INTRODUCTION

Multiple correlation method.

If one tries to understand the causation of business-cycle phenomena, one is almost invariably led to questions of the type: why did a given economic phenomenon — say, investment activity — fluctuate as it did? Most economists would agree that, generally, a number of "causes" are present, which may all be formulated as changes in some other phenomena. A fall in investment activity may be caused by a fall in profits, or an increase in interest rates, or a change in confidence, and so on. There is less unanimity about the relative strength of these causes in various circumstances. In attempting to find evidence on this relative strength, economic reasoning may be helped and completed by statistical analysis. The ordinary elementary methods of statistical analysis are, however, sufficient for this task only in special cases. One of these ordinary methods consists in looking for months or quarters or years in which only one of the assumed causes has shown a large change, the others remaining about constant. This is, however, a very uncommon case which seldom occurs. Another elementary method is the splitting-up of figures into partial figures — say, general investment activity into investment activity in special branches. The applicability of this method of course depends very much on the statistical material available. But, even apart from that, the splitting-up of the material, however useful, is not sufficient in a considerable number of circumstances. It very often happens that two or more causes are at work even in every subdivision of a phenomenon. In such circumstances, this method is clearly insufficient.

In addition to these elementary methods, a more advanced one — the method of multiple correlation analysis — is available which enables the investigator to find out, in a number of cases, the relative strength of various influences working on the same variable. If, for example, economic reasoning suggests that the fluctuations in variable v (investment activity) depend on fluctuations in the "explanatory" variables Z (profits), q (price of investment goods), m (interest rate) and l (wage rate), then by this method the so-called regression coefficients φ_1, φ_2, etc., can be found by which the variables Z, q, m, etc., have to be multiplied in order to get an expression $v^* = \varphi_1 Z + \varphi_2 q + \varphi_3 m + \varphi_4 l$,[1] the fluctuations of which come as near as possible to those of v. These coefficients can be determined only approximately, their accuracy or significance depending on a number of circumstances which we cannot enumerate now. The details of this method have been given in the preceding publication in this series[2] and need not be repeated here. Numerous results are discussed in the following chapters, and these will serve as examples of the method.

The following features may, however, be shortly recapitulated with a view to a proper understanding of our work.

1. A priori *considerations.*

The method essentially starts with *a priori* considerations about what explanatory variables are to be included. This choice must be based on economic theory or common sense. If *a priori* knowledge regarding the lags to be taken is available, these may be specified also. In many cases, for example, reactions are so quick that only lags of zero length are acceptable. If no such *a priori* knowledge is available, lags may be tried according to the same principle as coefficients — *i.e.*, by finding what lags give the highest correlation. This may be done either by trial and error — when the number of possibilities is quite small — or systematically, by introducing lagged and unlagged explanatory variables (*e.g.*, Z_{-1} and Z) and finding the regression coefficients for these two variables. The relative magnitude of the coefficients will characterise the relative importance of lagged and unlagged influences.

[1] As all variables will be expressed in terms of deviations from their average value over a certain period (in our case 1919-1932), no constant term is needed.

[2] *Statistical Testing of Business-cycle Theories*, I: *A Method and its Application to Investment Activity*, League of Nations, Geneva, 1939.

2. Lags and annual figures.

It has sometimes been doubted whether short lags can be determined at all from *annual figures*. It is maintained that quarterly or monthly figures would be better or even indispensable. No doubt the latter, when available, contain more information. Since, however, for the most relevant variables the more frequent figures are far less complete than the annual ones (*e.g.*, for profits, investment activity, consumption outlay, capital gains, etc.), and are in addition seasonal, and since some of the accidental movements have already been automatically smoothed out in the annual figures, the latter have been thought preferable. The determination of lags shorter than one year is still possible if the chief fluctuations of the series show periods materially longer than one year. For business cycles this is clearly the case. Further, it must not be overlooked that most lags are by their nature averages of distributed lags,[1] and that the use of annual figures rightly brings in — although admittedly in a rough way — an influence of more remote events. In any case, the significance of the average lags found to exist may be tested along much the same lines as that of single regression coefficients; and in many cases they are found to be significant within the limit of a few months. Only in the case of strongly curvilinear relations will the procedure become inaccurate for years of extreme values.

3. Constant regression coefficients.

Except in a few cases, the equations have been chosen linear, with coefficients that are constant in the course of time. The use of linear relations means much less loss of generality than is sometimes believed. In the case of small variations in variables (v, Z, q, m and l in our example), it can even be proved mathematically that there is no loss of generality at all.[2] In the case of bigger variations, however, it is possible to refine the method

[1] A notion introduced by Professor Irving FISHER.

[2] It is a well-known mathematical proposition that almost any function $f(Z, q, m, l)$ may be developed in a series which, for small intervals of the variables, can be reduced to a linear expression. And if the coefficient φ_1, with which Z acts on v, itself depends on a new variable x, then it follows that $\varphi_1(x)Z$ may, for small intervals of the variables, also be developed into a linear expression in x and Z.

when necessary. This may be done by introducing as new variables any functions of the explanatory variables, *e.g.*, m^2 or $\frac{l}{m}$ or $\frac{Z}{m}$, according to what the economist would expect to be the relevant combination.[1] On the other hand, it is interesting to note that there are astonishing examples [2] of good fits obtained with constant coefficients and linear equations, which suggest that this type of relation is more frequent than is often believed.

It goes without saying that any regression coefficient found for a market or a group of markets represents only an average for all individuals included, and cannot be applied to problems concerning one individual.

4. Statistical significance.

In order to test the accuracy of results, *statistical tests of significance* must be applied. These have been discussed in the preceding volume in this series, quoted above. The danger threatening the accuracy of our results is especially that of multicollinearity. The simplest form of multicollinearity consists of a high degree of parallelism between two of the explanatory series. In more complicated cases, it may consist of a high correlation between any one of these series and a combination of some others. If such a situation occurs, the separate regression coefficients cannot be determined, though certain combinations of coefficients will still be determinable.[3] The opinion is often expressed that

[1] Examples of curvilinear dependence will be found in sections 3.5, 4.4 and 4.8 of this study.

[2] *E.g.*, the relation between unemployment and marriages (1870-1913) in *Vierteljahrshefte zur Konjunkturforschung*, Sonderheft 21, Berlin, 1931 (P. Lorenz, " Der Trend "), page 18; the demand curve for beef in the Netherlands (1876-1912), in H. W. Methorst and J. Tinbergen, " Les recherches relatives à la conjoncture au Bureau Central de Statistique des Pays-Bas ", *Revue de l'Institut international de Statistique*, 1934, I, page 37; the "explanation" of interest rates in the United States before the war in Warren M. Persons, " Cyclical Fluctuations of the Ratio of Bank Loans to Deposits ", *Review of Economic Statistics* 1924 (VI), page 260; the "explanation" of world shipping freight rates from 1880 to 1911 in J. Tinbergen, " Scheepsruimte en vrachten ", *De Nederlandsche Conjunctuur*, March 1934, page 23.

[3] *Cf.* equation (2.1). It may be noted that the knowledge of such combinations is helpful only in so far as periods are analysed in which these intercorrelations are present (*cf.* Vol. I, page 32).

these cases must be frequent in business-cycle research, since all relevant variables show more or less parallel cycles.[1] In the United States, in the period studied here, this was not the case. Some of the reasons for this lack of parallelism are:

(*a*) Interest rates and some other monetary series are much influenced by gold stock fluctuations which are not at all parallel to the general cycle;

(*b*) Commodity prices seem to have come into the region of inelastic supply much more in 1920 than in 1929; they showed very high peaks in 1920, but not in 1929;

(*c*) Share prices showed the reverse behaviour: they were very high in 1929, but not in 1920.

It goes without saying that if some explanatory factor has not changed at all in the period studied, its influence cannot be determined. If it changed only slightly, its regression coefficient may be uncertain. Extrapolation of such results for large variations in the factors concerned is therefore not permitted. For problems of stabilisation, where the aim is to obtain smaller fluctuations, this does not seem to be a serious restriction.

5. Economic significance. Apart from the purely statistical tests, there are *economic tests* of the significance of the coefficients. The most important one is that of their algebraic sign, which in most cases the economist knows on *a priori* grounds. Sometimes further tests are available concerning the absolute magnitude of one coefficient or the relative magnitudes of several coefficients, occasionally even of different equations. Examples will be found in sections (2.1), (3.3), (3.4), (3.5), (4.3), (4.6).

6. Direct relations. The word "cause" has been used in the preceding paragraphs to indicate proximate causes only. This means that the economic considerations upon which the relation tested is based must be directed towards finding, as far as possible, "*direct causal relationships*". The variables in the relation must be directly connected either in the

[1] The author is indebted to Professor R. Frisch of Oslo University for a number of important remarks on this matter, some of which have been used in what follows.

minds of some persons (*e.g.*, through the reaction of the consumer to a given income and price) or by some definition (*e.g.*, value of sales equals volume times price). This is not always possible if the strictest sense of "direct" is kept to. Investment activity may be linked up directly with profit expectations, and these are hardly measurable. The next step connecting profit expectations with actual profits and some other variables may then also be included, and investment activity may be "explained" both by actual profits and by some other variables. The more, however, such combinations of successive steps can be avoided in the formulation of relations, the better. This combination may always be undertaken afterwards — in fact, it forms the very important next step in our work — but the more explicitly it is done, the better. By keeping to this principle, one obtains relations with what Professor FRISCH calls[1] the maximum degree of "*autonomy*" — *i.e.*, relations which are as little as possible affected by structural changes in departments of economic life other than the one they belong to. It is clearly the task of economic analysis to indicate the nature of those direct causal relationships.

Complete system of equations.

Returning to the example chosen as our starting-point, it will be clear that, in order to understand the mechanism of business cycles, further steps are necessary. Suppose, for example, that a successful application of multiple correlation analysis shows that the main cause of a given decrease in investment activity was a decrease of 20% in profits, we shall then want to know what caused this decrease in profits. We shall want to find an indirect, a "deeper", cause of the fall in investment activity, which at the same time is a proximate cause of profit fluctuations. This could be done by applying the same method to profits (Z) as to investment activity (v). Still further steps may be necessary: Z may depend partly on the value of total consumption (U), and U must therefore be investigated. If the method can be applied in all cases in which we are interested,

[1] In private correspondence with the author.

we get an increasing number of relations, representing the network of causal connections forming the business-cycle mechanism, with an increasing number of variables (*i.e.*, time series representing economic phenomena). If we are to understand the mechanism as a whole, we must continue this procedure until the number of relations obtained equals the number of phenomena the course of which we want to explain. We should not be able to calculate, say, n variables if we had only $n-2$ or $n-1$ relations; we need exactly n. Such a system of as many relations as there are variables to be explained may be called a *complete system.* The equations composing it may be called the *elementary equations.* The word "complete" need not be interpreted in the sense that every detail in the complicated economic organism is described. This would be an impossible task which, moreover, no business-cycle theorist has ever considered as necessary. By increasing or decreasing the number of phenomena, a more refined or a rougher picture or "model" of reality may be obtained; in this respect, the economist is at liberty to exercise his judgment. A conclusion about the character of cyclic movements is, however, possible only if the number of relations equals the number of phenomena (variables) included. (The remark may be made here that there is no separate or special variable representing "the cycle" which has to be included in the elementary relations. It is by the mechanism itself that all variables included are compelled to perform cyclic changes.)

An example. It is perhaps useful at this point to add a few remarks on the nature of a complete system of relations which has to explain business cycles. These remarks can best be made in connection with the concrete example of a very simple system.

Suppose, first, that the value, V_t, of investment goods produced during the period t depends in a linear way on profits one time period (of four months) earlier, Z_{t-1}:

$$V_t = \beta Z_{t-1} \tag{0.1}$$

Both variables are measured as deviations from some "normal", and β is a constant.

Suppose, further, that consumption outlay U_t is the total of:

(i) total wages L_t;

(ii) a term $\varepsilon_1 Z_{t-1}$, indicating that profits Z_t are only partly consumed, the marginal propensity to consume ε_1 being a constant, while there is a lag of four months;

(iii) a term $\varepsilon_2 (Z_{t-1} - Z_{t-2})$, indicating that speculative gains also influence consumption outlay. Speculative gains are supposed to be proportional to the rate of increase $Z_{t-1} - Z_{t-2}$, since share prices are assumed to be a linear function of Z_t and since a lag is again assumed to exist.[1]

We therefore get a second equation:

$$U_t = L_t + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2}) \qquad (0.2).$$

Finally, there is an equation telling how profits Z_t are calculated:

$$Z_t = U_t + V_t - L_t \qquad (0.3).$$

Dynamic features. Now the system of the three equations (0.1), (0.2) and (0.3) is a "dynamic" system in FRISCH's sense, since, in some of the relations ((0.1) and (0.2)), variables appear relating to different time periods. If this were not so — *i.e.*, if all lags were zero and the "speculative" term in (0.2) did not exist — no endogenous cycles could occur. In fact, in such circumstances, the system would be:

$$V_t = \beta Z_t \qquad (0.1').$$

$$U_t = L_t + \varepsilon_1 Z_t \qquad (0.2').$$

$$Z_t = U_t + V_t - L_t, \qquad (0.3').$$

which, after substitution of (0.1′) and (0.2′) in (0.3), gives the equation:

$$Z_t = (L_t + \varepsilon_1 Z_t) + \beta Z_t - L_t$$

or:

$$Z_t (1 - \varepsilon_1 - \beta) = 0 \qquad (0.4').$$

[1] Since it is only an example we are giving here, details need not be discussed. By comparison with our results in the following chapters, it will be found, however, that in many respects our assumptions are near to reality.

Since ε_1 and β are constants and $\varepsilon_1 + \beta \neq 1$, the only solution is $Z_t = 0$, meaning that the system always shows the same value of profits (Z_t being the deviation of profits from some "normal") and, through (0.1′) and (0.2′), of V_t and $U_t - L_t$ also. No cycles would occur unless the extra-economic "data" determining the "normal" levels showed cycles.

It is quite different, however, in the case of the "dynamic" system (0.1), (0.2), (0.3). The simple structure of the equations still easily permits a substitution of (0.1) and (0.2) in (0.3), leading to a final equation:

$$Z_t = (\beta + \varepsilon_1)\, Z_{t-1} + \varepsilon_2\, (Z_{t-1} - Z_{t-2}),$$

which may be written:

$$Z_t = (\beta + \varepsilon_1 + \varepsilon_2)\, Z_{t-1} - \varepsilon_2 Z_{t-2}.$$

Realistic values for β, ε_1 and ε_2 being 0.2, 0.4 and 1, respectively, we get:

$$Z_t = 1.6\, Z_{t-1} - Z_{t-2} \qquad (0.4).$$

Determinants of the movements.

This equation is of quite a different type from (0.4′). It enables us to calculate Z_t once we are given the values for Z_{t-1} and Z_{t-2}. But then, knowing Z_t and Z_{t-1}, we are again able to calculate Z_{t+1}, and so on. The following table is an example, where Z_0 and Z_1 have been chosen as 0 and + 5 respectively:

t =	0	1	2	3	4	5	6	7	8	9	10
Z_t =	0	+5	+8	+7.8	+4.5	−0.6	−5.5	−8.2	−7.6	−4	+1.2

The movements we find for Z_t appear to be cyclic. It can easily be ascertained that the actual movement depends on two sorts of given numbers:

(i) the "initial" values of Z_t, in our case Z_0 and Z_1;

(ii) the coefficients of the final equation (0.4), in our case $(\beta + \varepsilon_1 + \varepsilon_2)$ and $-\varepsilon_2$.

The initial values more or less represent what are usually called disturbances from equilibrium; and the coefficients the structure of society. A change in consumption habits would affect ε_1 and ε_2; a change in investment attitude would change β, as would changes in the relative importance of, for example, investment and consumption as a consequence of technical progress. It should be added that the coefficients may also be changed as a consequence of policy, and the problem of finding the best stabilising policy would consist in finding such values for the coefficients as would damp down the movements as much as possible. This will be attained, for example, by small values for the coefficients. *Thc outstanding importance of the numerical values of the coefficients may be clear from these few considerations.* In fact, it seems difficult to prove by pure reasoning alone — *i.e.*, without knowing anything about the numerical values of the coefficients — whether or not any given theory explains or does not explain cyclic movements. This may be demonstrated by two further numerical examples:

Example A: $\beta = 0.6$, $\varepsilon_1 = 0.8$, $\varepsilon_2 = 1$.
Final equation: $Z_t = 2.4\, Z_{t-1} - Z_{t-2}$.

The type of movement found for any initial value of Z_0 and Z_1 is non-cyclic, with values of Z at an increasing distance from the original values.

Example B: $\beta = 0.2$, $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.1$.
Final equation: $Z_t = 0.9\, Z_{t-1} - 0.1\, Z_{t-2}$.

The type of movement is non-cyclic, with a tendency to return to values $Z_t = 0$ after a short time.

No theory is therefore determinate unless the values of the coefficients in a complete system of equations describing it are known, at least approximately.

Testing of theories.

How, then, can business-cycle theories be tested statistically with the aid of the technique just described? The procedure consists of at least two stages: First, the explanation that a given theory provides for each of the variables of the economic system may be tested by the method of multiple correlation analysis, and secondly, it may be tested whether the system of numerical values found for the "direct causal relations" (or what comes nearest to them) really yields a cyclic movement when used in the final equation.

This may be clarified by indicating the two ways in which an unfavourable result for any theory may be found. First, it is possible that the explanation given for the fluctuations of any of the variables might prove to be poor; and, secondly, it might happen that, although these explanations were not too bad, the combination of the elementary equations would not lead to a cyclical movement.

Apart from these two ways in which a theory may fail, there is the third — already mentioned above — that the theory may prove to be incomplete — *i.e.*, that it contains less relations than variables to be explained — or indeterminate, in that it does not indicate from what other variables each variable depends and in what way.

Strictly speaking, there are very few, if any, "literary" theories that are complete and determinate in the above sense. Most of them — as will be seen from Professor HABERLER's study — emphasise some special relations, often without dealing with most of the others. Practically no single theory can therefore be used for a joint explanation of all the variables included in this statistical study. Nevertheless, many of these "literary" theories may prove highly useful in that they throw light on one detail or a number of details which are indispensable for a right understanding of the business-cycle phenomenon. They must, however, be combined, as Professor HABERLER also points out, and the most efficient way would seem to be to combine all theories open to statistical testing and to test them by means of the system of relations just described.

The present publication is one of the first attempts to construct such a complete system on a statistical basis.[1] The

[1] In recent times, a number of models of the sort discussed have been constructed (*e.g.*, by AMOROSO, CHAIT, FRISCH, KALECKI, LUNDBERG, Roos and others); but they have not been based on statistically tested relations, except in part. Models based on statistically determined relations are to be found in J. TINBERGEN, *An Econometric Approach to Business-cycle Problems*, Paris, 1937, and in E. A. RADICE, "A Dynamic Scheme for the British Trade Cycle, 1929-1937", *Econometrica*, January 1939.

character of the work involved is necessarily twofold. It is chiefly of a statistical nature, and to that extent consists in finding the quantitative importance of the chief factors causing fluctuations in each of the variables studied. Carrying out this task presupposes, however, that economic theory — or perhaps several competing economic theories — has indicated what the chief factors are. In this field much still remains to be done. The indispensable minimum of this work which is required in order to make the statistical part of the enquiry possible at all has also been included in this report. This may have led, at some points, to a choice which would not be approved by all economists. Clearly, this cannot be avoided, and the only excuse is that all details of the analysis have been indicated exactly.

Plan of the work.

The first five chapters will contain the description and justification of the relations assumed, and tested statistically, between the phenomena considered as important. In order to treat the matter systematically, these relations are, as far as possible, subdivided into four types, well known in economic theory:

Definitional relations;
Demand equations;
Supply equations;
Income formation equations.

Some relations of another type will be added at suitable places.

Chapters VI and VII are concerned with the resolution of the complete system of equations and the conclusions which can be drawn therefrom, in respect of certain theories and of general characteristics of the business cycle.

The period studied is 1919-1932.