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COMMENT

Further analysis of cross-country comparison of consumer expenditure patterns

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In the April 1973 issue of the European Economic Review (vol. 4, no. 1), Dr. Th. Gamaletsos enriched the literature on international comparisons of consumption patterns by his contribution 'Further analysis of cross-country comparison of consumer expenditure patterns'. In view of the interesting results of this analysis, a few critical comments seem to be in order. In particular, these results do not seem to justify the author's conclusion that 'The GLES model while not without its share of weaknesses seems to be more attractive than the LES and IAES models from the theoretical and empirical points of view' (l.c., p. 19).

The alleged theoretical superiority of GLES (Generalized Linear Expenditure System) over IAES (Indirect Addilog Expenditure System) appears to be based on two arguments, viz.:

(a) 'that the GLES model is a sharply defined theoretical model, in the sense that its direct and indirect utility functions are known explicitly', and

(b) that 'the GLES model, in contrast with the LES and IAES models, permits the own-price elasticities to take any negative values' (l.c., p. 19).

The first argument would be valid only if the (G)LES functions are allowed to assume negative values, i.e., enabling the consumers to become sellers instead of merely buyers (this might be one of the 'weaknesses' of GLES to which Dr. Gamaletsos alluded in the quotation above!). In the opposite case, restricting consumer expenditure to non-negative values, i.e., $q_i \geq 0$ in eqs. (1) and (2), and $e_i \geq 0$ in eq. (4), the latter functions would have to be replaced by

$$e_i = \tfrac{1}{2}(z_i + |z_i|) \qquad \text{(4a)}$$

$$= z_i \text{ or } 0,$$

according as

$$z_i = p_i\gamma_i + \beta_i\left(y - \sum_{j=1}^{n} p_j\gamma_j\right)$$

$$\geq 0 \text{ or } \leq 0, \text{ respectively.}$$

As long as no analytical expression has been found for a utility function underlying the non-negative consumer expenditure functions (4a), they 'suffer' from the same 'defect' (a) above, attributed to the IAES – presumably even more so, since for IAES at least the indirect utility function is known explicitly, viz. as eq. (10). Moreover, as shown in a recent report of the Econometric Institute,[1] for particular specifications of the IAES, direct utility functions *can* be expressed analytically.

For the rest, explicitness of utility functions does not seem to be of great importance, especially since utility is an ordinal rather than a cardinal concept; anyhow, the behavioural relationships derived from its maximization subject to one or more constraints are invariant against any monotonically increasing transformation (barring uncertainty).[2]

The second objection (b) to IAES rests on a misunderstanding, viz. that the β_i in the latter – indirect utility – functions should be negative, and that the same should apply to the α_i. Actually, the IAES functions (11) merely require that $\gamma_i = \alpha_i\beta_i \geqq 0$, i.e., that α_i and β_i should be of the same sign: either both negative or both positive, or that at least one of them be zero (requirement $-\Sigma_{i=1}^{n}\alpha_i = 1$, posited by Dr. Gamaletsos, is already redundant because of the monotonically increasing transformability of the utility functions). The single remaining *essential* restriction to be retained is $\beta_i > -1$, necessary and sufficient in order to ensure a proper (restricted) utility maximum. Consequently, direct price elasticities,

$$\eta_{ii} = -(1+\beta_i)+\beta_i w_i = -1-(1-w_i)\beta_i, \tag{14}$$

may assume *any* negative values. Incidentally, the same applies to the LES direct price elasticities,

$$\eta_{ii} = -1+(1-\beta_i)\gamma_i q_i^{-1}, \tag{6'}$$

notwithstanding Dr. Gamaletsos' assertion to the contrary. In his own GLES-model, however, he has to accept the non-negativity of the γ_i because he has to impose the non-negativity condition on the δ_i, otherwise $\Sigma_i\delta_i p_i^{\xi} = 0$ for particular sets of price vectors cannot be avoided, and in order to satisfy the second-order conditions for a (constrained) utility maximum – not considered by Dr. Gamaletsos in this article of his.

[1] W.H Somermeyer, Analytical utility functions underlying particular specifications of the expenditure allocation model, Report 7315 of the Econometric Institute, Netherlands School of Economics, Rotterdam.

[2] Dr. Gamaletsos made use of this expedient himself, by deriving his utility functions (1) from (2) by means of the transformation function:
$u = V^{\rho}$ (or rather v^{ρ}). He seems to have overlooked, however, that for $\rho < 0$, the transformation function should be: $v^{-\rho}$, implying that the minus sign preceding the summation sign in the 'middle' function (2) should be deleted.

Because of this invariance of the necessary and sufficient conditions for a utility maximum against *any* monotonically increasing transformation the requirement u_{ii} (or v_{ii}) > 0, in his footnote 2, is unnecessary and untenable: to be sure the negativeness of the corresponding $u_{ii} = f'v_{ii}+f''v_i^2$, is ensured only if $f'' = \partial^2 f/\partial u^2 \leq 0$, with $f' = \partial f/\partial u > 0$.

It should also be noted that raising the restriction $\beta_i < 0$ widens the range of IAES income elasticities to $(-\infty, \infty)$.

Consequently, Dr. Gamaletsos' case against IAES *from a theoretical point of view* collapses, while the advantage of GLES over LES is less than he makes out.

Dropping the unnecessary restriction $\beta_i < 0$ weakens Dr. Gamaletsos conclusions in disfavour of IAES *also from the empirical point of view.* In Dr. Gamaletsos' own words 'Of 55 compensated own-price elasticities, 10 have a wrong sign in the LES model, 20 have a wrong sign in the GLES model, and 9 with a wrong sign appear in the IAES model. A further 7 elasticities have the right sign but exceed the limits given by the theoretical IAES model' (l.c., p.16). Actually, the latter '7' should be reduced to '3' (cases in which $\beta_i < -1$ but the compensated own-price elasticities $\eta_{ii}^* > 0$, in addition to the 9 cases in which both $\beta_i < -1$ and $\eta_{ii} > 0$); still, even according to Dr. Gamaletsos' own (wrong) count, IAES-estimates show a smaller number of wrong signs (viz. $9+7 = 16$) than GLES (20). This holds a fortiori with the right count of wrong IAES-estimates ($9+3 = 12$). Anyhow, according to this criterion, GLES evidently performs worse instead of better than IAES, let alone LES. For the rest, Dr. Gamaletsos does not inform us about his rate of success in obtaining theoretically required positive values for his δ-estimates.

Although it might not make much difference in the present case, it should be noted – just for the record – that the number of mutually independent parameters to be estimated for the IAES and LES models, viz. $2n-1$ ($= 9$, with 5 budget categories distinguished) is slightly less than the corresponding number in the GLES model, viz. $2n$ ($= 10$, i.e., 5δ's, 5γ's, one τ minus 1), i.e., leaving 71 against 70 degrees of freedom (for which apparently no correction has been made).

Furthermore, the estimation procedures adopted are not the most efficient ones, according to Dr. Gamaletsos' own confession (p. 8). One might counter that this neglect of the disturbance covariance and heteroskedasticity affects GLES as well as LES and IAES. The stochastic specification of the models by means of additive errors is less inappropriate, however, for the *linear* GLES and LES models than for the 'ratio'- or 'allocation'-IAES models; for this reason, the numerator and the summands in the denominator of the fractions representing the IAES-functions (11) were multiplied by $\exp(\varepsilon_{it})$ and $\exp(\varepsilon_{jt})$, respectively, in the studies by Somermeyer, and Wit (1956), (incompletely)[3] cited by Dr. Gamaletsos.

In order to avoid the bias in the conclusions at least suggested if not intended by Dr. Gamaletsos, it seems fair to point out desirable properties of the IAES – lacking in its 'competitors'.

[3]This study was not mimeographed, but appeared as an article in: Statistical Studies 13, 30–53, issued by the Netherlands Central Bureau of Statistics.

As set out in our article,[4] the IAES model is well-behaved even under extreme conditions (very low or high values of income and prices); unfortunately, this cannot be said of the (G)LES-models. In contradistinction to the latter 'monotonic' models, IAES allows for possible saturation and over-saturation, implying 'inferiority' precluded by the others. This points to the flexibility of the model, notwithstanding its relatively small number of parameters. Moreover, it has the advantage that its parameters γ_i and β_i can be identified in a natural way, viz. as 'preference coefficients' and 'reaction intensities', respectively. In turn, such an interpretation contributed further to the adaptability of the model, by allowing it to be generalized; in particular, the γ_i may be made dependent on other variables, such as size of the household, inventories, past income, etc.[5] More in line with Dr. Gamaletsos generalization of the LES-model, the IAES-model might also be extended by replacing the expressions $\gamma_i(y/p_i)^{\beta_i}$ in formula (11) by (say)

$$\gamma_i(y/p_i)^{\beta_1} \exp\{\beta_2(y/p_i)\}.^6$$

Finally, the IAES-model may be considered as a specimen of an even more general 'allocation model':

$$e_i/y = f_i(y/p_i) \Big/ \sum_{j=1}^{n} f_j(y/p_j), \tag{11a}$$

ensuring compliance with the additivity condition

$$\sum_{i=1}^{n} e_i/y = 1.$$

Evidently, a similar idea underlies Dr. Gamaletsos' generalization of β_i as a function of all p_j, likewise adding up to 1.[7] At least, we appear to agree on the virtues of built-in consistencies!

[4]W.H. Somermeyer and A. Langhout, 1972, Shapes of Engel curves and demand curves: Implications of the expenditure allocation model applied to Dutch data, European Economic Review 3, no. 3, 351–386.

[5]To be fair, such a generalization might perhaps be grafted on the δ in the GLES-model as well. Again, an economic interpretation might be attached to the τ (or ρ) and δ_i parameters in the GLES model (and/or its underlying utility function), although it seems to be more difficult than in the case of the AIES.

[6]This generalized version of the IAES-model has been developed and applied to household expenditure data for Greece, by D. Athanasopoulos, 1962, Study on income elasticities, mimeographed paper, Institute of Social Studies, The Hague.

[7]For a fuller account of possibilities of generalizing the linear expenditure function, cf. W.H. Somermeyer, Delimitation of the class of utility maximizing partial linear consumer expenditure functions, Report 7316 of the Econometric Institute, Netherlands School of Economics, Rotterdam.