Simple heuristics for push and pull remanufacturing policies

Erwin A. van der Laan*
Fac. of Business Administration
Erasmus University Rotterdam
PO Box 1738
Burg. Oudlaan 50, 3000 DR Rotterdam
The Netherlands
tel/fax: +31-1-4082763/9010
e-mail: elaan@fbk.eur.nl

Ruud H. Teunter
Econometric institute
Erasmus University Rotterdam
PO Box 1738
Burg. Oudlaan 50, 3000 DR Rotterdam
The Netherlands
tel/fax: +31-1-4081523/9162
e-mail: teunter@few.eur.nl

August 4, 2004

Econometric Institute Report EI 2004-34

Abstract

Inventory policies for joint remanufacturing and manufacturing have recently received much attention. Most efforts, though, were related to (optimal) policy structures and numerical optimization, rather than closed form expressions for calculating near optimal policy parameters. The focus of this paper is on the latter. We analyze an inventory system with unit product returns and demands where remanufacturing is the cheaper alternative for manufacturing. Manufacturing is also needed, however, since there are less returns than demands. The cost structure consists of setup costs, holding costs, and backorder costs. Manufacturing and remanufacturing orders have non-zero lead times. To control the system we use certain extensions of the familiar (s, Q) policy, called push and pull remanufacturing policies. For all policies we present simple, closed form formulae for approximating the optimal policy parameters under a cost minimization objective. In an extensive numerical study we show that the proposed formulae lead to near-optimal policy parameters.

Keywords: Inventory control, remanufacturing, heuristics.

*corresponding author

1 Introduction

Environmental considerations, government regulations, and economic incentives motivate many businesses to engage in recovery activities (Fleischmann [4]). Recycling of materials is a well-known example. In recent years, more and more companies have initiated *value-added* recovery operations such as remanufacturing. Remanufacturing brings a product or product part up to an 'as-new' quality. Since remanufacturing is often cheaper than manufacturing, this type of recovery can lead to considerable cost savings. Items that are remanufactured nowadays include machine tools, medical instruments, copiers, automobile parts, computers, office furniture, mass transit, aircraft, aviation equipment, telephone equipment and tires (see Ayres *et al.* [1], Ferrer [3] [2], Graedel and Allenby [6], Guide [7], Heyman [8], Kandebo [10], Lund [12], Schrady [15], Sivinski and Meegan [18], Sprow [19], and Thierry *et al.* [22]).

A typical example of remanufacturing operations that need to be coordinated with manufacturing operations is the management of spare car parts. Volkswagen, for instance, retrieves and remanufactures used car parts and resells them as spare parts (Van der Laan [23]). Remanufactured parts come with the same quality and warranty as new parts, but are produced for less than halve of the cost. The availability of remanufacturable parts varies through time, so occasionally the stocks of serviceable spare parts have to be replenished by (expensive) newly manufactured parts. Both types are sold for the same price, though.

Figure 1 gives a graphical representation of the inventory system in the above practical situation. Note that since remanufactured items have the same quality as manufactured items and are sold for the same price in the same market, we do not need to distinguish between the two. Both types are serviceable and are used to satisfy the same customer demands. Clearly, in order to control such a system efficiently, manufacturing and remanufacturing decisions have to be coordinated.

INSERT FIGURE 1

There has been a considerable number of contributions dealing with inventory control for joint manufacturing and remanufacturing. Reviews are provided by Fleischmann *et al.* [5] and, more recent, by Van der Laan *et al.* [25]. However, only a few authors have proposed heuristic procedures for approximating optimal policy parameters.

In an early account, Simpson [17] studies a periodic review inventory system with general demand and return processes. The remanufacturing process is not modelled explicitly, but the probability density function for the remanufacturing output per time unit is assumed to be fixed and known. Manufacturing lead times are stochastic. The inventory policy is characterized by the order-up-to level for manufacturing. Under both a service objective (minimize holding costs under a fill rate constraint) and a cost objective (minimize holding and backorder costs), Simpson derives a simple newsboy equation for the order-up-to level.

Mahadevan *et al.* [13] study a slightly modified version of Simpson's model. Here, returns are remanufactured only at the time of review. The remanufacturing lead time is non-zero, but constant as is the manufacturing lead time. The analysis is restricted to cases with Poisson demand and return processes. Three heuristics for determining the order-up-to level are proposed and tested.

Muckstadt & Isaac [14] model the remanufacturing process explicitly as a queueing system with Poisson arrivals (the returns) and general service times. Manufacturing orders are triggered by a standard (s, Q) policy (reorder Q when inventory position drops to s). The objective is to minimize the average total cost, consisting of manufacturing setup costs, holding costs for serviceable inventory, and backorder costs. Using Markov Chain analysis and approximating the distribution of net inventory with a normal distribution, closed-form formulae for the order level and order quantity are derived.

Van der Laan *et al.* [24] extend the model of Muckstadt & Isaac by including a disposal option for returned items. A returned item is disposed of if there is already a certain number (the dispose-down-to level) of other returns waiting to be remanufactured. They propose an iterative algorithm for calculating a near-optimal order level, order-up-to level, and dispose-down-to level.

The above approaches have one thing in common: the remanufacturing process is autonomous and is not controlled through an inventory policy. In contrast, Kiesmüller and Minner [11] study a discrete time, continuous review inventory system with constant lead times for manufacturing and remanufacturing in which both processes are controlled by order-up-to policies. The objective is to minimize the total average costs, including holding costs and backorder costs, but no fixed costs. Closed-form formulae for the order-up-to levels are derived for cases where (1) lead times are equal, (2) the manufacturing lead time is larger, and (3) the remanufacturing lead time is larger.

We present a heuristical approach for general demand and return processes. The remanufacturing process and remanufacturable inventory are explicitly modeled. Lead times for (re-)manufacturing are equal, non-zero constants. Aside from holding costs, backorder costs and set-up costs for manufacturing, we also include set-up costs for remanufacturing. In order to control such an intricate system, we consider more versatile inventory control policies than the ones mentioned above. Multiple types of policies are analyzed. All use the same (s, Q) type policy for manufacturing, but they differ in the way that they push or pull remanufacturing orders. For all policies we present simple, closed form formulae for approximating the optimal policy parameters under a cost minimization objective. In an extensive numerical study we show that the proposed formulae lead to near-optimal policy parameters.

The remainder of the paper is organized as follows. In Section 2, the inventory system and the policies are described in detail. The formulae for approximating the optimal order levels and order quantities for the push and pull policies are developed in Sections 3, 4, and 4.3, respectively, and tested numerically in Section 5. We end with a summary and conclusions in Section 6.

2 System, policies, and notation

The inventory system is as depicted in Figure 1. Manufacturing and remanufacturing have the same lead time distribution with mean L. Demand and return are driven by independent continuous stochastic processes with (average) rates λ and γ, respectively. The density function and distribution function of demand during the lead time are denoted by f_D and F_D, respectively.

The objective is to minimize the average total cost over an infinite planning horizon, that is, to minimize the average total steady state cost. The following costs are incurred.

- *Manufacturing cost*: There is a set-up cost K_m for each manufacturing order.
- *Remanufacturing cost*: There is a set-up cost K_r for each remanufacturing order.
- *Backorder cost:* There is a fixed cost b for each backordered demand, i.e. for each demand not met directly.
- *Holding cost*: There are holding costs h_n and h_s per item per time unit for (returned) non-serviceable and (manufactured/remanufactured) serviceable items, respectively.

To control the inventory system we consider three policies. All use the same (s, Q) control for manufacturing, but the policies differ in how remanufacturing orders are triggered.

The PUSH policy is defined is follows. Whenever the stock of returned items reaches Q_r, those items are remanufactured. Whenever the serviceable inventory position (inventory on hand + everything on order) drops to s_m, a batch of size Q_m is manufactured. This is illustrated in Figure 2.

INSERT FIGURE 2

A disadvantage of the PUSH policy is that it can cause large stocks of serviceable items, especially in situations with high return rates where periods with more returns than demands can occur. Pull policies prevent large stocks by using an order level for remanufacturing. We consider two types of pull policies. The *simple PULL policy* uses the same order level for manufacturing and remanufacturing. The *general PULL policy* allows different order levels.

The simple PULL policy is defined as follows. Whenever the serviceable inventory position drops to the common order level s, a batch of size Q_r is remanufactured if enough returned items are available and a batch of size Q_m is manufactured otherwise. This is illustrated in Figure 3.

INSERT FIGURE 3

The general PULL policy is defined in a similar way, but uses separate order levels s_m for manufacturing and s_r for remanufacturing. See Figure 4.

INSERT FIGURE 4

The added flexibility of using different order levels sometimes leads to a reduction in the average total cost, as we will see in Section 5. On the other hand, applying a policy with separate order levels is more complex. Furthermore, the choice of order levels is not unrestricted. The condition $s_m \leq s_r \leq s_m + Q_m$ is needed to ensure that the remanufacturing order level can actually be reached (see Teunter *et al.* [21] for a detailed discussion). Push and pull policies were first proposed and analyzed by Van der Laan *et al.* [23], and some variations have since been studied by a number of authors.

Table 1 lists the notations that are used in the remainder of the paper.

INSERT TABLE 1

3 Heuristics for push control

A stochastic analysis of the system at hand is rather complicated, since we have to deal with a two-dimensional state space (inventory position and remanufacturable inventory) that are mutually dependent. Since the classic EOQ formula has proved to be very robust in stochastic settings, we propose a similar approach and analyse a deterministic model in order to approximate the optimal order quantities (Section 3.1). Given these order quantities, in Section 3.2, we approximate the optimal order level in the original stochastic setting.

3.1 Approximately optimal order quantities

In order to derive EOQ formulae for Q_r and Q_m, we consider a deterministic system with *continuous* demand and return flows with rates λ and γ, respectively. The derivation is not as straightforward as that of the traditional EOQ without returns since, as we will show, different patterns of manufacturing and remanufacturing batches lead to different formulae. We will consider two distinct patterns, (Case A) one manufacturing batch is followed by multiple remanufacturing batches and (Case B) one remanufacturing batch is followed by multiple manufacturing batches. Though other patterns can also occur due to the stochasticity in demand and return, these are most likely since there are either more remanufacturing batches or more manufacturing batches on average. For both cases A and B, we will derive a pair of EOQ formulae. For practical reasons, rather than to propose conditions under which either pair should be used, we then combine the two pairs into a single one that is to be used in all situations.

Case A and Case B are illustrated in Figures 5 and 6, respectively, for $L = 0$. Note that the lead time does not influence the optimization in any way. Note further that in order to obtain a stationary policy, manufacturing and remanufacturing are synchronized so that either the time $T_m = Q_m/(\lambda - \gamma)$ between two successive manufacturing batches is an integer multiple of the average time $T_r = Q_r/\gamma$ between two successive remanufacturing batches (Case A), or T_r is an integer multiple of T_m (Case B). Moreover, the serviceable inventory is minimized by letting it drop to 0 just before a manufacturing batch starts, and also just before a remanufacturing batch that is followed by a manufacturing batch starts.

INSERT FIGURES 5 AND 6

Case A: T_m is an integer multiple of T_r

By adding some help-lines to Figure 5, as is done in Figure 7, it is easy to see that the average serviceable inventory is

$$\frac{Q_m + Q_r}{2} - \frac{Q_m(Q_r/\lambda)}{T_m} = \frac{Q_m + Q_r}{2} - \frac{Q_m(Q_r/\lambda)}{Q_m/(\lambda-\gamma)} = \frac{Q_m + Q_r}{2} - \frac{Q_r(\lambda-\gamma)}{\lambda}.$$

INSERT FIGURE 7

So the total average inventory plus ordering costs costs as a function of Q_r and Q_m reads

$$TC^{\mathrm{A}}(Q_m, Q_r) = \frac{(\lambda-\gamma)K_m}{Q_m} + \frac{\gamma K_r}{Q_r} + h_s\left(\frac{Q_m + Q_r}{2} - \frac{Q_r(\lambda-\gamma)}{\lambda}\right) + h_r\left(\frac{Q_r}{2}\right).$$

Though this expression is simple, the restriction that $T_m = Q_m/(\lambda-\gamma)$ is an integer multiple of $T_r = Q_r/\gamma$ prevents an easy determination of the optimal values for Q_m and Q_r. However, the cost expression does hold approximately if the restriction is not satisfied. Since our objective is to find simple heuristics, we will therefore ignore the restriction. Minimizing $TC^{\mathrm{A}}(Q_m, Q_r)$ then gives the following EOQ-type formulae for Q_m and Q_r.

$$Q_m^{\mathrm{A}} = \sqrt{\frac{2K_m(\lambda-\gamma)}{h_s}} \quad , \quad Q_r^{\mathrm{A}} = \sqrt{\frac{2K_r\gamma}{h_r - h_s + (2\gamma/\lambda)h_s}}. \tag{1}$$

Note that Q_m^{A} reduces to the traditional EOQ formula if $\gamma = 0$ (manufacturing only), and that Q_r^{A} reduces to the traditional EOQ formula if $\gamma = 1$ (remanufacturing only).

Case B: T_r is an integer multiple of T_m

It is easy to show (see Teunter, 2001) that the total average inventory plus ordering costs costs as a function of Q_r and Q_m reads

$$TC^{\mathrm{B}}(Q_m, Q_r) = \frac{(\lambda-\gamma)K_m}{Q_m} + \frac{\gamma K_r}{Q_r} + h_s\left[\left(1-\frac{\gamma}{\lambda}\right)\left(\frac{Q_m}{2}\right) + \left(\frac{\gamma}{\lambda}\right)\left(\frac{Q_r}{2}\right)\right] + h_r\left(\frac{Q_r}{2}\right).$$

Similar to Case A, we ignore the restriction that $T_r = Q_r/\gamma$ is an integer multiple of $T_m = Q_m/(\lambda-\gamma)$, and determine Q_m and Q_r that minimize $TC^{\mathrm{B}}(Q_m, Q_r)$. This gives the following EOQ-type formulae.

$$Q_m^{\mathrm{B}} = \sqrt{\frac{2K_m\lambda}{h_s}} \quad , \quad Q_r^{\mathrm{B}} = \sqrt{\frac{2K_r\gamma}{h_s\gamma/\lambda + h_r}} \tag{2}$$

Again, the EOQ formulae for manufacturing and remanufacturing reduce to the traditional EOQ formula if $\gamma = 0$ or $\gamma = \lambda$, respectively.

One set of approximate formulae for Cases A and B

The preferred type of policy and the associated pair of EOQ formulae (associated with Case A or Case B) can be determined by comparing $TC^{\mathrm{A}}(Q_m^{\mathrm{A}}, Q_r^{\mathrm{A}})$ with $TC^{\mathrm{B}}(Q_m^{\mathrm{B}}, Q_r^{\mathrm{B}})$. It is simpler and therefore more practical, however, to work with a single pair of EOQ-formulae without having to calculate the associated costs. Moreover, as will be discussed in Section 3.2, in a real life setting with stochastic demand and return, for which the EOQ formulae are intended, cases A and B cannot be distinguished anyway. In the remainder of this section we will therefore propose a single pair of approximate EOQ formulae, based on the above results, to be used in all situations. The arguments that we use are similar to those in Teunter [20].

It is easy to see that Q_m^{A} and Q_m^{B} are approximately equal if the return rate γ/λ is small. They only differ considerably if the return rate is large. In situations with a high return rate, policies with multiple remanufacturing batches per manufacturing batch (Case A) are often preferable. Therefore, we propose to use the EOQ formula for push remanufacturing from Case A in all situations.

$$Q_m^* := Q_m^{\mathrm{A}} = \sqrt{\frac{2K_m(\lambda-\gamma)}{h_s}}$$

Similarly, Q_r^{A} and Q_r^{B} are approximately equal if the return rate γ/λ is large. They only differ considerably if the return rate is small. In situations with a small return rate, policies with multiple manufacturing batches per remanufacturing batch (Case B) are often preferable.

Therefore, we propose to use the EOQ formula for push remanufacturing from Case B in all situations.

$$Q_r^* := Q_r^{\mathrm{B}} = \sqrt{\frac{2K_r\gamma}{h_s\gamma/\lambda + h_r}}$$

3.2 Approximately optimal order level

In the previous section, we proposed a pair of EOQ formulae for determining batch quantities for manufacturing and remanufacturing. In this section we assume that the batch quantities are fixed to their EOQ values and focus on the manufacturing order level s. It is clear that this should be done in a stochastic setting, i.e., with a stochastic demand and return process. See Figure 2 and note that the time between manufacturing batches and the time between remanufacturing batches is no longer constant due to the stochastic nature of the demands and returns. The derivation of the optimal s does not depend on the actual pattern of remanufacturing batches. Therefore, it is not necessary to consider two different cases, as was done in Section 3.1. In fact, the time between successive (re)manufacturing batches varies, and therefore such cases cannot be distinguished.

In order to determine the optimal s, we use the following two approximations.

A1 The expected on-hand inventory is approximately equal to the expected net inventory.

A2 The probability that a remanufacturing batch arrives too late is negligible.

A1 is a standard approximation for deriving the optimal order level (see e.g. Silver *et al.* [16]) in systems without product returns. Obviously, the approximation works best if the average backorder position is small. If A2 holds then there are no backorders outstanding at the time that a remanufacturing order comes in. For the backorder position we only have to focus on the incoming manufacturing orders. The justification for approximation A2 is that the inventory position is always (well) above s when a remanufacturing batch is started (see Figure 2).

Similar to Silver *et al.* [16], it is easy to see that the expected net inventory is equal to the sum of the safety stock SS (the expected inventory just before a manufacturing order arrives), which depends on s, and the expected cycle stock CS, which is a constant given Q_m and Q_r. Since all orders that arrive within leadtime L are contained in the inventory position , the safety stock is just given as the inventory position at the time of a manufacturing order (s) minus the demand during the lead time: $SS = s - L\lambda$.

Under assumption A2, the average number of backorders per time unit is approximately

$$\frac{1}{T_m}\int_s^\infty (x-s)f(x)\mathrm{d}x \tag{3}$$

Hence, under assumptions A1 and A2, the total cost per time unit is approximated as

$$h_s\left(s - L\lambda + CS\right) + \frac{b}{T_m}\int_s^\infty (x-s)f(x)\mathrm{d}x$$

Taking the derivative w.r.t. s and equating the result to zero gives

$$h_s - \frac{b}{T_m}(1-F(s)) = 0.$$

So the optimal value for s is approximately

$$s^* = F^{-1}\left(1 - \left(\frac{h_s}{b}\right)\left(\frac{Q_m}{\lambda-\gamma}\right)\right) \tag{4}$$

Note that for $\gamma = 0$ this reduces to the formula that Silver *et al.* [16] derive for the classical model without returns.

4 Heuristics for pull control

As was done for push control in the previous section, we first approximate the optimal order quantities under the assumption that demand and return are deterministic (Section 4.1) and then approximate the optimal order level in a stochastic setting (Section 4.2).

4.1 Approximately optimal order quantities

The analysis is similar to that for push control in Section 3.1. We consider a deterministic system with *continuous* demand and return flows with rates λ and γ, respectively. Since the lead time and the setting of the reorder levels do not affect the optimization, we can just consider $L = 0$ and do not need to distinguish between the simple and general PULL policies.

Again case A (T_m is an integer multiple of T_r) and case B (T_r is an integer multiple of T_m) are considered, which are illustrated in Figures 8 and 9, respectively. Note that the inventories in Figure 9 are identical to those in Figure 6, but the interpretation is slightly different (push vs pull).

INSERT FIGURES 8 AND 9

Case A: T_m is an integer multiple of T_r

The total average inventory plus ordering costs costs as a function of Q_r and Q_m reads (see Schrady [15])

$$
\begin{aligned}
& TC^{\mathrm{A}}(Q_m, Q_r) \\
= \; & \frac{(\lambda-\gamma)K_m}{Q_m} + \frac{\gamma K_r}{Q_r} + h_s\left(\left(1-\frac{\gamma}{\lambda}\right)\frac{Q_m}{2} + \left(\frac{\gamma}{\lambda}\right)\frac{Q_r}{2}\right) + h_r\left(\frac{\gamma}{\lambda}\right)\left(\frac{Q_m+Q_r}{2}\right).
\end{aligned} \tag{5}
$$

Optimizing for Q_m and Q_r leads to the following EOQ-type formulae.

$$
Q_m^{\mathrm{A}} = \sqrt{\frac{2K_m(\lambda-\gamma)}{\gamma/\lambda h_r + (1-\gamma/\lambda)h_s}} \quad , \quad Q_r^{\mathrm{A}} = \sqrt{\frac{2K_r\gamma}{h_r+h_s}}. \tag{6}
$$

Case B: T_r is an integer multiple of T_m

As remarked before, the cycle inventories for this case are the same as for case B with push control. Therefore, the optimal ordering sizes are equivalent to the ones for push control:

$$
Q_m^{\mathrm{B}} = \sqrt{\frac{2K_m\lambda}{h_s}} \quad , \quad Q_r^{\mathrm{B}} = \sqrt{\frac{2K_r\gamma}{h_r+h_s\gamma/\lambda}}. \tag{7}
$$

One set of approximate formulae for Cases A and B

For practicality, we prefer a single set of EOQ formulae that can be used in all situations. Using similar arguments as in Section 3.1, we choose

$$
Q_m^* := Q_m^{\mathrm{A}} = \sqrt{\frac{2K_m(\lambda-\gamma)}{\gamma/\lambda h_r + (1-\gamma/\lambda)h_s}} \quad \text{and} \quad Q_r^* := Q_r^{\mathrm{B}} = \sqrt{\frac{2K_r\gamma}{h_r+h_s\gamma/\lambda}}\,.
$$

4.2 Approximately optimal order level for simple PULL

The analysis is similar to that in Section 3.2. But since remanufacturing batches are pulled now, we no longer assume that the probability that a remanufacturing batch arrives too late is negligible (A2). Indeed, remanufacturing batches are just as likely to be late as manufacturing batches (see also Figure 3). The expected number of backorders per time unit is obtained by dividing the backorder position per (re)manufacturing cycle, $\int_s^\infty (x-s)f(x)\mathrm{d}x$, by the expected length of a (re)manufacturing cycle. Approximating the expected on-hand inventory by the expected net inventory (A1), we get the following approximate total cost:

$$
h_s\left(s - L\lambda + CS\right) + b\left(\frac{1}{T_m}+\frac{1}{T_r}\right)\int_s^\infty (x-s)f(x)\mathrm{d}x,
$$

where the cycle stock CS does not depend on order level s. Taking the derivative w.r.t. s and equating the result to zero gives

$$h_s - b\left(\frac{1}{T_m} + \frac{1}{T_r}\right)(1 - F(s)) = 0.$$

So the optimal value for s is approximately

$$s^* = F^{-1}\left(1 - \frac{h_s}{b\left(\frac{\lambda-\gamma}{Q_m} + \frac{\gamma}{Q_r}\right)}\right). \tag{8}$$

Note that for $\gamma = 0$ this again reduces to the formula that Silver *et al.* [16] derive for the classical model without returns.

4.3 Approximately optimal order level for general PULL

The average net inventory in a period between two successive times that the inventory position crosses level s_r is (see also Figure 4) $s_m - L\lambda + Q_m/2$ if a manufacturing order is placed in that period and $s_r - L\lambda + Q_r/2$ if a remanufacturing order is placed. Approximating the expected on-hand inventory by the expected net inventory as before (approximation A1), the expected total cost becomes

$$\frac{\lambda-\gamma}{\lambda}h(s_m - L\lambda + \frac{Q_m}{2}) + \frac{\gamma}{\lambda}h(s_r - L\lambda + \frac{Q_r}{2}) + b\frac{\lambda-\gamma}{Q_m}\int_{s_m}^{\infty}(x - s_m)f(x)\mathrm{d}x + b\frac{\gamma}{Q_r}\int_{s_r}^{\infty}(x - s_r)f(x)\mathrm{d}x.$$

This leads to the approximations

$$s_m^* = F^{-1}\left(1 - \frac{h_s Q_m}{b\lambda}\right), \qquad s_r^* = F^{-1}\left(1 - \frac{h_s Q_r}{b\lambda}\right). \tag{9}$$

Note that these are both applications of the traditional formula that Silver *et al.* [16] derive for the approximately optimal order level.

Recall from Section 2 that the heuristic general PULL policy can only be implemented if $s_m^* \leq s_r^* \leq s_m^* + Q_m^*$. If this condition is not satisfied, then the simple PULL policy should be used instead.

5 Numerical evaluation

The above analysis has lead to the following heuristic expressions.

policy	Q_m^*	Q_r^*	$1-F(s_m^*)$	$1-F(s_r^*)$
PUSH	$\sqrt{\frac{2K_m(\lambda-\gamma)}{h_s}}$	$\sqrt{\frac{2K_r\gamma}{h_r\gamma/\lambda+h_s}}$	$\frac{h_sQ_m}{b(\lambda-\gamma)}$	–
simple PULL	$\sqrt{\frac{2K_m(\lambda-\gamma)}{\gamma/\lambda h_r+(1-\gamma/\lambda)h_s}}$	$\sqrt{\frac{2K_r\gamma}{h_r\gamma/\lambda+h_s}}$	$\frac{h_s}{b\left(\frac{\lambda-\gamma}{Q_m}+\frac{\gamma}{Q_r}\right)}$	$\frac{h_s}{b\left(\frac{\lambda-\gamma}{Q_m}+\frac{\gamma}{Q_r}\right)}$
general PULL†	$\sqrt{\frac{2K_m(\lambda-\gamma)}{\gamma/\lambda h_r+(1-\gamma/\lambda)h_s}}$	$\sqrt{\frac{2K_r\gamma}{h_r\gamma/\lambda+h_s}}$	$\frac{h_sQ_m}{b\lambda}$	$\frac{h_sQ_r}{b\lambda}$

† provided that $s_m^* \leq s_r^* \leq s_m^* + Q_m^*$.

Studying the table above we observe the following similarities and differences among the heuristics.

- The order quantity for remanufacturing is the same for all push and pull policies.

- The manufacturing order quantity for PUSH does not depend on the remanufacturable holding cost rate, in contrast with the manufacturing order quantities of both pull policies.

- With respect to the order quantities there is no distinction between the simple and general PULL heuristic.

- The reorder levels differ for all three policies

For the numerical evaluation of the various heuristics we generate a large set of scenarios by varying six parameters according to the following values: $\gamma \in \{3, 5, 7\}$, $L \in \{2, 4, 6\}$, $h_r \in \{0, .5, 1\}$, $b \in \{10, 50, 100\}$, $K_m \in \{10, 30, 100\}$, $K_r \in \{10, 30, 100\}$. These values can be interpreted relative to those of λ and h_s, which remain constant at the values of 10 and 1, respectively, for all scenarios. We employ a full factorial design, which results in $3^6 = 729$ scenarios. Since each heuristic is tested in each scenario, the total number of experiments is 2187. For all experiments the long run average costs are evaluated by analyzing an appropriate Markov chain along the lines of Van der Laan et al. (1999). These costs are compared to the costs associated with the *optimal* policy parameter settings obtained through a search of the decision variables in an appropriate subspace.

We use the relative error with respect to the optimum as an evaluation criterium. For example, the relative error in the long run average cost is defined as follows.

$$\textbf{Relative cost error} = \left(\frac{\text{long run average costs of heuristic}}{\text{optimal long run average costs}} - 1\right) \times 100\%$$

Figure 10 summarizes the results for all scenarios together. Each vertical line displays the minimum, maximum, and average relative error observed for the costs and policy parameters.

INSERT FIGURE 10

On average, the relative cost error for each heuristic is well below 1.5%, even below 1% for the pull policies. The PUSH heuristic has a worst case error of 18.4%, while the pull heuristics always score lower than 2.6%. We may conclude that the pull heuristics show an excellent performance both with respect to average and worst case behavior. The poorer performance of the push heuristic can be explained by looking at the relative errors in the policy parameters. While the order levels are 'unbiased' in the sense that the average error is close to 0% for all three heuristics, the maximum error is larger for the push heuristic. Moreover, the largest errors for the PUSH heuristic are negative, implying that the push heuristic underestimates the optimal order levels. The cost function is more sensitive to an underestimation than to an overestimation of the order level, since the backorder cost is large relative to the holding cost rate.

Figure 10 also shows that each heuristic always underestimates the order quantities. This is expected, since in calculating the order quantities the heuristic ignores the (expected) backorder cost. It does not take into account that increasing the order quantity leads to less orders and therefore less backorder cost per time unit.

Decreasing the backorder cost, b, results in an increased expected number of backorders in the optimal policy. This deteriorates the performance of all the heuristics in two ways. First, the order quantities are further from optimal, since backorder costs are ignored in their determination. Second, the error in the order level is generally larger, since the backorder position is ignored in the calculation of the on-hand inventory. The combined result of these two effects is that the heuristics perform best for large b (Figure 11a). The PUSH policy is particularly sensitive with respect to b, since approximation A2 is less appropriate if b is small.

INSERT FIGURE 11

Remark: The PUSH policy's large worst case peak for $b = 10$ makes it hard to do a (graphical) comparison among the different policies. Therefore, in Figure 11c–g we leave out the scenarios related to $b = 10$. While this eliminates the outliers, it hardly affects the analysis.

Like decreasing the backorder cost, increasing the lead time results in an increased expected number of backorders for the optimal policy. So, this deteriorates the performance of all the heuristics in the same way as for changes in the backorder cost (Figure 11b). Again, the PUSH heuristic is the more sensitive to these changes. Note that dropping the scenarios with $b = 10$ dramatically improves the worst case performance of PUSH: All cost errors are now below 2.5%.

The performance of the simple PULL heuristic is rather robust regarding changes in the return rate (Figure 11c). Increasing the return rate slightly decreases the performance of the other two heuristics, while the general PULL heuristic appears to be a bit more sensitive to changes than the push heuristic.

An increase in the holding cost rate for remanufacturables, h_r, hardly affects the performance of the push heuristic (Figure 11d). This is understandable, since the computation of the average remanufacturable inventory for the push heuristic is exact. This does not hold for the pull heuristics, which are clearly affected by changes in h_r.

Figures 11e–f show that the performance of the heuristics improves when the fixed manufacturing cost or remanufacturing cost increases. This explained through Figure 10, which shows that an increase in the fixed manufacturing/remanufacturing cost reduces the relative error in the manufacturing/remanufacturing order quantity.

5.1 Comparison of optimal policies

Although the main objective of this paper is to develop the push and pull heuristics and study their performance, an important issue is the comparison of the policy types. The use of a more complex policy, like general PULL, only pays off if there is a considerable reduction in cost. Figure 12 compares the cost of the optimal PUSH policy and optimal simple PULL policy to that of the optimal general PULL policy.

INSERT FIGURE 12

Since the simple PULL policy is a special case of the general PULL policy, optimal general PULL never performs worse than optimal simple PULL. Figure 12 shows that the relative cost difference is however small, never more than 3.2%. For the PUSH policy the cost difference is more than 5% higher for 250 out of the 729 scenarios and can be as large as 29.3%. There are, however, 251 cases where PUSH performs better (up to 10.8%).

6 Summary and conclusion

We analyzed an inventory system with product returns, where remanufacturing is an alternative for manufacturing and both have the same lead time. Heuristics were developed for three types of inventory policies: push, simple PULL, and general PULL. The push policy remanufactures Q_r items as soon as they are available. It manufactures Q_m whenever the serviceable inventory position drops to s_m. The simple PULL policy starts a lot if the serviceable inventory position drops to s. A batch of size Q_r is remanufactured if enough returned items are available and a batch of size Q_m is manufactured otherwise. The general PULL policy is similar, but uses separate order levels s_m for manufacturing and s_r for remanufacturing.

Following the common approach for traditional inventory systems without returns, we first developed order size formulae based on a deterministic model, and then developed order level formula(e) in a stochastic setting. The heuristics were tested in an extensive numerical study. It turns out that all three heuristics perform well, with an average cost increase (compared to the optimal policy of the same type) of less than 1.3%. The pull heuristics perform especially well, with a maximum cost increase of 2.6% (for PUSH 18.4%).

The numerical experiments also reveal that the optimal general PULL policy performs only slightly better (at most 3.2%) than the simple PULL policy, but that it can outperform the PUSH policy by up to 30%. Combined with the simple structure and excellent performance of the simple PULL heuristic, we recommend its use in practise.

A limitation of this study is the assumption that the lead times for manufacturing and remanufacturing are equal. The inventory position is then defined in the traditional way: on hand + on order - backorders. For the case that the lead times are different, it is not clear how the inventory position should be defined (see Kiesmüller and Minner [11], Inderfurth, and Van der Laan [9], and Teunter *et al.* [21]). We remark that numerical results in the latter two papers indicate that in situations with comparable lead times for manufacturing and remanufacturing (not differing by a factor more than 2), the performance of order level, order quantity policies with the traditional definition of inventory position is usually improved by adjusting the smaller lead time to the larger one. With such an adjustment, our heuristical approach can also be applied to cases with different lead times for manufacturing and remanufacturing as long as they do not differ too much. However, more research into this issue is needed. If lead times differ by a factor more than 2, then policies with separate order levels for manufacturing and remanufacturing, such as the general PULL policy, seem more appropriate. Further research is needed for these situations with unequal lead times. That research should also address the complicated issue of how to define the inventory position.

Other directions for future research are to consider stochastic lead times and to consider a service objective instead of a cost objective. Similar to the model with deterministic lead times and a cost objective that we studied, heuristical results/arguments for the traditional model without returns can be extended to the situation with product recovery.

References

[1] R. Ayres, G. Ferrer, and T. van Leynselee. Eco-efficiency, asset recovery and remanufacturing. *European Management Journal*, 15(5):557–574, 1997.

[2] G. Ferrer. The economics of personal computer remanufacturing. *Resources, Conservation and Recycling*, 21:79–108, 1997.

[3] G. Ferrer. The economics of tire remanufacturing. *Resources, Conservation and Recycling*, 19:221–255, 1997.

[4] M. Fleischmann. *Quantitative models for reverse logistics*. PhD thesis, Erasmus University Rotterdam, the Netherlands, 2000.

[5] M. Fleischmann, J.M. Bloemhof-Ruwaard, R. Dekker, E. Van der Laan J.A.E.E. van Nunen, and L.N. van Wassenhove. Quantitative models for reverse logistics: a review. *European Journal of Operational Research*, 103:1–17, 1997.

[6] T.E. Graedel and B.R. Allensby. *Industrial Ecology*. Prentice-Hall, Englewood Cliffs, New York, 1995.

[7] V.D.R. Guide, Jr. Scheduling using drum-buffer-rope in a remanufacturing environment. *International Journal of Production Research*, 34(9):1081–1091, 1996.

[8] D.P. Heyman. Optimal disposal policies for a single-item inventory system with returns. *Naval Research Logistics Quarterly*, 24:385–405, 1977.

[9] K. Inderfurth and E. Van der Laan. Leadtimes effects and policy improvement for stochastic inventory control with remanufacturing. *Preprint No. 22, Faculty of Economics and Management, Otto-von-Guericke University of Magdeburg, Magdeburg, Germany*, 1998.

[10] S.W. Kandebo. Grumman, U.S. navy under way in f-14 remanufacturing program. *Aviation Week and Space Technology*, December 17th:44–45, 19.

[11] G.P. Kiesmüller and S. Minner. Simple expressions for finding recovery system inventory control parameter values. *Journal of the Operational Research Society*, 53:83–88, 2003.

[12] R. Lund. Remanufacturing. *Technology Review*, 87(2):18–23, 1984.

[13] B. Mahadevan, D.F. Pyke, and M. Fleischmann. Periodic review, push inventory policies for remanufacturing. *European Journal of Operational Research*, 151:536551, 2003.

[14] J. A. Muckstadt and M. H. Isaac. An analysis of single item inventory system with returns. *Naval Research Logistics Quarterly*, 28:237–254, 1981.

[15] D.A. Schrady. A deterministic inventory model for repairable items. *Naval Research Logistics Quarterly*, 14:391–398, 1967.

[16] E.A. Silver, D.F. Pyke, and R. Peterson. *Inventory Management and Production Planning and Scheduling*. John Wiley and Sons, New York, 1998.

[17] V. P. Simpson. An ordering model for recoverable stock items. *AIIE Transactions*, II(4):315–320, 1970.

[18] J.A. Sivinski and S. Meegan. Case study: Abbott labs formalized approach to remanufacturing. In *APICS Remanufacturing Seminar Proceedings*, pages 27–30, 1993.

[19] E. Sprow. The mechanics of remanufacture. *Manufacturing Engineering*, pages 38–45, 1992.

[20] R.H. Teunter. Economic ordering quantities for stochastic discounted cost inventory systems with remanufacturing. *International Journal of Logistics*, 5(2):161–175, 2002.

[21] R.H. Teunter, E.A. van der Laan, and D. Vlachos. Inventory strategies for systems with fast remanufacturing. *Journal of the Operational Research Society*, 55(5):475–484, 2004.

[22] M. Thierry, M. Salomon, J. van Nunen, and L. van Wassenhove. Strategic issues in product recovery management. *California Management Review*, 37(2):114–135, 1995.

[23] E.A. Van der Laan. *The effects of remanufacturing on inventory control.* PhD thesis, Erasmus University Rotterdam, the Netherlands, 1997.

[24] E.A. Van der Laan, R. Dekker, A. Ridder, and M. Salomon. An (s, q) inventory model with remanufacturing and disposal. *International Journal of Production Economics*, 46-47:339–350., 1996.

[25] E.A. Van der Laan, R. Kuik, G. Kiesmüller, D. Vlachos, and R. Dekker. Managing product returns. In R. Dekker, M. Fleischmann, K. Inderfurth, and L.N. Van Van Wassenhove, editors, *Reverse Logistics: Quantitative models in Closed-Loop Supply Chains.* Springer, 2004.

Acknowledgement: The research of Dr. Ruud H. Teunter has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences.

D		Demand per time unit with mean λ
R		Returns per time unit with mean γ
L		Lead time
K_m	(K_r)	Fixed manufacturing (remanufacturing) cost
h_s	(h_r)	Holding cost rate for serviceables (remanufacturables)
b		Backorder cost per occurrence
Q_m	(Q_r)	Order quantity for manufacturing (remanufacturing)
s_m	(s_r)	Order level for manufacturing (remanufacturing)
s		Common order level for manufacturing and remanufacturing (simple PULL policy)
T_m	(T_r)	Average time between two manufacturing (remanufacturing) orders
$SS(s)$		Safety stock for order level s
$CS(Q_m, Q_r)$		Expected cycle stock for order quantities Q_m and Q_r
$f(.)$		Probability density function for lead time demand
$F(.)$		Probability distribution function for lead time demand

Table 1: Notation.

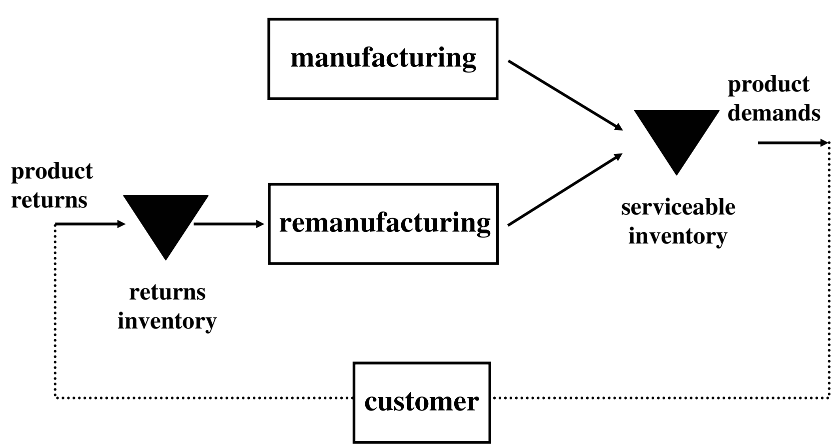

Figure 1: Inventory system with remanufacturing.

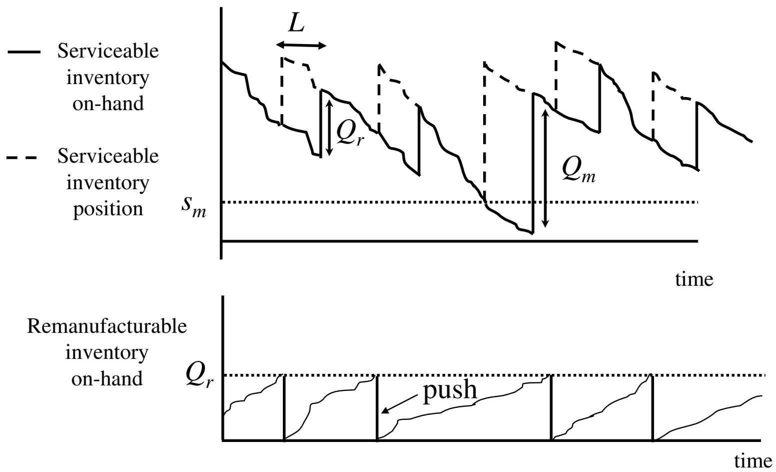

Figure 2: Inventories with push control for stochastic demand and return.

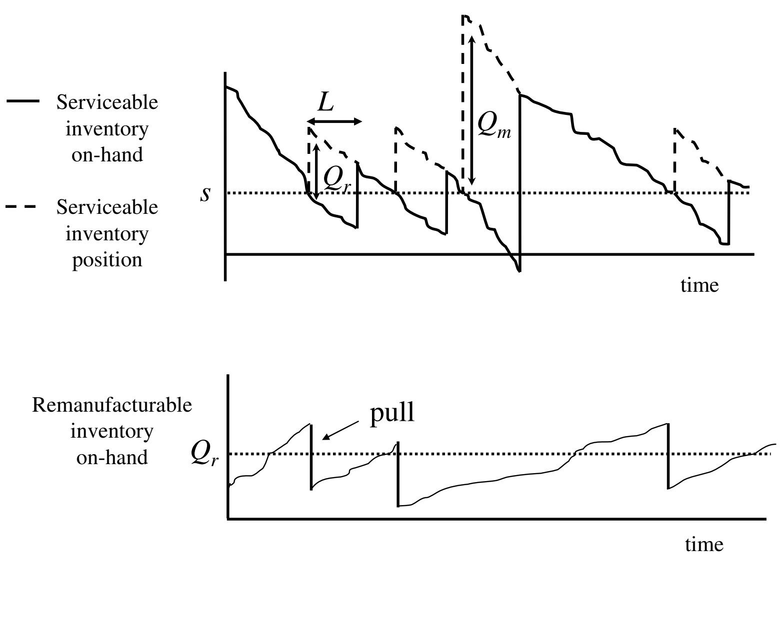

Figure 3: Inventories with simple PULL control for stochastic demand and return.

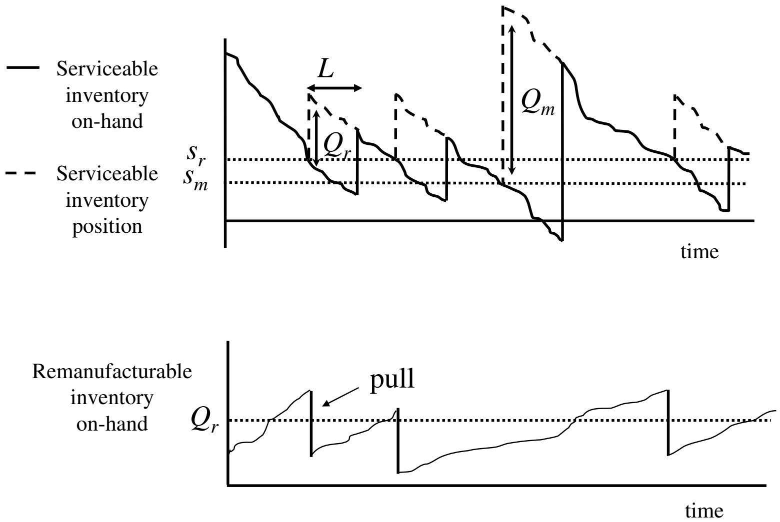

Figure 4: Inventories with general PULL control for stochastic demand and return.

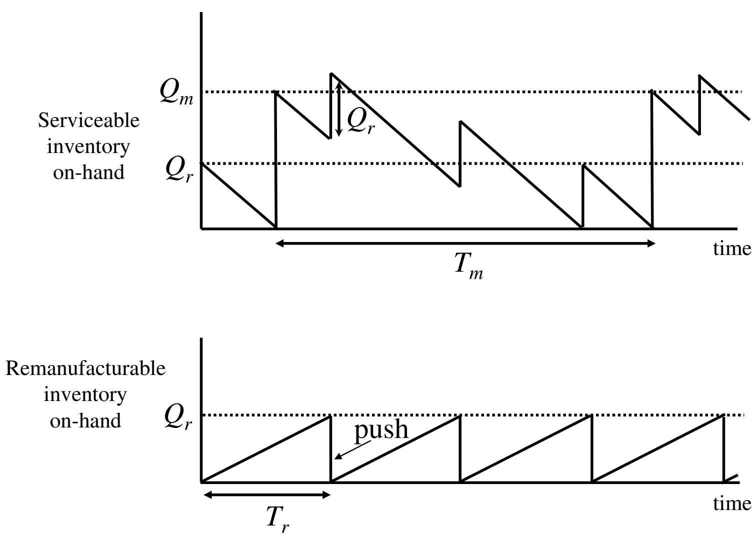

Figure 5: Inventories with push control for deterministic demand and return. Case A: T_m integer multiple of T_r.

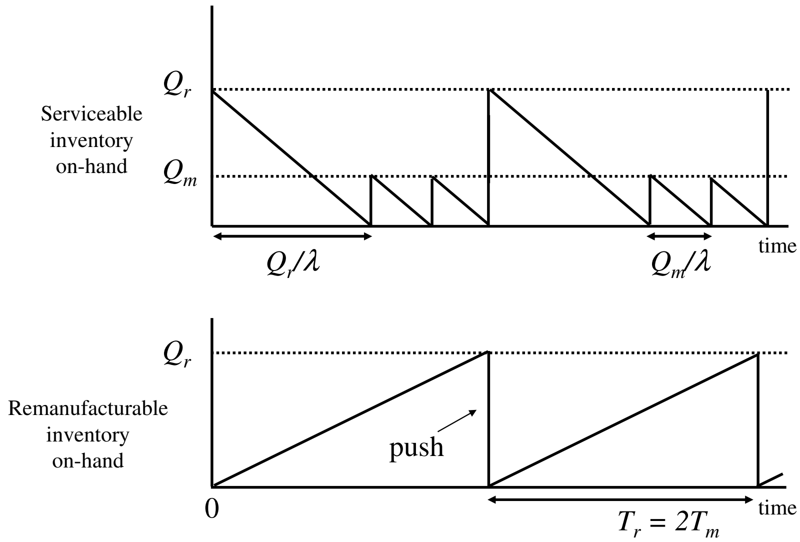

Figure 6: Inventories with push control for deterministic demand and return. Case B: T_r integer multiple of T_m.

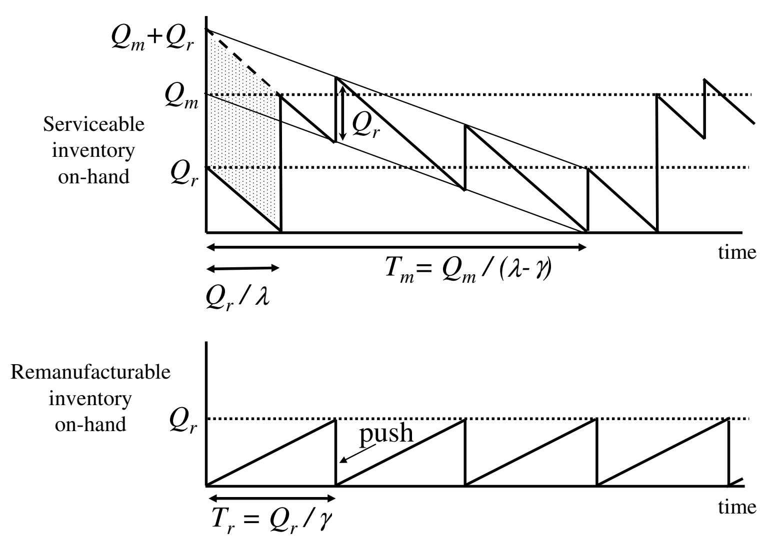

Figure 7: Inventories with push control for deterministic demand and return. Case A: T_m integer multiple of T_r. Help-lines are added for determining the average serviceable inventory.

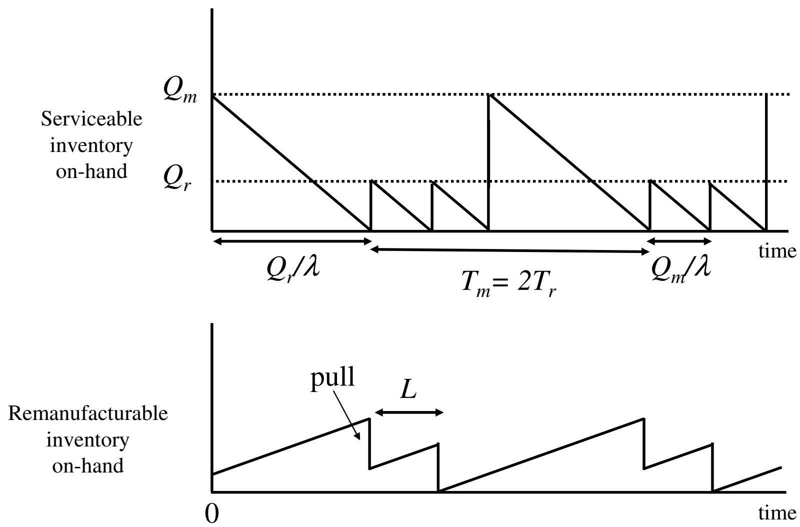

Figure 8: Inventories with pull control for deterministic demand and return. Case A: T_m integer multiple of T_r.

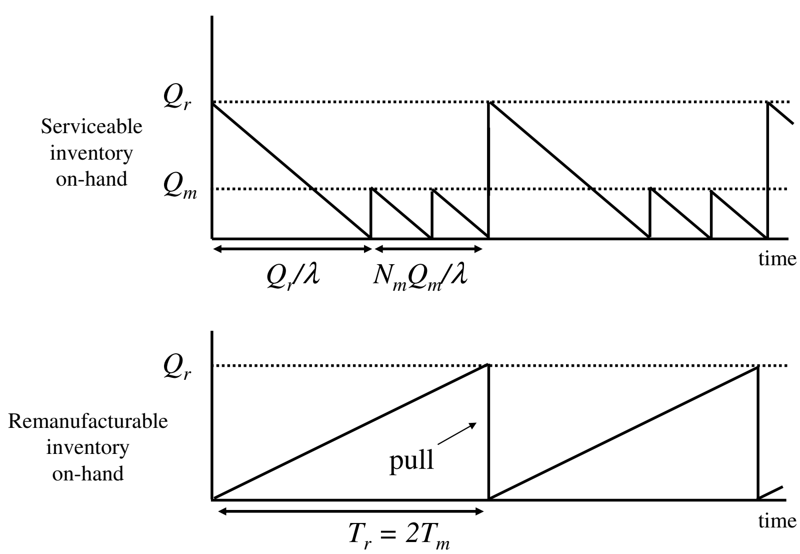

Figure 9: Inventories with pull control for deterministic demand and return. Case B: T_r integer multiple of T_m.

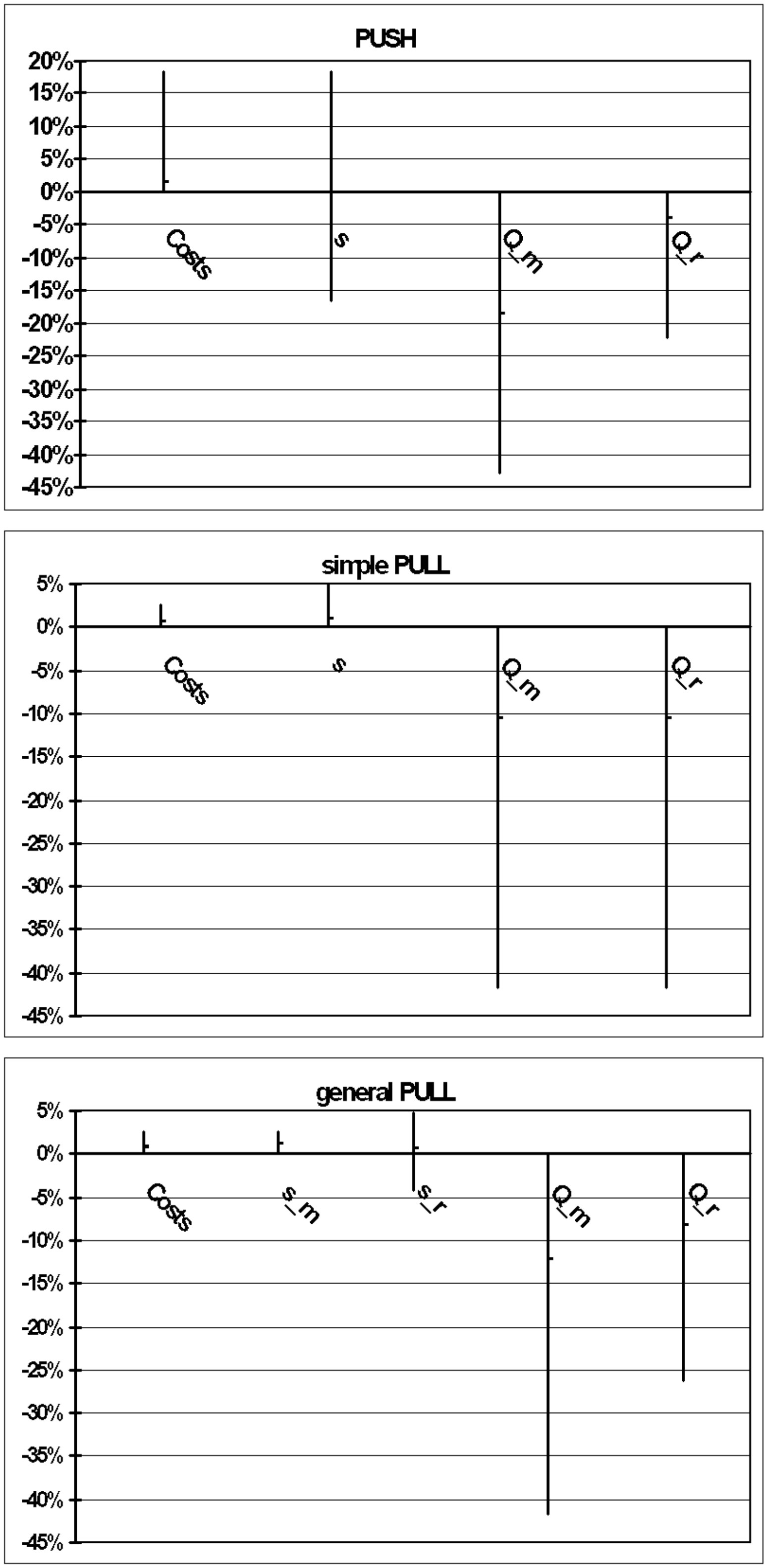

Figure 10: Policy performance over all scenarios.

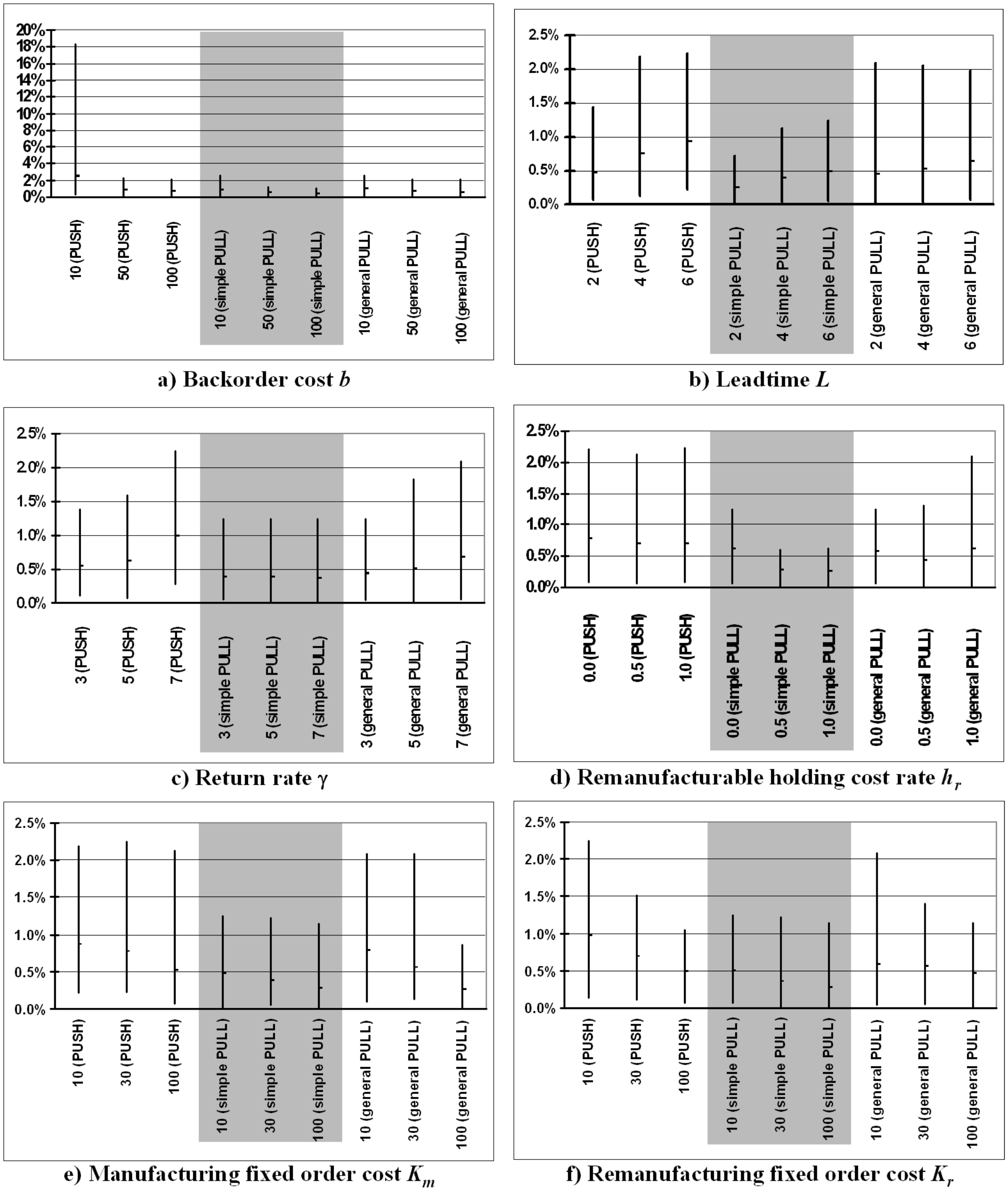

Figure 11: Model parameter sensitivity analysis.

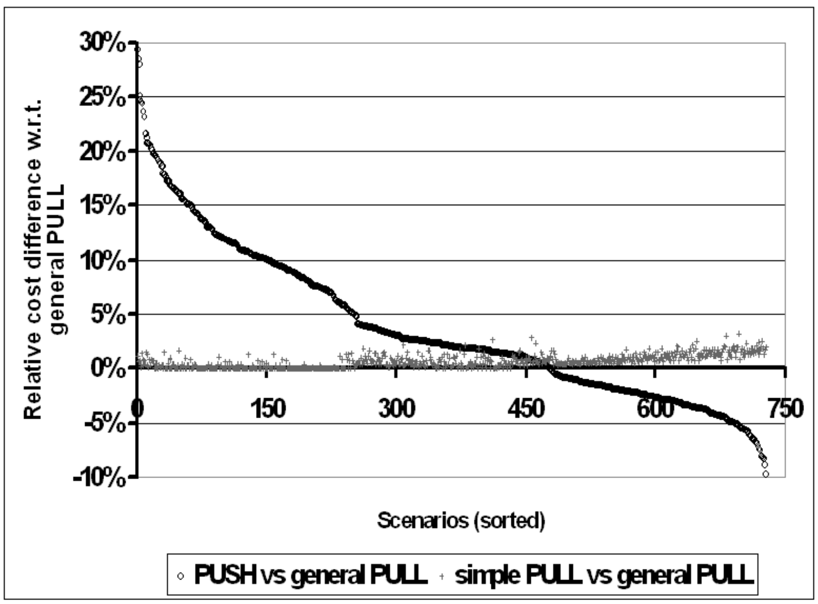

Figure 12: Cost comparison of the optimal PUSH, simple PULL, and general PULL policies.