \qquad (6.61).$$

For that case, the following equation for H was found:

$$H = -0.3\, Z^c \qquad (6.62).$$

By the substitution of this relation for the H-terms in the final equation, we get a new final equation which describes the movements of the system in which hoarding has developed, as long as (6.61) is satisfied:

$$Z^c = 0.206\, Z^c_{-1} - 0.508\, Z^c_{-2} + 0.030\, Z^c_{-3} + 0.062\, Z^c_{-4} \qquad (6.63).$$

We may, however, take account of the fact that the coefficient for H in equation (4.91) is not very certain (*cf.* the table on page 113). For this purpose we may study the effects of varying the coefficient of Z^c in equation (6.62) which "explains" H.[1] Graph 6.61 shows the effects on the damping ratio and the period of the cyclical solution of the resulting final equation for the values of this coefficient running from 0 (the general final equation) to -0.5.[2] It is seen that the period is shortened over the whole range. Similarly, the damping ratio decreases, initially at an approximately constant rate.

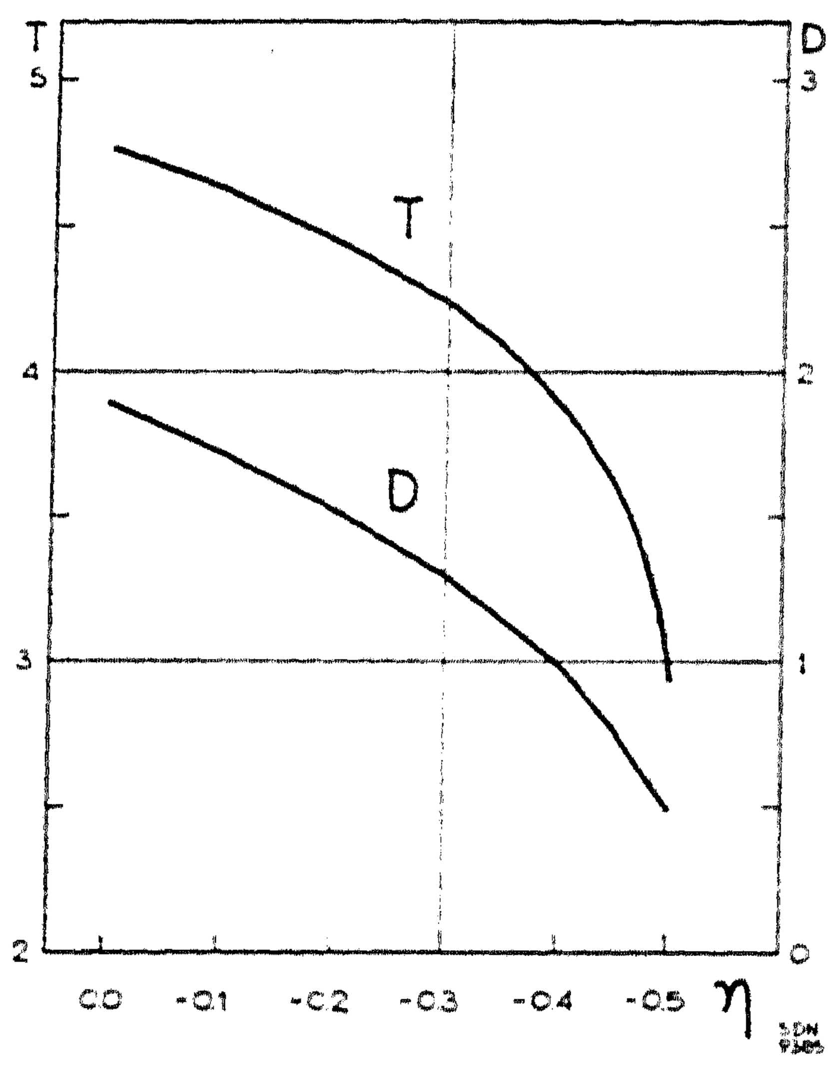
At about $H = -0.4\, Z^c$, the damping ratio becomes 1; *i.e.*, at values of $H < -0.4\, Z^c$ the cycle becomes anti-damped.

It follows from these figures that the more intensively hoarding occurs, the less the cycle is damped, and it would finally

Graph 6.61.

DAMPING RATIO (D) AND PERIOD (T) as a function of the INTENSITY OF HOARDING.

$H = \eta\, Z^c$,
where $\eta = 0.0, -0.1 \ldots -0.5$.



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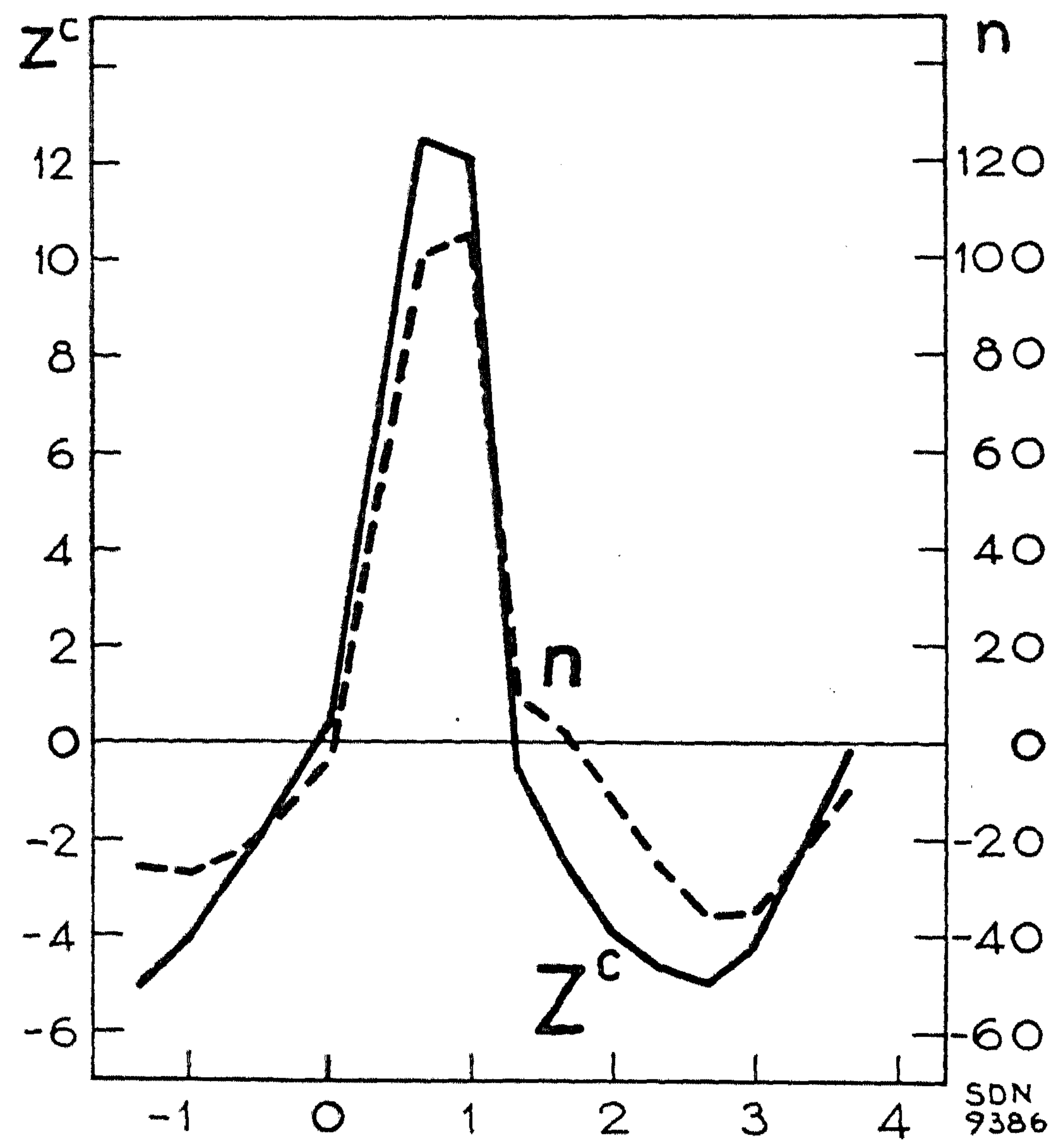
[1] Neglecting the relatively small changes in the other coefficients in these alternative cases.

[2] The calculations are not extended further, since it seems doubtful whether the damping ratio and the period found for $H < -0.5\, Z^c$ have any economic significance.

even become anti-damped when hoarding came to exceed a certain intensity. It should be added that, in any case, the development of Z^c would be described by the final equation (6.63) (or its variant according to other values than — 0.3 in (6.62)) for one or two years at the most: then the depression will be over (*cf.* graph 6.61) and the hoarding condition (6.61) will no longer be fulfilled.

Graph 6.62.

Movements of Corporation Profits (Z^c) and Share Prices (n), under the action of a Speculative Boom, and of Hoarding.



The heavy damping of the system makes it improbable — unless when very large shocks occur — that the movement of Z^c will pass from the normal interval into the hoarding interval. But it is, on the other hand, quite probable that such a heavy fall will occur after a large positive deviation from the average due to a stock-exchange boom.

The possibility of this complication has not been taken into account in section (6.5). We may do this now, and introduce into the numerical solution executed in that section the property that H is replaced by $-0.3\,Z^c$ as soon as, and as long as, $Z^c_m - Z^c > 7$. The result of this changed calculation is shown in graph 6.62.

The period is slightly shortened, to about four years. The downswing is much more rapid than in graph 6.52, and the movement is therefore not unlikely [1] to be swung back into the speculative zone. It does not, however, seem justified to continue the calculation after the depression (the fourth year). Both hoarding and speculation are phenomena in which the psychological factors may change from one cycle to another. Therefore, coefficients determined for past events cannot safely be assumed to hold with about the same values for the future.

In the interpretation of these results, it should be borne in mind that currency-hoarding only has a considerable influence on economic life when, on account of its magnitude in relation to the other items of the balance-sheet of the Federal Reserve Banks, it forces the member banks into debt with the Federal Reserve Banks (*cf.* section (4.4), and especially graph 4.421/2). The continuance of hoarding during the years 1933 to 1937, when it did not prevent the accumulation of excess reserves, cannot, according to the line of our deduction, be held responsible for any considerable depressive influences.[2]

The above treatment of the phenomena of hoarding and stock-exchange speculation may show to what extent the apparatus used in this study is capable, on the basis of an analysis of the elementary relations, of a gradual approach to the complicated and seemingly irregular reality of actual business cycles. In particular, it shows how an illuminating synthesis may be established between these two extreme positions with

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[1] This conclusion must evidently be confined to the indication of certain chances. Whether the actual movement swings back to the speculative interval depends not only on the internal propagation of past shocks according to the final equation, but also on the occurrence and the direction of new shocks.

[2] In this connection, the importance for the United States of the huge gold imports becomes clear (*cf.* section (4.4)). Net imports of gold amounted to nearly \$6 milliard between the depreciation of the dollar and the end of 1937; at that date the excess reserves figured only at \$1 milliard (or, calculated according to the pre-1936 reserve percentages, \$4 milliard).

regard to business cycles: the denial of any regularity in the movements on the one hand, and the assumption of cycles that are strictly regular with regard to period and damping ratio (and perhaps even amplitude) on the other hand. It is especially the rôle attributed to the shocks and the incorporation in the system of a few, but important, non-linear equations which make it possible that one cycle is completely different from another, and that it is yet, in both, one and the same mechanism that links the variables together.

(6.7) Building Activity and General Activity; the Building Cycle; a New "Multiplier" Concept

The peculiar rôle played by the housing market in the general economic system has already been mentioned (section (6.3)). Some attention may now be given to the so called "housing cycle". The general business-cycle position, of course, influences house building to some extent, but only in a rather small degree, as a glance at graph 2.5 shows. But the chief factor for house building seems to be the relative shortage or abundance of houses some four years back (of course it is a distributed lag which must be assumed here); and as far as rents are another determinant, these themselves are also influenced to some extent by that shortage or abundance. Putting aside for a moment the smaller factors, we are left with a skeleton of relations:

$$v_B = -0.30\,h_{-4} \tag{6.71}$$

$$\Delta h = h - h_{-1} = 0.92\,v_B \tag{6.72}.$$

It is easily found — as well by simply trying out with arbitrary numbers as by rigorous mathematical treatment — that this set of relations leads to cycles of a period of about sixteen years, which fits fairly well with observations even over a longer period.[1] Although this cycle existed long before

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[1] *Cf.* Roos: *loc. cit.*; Newman: *The Building Industry and Business Cycles*, Chicago, 1935. The method of computation of the period is the one indicated in J. Tinbergen: "Ein Schiffbauzyklus ?", *Weltwirtschaftliches Archiv* (34), 1931, II, page 151.

The explanation given by Dr. Roos, *loc. cit.*, page 78, seems to lead to the same conclusion. Dr. Roos explains residential building the war and was remarkably unaffected by the ordinary business cycle, the exceptional reduction of the house-building level during the war accentuated the amplitude of this cycle, and must be held responsible for the exceptional high in 1925 and the exceptional low in 1933. Of course other causes existed as well for the latter low point; but it is remarkable that recovery in private building was so slow in 1934, a fact which fits perfectly well into our present representation. It is very doubtful whether such general statements as "building precedes the business cycle" and "building moves against the cycle" are justified. It all depends on the year observed; in general the direct connection is rather weak.

The fact that v_B shows largely autonomous movements has been one reason for treating it as an external variable. Another is that this procedure enables us to find out the influence of any autonomous increase in building activity on Z^c, and on any other economic variable as well. The most interesting case will be the influence on total volume of production, as this is the well-known problem of the "Multiplier".[1] In order to find the influence of a given increase in building volume v_B on total volume of production $u+v$, this latter variable has been expressed first in terms of Z^c, Z^c_{-1}, Z^c_{-2}, etc., and the external factors:[2]

$$u+v = 1.551\,Z^c + 0.417\,Z^c_{-1} - 0.153\,Z^c_{-2} - 0.097\,Z^c_{-3} + 0.022\,Z^c_{-4} + 1.411\,v_B - 0.257\,(v_B)_{-1} \qquad (6.73).$$

activity chiefly by the fluctuations in foreclosure rates two and a half years before. These foreclosure rates themselves seem, however, to be mainly dependent on the relative shortage or abundance of houses a short time before, which influences the profitability of owning houses.

[1] The multiplier concept may be applied as well to employment, income or production, but in any case it should, of course, be a comparison between two phenomena of the same sort. Since v_B has the "dimension" of a volume of production, we have here to apply it to volumes of production.

[2] The other exogenous terms are of no importance in this connection, and are therefore omitted. This is also the case for the terms with h_{-2} and h_{-3}, which relate to influence *via* rents. Since, in this context, v_B represents additional investment activity in general, the repercussions that are particular to investment in dwelling-houses are to be disregarded.

In this expression, Z^c may be replaced by the expression found in the final equation (6.30)[1] and so expressed in terms of Z^c_{-1}, Z^c_{-2}, etc. and v_B[2]. This may be repeated. Owing to the fact that the coefficients of Z^c_{-1}, Z^c_{-2}, etc., in the final equation are smaller than that of Z^c, this procedure leads to ever-decreasing coefficients which after some time become negligible. Thus the procedure finds a natural end, leaving the following formula:

$$u + v = 4.44 v_B + 1.59 (v_B)_{-1} - 0.23 (v_B)_{-2} - 0.57 (v_B)_{-3}$$
$$- 0.02 (v_B)_{-4} + 0.15 (v_B)_{-5} + 0.04 (v_B)_{-6} - 0.03 (v_B)_{-7} + \dots$$

From this we find that the consequences for total employment of an addition of one unit to building activity in year 1 are, in consecutive years, of the following size:

Year	1	2	3	4	5	6	7
	4.44	1.59	− 0.23	− 0.57	− 0.02	0.15	0.04
Year	8	9	10	11	12	13	
	− 0.03	− 0.02	0.00	0.01	0.00	0.00	

(6.74).

The total of this series, 5.36, is comparable with the old concept of the multiplier.[2] Yet it differs from it in many respects.

(i) The figure represents the ratio between the total effect on production and an initial increase in investment (building activity); this effect consists of u and v, and hence partly of new investment. In the theory of Mr. KEYNES,[3] on the other hand, the multiplier is defined with *total* (and not *initial*) net investment as the denominator.

(ii) The method here employed enables us to take account of a great number of repercussions, *via* many more variables

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[1] After transformation of the term with h_{-4} to v_B and Z^c (equation (2.5); the other terms in this equation may be neglected). The terms with h_{-2} and h_{-3} are disregarded (*cf.* page 163, note 2). The omittance of the q_B-term in (2.5) may be interpreted that it is cancelled out by the q_B-term in (1.16) (*cf.* section (3.6)); it follows, then, that the results found refer not to the execution of a certain *volume* of "public works", but to the spending of a certain *sum*.

[2] A calculation along the lines indicated by Mr. KEYNES would give about 6 (Method of calculation, *cf.* POLAK, *loc. cit.*, page 10).

[3] *The General Theory of Employment, Interest and Money*, London, 1936.

than can be allowed for by a procedure such as that first developed by Mr. R. F. Kahn.[1]

(iii) Previous calculations found the multiplier as the sum of the terms of a geometric series with a positive ratio; no negative terms occurred in this sum. The series (6.74), on the other hand, shows an alternation of positive and negative terms. This is due to the fact that it may be considered as the sum of:

(*a*) $1.411 v_B - 0.257 (v_B)_{-1}$ in (6.73);

(*b*) Four superposed damped cycles of the period and damping of the final equation (*cf.* graph 6.32), weighted with coefficients 1.551, 0.417, etc., and with a phase-difference of one year between any two cycles.

It will be clear, then, not only that part (*b*) contains certain negative figures, but also that its net sum over an infinitely long period of time is only positive when the cycle is damped and the sum of the weights is > 0. If the cycle had been undamped, the sum of (*b*) over a certain number of years would be alternatively increasing and decreasing, passing through 0 after any 4.8 years.

(iv) It is clearly shown in this approach that the value found for the multiplier is subject to a certain restriction. We have used the final equation for Z^c based on the "normal" interval in the *n*-equation. Since the multiplier concept has been generally used for depression periods, this is justifiable; but it means that, if by the autonomous addition to building volume a boom develops, the formula is no longer valid. It would lead too far to go into the problems involved.[2]

Another restriction to be made is that, if the "public works" represented here by v_B were to consist in the building of residences, we should have to take account of the fact that, according to equation (2.5), to-day's building "spoils the market" for the future. This would give additional negative terms in (6.74) after the third year.

[1] "The Relation of Home Investment to Unemployment", *Economic Journal*, XLI (1931), page 173.

[2] *Cf.* also J. Tinbergen: "Über die Sekundärwirkungen zusätzlicher Investitionen", *Weltwirtschaftliches Archiv* (45), 1937, I, page 39.

(6.8) The Formal Characteristics of Business-cycle Policy

In the preceding sections, certain characteristics, in different circumstances, of the business cycle of the United States were discussed. We may now turn to the analysis of the effects on the character of this cyclic movement of measures of policy and of some much-discussed changes in the economic structure.

For a good understanding of the problems to be treated here, some methodological problems must again be touched upon.

(i) Measures of policy may, in accordance with the analysis in section (6.3), be grouped as:

(*a*) Changes in the coefficients or lags, or both, *i.e.*, changes in the economic structure of society — *e.g.*, price-stabilising measures;

(*b*) Shocks — *e.g.*, the Veterans' bonus payment in the United States in 1936;

(*c*) Changes in the average level of some variable — *e.g.*, minimum wage legislation. The effects of this last type of change are not taken into account in this study, which is concerned specially with the problem of business cycles.

The apparatus for analysing the effects of shocks has been discussed in section (6.7), where a more refined and qualified multiplier-concept has been elaborated. Nothing more need be said about these here, especially since a policy that takes the form of irregular disturbances will in general not be considered desirable, since it would increase uncertainty instead of decreasing it.

In this section we shall therefore restrict the analysis to the effects of changes in coefficients and lags.

(ii) Any measure, therefore, which is not defined in sufficient detail to be translated into a definite change in some definite coefficient cannot be discussed at all. As an example, take a proposal to make consumption less dependent on the business cycle; *e.g.*, by higher taxes during the boom, lower taxes during depression, and more stabilised ordinary expenditure by the State. Since we have seen (table, section (6.4)) that consumption outlay depends on Z^c, Z^c_{-1}, Z^c_{-2}, Z^c_{-3} and Z^c_{-4}, five coefficients might be affected by this proposal. It makes some difference which of these coefficients is changed most; measures of a given intensity but with different lags may have quite different results.[1]

(iii) In the study of the effects of a change in one coefficient, it must not be forgotten that we have not analysed why the coefficients are as large as we have found them to be; we have simply determined their magnitude by the multiple correlation technique. Hence it is not sure beforehand that a change in one coefficient will leave all other coefficients as they are: some coefficients may be linked to one another by relations into which we did not enquire: *e.g.*, a stabilisation of dividends may influence the way in which shareholders appreciate a certain value of d — *i.e.*, the coefficient ν_1 in equation (4.81). The so-called "variation problem" to be attacked now must therefore be handled with care in this respect.

(iv) The result of these variation calculations has various aspects. Any cyclic movement is characterised by period, damping degree, phase and amplitude, and each of these four may be changed. In some discussions too little care seems to have been taken to distinguish between these types of change. Such is the case in the well-known question whether the boom can be continued (once a peak level has been reached) by increased consumption or increased saving. Such a discussion generally bears upon an incidental lengthening of the boom; and such a lengthening may be the consequence of a change in phase as well as a change in period, damping degree or amplitude. The distinction between these has seldom been discussed. Nevertheless, it seems necessary to do so since, *e.g.*, changes in phase are far less important than changes in damping degree and amplitude. It is especially these latter which a systematic stabilisation tries to change. In the following pages only the

[1] The more important distinction, between changing the average level of consumption and changing the fluctuations in consumption, is often forgotten in more popular discussions. Here, of course, we are speaking only of the latter.

very first beginnings of the questions involved are indicated. Much more space would be required — and in fact it would involve a new subject — if these questions were to be dealt with completely.

We shall treat the variation problem in two ways. In the first place it will be assumed that for a certain variable *all* fluctuations are excluded, *i.e.*, that all coefficients in the equation "explaining" this variable are taken as zero. Secondly, the effects of relatively small changes in a number of important coefficients and lags will be taken up one by one.

It should be noted that these calculations not only give an estimate of what would be the characteristics of the American business cycle when certain changes in the equations are brought about (the problem of policy), but they also tell us to what extent our idea of the present mechanism may be falsified by the fact that some of our equations afford an inaccurate picture of reality (margins of error). The same effects on our final equation are caused by a certain coefficient's *being* in reality 5 % lower than it *seems* to be, or by its *becoming* 5 % lower than it *is*.

For reasons of exposition we start again with what has been called the "normal interval", *i.e.*, we assume the absence of a stock-exchange boom and of exceptional hoarding. Given the importance of speculation or hoarding for economic life as a whole, this point must not be neglected.

(6.9) THE EFFECT OF SOME MEASURES OF BUSINESS-CYCLE POLICY OR CHANGES IN THE ECONOMIC STRUCTURE ON THE CHARACTER OF THE CYCLIC MOVEMENT

I. *All Coefficients in one Equation taken as Zero.*

First, let us consider the consequences of a complete stabilisation [1] of investment activity, *e.g.*, by compensatory public investment.

[1] This term must be understood not to mean that investment outlay would always have the same constant value, but that it would have some smoothly increasing ("trend") value. For some of the implications involved *cf.* J. TINBERGEN: *Fondements mathématiques de la stabilisation des affaires* (Hermann, Paris, 1938).

Such a stabilisation of investment outlay assumes that all coefficients in the expression determining v are zero. As, in formula 216 (Appendix B, table V), v is still included explicitly, the corresponding final equation is easily calculated.

This final equation would run (apart from external terms):

$$\begin{aligned} 0 &= -0.76Z^c - 0.26Z^c_{-1} - 0.15Z^c_{-2} - 0.21Z^c_{-3} \text{ or} \\ Z^c &= -0.34Z^c_{-1} - 0.20Z^c_{-2} - 0.03Z^c_{-3}, \end{aligned} \tag{6.91}$$

and the corresponding cycles would be substantially more damped than the original cycles, whereas the period would be shorter (*cf.* table 6.91).

Table 6.91.

THE EFFECT OF SOME MEASURES OF POLICY ON THE DAMPING RATIO AND PERIOD OF THE CYCLE.

Case	Period in years	Damping ratio
No policy	4.8	1.89
Stable investment outlay . .	3.8	2.41
Stable consumption outlay .	3.2	2.62
Rigid wages	4.7	1.85
Rigid prices	4.3	1.71

Second, let us consider a stabilisation of consumption. This might be the consequence of a change in human attitude, leading to an increased rate of savings in boom periods and a decreased rate of savings in depressions. It might also be the consequence of Government policy, as has been observed already — *e.g.*, by "compensating taxes" and a stabilisation of ordinary State expenditure. Finally, it might also partially be obtained by a less unequal distribution of incomes. The consequences can be found by much the same method as used in the case of investment stabilisation.

The final equation therefore becomes:

$$\begin{aligned} 0 &= -1.41Z^c - 0.17Z^c_{-1} - 0.16Z^c_{-2} - 0.03Z^c_{-3} \text{ or} \\ Z^c &= -0.12Z^c_{-1} - 0.12Z^c_{-2} + 0.02Z^c_{-3}, \end{aligned} \tag{6.92}$$

and the corresponding movements are even more damped than those obtained in the previous case.

In the third place, the consequences of changes in the flexibility of wages, and in the fourth place those of changes in the flexibility of prices, will be considered. Both problems have attracted a good deal of attention in economic literature. It has been held that the rigidity of wages and prices is responsible for the increased amplitude of business cycles in recent times. On the other hand, stabilisation of prices has been advocated as a means of stabilising general activity. The treatment of these problems seemed easiest when (i) wage rates l_w and (ii) all prices (p, p', p^f, q, q_B and m_r) were assumed to be absolutely rigid, and the final equations were recalculated on this basis.

They run:

$$\text{Wage rigidity: } Z^c = 0.14Z^c_{-1} - 0.25Z^c_{-2} - 0.04Z^c_{-3} \qquad (6.93)$$

$$\text{Price rigidity: } Z^c = - 0.01Z^c_{-1} - 0.35Z^c_{-2} - 0.05Z^c_{-3} \qquad (6.94)$$

as against the normal case:

$$Z^c = 0.40Z^c_{-1} - 0.22Z^c_{-2} + 0.01Z^c_{-3} + 0.03Z^c_{-4} \qquad (6.36).$$

The case of wage rigidity hardly differs from the normal case with respect to damping ratio and period. This means that, at least in the United States, wage rigidity is not so detrimental to a stabilisation of cyclic movements as has sometimes been believed.

According to these calculations, price rigidity or price stabilisation [1] would have had a somewhat anti-damping effect. This result should, however, be accepted with some caution,

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[1] These two words represent, of course, two very different types of policy. Price rigidity is commonly understood to be caused by monopolistic tendencies, and to be a form of policy pursued by private concerns or groups. Price stabilisation covers a far wider field; it may be used for the same private ends, but also for governmental action in which varying instruments are brought into play in order to attain a stable price level. Of this latter class, only such measures are meant here as act directly on prices themselves; *e.g.*, a policy of holding stocks of raw materials, or using raw materials as cover for note circulation,

since the coefficients for the price-variables are rather uncertain in some of the equations (*e.g.*, (2.1) and (2.4)).

Finally, it must not be overlooked that the foregoing calculations are all only valid for a period with no stock-exchange boom. Although there is no doubt that a stabilised economy would offer fewer opportunities for the development of a stock-exchange boom, it is still possible that even with a perfectly stabilised endogenous development some incidental cause may lead to a speculative boom, as described in section (6.5). Unless, therefore, the price-formation of shares is changed considerably, this part of the mechanism will continue to be a threat to stability in economic life. It follows from the above that those proposals for stabilising the so-called "general price level", in which share prices are also included, may be of still more importance than those that aim only at stabilising the prices of goods and services — provided that the method for obtaining stabilisation is indicated clearly.

II. *Changes in Coefficients.*

The most important coefficients, the effects of changes in which we wish to study particularly, occur in the equations determining

(2.1) Consumption;
(2.4) Investment;
(2.6) Stocks;
(3.5) Prices of capital goods;
(4.8) Share prices;
(5.1) Dividends;
(5.2) Entrepreneurial withdrawals;
(5.3) Capital gains;

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or, finally, regulating prices by decree or by subsidies. Such more indirect measures of price stabilisation as act through the volume of credit, for example, are not included. They could better be designated by terms indicating what instrument is used (*e.g.*, credit rationing, discount-rate policy, etc.); and in order to discover their consequences other calculations than those made here are necessary.

which largely coincide with the equations indicated above as "strategic" equations.

We may write these equations as follows (dots indicating the terms in each equation in which no changes are made):[1]

$$
\begin{aligned}
&(\text{V2.1}) \quad U' = (0.95 + \upsilon^{L})(L_w + L_s) + (0.77 + \upsilon^{E})\,E + E'_F \\
&\qquad\qquad + 0.28G\,\text{[2]} + (0.049 + \upsilon^{\Delta})\,\Delta p^f + (0.03 + \upsilon^{p})\,p \\
&(\text{V2.4}) \quad v' = (0.33 + \varphi^{z})\,Z^c + (0.33 + \varphi_1^{z})\,Z^c_{-1} - (0.47 + \varphi^{m}) \\
&\qquad\qquad [m_{Ls} + (m_{Ls})_{-1}] - \ldots \\
&(\text{V2.6}) \quad w = (0.105 + F)\,u' + (0.047 + F_1)\,u'_{-1} \\
&(\text{V3.5}) \quad q = 0.35\,l + (1.29 + \chi)\,v'_{-0.46} \\
&(\text{V4.8}) \quad n = (20.6) + \nu)\,d - \ldots \\
&(\text{V5.1}) \quad D = (0.151 + \delta)\,Z^c + (0.083 + \delta_1)\,Z^c_{-1} + \ldots \\
&(\text{V5.2}) \quad E_E = (0.110 + \varepsilon)\,Z^c + (0.066 + \varepsilon_1)\,Z^c_{-1} + \ldots \\
&(\text{V5.3}) \quad G = (0.200 + \gamma)\,\dot{n}_{-\frac{1}{2}}
\end{aligned}
$$

The Greek letters υ^L, υ^E ..., etc., represent relatively small variations in the coefficients together with which they occur. The elimination process may now be repeated with the changed coefficients, *i.e.*, carrying on the variations to the coefficients as algebraic symbols[3] throughout the process. In this way, a final equation[4] is obtained, which contains terms with $\upsilon^L Z^c$, $\upsilon^L Z^c_{-1}$, ... $\upsilon^E Z^c$, $\upsilon^E Z^c_{-1}$ ... etc.

From this equation we may study alternatively the dependence of damping ratio and period of the resulting cyclical

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[1] The V before the numbers indicates that the equations are obtained by *variation* of (2.1), etc.

[2] It is not necessary to treat separately the coefficient for G in this equation, since the effects of a certain change in it are equivalent to those of a change of the same relative magnitude in the coefficient γ (equation (V5.3)).

[3] As the effect of small variations only is to be studied, products or second and higher powers of variations in coefficients have been neglected.

[4] The laborious process of differentiating each intermediate coefficient in the whole elimination process with respect to these sixteen variables results in a mass of figures which it would require too much space to publish.

movement on small variations in each of the coefficients chosen (by equating to zero all but one of the variations). The response to a 10% change in each coefficient is given in the following table:

Table 6.92.

EFFECT ON DAMPING RATIO AND PERIOD OF A 10% INCREASE IN CERTAIN COEFFICIENTS.

Equation	Determining	Coefficient of:	Magnitude in equation	Change in: Damping ratio	Change in: Period (years)
(2.1)	U′	$(L_w + L_s)$	0.95	− 0.147	0.56
		E	0.77	− 0.033	0.18
		G	0.28	− 0.134	− 0.07
		Δpf	0.049	− 0.039	0.09
		p	0.03*	0.072*	0.27*
(2.4)	v'	Z^c	0.33	− 0.018	0.02
		Z^c_{-1}	0.33	0.011	0.07
		Lag of Z^c	*0.50*	*0.029*	*0.05*
		$m_{Ls} + (m_{Ls})_{-1}$	− 0.47	− 0.0004	0.02
(2.6)	w	u'	0.105	− 0.051	− 0.15
		u'_{-1}	0.047	− 0.069	0.05
		Lag of u'	*0.31*	*− 0.046*	*0.12*
(3.5)	q	$v'_{-0.46}$	1.29	− 0.010	− 0.01
(4.8)	n	d	2.06	− 0.119	0.02
(5.1)	D	Z^c	0.151	− 0.082	− 0.11
		Z^c_{-1}	0.083	− 0.082	0.13
		Lag of Z^c	*0.35*	*− 0.037*	*0.19*
(5.2)	E_E	Z^c	0.110	— 0.019	0.01
		Z^c_{-1}	0.066	0.006	0.03
		Lag of Z^c	*0.38*	*0.017*	*0.02*
(5.3)	G	$\dot{n}_{-\frac{1}{2}}$	0.200	− 0.134	− 0.07

* Since this coefficient represents a minimum, rather than an average value, the variation has been calculated for a 100% increase.

For the treatment of this variation problem, the variant (5.7′) has been chosen for the L_s-equation (*cf.* pages 120 and 146). This choice was due to the appearance of the somewhat improbable values (due to the term with Z^c_{-2} in (5.7)), in table 6.93 which are given below as they would have been with equation (5.7) instead of (5.7′), as far as the difference is considerable (> 10% of the values of (6.92)).

Again, values corresponding to any intermediate equation for L_s, between (5.7) and (5.7′), may be approximately calculated by interpolation between the figures of tables 6.92 and 6.93.

It is, however, believed, in particular on the ground of the sign attaching to the influence of υ^L on the damping ratio in table 6.93, that the figures of table 6.92 are nearer to reality.

Table 6.93.

Equation	Coefficients of:	Change in:	
		Damping ratio	Period
(2.1)	$L_w + L_s$	0.051	0.49
	p	0.105	0.11
	$\Delta p'$	— 0.020	0.08
(2.6)	u'	— 0.063	— 0.07
	u'_{-1}	— 0.046	0.08
	Lag in u'	*— 0.018*	*0.11*

Returning to table 6.92, the general impression is that an increase in one of the coefficients in nearly all cases causes (i) a decrease in the damping ratio, and (ii) an increase in the period.

The table enables us to see what coefficients are of the greatest importance for the characteristics of the cyclical mechanism.

(2.1) Most coefficients in the consumption equation prove to be important; especially those of the series L_w+L_s (with the largest standard deviation) and of G. It should be emphasised that the latter is one of the few coefficients, an increase in which causes a decrease in the period. The figures confirm the theory that lower marginal propensities to consume are an important objective for a stabilisation policy.

(2.2) Changes of the coefficients of Z^c and Z^c_{-1} in the same direction have opposite effects on the damping ratio. Hence the relative importance of a change in the lag. An increase in the lag may be represented by a decrease of the coefficient of Z^c, combined with an increase of the same magnitude in the coefficient of Z^c_{-1}; these two changes affect the damping ratio in the same direction, so as to make a larger time-lag in the investment decisions of entrepreneurs conducive to a reduction in cyclical fluctuations.

The results found with respect to this equation are, again, in harmony with what we found above — viz., that an increase in the fluctuations in investment activity intensifies the cycle.[1] The influence of changes in the m_{Ls}- coefficient is very small.

(2.6) The rôle of stocks of consumption goods proves to be rather important; if stocks were constant, and, hence, the coefficients in equation (2.6) were both zero, the damping ratio would be larger by $10 \times (0.051 + 0.069) = 1.20$.

(3.5) The coefficient of 1.29 in this equation is an inverted measure of the elasticity of supply. By varying this coefficient, we may find out how the cycle is changed by a change in the elasticity of supply of capital goods — *e.g.*, as a consequence of a change in the organisation of the market — or when bottlenecks occur.

It was estimated in section (3.5) that, owing to the first of these two events, the coefficient for v' in the "explanation" of q had been about 3×1.29 in the years 1919, 1921 and 1922, and that it had been considerably larger in the bottle-neck years 1920 and 1923. If we take as a rough figure a 500% increase in the coefficient as representative of this latter situation, we find :

[1] The figures show that this statement is only true for investments that are less than about eight months lagged behind Z^c ($0.18 \div 0.29 \times 12$ months). This would imply that public works, even if executed without a purposive policy of timing, would in fact have a certain damping effect, provided that they lagged sufficiently behind profits in private enterprise (which will often be the case). It will, of course, be clear that this effect may be greatly increased by well-balanced timing of public works.

In the case of	Coefficient increased by	Damping ratio	Period
Less monopolistic organisation .	200%	1.69	4.6
Bottle-necks	500%	1.39	4.3

It will be seen that a decrease in the elasticity of supply of capital goods diminishes the damping of the cycle; in the case of very serious bottle-necks, the cycle may even become anti-damped.

(4.8) Given the large rôle played by the share price in the system of equations, it is no surprise to find that a 10% increase in the coefficient of its most important determinant, *d*, has an appreciable negative influence on the damping of the system. Since, however, this coefficient is known with a considerable degree of precision (*cf.* table on page 113), there is not much danger of a serious error in the damping ratio on its account.

(5.1) The influence found for a 10% change in the dependence of dividends on profits (current and for the preceding year) is evidently still greater than that of an equally large variation in the *d*-coefficient in (4.8), where the effects via consumption are not included. The figures found seem especially interesting with regard to measures of policy: if the distribution of dividends could be made to be more stable, say by 25%, the damping ratio of the cycle would increase by $2.5 \times (0.082 + 0.082)$ or 0.410. The period proves to be very sensitive to changes in the lag of dividends behind profits. It may be deduced that the quick reaction of dividends to changes in profits (with an average lag of 0.35 of a year) might well be one of the main factors making for the difference in period between the European and the American cycle. If the lag of dividends behind profits were twice as large — *i.e.*, eight to nine months, or about at the magnitude it probably has in most countries in Europe — the period would be about two years longer.[1]

[1] To check whether the use of the figures in table 6.92 is, in this case, legitimate for so large a change in the coefficients, the elimination process has been repeated with an equation for D with a lag of nine months: $D = 0.059\,Z^c + 0.175\,Z^c_{-1}$. This calculation confirmed the prolongation of the period.

(5.2) With regard to this equation, which "explains" E_E, the influence of the lag on the characteristics of the cycle is of particular importance. According to the coefficients in equations (5.1), (5.2), (5.6), $3(E_E - E'_F - E''_F)$ is about equal to $E_E - E'_F - E''_F + D + L_c$, the constituents of which form the most fluctuating items in E (equation (1.3)). Hence an increase of one month in the lag of entrepreneurial withdrawals behind profits is, in its effects on damping and period, equivalent to an increase in the lag of consumption due to business income $E_E - E'_F - E''_F + D + L_c$ behind these incomes by about one third of a month. In this way, the calculated effects of the variation in the lag in equation (5.2) may serve to determine whether a possible small lag of consumption of non-workers behind their incomes, of which we found no evidence in section (2.1), might have had an appreciable effect on the damping ratio and the period. It appears that, if the incomes $E_E - E'_F - E''_F + D + L_c$ entered in the consumption equation with a lag of three months, the damping ratio would be 0.35 higher and the period half a year longer.

(5.3) Finally, the coefficient for $\dot{n}_{-\frac{1}{2}}$ in the equation "explaining" G proves to be of very great importance. If the speculative income arising from a given rise in share prices became twice as large — or if consumption reacted with double intensity to speculative gains [1] — the cycle would become heavily anti-damped. It follows, as it does from the features of the "speculative interval" described above (6.5), that, in the period and country under review, a policy directed to diminish speculation would have a stabilising effect.

The discussion of the effects of the variation of individual coefficients on the damping ratio and the period of the cycle may also serve to determine the uncertainty of the figures found for these magnitudes (1.89 and 4.8 years). Here a possible offsetting of the effects of various coefficients in one equation must be taken into account. Let us consider the consumption-

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[1] This comes to the same, since the only place in the equation system where G occurs is (2.1). Hence the variations with respect to (5.3) could directly be applied to the G-coefficient in (2.1).

equation (2.1). From a comparison [1] between cases 1*a* and 1*b*, and 2*a* and 2*b*, we find that a decrease in the coefficient of $L_w + L_s$ is accompanied by twice as large an increase in the coefficient of E. Accordingly, when the coefficient of $L_w + L_s$ changes by 10% (the case treated in table 6.92), that for E must change in the opposite direction, and by 25%.[2] Similarly, a comparison of cases 4*b* and 1*b*, 6*a* and 2*a* and 6*b* and 2*b* shows that, at a given coefficient for $L_w + L_s$, an increase in the coefficient for p is accompanied by 2.5 times as large a fall in the coefficient for E; hence a 10% decrease in the latter is to be compared with a 100% increase in the former. The effects of these *combined variations* are shown below:

Coefficient of:	Change	Effect on: Damping ratio	Period
$L_w + L_s$	— 10%	0.147	— 0.56
E	+ 25%	— 0.083	0.45
Combined effect		0.064	— 0.11
E	— 10%	0.033	— 0.18
p	+ 100%	0.072	0.27
Combined effect		0.105	0.09

It will be seen that, whereas the effects of compensatory changes in the coefficients of $L_w + L_s$ and E cancel out to a great extent, this is not the case for the damping ratio when the coefficients of E and p (or of $L_w + L^s$ and p) are varied in this way.

To give an idea of the extent to which uncertainty with regard to the final equation is due to equation (2.1), the damping ratio and the period of the latter have been calculated for what would seem to be two extreme cases. The first (I) is case 2*a*,[3] the other (II) is derived from a comparison of cases 6*b* and 2*b*, but with a fixed coefficient of 0.30 for p (which means a price elasticity of demand for consumers' goods of $\frac{1}{2}$)[4]. The

[1] *Cf.* page 37. The figures in other cases are disturbed by the inclusion of other series, which lead to multicollinearity.
[2] $2 \times 10 \times 0.95 \div 0.77$. Changes in the other coefficients are for a moment left out of account.
[3] *Cf.* page 37.
[4] *Cf.* equation (1.10).

coefficients for the various variables and the corresponding damping ratio and period run as follows:

	Coefficient for:					Damping ratio	Period (years)
	$L_w + L_s$	E	G	D_p^f	p		
Case chosen . .	0.95	0.77	0.28	0.049	0.03	1.89	4.8
Extreme I . . .	1.00	0.75	0.27	0.046	—	1.82	4.7
Extreme II . . .	0.80	0.41	0.29	0.061	0.30	2.78	5.7

It follows that the errors which may be present in the consumption equation chosen would tend to cause too low a damping ratio and too short a period, rather than the opposite.

To be able to estimate the total probable error in the damping ratio and the period, we should know:

(i) The probable error in all elementary coefficients;

(ii) The degree of (positive or negative) interdependence between the probable errors of the coefficients within each equation;

(iii) The derivative of the damping ratio and the period with respect to all elementary coefficients.

Each of these three requirements is only partly fulfilled in the present investigation. Hence we cannot estimate the exact amount of the probable error in the final results. To arrive, however, at a figure from which an impression of the order of magnitude of these errors may be obtained, the error in the damping ratio and the period is calculated on these assumptions:

(i) The seventeen coefficients mentioned in table 6.92 (page 173) have each a standard error of $\pm$ 10% of the value of the coefficient;

(ii) These standard errors are independent;

(iii) The other coefficients are free of error.

On these assumptions we find:

for the damping ratio, 1.89 $\pm$ 0.32;
for the period, 4.8 $\pm$ 0.7.