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A Seasonal Periodic Long Memory Model for Monthly River Flows

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Abstract

Based on simple time series plots and periodic sample autocorrelations, we document that monthly river flow data display long memory, in addition to pronounced seasonality. In fact, it appears that the long memory characteristics vary with the season. To describe these two properties jointly, we propose a seasonal periodic long memory model and fit it to the well-known Fraser river data (to be obtained from Statlib at http://lib.stat.cmu.edu/datasets/). We provide a statistical analysis and provide impulse response functions to show that shocks in certain months of the year have a longer lasting impact than those in other months.

Keywords

Seasonal difference, Periodic model, Long Memory, PARFIMA, SPARFIMA

1 Introduction

It is well known since the early work by Hurst on Nile data that river flows show persistent fluctuations which may be characterized by long memory. Additional to long memory, most river flow data display pronounced seasonality, both in mean and in variance.

In this paper we propose a new periodic model where long memory characteristics at yearly lags, so called seasonal long memory, vary from month to month. The model specification is motivated by examining sample periodic autocorrelation functions for monthly river flows at long yearly lags.

Lawrance and Kottegada (1977) presented an overview of early results on the statistical modeling of river flows. One of the main objectives of these modeling efforts is to develop simulation models which can be used for the design and operation of reservoirs. Brochu (1978) noticed that even modest improvements in the operation of large reservoir systems can result in multi-million dollar savings per year. The time series analysis of river flow data has remained an innovating research area. Novel statistical models for simulation, forecasting and diagnostic analysis have been introduced for river flow data and other new methods have been tried on river flow data soon after their introduction, see e.g. Lawrance and Kottegada (1977), McLeod and Hipel (1978), and Hipel and McLeod (1978ab).

Noakes, Hipel, McLeod, Jimenez and Yakowitz (1988) compared the one-year-ahead forecasting ability of ARMA models, fractional Gaussian noise models, Fractional ARMA models, Markov type models and nonparametric regression models for four yearly river flow series. These series were analyzed earlier by McLeod and Hipel (1978). They showed that it is hard to find significant differences between the models, but the simple fractional models seemed too restrictive for the four series they studied.

The evidence on the adequacy of statistical models for seasonality is much clearer. Not only the mean and variance of monthly river flows depend on the season. Other characteristics like skewness and autocorrelation do as well, as shown by Moss and Bryson (1974). Moreover, on average, monthly lag 1 correlations tend to be larger than yearly lag 1 correlations.

The skewness is usually taken care of by an a priori log transformation of the series. The seasonally dependent autocorrelations are successfully modeled using periodic autoregressive moving average (PARMA) models, see e.g. Vecchia and Ballerini (1991).

Periodic autoregressive (PAR) models have definite computational advantages over PARMA models. PAR models are easy to identify using periodic partial autocorrelation functions, and they are easy to estimate using least squares, using Yule-Walker equations, or by Maximum Likelihood, see e.g. McLeod (1994). PAR models for monthly riverflow modeling and simulation were originally introduced by Thomas and Fiering (1962) .

Noakes, McLeod and Hipel (1985) compared the short-term forecasting ability of seasonal ARIMA models, deseasonalized ARMA models and periodic autoregressive (PAR) models on 30 monthly river flow series. The results clearly suggest that periodic autoregressive models, identified by the partial autocorrelation function, provided the most accurate forecasts. They also established the superiority of the natural log transformation over other Box-Cox transformations, Box and Cox (1964), in a classical likelihood framework.

Although the explicit modeling of long range dependence may not be too useful for point-forecasting, especially if the process is stationary, it can still be important for confidence interval forecasting, see e.g., Ray (1993). It also plays a decisive role in hypothesis testing and in the development of simulation models. It is, e.g., important to take account of long range dependence if one wants to do inference on the (seasonal) long run mean of a process, as emphasized by Beran (1994) in the first chapter of his monograph on "Statistics for Long-Memory Processes". Neglecting long-range dependence may result in gross downward biases in estimates of the uncertainty about the mean. This is especially relevant if one wants to test for structural stability of the correlation structure and the mean, where proper estimation of the variance of the (sub)sample means is crucial.

Beran and Terrin (1996) reanalyzed yearly minimum water levels for the river Nile (622-1281) using an ARFIMA(0,d,0) model and found significant changes in the correlation structure over time. The analysis of structural change in long geophysical time series is particularly interesting for climate research. Atkinson, Koopman and Shephard (1997) used recent structural time series for the annual flow of the Nile (1871-1970) to illustrate new tests for structural breaks and found that the process could well be described by white noise allowing for a couple of additive outliers and a structural break due to the building of the Aswan dam in 1899. MacNeill, Tang and Jandhyala (1991) surveyed earlier analyses of those Nile data.

A class of models, which seems to have been overlooked in the literature on river flow modeling is the class of seasonal long memory models, introduced by Carlin, Dempster and Jonas (1985) for economic time series. These models are used to describe long range depedence in the seasonal pattern of time series, and focus on the correlation structure at yearly intervals. Our model focuses on this aspect as well. We discuss the relationship between our model and other seasonal long memory models in more detail in section 3.1 below. In our model we combine seasonal long memory allowing for the well established periodic variation in the autocovariance function of monthly river flows. The combination of these two features may explain both the long memory apparent in yearly series of minima of river flows, the so-called Joseph effect, and the absence of long memory in aggregate yearly river flow data. In this paper we specify and estimate such a seasonal periodic model for monthly river flows.

Droughts and floods are phenomena that are typical for special seasons of the year. If we look at autocorrelations for data of a specific month we may notice the long nonperiodic cycles, whereas we overlook them if we aggregate flow data over the year. This is of course more likely to happen if the seasonal long range dependence occurs for months with relatively small flows.

There seems to be a misunderstanding among practitioners that seasonal (fractional) differencing as applied in seasonal AR(F)IMA modeling, and periodic modeling, as in PAR models, are substitutes for describing seasonal phenomena. Our application shows that they are complements. In fact, seasonal parameters can be periodic as well.

The specification of our model does not involve new statistical problems. The model can easily be estimated using existing software for ARFIMA analysis. We basically extend the periodic AR(3) model of McLeod (1994) introducing error terms which display seasonal fractional integration which varies from month to month.

For application we consider the monthly Fraser river flow data at Hope, B.C., made available on Statlib at `http://lib.stat.cmu.edu/datasets/` by Ian McLeod.

We show how we can capture the interesting long memory characteristic, which appears evident from the periodic autocorrelation functions at long yearly lags. Statistical analysis shows seasonal long memory to be significant, especially for the month of March. Our statistical analysis provides an additional test on model adequacy and can be used as a parametric complement to the residual serial correlation tests for periodic models developed by Vecchia and Ballerini (1991), McLeod (1994) and Franses (1996) and residual correlation tests for long memory models by Beran (1992) and Robinson (1994).

The outline of our paper is as follows. In section 2 we present the relevant characteristics of the monthly Fraser river flow data. In section 3 we propose the novel seasonal periodic long memory model, compare it with related models and discuss estimation issues and available software. Section 4 provides the empirical analysis and section 5 concludes.

2 Data and memory characteristics

Let y_t denote a monthly time series, $t = 1, 2, \ldots, n,$. In our case y_t concerns log-transformed data of the monthly mean river flows in cubic feet per second, following the analyses in Vecchia and Ballerini (1991) and McLeod (1994). Let $Y_{m,T}$ denote these observations, $m = 1, 2, 3, \ldots, 12$ and $T = 1, 2, \ldots, N,$ so that m denotes the number of the month and T denotes the number of the year. We have N years of subsequent observations with monthly data. To simplify notation we only use complete years with observations starting in month 1, so that $t = m+12\cdot(T-1)$. Note that the Fraser river flow data in our analysis start in January 1914, whereas Vecchia and Ballerini (1991) use index 1,1 for October 1912 and McLeod (1994) denotes March 1913 by 1,1. We use the natural logarithmic transformation. Vecchia and Ballerini (1991) seem to have used the $^{2}\log$-function. This matters for the periodic means of the series, $E(Y_{m,T})$. Our sample mean for June is 8.84, see Table 1 below, whereas Vecchia and Ballerini (1991, Table 3) obtain a value of 12.408. The basis of the log-transformation does not change the periodic variances and autocovariances which we define as

$$\gamma_{t,l,m} = \mathrm{cov}(y_t, y_{t-l}) = \gamma_{l,m}, \tag{1}$$

following McLeod (1994). Throughout we assume that y_t is periodic stationary: $\gamma_{l,m}$ depends only on the lag l between y_t and y_{t-l}, and on m, the index of the "leading" month y_t. For example, we assume that the lag 1 autocovariance for June, $\gamma_{1,6} = \mathrm{cov}(Y_{6,T}, Y_{5,T})$ does not change over time T.

McLeod (1994) presents the sample information on these covariances in scatterplots. Such scatterplots present additional evidence on the adequacy of the log-tranformation to obtain approximate (multivariate) normality. Note that Vecchia and Ballerini (1991) use periodic lead l autocovariances, indexed according to the "lagging" month $\gamma^*_{l,m}$ =cov(y_t, y_{t+l}), so $\gamma^*_{1,6}$=cov$(Y_{6,T}, Y_{7,T})$. The lead l and lag l autocovariances differ for periodic processes! Throughout we will use (1).

Figures 1 and 2 show the pronounced periodicity of the process. The scales on the vertical axes of Figure 1 show the variation in the mean and the changes in variability of the log mean river flows from month to month. The mean varies from 6.71 in March to 8.84 in June. The standard deviation varies from 0.20 in August to 0.36 in April, see also Table 1 below. One can interpret the standard deviations approximately as relative errors for the untransformed mean river flows.

Our study is motivated by inspection of the sample periodic autocorrelation function at longer lags. McLeod (1994, equations (2.1)-(2.2)) gives a definition of the sample periodic autocorrelation function. The periodic sample autcorrelation functions are used in the identification of the submodels for each month, in a way similar to model identification for nonperiodic time series models. Vecchia and Ballerini (1991) and McLeod (1994) present an extensive data analysis including time series plots of untransformed and logarithmic data, and plots of periodic sample autocorrelations at monthly lags of order 1 to 6.

The inspection of sample periodic autocorrelation functions, PeACFs, and sample periodic partial autcorrelation functions, PePACFs, led McLeod to the specification of periodic stationary AR model of order 3 in June and October, of order 2 in July and of order 1 in the other months of the year, i.e., a PAR(1,1,1,1,1,3,2,1,1,3,1,1) model. Vecchia and Ballerini (1991) specified a PARMA(1,1) model using the same orders for each month. We present the residual standard errors of the respective models in Table 1 below. McLeod (1994, Table III, last row) uses a method of indexing that is incompatible with the one used by Vecchia and Ballerini (1991), leading to huge differences in measures of fit.

The periodic sample autocorrelations tend to zero first as shown in Figure 3 of McLeod (1994), who did not show that they increase again for some months to reach local maxima at seasonal lags 12, 24 etc. The lag 1 autocorrelations are relatively small for May and June, 0.2-0.3, compared to 0.6-0.8 for the other months. Figure 2 shows that lag 12 autocorrelations, i.e. one year autocorrelations, vary from about zero for January and April to 0.3 for February and March. So there is significant periodicity in the autocorrelation function at seasonal lags as well.

For the month of March the oscillations retain a significant amplitude up to 120 months, as can be seen from Figure 2. Since we want to focus on seasonal modeling we present sample periodic autocorrelations at yearly lags in Figure 2. This analysis does not require special software for periodic analysis: the correlations in Figure 2 are simply sample autocorrelations of the monthly subseries $Y_{m,T}$ presented in Figure 1. The corresponding spectral density estimates are also presented in Figure 2 for additional interpretation in the frequency domain. We used GiveWin, Doornik and Hendry, (1996) for the computations and chose a maximum yearly lag of 25, i.e. a monthly lag of 300.

Figure 1 already shows marked differences in the "trending" behavior for the different months. February seems to display two long cycles and March appears to show an upward trend. May shows neither a trend nor a cycle. The plots do not show severe outliers.

The sample periodic autocorrelation functions and spectral densities in Figure 2 confirm the need for the extra periodic modeling at yearly lags. The standard error of the autocorrelation estimates is about 0.1 under the white noise assumption, so many autocorrelations are statistically significant. One would expect white noise to be a good approximation for all the monthly subseries if a short memory low order PAR model would apply. However, they deviate systematically from zero for some months. Moreover, the autocorrelation patterns differ substantially from month to month. The null hypothesis of white noise is clearly inappropriate.

The data for March deserve a closer look. The autocorrelation function for March seems to display the typical characteristics of a long memory process: it dies off very slowly and stays positive for high lags. A simple Dickey-Fuller test for a (yearly) autoregressive unit root would not even reject the null that the March series follows a random walk, although an augmented test does reject the unit root hypothesis, see Dickey and Fuller (1979). Dickey-Fuller tests, including the computation of their p-values, have become a standard feature of econometric software packages like PcGive. Hassler and Wolters (1995) discuss the use of Dickey-Fuller tests in the presence of long-memory processes in more detail. The spectral representation of the March autocorrelation function shows a prominent peak near zero, which may indicate the pole in the spectral density that characterizes long memory processes.

The data analysis confirms the need for more careful modeling of the periodic autocorrelations at seasonal lags, where long memory may be in order. The time series plots and autocorrelation functions show that the seasonal pattern in the monthly river flows changes persistently over time, but these changes are most marked in the winter months where a long period with large river flows is observed in the sixties and in the beginning of the seventies. In the next section we develop a statistical model to capture these characteristics.

3 A Seasonal Periodic Long Memory model

The basic idea of our model is very simple. It is a standard multiplicative (Seasonal) Autoregressive Integrated Moving Average model, (S)ARIMA $(p,0,0)\times(0,D,0)_{12}$, where the integration parameter d and the MA order for monthly lags, q are zero and where the other parameters of the model, i.e. the seasonal AR and MA parameters, the seasonal integration parameter D and the AR parameters for the monthly lags are allowed to vary from month to month. Furthermore, we allow for fractional D, so that the seasonal pattern can be periodic long memory without being periodic nonstationary, i.e., the periodic autocovariances at seasonal lags may not be absolutely summable (seasonal long range dependence), but the unconditional periodic variances of the undifferenced series still exist.

Our point of departure is a periodic AR model of order p used by McLeod (1994), i.e.

$$y_t-\mu_m-\phi_{1,m}y_{t-1}-\phi_{2,m}y_{t-2}-\cdots-\phi_{pm}y_{t-p}=\eta_t,\quad t=1,2,\ldots,n, \tag{2}$$

with η_t a white noise process. The AR parameters $\phi_{i,m}$ vary with the months, $i=1,2,\ldots,p$, $m=1,2,\ldots,12$. We condition on the starting values $y_0,y_{-1},\ldots,y_{-p}$. In order to capture the long memory characteristics we extend the model of McLeod (1994) by representing η_t as follows:

$$\eta_t=(1-L^{12})^{-D_m}\varepsilon_t, \tag{3}$$

where $(1-L^{12})^{-D_m}$ is defined by its binomial expansion

$$(1-L^{12})^{-D_m}=\sum_{k=0}^{\infty}\begin{pmatrix}-D_m\\ k\end{pmatrix}(-L^{12})^k=1+D_mL^{12}+\frac{1}{2}D_m(1+D_m)L^{24}+\ldots \tag{4}$$

involving the lag operator L, $L^ky_t=y_{t-k}$. This definition is analogous to the fractional time series differencing operator introduced by Granger and Joyeux (1980) and Hosking (1981). We assume $-\frac{1}{2}<D_m<\frac{1}{2}$. We say that η_t is integrated of order D_m in month m. The innovations ε_t have a seasonally varying variance σ_m^2.

For the statistical analysis of this model it is useful to put it in a *companion form* which explains the values of the different months $Y_{m,T}$. See Pagano (1978), Tiao and Grupe (1980) and Lütkepohl (1991) for detailed descriptions of this procedure. Recall that the time series plots in Figure 1 represent these different months. We use the following vector notation, thereby extending the notation for the periodic autoregressive model used by Ooms and Franses (1997). Equations (2)-(3) can be written as

$$A_0 Y_T = A_1 Y_{T-1} + \cdots + A_P Y_{T-P} + \mu + D(L) E_T, \quad (5)$$

with $Y_T = (Y_{1,T}, Y_{2,T}, \ldots, Y_{12,T})'$, $A_{0ii} = 1$, $A_{0ij} = 0$, $j > i$, $A_{0ij} = -\phi_{i-j,j}$, $j < i$, $A_{kij} = \phi_{i+12k-j,j}$, $i = 1, 2, \ldots, 12$, $j = 1, 2, \ldots, 12$, $k = 1, 2, \ldots, P$, $P = 1 + [(p-1)/12]$ if $p > 0$, and μ is a 12×1 vector of constants, where the expressions with index m obey the modulo-12 arithmetic, i.e. $\phi_m = \phi_{m+12k}$, $k = \ldots, -2, -1, 0, 1, 2, \ldots$ $E_T = (E_{1T}, E_{2T}, \ldots, E_{12T})'$ denotes the zero mean white noise innovations corresponding to Y_T'. $D(L)$ is a diagonal matrix $\{(1-L)^{-D_1}, \ldots, (1-L)^{-D_{12}}\}$, where the lag operator now shifts observations by a year. Note that periodic AR models up to order $p = 12$ as in (2) can be captured in a VAR model of order $P = 1$, as in our application, where $p = 3$. Note further that the constants μ_m in (2) and in (5) do not measure the monthly means of y_t, but rather the means of the monthly long memory innovations, η_t. The monthly means of y_t can easily be derived from μ and $A(L)$ under our assumption of periodic stationarity.

In contrast to Ooms and Franses (1997) we assume the roots of $\det(A_0 - A_1 z) = 0$ to lie strictly outside the unit circle, so we assume a priori there is no periodic integration. The VAR-part of the model does not lead to nonstationarity. Therefore, all long memory properties of the model are captured in $D(L)$. Given these assumptions the multivariate model is covariance stationary if $D_m < 0.5, m = 1, 2, \ldots, 12$. Covariance stationarity of $D(L)E_T$ is a necessary condition for Gaussian maximum likelihood estimation of $D_m, m = 1, \ldots, 12$.

The model is invertible, in the sense that the coefficients of the VAR(∞) representation of (5) die off to zero at long lags, if $D_m > -0.5, m = 1, 2, \ldots, 12$. Estimation using nonlinear least squares methods following Beran (1995) is a possibility for invertible cases. See Odaki (1993) for other definitions and conditions regarding invertibility in long memory models.

3.1 Relations with other seasonal long memory models,

Under the nonperiodicity restrictions $\phi_{i,m} = \phi_i$, $D_m = D$, $\sigma_m^2 = \sigma^2$, the model reduces to the seasonal fractionally differenced model applied by e.g. Porter-Hudak (1990).

$$(1 - L^{12})^D (1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p) y_t = \varepsilon_t \quad (6)$$

This model and its more flexible extensions do not seem to have been used for monthly river flows. See Ooms (1995) for a survey of other seasonal long memory models. In the econometric literature tests for the adequacy of the unit autoregressive seasonal differencing operator $(1 - L^{12})^1$ have received a lot of attention. Dickey, Hasza, and Fuller (1984) developed a likelihood ratio test against short memory alternatives like $(1 - \rho L^{12}), 0 < \rho < 1$. Hylleberg, Engle, Granger, and Yoo (1990) extended the so-called seasonal unit root tests to a test for the adequacy of the separate factors, $(1 - \exp(2\pi ik/12)L), k = 1, \ldots, 12, \; i = \sqrt{-1}$, of the operator $(1 - L^{12})$.

Our model can also be used to test the adequacy of the (unit) seasonal difference operator, in our case against fractional alternatives. There are three important differences with the seasonal unit root tests of Dickey, Hasza and Fuller (1984). First, the null and the alternative hypothesis for the integration parameters are not restricted to 1 and zero, respectively. Second, the LM, likelihood ratio and Wald tests follow similar standard chi-squared limiting distributions under the null, whereas seasonal unit root tests have non-standard asymptotic distributions that depend on the design of the regressor set of explanatory variables, even asymptocially, see Robinson (1994). Third and most important, the tests do not require the assumption of an identical dynamic model for each month of the year, an assumption that is clearly inappropriate for monthly river flows.

We do not test for the separate factors of the seasonal differencing operator in the context of our periodic models following Hylleberg et al. (1990), although this could be done in principle using standard inference in approximate maximum likelihood methods, see Ooms (1995) for an application in nonperiodic seasonal ARFIMA models.

Our model differs crucially from the periodic autoregressive fractionally integrated model, PARFIMA(p_m, d_m, q_m), suggested by Hui and Li (1995) and Franses and Ooms (1997) in the unit of the lag to which the fractional difference operator is applied. In those articles the fractional difference operator was applied to weekly and quarterly lags, respectively, corresponding to the basic time interval of the time series analyzed. Here we apply the operator to yearly lags, which is the seasonal lag for our time series. We therefore label our model SPARFIMA $(p_m, d_m, q_m) \times (P_m, D_m, Q_m)_{12}$ to distinguish it from the PARFIMA(p_m, d_m, q_m) model. As indicated at the beginning of this section the last three "uppercase" parameters deal with the seasonal lag operator, and the first three concern the lag operator at the basic frequency of the time series.

In this paper we put $P_m = Q_m = d_m = q_m = 0$. Extension of the statistical analysis and the estimation procedures to models where P_m and Q_m differ from zero is straightforward as long as $p_m < 12$. Only equation (3) changes and one simply obtains ARFIMA models for each month of the year, where the preceding months are used as explanatory variables.

Generalization to cases where d_m differs from zero is not as easy. General conditions for stationarity are not known and simulation and identification procedures still have to be developed. Estimation and forecasting has been done for simple cases however, although not for monthly river flows. Franses and Ooms (1997) and Hui and Li (1995) used nonlinear least squares on the AR(∞)-representation to estimate PARFIMA($p_m, d_m, 0$) models. The negative of the minimand of this procedure can be viewed as an approximate likelihood function following Beran (1995). Stationarity conditions on the integration parameters d_m for (PARFIMA($0, d_m, 0$) models are still a subject of research. It is, e.g., obvious that the existence of one $d_m = 0$ is a sufficient condition for short memory. Is is therefore not necessary for all the individual d_ms to be smaller than $\frac{1}{2}$ in order to get periodic stationarity. This is a marked difference with the periodic stationarity conditions for the seasonal fractional integrations parameters D_m.

3.2 Estimation and Software

Under the assumption of covariance stationarity we can easily estimate the model using Gaussian maximum likelihood for the 12 equations for the yearly data for the separate months, i.e., the 12 rows of (5). This amounts to estimating an ARFIMA($0, D, 0$) model where the mean depends on regressor variables.

The monthly lags are treated as regressors, as is customary in least squares estimation of periodic autoregressive models. Pagano (1978) showed that Yule-Walker estimates for parameters for the different periods in PAR models are asymptotically uncorrelated, so that the information matrix is block diagonal. The same property holds for the asymptotically equivalent least squares estimates. There is no reason to believe that this property does not hold for our estimates which are asymptotically equivalent to generalized least squares estimates of the parameters $\phi_{i,m}$, c.f. Dahlhaus (1995). Efficient estimation can therefore proceed equation by equation. One can also look at the set of equations (5) as an econometric dynamic simultaneous equations model. Under the restriction $D_m = 0, m = 1, \ldots, 12$, (5) reduces to an (exactly identified) recursive dynamic simultaneous equations model, for which it is well known that equation-by-equation least squares estimation is asymptotically efficient, see e.g. Spanos (1986, §25.2). Since our dynamic error specification excludes cross-equation-effects, equation-by-equation generalized least squares estimation should also be asymptotically efficient in our case where the errors are serially correlated.

We employ the ARFIMA-package of Doornik and Ooms (1996), now written in Ox 2.0 by Doornik (1998).

The package is freely available from `http://www.nuff.ox.ac.uk/Users/Doornik` for academic users. Ox is an object oriented matrix language with a large number of econometric and statistical procedures. The ARFIMA-package is a class of procedures derived from the database class available in Ox. Data can be imported from many software packages like Excel, Gauss, text-files etc. The ARFIMA-package implements the basic algorithm of Sowell (1992), with several improvements in memory use and numerical stability, which makes the package also suitable for bootstrapping exercises, see Ooms and Doornik (1998). The ARFIMA package implements also nonlinear least squares estimation. It allows for the simultaneous estimation of the coefficients of regressor variables, which makes it ideally suited for the problem at hand. Graphs can be exported in several formats like encapsulated postscript. There is a special link with GiveWin (Doornik and Hendry(1996)) which can be used to show graphs online. It also offers the possibility to export graphs in Windows metafile format to other packages which are compatible with Windows 95/98 or Windows NT. The graphs in Figures 1 and 2 are in standard GiveWin format.

Given the estimates of $\phi_{i,m}$ and D_m one can simply invert (5) to obtain periodic impulse response functions. This is easy to program in a matrix-language like Ox. These impulse response functions represent the effect of an innovation in month m_0 in year T_0 on a later month m_1 in year T_1. The impulse response function estimates can also be used to obtain approximate confidence intervals for out-of-sample forecasting. This requires a little more programming but virtually no computation time. That is an important advantage of the linearity of the model. In the next section we present estimation results for the parameters and impulse responses.

4 Empirical Results

We first check McLeod's periodic AR(3) model using the familiar diagnostic regression tests implemented in PcGive 9.0, Hendry and Doornik (1996). We simply regressed the April series on a constant and the series for March, February and January. The May series were also regressed on the three previous months and so forth. In contrast to McLeod (1994) we used the same AR order for all months. These regressions, which are not reproduced here, showed no substantial problems with the model specification at yearly lags, with regard to nonnormality and short run serial correlation. Only the March and September equations showed a little evidence of residual serial correlation with p-values of second order serial correlation tests of 0.03 and 0.04, respectively. McLeod (1994) already checked the specification for residual serial correlation at monthly lags. Testing is particularly easy because of the absence of periodic MA parameters in the model, cf. McLeod (1994). Our regression results for June and October are qualitatively similar to the Yule-Walker estimates obtained by McLeod (1994). Our regression standard errors are only marginally larger. Diagnostics for (yearly) residual serial correlation are extremely easy to compute in all leading econometric packages for the analysis of time series. Since least squares is more akin to maximum likelihood than Yule-Walker estimation, we prefer (generalized) least squares. See Brockwell and Davis (1991) for a thorough comparison of different estimation methods for ARMA-models.

Figure 2 indicated that McLeod's model should be extended with a periodic seasonal long memory part. Two long memory parameters are statistically significant when we apply Wald-tests in the SPARFIMA model. We present our equation-by-equation maximum likelihood estimation results in Table 1. It turns out that the long memory parameters for March and September are indeed significantly larger than zero, whereas the long memory parameters for the other months are not. The residual variance for March changes from 0.029 in the AR(3) to 0.025 in the long memory model. For September it decreases from 0.025 to 0.023. In terms of reducing one-step-ahead forecast error variance the progress is not so impressive. The last rows in Table 1 show comparisons in goodness-of-fit with the models of McLeod (1994) and Vecchia and Ballerini (1991). The differences between the models are much smaller than indicated by McLeod (1991, Table III). Note also that Vecchia and Ballerini used a shorter sample. The introduction of the long memory component does not influence the AR parameter estimates and their standard errors significantly. For these type of data one can therefore start model identification with the specification of the AR part, ignoring long memory, without making gross mistakes.

The AR-parameters do not have a clear structural interpretation, but one sees that the river's new year starts around May, June, where the links with river flows of the previous season are weakest. The R^2 is only about 0.10 for May and 0.25 for June. This phenomenon also showed in the periodic autocorrelation functions for low lags, clearly presented by Vecchia and Ballerini (1991). Overall, one-month-ahead predictability is rather high.

A more systematic understanding of the properties of the model can be obtained from the impulse responses, which we present in Tables 2 and 3. The impulse responses are most easily interpreted using the multivariate formulation (5) of the model. The columns in the tables present the effect of white noise innovations, affecting the system for the first time in the corresponding month. Remember that the size of the innovations is measured approximately in relative terms because of the initial log transformation of the monthly river flows. Note further that the innovations are orthogonal by construction.

Let us first look at short run effects in the upper panel of Table 2. The weakness of the links between May river flows and previous months is seen in the rows labeled May. The largest effect, 0.216, is from a one percent shock in the March river flow of the same year. The short term "pipeline" effects on other months are much larger, which is seen in the off-diagonal elements in the upper panel. The effect of June innovations seems to last longest during the first year, even the following April shows an effect of 0.133.

Seasonal effects appear in the diagonals of the following panels. The diagonal elements of the second panel correspond quite closely to the periodic seasonal fractional integration parameters D_m, c.f. (3). The AR parameters do not affect the impulse responses at lag 12 very much. Comparing the diagonals of subsequent panels, one notices the slow decay of the periodic impulse response function due to the long memory character of the model. An exponential decay would result in a short memory model, and this would not correspond to the periodic correlations observed in Figure 2.

It is seen at the bottom of Table 2 that the March and September innovations have a long lasting effect, whereas this effect is substantially smaller for the other months. Again, this corresponds to the relatively high estimates for D_m in those months. The March innovations influence future March, April and May observations in particular.

The effect of the September innovations die out more slowly within the year.

Long non-periodic cycles present in the winter precipitation and temperatures could be the cause of the March long memory innovations. These cycles have only a limited effect on the summer and autumn months. The long memory parameter estimate for September is harder to interpret. It is not as clearly identified from Figure 2 as the parameter for March. Only the first two yearly autocorrelations are positive. The September estimate might pick up short memory correlation at yearly lags and an ARIMA $(p, 0, 0) \times (P, 0, Q)$ might be more appropriate for September than the current $(3, 0, 0) \times (0, D, 0)$ specification. The development of a more specific model identification strategy is an interesting exercise, which we leave for future research.

5 Conclusion

We extended the periodic AR model developed by McLeod (1994) for the monthly data for the logs of the Fraser river flow, with a periodic seasonal long memory innovation process. Our statistical likelihood based procedure detected the presence of long non-periodic cycles which are also evident in sample periodic correlations for the data for the month of March. We estimated the model using Gaussian Maximum Likelihood month-by-month.

The linear Gaussian specification can easily be used for tests for structural breaks, for simulation and for point and (cumulative) interval forecasting. These are all important areas of research in river flow modeling, as we indicated in our introduction.

Other estimation methods could apply in different situations. If the long memory is not too pronounced and the MA-part of the model is clearly invertible one can use the PAR(∞) expansion of the model and estimate this using nonlinear least squares methods following Beran (1995), Franses and Ooms (1997) and Baillie, Chung and Tieslau (1996). In that way one could estimate other extended versions of the model allowing for GARCH-errors, non-normal innovations and so on. This could robustify the results in the presence of outliers and serially dependent innovation volatility and make the model suited for a wider range of data sets.

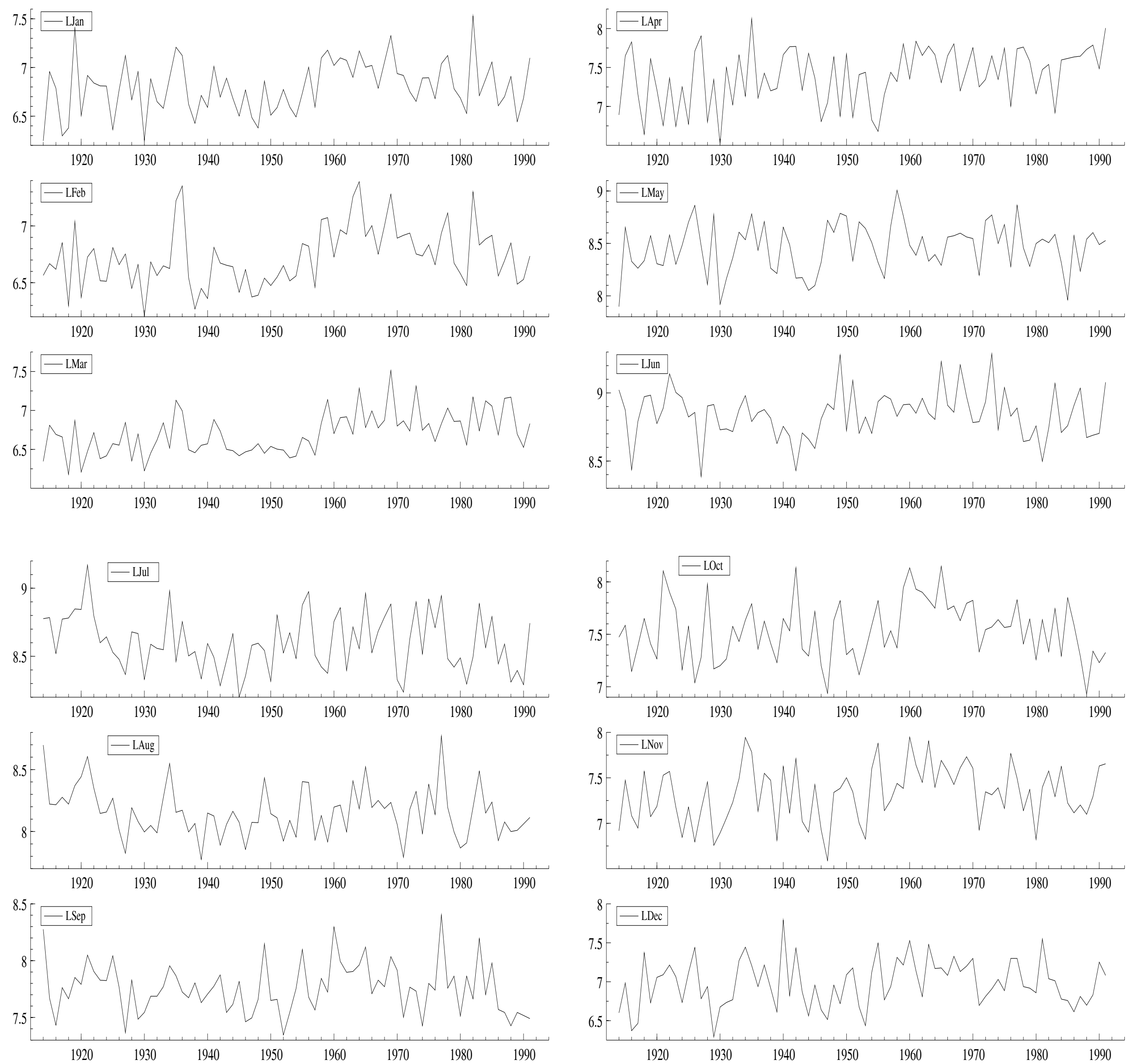

Figure 1: The Fraser River flow series 1914-1991 in each month (in logs).

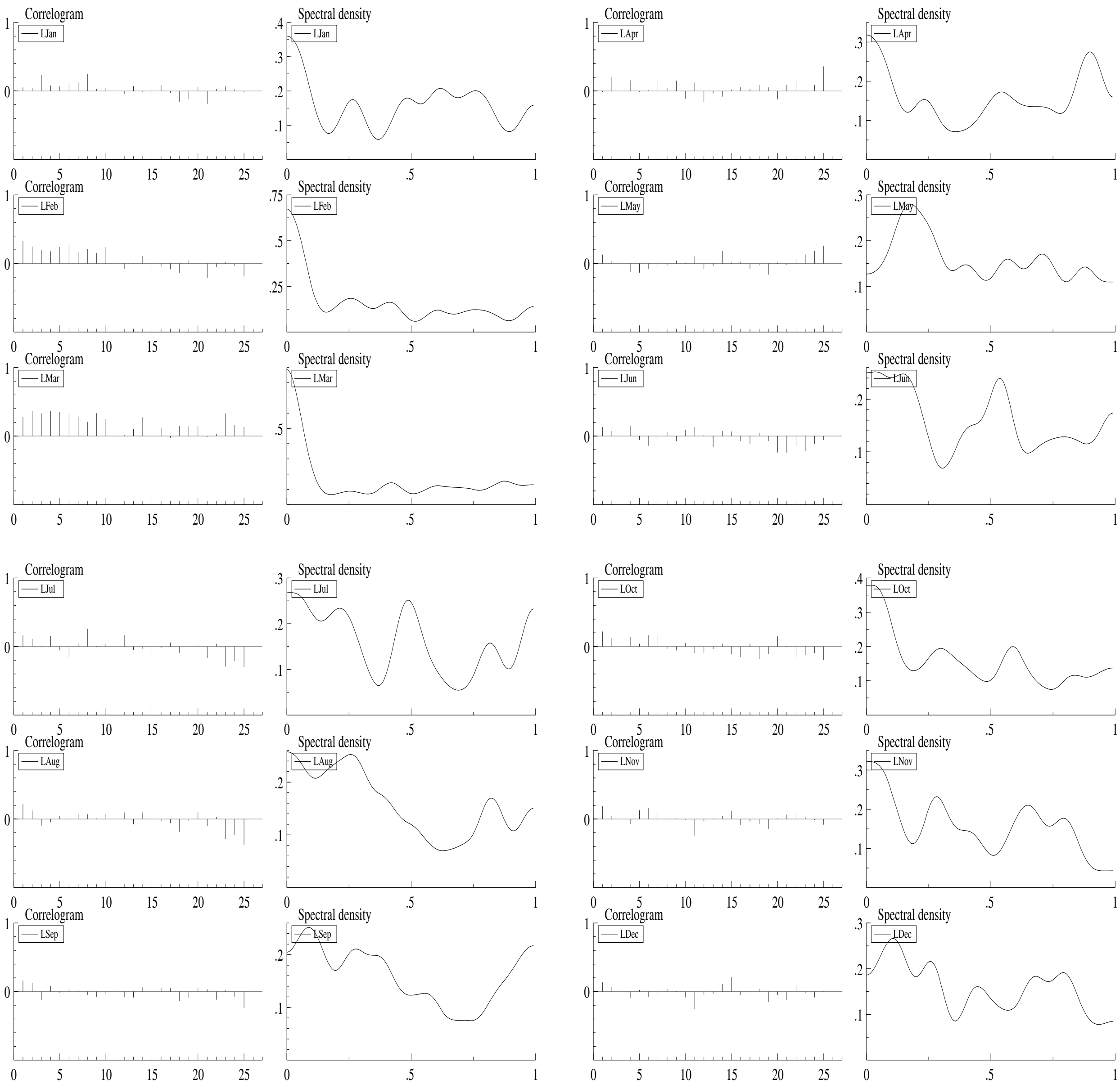

Figure 2: Sample autocorrelations functions and spectral densities for each month.

Table 1: *SPARFIMA* $(3,0,0)\times(0,D_m,0)$ *model estimates for log monthly river flows of Fraser River and Comparisons with PAR model of McLeod (1991) and PARMA models of Vecchia and Ballerini (1991)*

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
D	0.027	0.119	**0.218**	-0.210	0.138	0.031	0.160	-0.128	**0.217**	0.019	0.069	-0.005
	(.09)	(.12)	(.08)	(.11)	(.12)	(.10)	(.10)	(.14)	(.10)	(.10)	(.10)	(.13)
μ	**1.959**	**1.424**	**1.458**	**2.217**	**6.678**	**6.938**	**2.897**	1.141	1.526	0.588	0.608	0.387
	(.56)	(.58)	(.53)	(.78)	(.74)	(.75)	(1.14)	(.73)	(.94)	(1.05)	(1.13)	(.83)
ϕ_1	**0.579**	**0.780**	**0.659**	**0.965**	**0.184**	**0.299**	**0.802**	**0.765**	**0.921**	**1.169**	**0.755**	**0.701**
	(.10)	(.11)	(.12)	(.16)	(.08)	(.08)	(.12)	(.08)	(.14)	(.14)	(.13)	(.10)
ϕ_2	0.051	-0.211	0.183	**-0.550**	0.039	**-0.255**	**-0.200**	-0.023	-0.247	**-0.553**	-0.164	-0.112
	(.12)	(.11)	(.14)	(.23)	(.17)	(.06)	(.09)	(.10)	(.14)	(.22)	(.22)	(.13)
ϕ_3	0.058	**0.200**	-0.060	0.353	0.026	**0.187**	0.042	0.074	0.094	0.278	0.283	0.298
	(.10)	(.09)	(.09)	(.21)	(.15)	(.08)	(.06)	(.06)	(.12)	(.18)	(.18)	(.16)
σ^2	0.028	0.023	0.025	0.081	0.044	0.023	0.025	0.013	0.023	0.035	0.049	0.037
σ^2_M	0.036	0.026	0.037	0.088	0.047	0.023	0.025	0.016	0.025	0.036	0.052	0.038
σ^2_{VB}	0.030	0.027	0.024	0.088	0.051	0.023	0.026	0.015	0.023	0.038	0.045	0.040
mean	6.80	6.73	6.71	7.39	8.47	8.84	8.60	8.16	7.76	7.54	7.32	6.99
s.e.	0.27	0.26	0.27	0.36	0.23	0.18	0.21	0.20	0.22	0.28	0.31	0.30

NOTES: Model: $y_t - \phi_{1,m}y_{t-1} - \phi_{2,m}y_{t-2} - \phi_{3,m}y_{t-3} = \mu_m + (1-L^{12})^{-D_m}\epsilon_t$, $\epsilon_t \sim NID(0,\sigma^2_m)$. Sample: 1914.1-1991.12. Bold numbers indicate significance at the 5% level. σ^2_M: residual variances from McLeod (1994) for PAR(1,1,1,1,1,3,2,1,1,3,1,1)-model. σ^2_{VB}: residual variances from Vecchia and Ballerini from PARMA(1,1) model. Standard errors appear in parentheses. mean and s.e.: sample means and standard deviations. These correspond to the results obtained by Vecchia in Ballerini for the sample 1912/13-1983, except for their value for the mean in October (month 1 in their notation).

Table 2: *Effect of Shock in month labeled in column on month labeled in row in year* $T+i$

T+0	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	1.000											
Feb	0.780	1.000										
Mar	0.697	0.659	1.000									
Apr	0.596	0.086	0.965	1.000								
May	0.157	0.067	0.216	0.184	1.000							
Jun	0.025	0.122	0.006	-0.200	0.299	1.000						
Jul	0.014	0.088	0.002	-0.155	0.040	0.802	1.000					
Aug	0.022	0.069	0.017	-0.100	0.098	0.591	0.765	1.000				
Sep	0.019	0.054	0.016	-0.073	0.109	0.440	0.458	0.921	1.000			
Oct	0.014	0.049	0.010	-0.073	0.084	0.411	0.390	0.523	1.169	1.000		
Nov	0.014	0.048	0.010	-0.072	0.073	0.405	0.436	0.527	0.718	0.755	1.000	
Dec	0.014	0.044	0.010	-0.064	0.074	0.369	0.398	0.585	0.671	0.418	0.701	1.000

T+1	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.036	0.031	0.007	-0.045	0.052	0.258	0.275	0.396	0.492	0.338	0.457	0.579
Feb	0.028	0.143	0.005	-0.036	0.039	0.205	0.218	0.291	0.386	0.327	0.409	0.241
Mar	0.025	0.097	0.222	-0.028	0.031	0.160	0.170	0.229	0.304	0.252	0.311	0.204
Apr	0.021	0.026	0.214	-0.233	0.026	0.133	0.141	0.200	0.255	0.183	0.236	0.269
May	0.006	0.012	0.048	-0.045	0.145	0.036	0.038	0.053	0.069	0.052	0.066	0.064
Jun	0.001	0.015	0.001	0.041	0.043	0.037	0.007	0.008	0.012	0.016	0.018	-0.011
Jul	0.000	0.011	0.000	0.032	0.006	0.028	0.164	0.004	0.007	0.010	0.011	-0.011
Aug	0.001	0.009	0.004	0.020	0.015	0.023	0.128	-0.121	0.010	0.011	0.013	-0.003
Sep	0.001	0.007	0.004	0.015	0.016	0.018	0.078	-0.112	0.225	0.009	0.011	-0.001
Oct	0.000	0.006	0.002	0.015	0.012	0.016	0.066	-0.063	0.260	0.026	0.009	-0.003
Nov	0.000	0.006	0.002	0.014	0.011	0.016	0.073	-0.063	0.162	0.021	0.078	-0.003
Dec	0.000	0.006	0.002	0.013	0.011	0.015	0.067	-0.071	0.152	0.015	0.057	-0.007

T+2	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.014	0.004	0.002	0.009	0.008	0.010	0.046	-0.048	0.111	0.011	0.037	-0.004
Feb	0.011	0.070	0.001	0.007	0.006	0.008	0.037	-0.035	0.087	0.010	0.033	-0.002
Mar	0.010	0.046	0.134	0.006	0.005	0.006	0.029	-0.027	0.069	0.008	0.025	-0.002
Apr	0.008	0.008	0.129	-0.078	0.004	0.005	0.024	-0.024	0.057	0.006	0.019	-0.002
May	0.002	0.005	0.029	-0.014	0.080	0.001	0.006	-0.006	0.015	0.002	0.005	-0.001
Jun	0.000	0.008	0.001	0.017	0.024	0.016	0.001	-0.001	0.003	0.000	0.001	0.000
Jul	0.000	0.006	0.000	0.013	0.003	0.013	0.093	-0.000	0.002	0.000	0.001	0.000
Aug	0.000	0.005	0.002	0.009	0.008	0.010	0.072	-0.057	0.002	0.000	0.001	-0.000
Sep	0.000	0.004	0.002	0.006	0.009	0.007	0.043	-0.052	0.134	0.000	0.001	-0.000
Oct	0.000	0.003	0.001	0.006	0.007	0.007	0.037	-0.030	0.156	0.010	0.001	-0.000
Nov	0.000	0.003	0.001	0.006	0.006	0.007	0.041	-0.030	0.096	0.007	0.038	-0.000
Dec	0.000	0.003	0.001	0.005	0.006	0.006	0.038	-0.033	0.090	0.004	0.027	-0.002

T+3	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.010	0.002	0.001	0.004	0.004	0.004	0.026	-0.022	0.066	0.003	0.017	-0.001
Feb	0.007	0.049	0.001	0.003	0.003	0.003	0.021	-0.016	0.052	0.003	0.015	-0.001
Mar	0.007	0.032	0.099	0.002	0.002	0.003	0.016	-0.013	0.041	0.002	0.012	-0.000
Apr	0.006	0.005	0.095	-0.048	0.002	0.002	0.013	-0.011	0.034	0.002	0.009	-0.001
May	0.001	0.003	0.021	-0.009	0.057	0.001	0.004	-0.003	0.009	0.001	0.003	-0.000
Jun	0.000	0.006	0.001	0.010	0.017	0.011	0.001	-0.000	0.002	0.000	0.001	0.000
Jul	0.000	0.004	0.000	0.008	0.002	0.009	0.067	-0.000	0.001	0.000	0.000	0.000
Aug	0.000	0.003	0.002	0.005	0.006	0.006	0.052	-0.035	0.001	0.000	0.000	0.000
Sep	0.000	0.003	0.002	0.004	0.006	0.005	0.031	-0.032	0.099	0.000	0.000	0.000
Oct	0.000	0.002	0.001	0.004	0.005	0.004	0.026	-0.018	0.115	0.006	0.000	0.000
Nov	0.000	0.002	0.001	0.004	0.004	0.004	0.029	-0.019	0.071	0.005	0.026	0.000
Dec	0.000	0.002	0.001	0.003	0.004	0.004	0.027	-0.021	0.066	0.003	0.018	-0.002

T+4	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.007	0.001	0.001	0.002	0.003	0.003	0.019	-0.014	0.049	0.002	0.012	-0.001
Feb	0.006	0.038	0.001	0.002	0.002	0.002	0.015	-0.010	0.038	0.002	0.011	-0.000
Mar	0.005	0.025	0.079	0.001	0.002	0.002	0.011	-0.008	0.030	0.002	0.008	-0.000
Apr	0.004	0.004	0.076	-0.033	0.001	0.001	0.010	-0.007	0.025	0.001	0.006	-0.000
May	0.001	0.003	0.017	-0.006	0.044	0.000	0.003	-0.002	0.007	0.000	0.002	-0.000
Jun	0.000	0.004	0.000	0.007	0.013	0.008	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.003	0.000	0.005	0.002	0.007	0.053	-0.000	0.001	0.000	0.000	0.000
Aug	0.000	0.003	0.001	0.004	0.004	0.005	0.041	-0.025	0.001	0.000	0.000	0.000
Sep	0.000	0.002	0.001	0.003	0.005	0.004	0.024	-0.023	0.079	0.000	0.000	0.000
Oct	0.000	0.002	0.001	0.003	0.004	0.003	0.021	-0.013	0.092	0.005	0.000	0.000
Nov	0.000	0.002	0.001	0.003	0.003	0.003	0.023	-0.013	0.057	0.004	0.020	0.000
Dec	0.000	0.002	0.001	0.002	0.003	0.003	0.021	-0.015	0.053	0.002	0.014	-0.001

NOTE: Model: $(1-L^{12})^{D_m}[\phi_m(L)y_t-\mu_m]=\epsilon_t$.

Table 3: *Effect of Shocks in years* $T+i, i=5,\dots,9$

T+5	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.006	0.001	0.001	0.002	0.002	0.002	0.015	-0.010	0.039	0.002	0.009	-0.001
Feb	0.005	0.031	0.000	0.001	0.002	0.002	0.012	-0.007	0.031	0.002	0.008	-0.000
Mar	0.004	0.021	0.067	0.001	0.001	0.001	0.009	-0.006	0.024	0.001	0.006	-0.000
Apr	0.003	0.003	0.065	-0.025	0.001	0.001	0.008	-0.005	0.020	0.001	0.005	-0.000
May	0.001	0.002	0.014	-0.005	0.037	0.000	0.002	-0.001	0.005	0.000	0.001	-0.000
Jun	0.000	0.004	0.000	0.005	0.011	0.007	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.003	0.000	0.004	0.001	0.005	0.044	-0.000	0.001	0.000	0.000	0.000
Aug	0.000	0.002	0.001	0.003	0.004	0.004	0.034	-0.020	0.001	0.000	0.000	0.000
Sep	0.000	0.002	0.001	0.002	0.004	0.003	0.020	-0.018	0.067	0.000	0.000	0.000
Oct	0.000	0.001	0.001	0.002	0.003	0.003	0.017	-0.010	0.078	0.004	0.000	0.000
Nov	0.000	0.001	0.001	0.002	0.003	0.003	0.019	-0.010	0.048	0.003	0.016	0.000
Dec	0.000	0.001	0.001	0.002	0.003	0.002	0.018	-0.011	0.045	0.002	0.011	-0.001

T+6	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.005	0.001	0.000	0.001	0.002	0.002	0.012	-0.008	0.033	0.001	0.007	-0.001
Feb	0.004	0.026	0.000	0.001	0.001	0.001	0.010	-0.006	0.026	0.001	0.007	-0.000
Mar	0.003	0.018	0.058	0.001	0.001	0.001	0.008	-0.004	0.020	0.001	0.005	-0.000
Apr	0.003	0.003	0.056	-0.020	0.001	0.001	0.006	-0.004	0.017	0.001	0.004	-0.000
May	0.001	0.002	0.013	-0.004	0.031	0.000	0.002	-0.001	0.005	0.000	0.001	-0.000
Jun	0.000	0.003	0.000	0.004	0.009	0.005	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.002	0.000	0.003	0.001	0.004	0.038	-0.000	0.000	0.000	0.000	0.000
Aug	0.000	0.002	0.001	0.002	0.003	0.003	0.029	-0.016	0.001	0.000	0.000	0.000
Sep	0.000	0.001	0.001	0.002	0.003	0.002	0.017	-0.015	0.058	0.000	0.000	0.000
Oct	0.000	0.001	0.001	0.002	0.003	0.002	0.015	-0.008	0.068	0.003	0.000	0.000
Nov	0.000	0.001	0.001	0.002	0.002	0.002	0.017	-0.008	0.042	0.002	0.014	0.000
Dec	0.000	0.001	0.001	0.001	0.002	0.002	0.015	-0.009	0.039	0.001	0.010	-0.001

T+7	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.004	0.001	0.000	0.001	0.002	0.001	0.010	-0.006	0.029	0.001	0.006	-0.000
Feb	0.003	0.023	0.000	0.001	0.001	0.001	0.008	-0.005	0.022	0.001	0.006	-0.000
Mar	0.003	0.015	0.052	0.001	0.001	0.001	0.006	-0.004	0.018	0.001	0.004	-0.000
Apr	0.002	0.002	0.050	-0.017	0.001	0.001	0.005	-0.003	0.015	0.001	0.003	-0.000
May	0.001	0.002	0.011	-0.003	0.028	0.000	0.001	-0.001	0.004	0.000	0.001	-0.000
Jun	0.000	0.003	0.000	0.003	0.008	0.005	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.002	0.000	0.003	0.001	0.004	0.033	-0.000	0.000	0.000	0.000	0.000
Aug	0.000	0.002	0.001	0.002	0.003	0.003	0.026	-0.013	0.001	0.000	0.000	0.000
Sep	0.000	0.001	0.001	0.001	0.003	0.002	0.015	-0.012	0.052	0.000	0.000	0.000
Oct	0.000	0.001	0.001	0.001	0.002	0.002	0.013	-0.007	0.060	0.003	0.000	0.000
Nov	0.000	0.001	0.000	0.001	0.002	0.002	0.015	-0.007	0.037	0.002	0.012	0.000
Dec	0.000	0.001	0.001	0.001	0.002	0.002	0.013	-0.008	0.035	0.001	0.008	-0.001

T+8	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.004	0.001	0.000	0.001	0.001	0.001	0.009	-0.005	0.025	0.001	0.005	-0.000
Feb	0.003	0.021	0.000	0.001	0.001	0.001	0.007	-0.004	0.020	0.001	0.005	-0.000
Mar	0.003	0.014	0.047	0.000	0.001	0.001	0.006	-0.003	0.016	0.001	0.004	-0.000
Apr	0.002	0.002	0.045	-0.014	0.001	0.001	0.005	-0.003	0.013	0.001	0.003	-0.000
May	0.001	0.001	0.010	-0.003	0.025	0.000	0.001	-0.001	0.004	0.000	0.001	-0.000
Jun	0.000	0.002	0.000	0.003	0.007	0.004	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.002	0.000	0.002	0.001	0.003	0.030	-0.000	0.000	0.000	0.000	0.000
Aug	0.000	0.001	0.001	0.001	0.002	0.002	0.023	-0.011	0.001	0.000	0.000	0.000
Sep	0.000	0.001	0.001	0.001	0.003	0.002	0.014	-0.011	0.046	0.000	0.000	0.000
Oct	0.000	0.001	0.000	0.001	0.002	0.002	0.012	-0.006	0.054	0.002	0.000	0.000
Nov	0.000	0.001	0.000	0.001	0.002	0.002	0.013	-0.006	0.033	0.002	0.010	0.000
Dec	0.000	0.001	0.000	0.001	0.002	0.002	0.012	-0.007	0.031	0.001	0.007	-0.001

T+9	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	0.003	0.001	0.000	0.001	0.001	0.001	0.008	-0.005	0.023	0.001	0.005	-0.000
Feb	0.003	0.019	0.000	0.001	0.001	0.001	0.007	-0.003	0.018	0.001	0.004	-0.000
Mar	0.002	0.012	0.043	0.000	0.001	0.001	0.005	-0.003	0.014	0.001	0.003	-0.000
Apr	0.002	0.002	0.041	-0.012	0.001	0.001	0.004	-0.002	0.012	0.000	0.002	-0.000
May	0.001	0.001	0.009	-0.002	0.022	0.000	0.001	-0.001	0.003	0.000	0.001	-0.000
Jun	0.000	0.002	0.000	0.003	0.007	0.004	0.000	-0.000	0.001	0.000	0.000	0.000
Jul	0.000	0.002	0.000	0.002	0.001	0.003	0.027	-0.000	0.000	0.000	0.000	0.000
Aug	0.000	0.001	0.001	0.001	0.002	0.002	0.021	-0.010	0.000	0.000	0.000	0.000
Sep	0.000	0.001	0.001	0.001	0.002	0.002	0.012	-0.009	0.042	0.000	0.000	0.000
Oct	0.000	0.001	0.000	0.001	0.002	0.002	0.011	-0.005	0.049	0.002	0.000	0.000
Nov	0.000	0.001	0.000	0.001	0.002	0.002	0.012	-0.005	0.030	0.002	0.009	0.000
Dec	0.000	0.001	0.000	0.001	0.002	0.001	0.011	-0.006	0.028	0.001	0.007	-0.000

NOTE: see also Table 2

References

Atkinson, A.C., Koopman, S.J., and Shephard, N. (1997), "Detecting shocks: Outliers and breaks in time series," *Journal of Econometrics*, 80, 387–422.

Baillie, R.T., Chung, Ch.-F., and Tieslau (1996), "Analysing Inflation by the Fractionally Integrated ARFIMA-GARCH Model," *Journal of Applied Econometrics*, 11, 23–40.

Beran, J. (1992), "A Goodness-of-fit Test for Time Series with Long Range Dependence," *Journal of the Royal Statistical Society, Series B*, 54, 749–760.

——— (1994), *Statistics for Long-Memory Processes* , Chapman and Hall, ITP publishing.

——— (1995), "Maximum Likelihood Estimation of the Differencing Parameter for Invertible Short and Long Memory Autoregressive Integrated Moving Average Models," *Journal of the Royal Statistical Society*, 57, 659–672.

Beran, J., and Terrin, N. (1996), "Testing for a change of the long-memory parameter," *Biometrika*, 83, 627–638.

Box, G.E.P. and Cox, D.R. (1964), "An Analysis of Transformations," *Journal of the Royal Statistical Society, Series B*, 26, 211–252.

Brochu, M. (1978), "Computerized Rivers," *Science Dimension*, 10, 18–20.

Brockwell, P.J., and Davis, R.A. (1991/1993), *Time Series: Theory and Methods (2nd ed.)*. Springer-Verlag, New-York, USA.

Dahlhaus, R. (1995), "Efficient Location and Regression Estimation for Long Range Dependent Regression Models," *The Annals of Statistics*, 23, 1029–1047.

Dickey, D.A., and Fuller, W.A. (1979), "Distribution of the Estimators for Autoregressive Time Series With a Unit Root," *Journal of the American Statistical Association*, 74, 427–431.

Doornik, J.A. (1998), *Object-oriented Matrix Programming using Ox version 2.0.* Timberlake Consultants Limited, West Wickham, Kent, UK.

Doornik, J.A. and Hendry, D.F. (1996), *GiveWin, An Interface to Empirical Modelling.* International Thomson Business Press, London.

Doornik, J.A., and Ooms, M. (1996), "A Package for Estimating, Forecasting and Simulating Arfima Models," Discussion paper, Nuffield College, Oxford.

Franses, P.H. (1996), "Recent Advances in Modelling Seasonality," *Journal of Economic Surveys*, 10, 299–345.

Franses, P.H., and Ooms, M. (1997), "A Periodic Long Memory Model for Quarterly UK Inflation," *International Journal of Forecasting*, 13, 119–128.

Granger, C.W.J., and Joyeux, R. (1980), "An Introduction to Long-Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, 1, 15–29.

Hassler, U. and Wolters, J. (1994), "On the power of unit root tests against fractional alternatives," *Economics Letters*, 45, 1–6.

Hendry, D.F. and Doornik, J.A. (1996), *Empirical Econometric Modelling Using PcGive 9.0 for Windows.* International Thomson Business Press, London.

Hipel, K.W., and McLeod, A.I. (1978), "Preservation of the Rescaled Adjusted Range. 3. Fractional Gaussian Noise Algorithms," *Water Resources Research*, 14, 517–518.

——— (1978a), "Preservation of the Rescaled Adjusted Range. 2. Simulation Studies Using Box-Jenkins Models," *Water Resources Research*, 14, 509–516.

Hosking, J.R.M. (1981), "Fractional Differencing," *Biometrika*, 68, 165–176.

Hui, Y.V., and Li, W.K. (1995), "On Fractional Differenced Periodic Processes," *Sankhya: The Indian Journal of Statistics*, 57, 19–31.

Lawrance, A.J., and Kottegada, N.T. (1977), "Stochastic Modelling of Riverflow Time Series," *Journal of the Royal Statistical Society, Series A*, 140, 1–31.

Lütkepohl, H. (1991), *Introduction to Multiple Time Series Analysis.* Springer Verlag, Berlin, Germany.

MacNeill, I.B., Tang, S.M., and Jandhyala, V.K. (1991), "A Search for the source of the Nile's changepoints," *Environmetrics*, 2, 341–375.

McLeod, A.I. (1994), "Diagnostic Checking of Periodic Autoregression Models with Application," *Journal of Time Series Analysis*, 15, 221–233.

McLeod, A.I., and Hipel, K.W. (1978), "Preservation of the Rescaled Adjusted Range. 1. A Reassessment of the Hurst Phenomenon," *Water Resources Research*, 14, 491–508.

Moss, M.E., and Bryson, M.C. (1974), "Autocorrelation structure of monthly streamflows," *Water Resources Res.*, 10, 737–744.

Noakes, D.J., Hipel, K.W., McLeod, A.I., Jimenez, C., and Yakowitz, S. (1988), "Forecasting Annual Geophysical Time Series," *International Journal of Forecasting*, 4, 103–115.

Noakes, D.J., McLeod, A.I., and Hipel, W. (1985), "Forecasting Monthly Riverflow Time Series," *International Journal of Forecasting*, 1, 179–190.

Odaki, M. (1993), "On the invertibility of fractionally differenced ARIMA processes," *Biometrika*, 80, 703–709.

Ooms, M. (1995), "Flexible Seasonal Long Memory and Economic Time Series," Econometric Institute Report 9515/A, Erasmus University Rotterdam, presented at World Congress Econometric Society 1995, Tokyo, `http://www.eur.nl/few/ei/papers/`.

Ooms, M. and Doornik, J.A. (1998), "Estimation, Simulation and Forecasting for Fractional Autoregressive Integrated Moving Average Models," Discussion paper, Econometric Intitute, Erasmus University Rotterdam, presented at the fourth annual meeting of the Society for Computational Economics, June 30, 1998, Cambridge, UK.

Ooms, M., and Franses, P.H. (1997), "On Periodic Correlations Between Estimated Seasonal and Nonseasonal Components in German and U.S. Unemployment," *Journal of Business and Economic Statistics*, 15, 470–481.

Pagano, M. (1978), "Periodic and multiple autoregressions," *Annals of Statistics*, 6, 1310–1317.

Porter-Hudak, S. (1990), "An Application of the Seasonal Fractionally Differenced Model to the Monetary Aggregates," *Journal of the American Statistical Association*, 85, 338–344.

Ray, B.K. (1993), “Modeling Long-Memory Processes for Optimal Long-Range Prediction,” *Journal of Time Series Analysis*, 14, 511–525.

Robinson, P.M. (1994), “Efficient Tests of Nonstationary Hypotheses,” *Journal of the Americal Statistical Association*, 89, 1420–1437.

Sowell, F. (1992), “Maximum Likelihood estimation of stationary univariate fractionally integrated time series models,” *Journal of Econometrics*, 53, 165–188.

Spanos, A. (1986), *Statistical foundations of econometric modelling.* Cambridge University Press, Cambridge, UK.

Thomas, H.A., and Fiering, M.B. (1962), “Mathematical Synthesis of stream flow sequences for the analysis of river basins by simulation,” in *Design of Water Resources*, ed. by Maas, A., Hufschmidt, M.M., Dorfman, R., Thomas, H.A., Marglin, S.A. and Fair, G.M. Harvard University Press, Cambridge, MA, USA.

Tiao, G.C., and Grupe, M.R. (1980), “Hidden Periodic Autoregressive-Moving Average Models in Time Series Data,” *Biometrika*, 67, 365–373.

Vecchia, A.V., and Ballerini, R. (1991), “Testing for periodic autocorrelations in seasonal time series data,” *Biometrika*, 78, 53–63.