



0021-9290(94)00122-7

TECHNICAL NOTE

RECONSTRUCTION OF BONE LOADING CONDITIONS FROM *IN VIVO* STRAIN MEASUREMENTS

Harrie Weinans and Leendert Blankevoort

Section Biomechanics, Institute of Orthopaedics, University of Nijmegen, P.O. Box 9101, 6500 HB Nijmegen, The Netherlands

Abstract—A method is described whereby bone loading conditions can be reconstructed from *in vivo* strain measurements. The method uses *ex vivo* calibration measurements to find the relationship between the strain data and the loads applied to the bone. Using singular value decomposition, a transformation matrix is determined which provides the best linear relationship available between the measured strain data and the measured loading components in the calibration measurements. The transformation matrix can then be used to calculate the loads which correspond best with any given strain data set made with that specific bone and strain gages. In this manner, the applied loads of earlier performed *in vivo* strain measurements can be reconstructed. The method was tested for the reconstruction of the loads on a tibia of a goat. After determining the transformation matrix from a set of calibration measurements, the transformation matrix was used to reconstruct all loading components (three forces and three moments) of a set of test measurements whereby the applied loads were measured. It was found that the axial force and the torsional moment on the bone could be reproduced very accurately, showing a root mean square error (RMSE) of only 2% of the maximal load in the test. The reconstruction of the bending moments was slightly worse, showing a RMSE of 5–8% of the maximal moments. The reconstruction of the transverse force components proved less accurate and a RMSE up to 24% of the maximum was found. Accuracy can be improved by using weight factors for the loading conditions and a more accurate measurement of the location of the loads during the calibration measurements.

NOMENCLATURE

| | |
|----------|--|
| l | number of strain signals |
| m | number of loading components (maximal 6: three forces and three moments) |
| n | number of calibration measurements |
| B | matrix with applied loading components |
| B^* | calculated load matrix |
| B_{ij} | Loading components for experiment number i and component number j |
| S | matrix with strain data |
| S_{ik} | strain value of experiment number i and strain gage number k |
| T | transformation matrix (relationship between strains and loads) with l rows and m columns |
| S^+ | pseudo-inverse of matrix S ($S_{ik} T_{kj} = \sum_{k=1} S_{ik} T_{kj}$) |

INTRODUCTION

Strain gages are frequently used to determine *in vivo* loading conditions in terms of bone strains or, when the material properties of the bone are known, in terms of the bone stresses (e.g. Lanyon, 1981). In addition, the external loading conditions on a bone can be determined by *in vivo* strain measurements (Rybicki *et al.*, 1977; Carter *et al.*, 1981a,b). A representation of the external loading condition on a bone can be given by six individual loading components: three

forces and three moments defined in a Cartesian coordinate system. Hence, these six loads can only be reconstructed from at least six independent strain signals. Rybicki *et al.* (1977) and Carter *et al.* (1981b) used an analytical model to calculate the external loading components. The method requires an accurate characterization of the bone material properties and the geometry. A finite element model can be used instead of an analytical model when the geometry of the bone, is more complex (Carter *et al.*, 1981b). A disadvantage of this method is that the material properties and geometry of the bone as well as the direction of the strains must be determined accurately for the load reconstruction. Even cortical bone anisotropy must be accounted for (Carter, 1978). When these input data are not precise, the loading components as determined by this method become inaccurate. A more comprehensive method was introduced by Carter *et al.* (1981a), using a calibration method for the reconstruction of the axial force and bending moments in a long bone. However, this calibration method has similar disadvantage as the analytical method: it cannot handle a large number of strain gages and calibration measurements, since they will likely produce (to a certain extent) inconsistent data because of stochastic errors and small nonlinearities in the configuration. Another related problem with this calibration method is that specific loading cases produce strains at locations where they are not supposed to occur. For instance, a bending moment produces torsion when the force producing the bending moment is not precisely located at the center of the cross-section where the gages are applied. This will result in strains at all gages which are not precisely placed in the axial direction and subsequently an error is made in the reconstruction of the loading.

The present study introduces a method whereby the above-mentioned disadvantages are diminished. After *in vivo* animal experimental strain gage measurements on a bone, a set of *ex vivo* calibration measurements is performed. Instead of determining an exact relation between strain and the load, a fit is provided from a large number of calibration measurements. Then a transformation matrix is determined which gives the best linear relationship between the strain signals and the external loading components during the calibration measurements. Thereafter the loading components can be calculated from the product of the transformation matrix and the strain vector. In fact, the transformation matrix provides a least-squares fit between the applied and calculated loading components in the calibration experiment. The *in vivo* loading components, of which the strain was measured earlier, can now be determined without additional information with respect to geometry, material properties and strain gage direction.

METHODS

The configuration assumed for the analysis is a bone with l strain gages glued to its surface. The objective is to find the relationship between external loads and the corresponding strains recorded by the gages. Initially, the loads are accurately defined in a coordinate system. Subsequently, a number of loading cases are applied on to the bone and the corresponding strains are measured. In the present study a tibia of a goat is considered for the analysis (Fig. 1). In this way, a number of n calibration measurements are performed. The loading components and the strain gage data from these n different measurements are gathered in two matrices B and S . Matrix B contains the loading components and has n rows and m (the number of loading components) columns. Matrix S , the strain data matrix, has n rows and l (the number of strain signals) columns. Next, a transformation matrix T is defined, which relates matrix B to matrix S . Therefore, we would like to have a matrix T such that $ST = B$. However, it is likely that such a matrix T does not exist. The best we can do is find a transformation matrix T which provides a B^* by

$$ST = B^*, \quad (1)$$

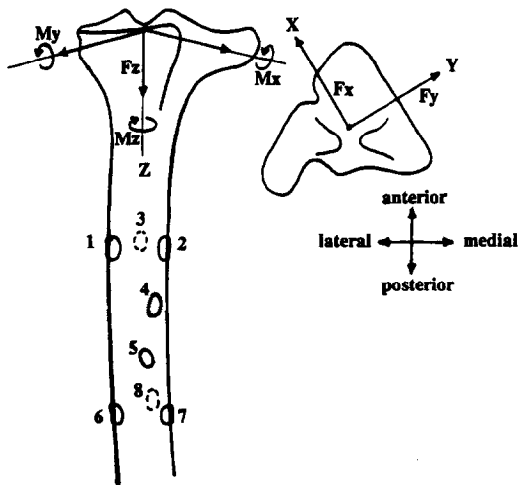


Fig. 1. Definition of the coordinate system at the left tibia of the goat and the location of the gages. All gages supply one signal whereby 1, 2, 3, 6, 7 and 8 were oriented approximately parallel to the Z-axis and gages 4 and 5 were oriented under $\pm 45^\circ$ with the Z-axis.

with B^* as close as possible to B . In principle, the matrix T follows from the relation $T = S^{-1} \cdot B$, where S^{-1} is the inverse matrix of S . However, S is not a square matrix, and therefore its inverse cannot be determined. The procedure applied in this study uses singular value decomposition to compute the 'pseudo-inverse' S^+ such that a transformation matrix T is given by $T = S^+ \cdot B$ (Lawson and Hanson, 1974). In matrix components this relationship yields

$$T_{kj} = S_{ki}^+ B_{ij}. \quad (2)$$

Many software libraries contain a singular value decomposition routine providing the pseudo-inverse S^+ . The transformation matrix T as found with this procedure provides the exact minimum error with

$$\text{error} = \sum_{j=1}^m \sum_{i=1}^n (S_{ik} T_{kj} - B_{ij})^2 \quad (3)$$

(IMSL math/library t.m., Houston, 1987). In fact a least-square, fit between reconstructed loads (from the product of ST) and measured loads, B , is produced. It is important that all m loading components to be reconstructed are used in the set of calibration measurements. These components determine the column space of B and in fact also the column space of S , because of the essentially linear relationship between S and B . In other words, the number of independent columns of matrix S (its rank) should, in theory, be equal to the number of independent loading components, m . However in reality, this is not the case. Due to stochastic errors in the calibration measurements and small deviations from the assumed linear elasticity, the strain responses in the numerous measurements is slightly inconsistent. Usually, the rank of matrix S is equal to the number of the columns of $S(l)$, and hence the number of the strain gages used. However, the rank of S is not representative for the independency of the rows. The singular values of matrix S give information about the amount of linear independence of the columns of S (Lawson and Hanson, 1974; Sullivan, 1984). When all singular values are of the same order of magnitude, the columns are clearly independent. Hence, more singular values than the number of loading components, all with the same order of magnitude, indicate a system under poor conditions. With l strain signals and m loading components, it is expected that there are m singular values of the same order of magnitude and $l-m$ singular values small relatively to the other singular values. In the computation process of the pseudo-inverse S^+ these small singular values should be set to zero in order to minimize round off errors. The IMSL routine as presently used contains a tolerance criterion, which sets all singular values that are negligible in the context of the computation to zero.

Eight strain gage units of 4 mm in diameter, all giving one strain signal, were glued to the mid/distal part of the tibia of a goat (41 kg weight), as described earlier by Roszek *et al.* (1993) in more detail (Fig. 1). *In vivo* strain measurements on the tibia were performed during several different gait trials and stored in a PC. After these strain measurements, the goat was killed and the tibia of the goat was removed leaving all strain gages intact. Within 24 h afterwards a set of *ex vivo* calibration measurements was performed. Muscles and soft tissue were stripped at the proximal and distal parts for about 3 cm. The distal part was fixed in acrylic cement and attached to the table of an MTS loading machine. A coordinate system was defined relative to the geometry of the tibia in a reproducible procedure (Roszek *et al.*, 1993), with the origin located at the intercondylar fossa, the z-axis in the axial direction and the x-axis parallel to the medial anterior rim of the tibia plateau (Fig. 1). Calibration measurements were performed such that each of the six loading components were applied individually, or in combination with one other component, as shown in Table 1. The magnitude of the loading components was chosen in the range of the anticipated *in vivo* loads. The transformation matrix between loads

Table 1. Some examples from the set of calibration measurements. The calibration loads (B) are the loads used in the calibration experiments and the reconstructed loads are calculated from the strain data using equation (1)

| Test no. | Calibration loads (B) | | | | | | Reconstructed loads (ST) | | | | | |
|----------|-----------------------|-----------|-----------|--------------|--------------|--------------|--------------------------|-----------|-----------|--------------|--------------|--------------|
| | F_x (N) | F_y (N) | F_z (N) | M_x (N cm) | M_y (N cm) | M_z (N cm) | F_x (N) | F_y (N) | F_z (N) | M_x (N cm) | M_y (N cm) | M_z (N cm) |
| 1 | 0.0 | 0.0 | 400.8 | 0.0 | 0.0 | 0.0 | 1.2 | 2.4 | 393.7 | 17.9 | 32.0 | 0.64 |
| 2 | 0.0 | 0.0 | 280.0 | 1002.4 | 0.0 | 0.0 | 0.5 | 2.7 | 279.6 | 967.0 | -17.8 | -0.5 |
| 3 | 0.0 | 0.0 | 272.4 | -975.2 | 0.0 | 0.0 | 0.2 | 2.1 | 272.2 | -987.9 | -10.8 | -0.4 |
| 4 | 0.0 | 0.0 | 330.0 | 0.0 | 1181.4 | 0.0 | 0.4 | -2.5 | 327.0 | 31.9 | 1163.3 | 1.4 |
| 5 | 0.0 | 0.0 | 371.6 | 0.0 | -1330.3 | 0.0 | -0.6 | -2.2 | 371.7 | -6.0 | -1301.1 | 0.4 |
| 6 | 25.6 | 0.0 | 0.0 | 0.0 | -102.4 | 0.0 | 24.6 | 0.1 | -9.5 | -9.9 | -126.9 | 0.9 |
| 7 | -31.0 | 0.0 | 0.0 | 0.0 | 124.0 | 0.0 | -30.5 | 0.38 | -12.9 | -6.2 | 60.4 | 1.1 |
| 8 | 0.0 | 25.6 | 0.0 | 102.4 | 0.0 | 0.0 | -0.2 | 17.8 | -6.0 | 79.9 | 9.4 | 0.2 |
| 9 | 0.0 | 25.6 | 0.0 | -102.4 | 0.0 | 0.0 | 0.4 | -21.4 | 3.0 | -90.1 | 5.3 | -1.0 |
| 10 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 212.2 | 0.3 | -0.1 | -3.6 | 41.5 | 15.7 | 210.5 |
| 11 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | -212.2 | 0.2 | -0.2 | -1.3 | 11.6 | 1.2 | -214.5 |

($m = 6$) and strain gage data ($l = 8$) was determined using the earlier described method. For each loading component j the root mean square error (RMSE) between reconstructed load (by the product ST) and applied load, B , was determined by

$$RMSE_j = \sqrt{\frac{1}{n} \sum_{i=1}^n (S_{ik} T_{kj} - B_{ij})^2} \quad (4)$$

Furthermore, we have determined the RMSE relative to the maximal loading component in the row concerned: $RMSE_j/B_j^{max}$, with B_j^{max} the maximal loading component of row j .

In addition, a number of test measurements were performed whereby an arbitrary combination of loading components was applied of which the magnitude was in the range of the calibration measurements. Again, the average and maximal deviations between reconstructed and measured loads were determined. Afterwards, the product of the *in vivo* strains (measured earlier) and the transformation matrix was used to obtain the *in vivo* loading conditions on the tibia of the goat.

RESULTS

In general, the loading reconstructions of the calibration measurements as provided by equation (1) were relatively accurate. The differences between the loads applied during the calibration experiments and the calculated loads from the strain data assembled during the same calibrations were all relatively small (Table 1). This is also shown by the RMSE of the different components which were low relative to the range of the specific loading component concerned. The maximum value for the RMSE was found to be only 10% of the maximum loading component for the reconstruction of the transverse force in the y -direction (Table 2).

The test or verification experiment showed that again the transverse forces gave a relatively poor reconstruction (error of 24%) when compared to the range of the applied transverse forces. However, most of the other test measurements gave a very good reconstruction of the applied forces for all loading components. This is demonstrated in Table 3 which shows six representative examples of the 45 test measurements. The RMSE of the transverse forces was about 6 N, which is 24% of the maximal values of the transverse forces, as shown in Table 4. Relative to the maximal loading components the RMSE was 2% for the axial force and the torsional moment and 5-8% for the bending moments (Table 4).

It was found that from the eight singular values (see methods section), two were rather small in the context of the present analysis, and hence from the eight strain gages applied two strain values are more or less dependent on the

Table 2. Deviations between the applied loading components (in 153 calibration measurements) and the reconstructed ones from the multiplication of the measured strain and the transformation matrix (ST) given in route mean square error (RMSE), and the percentage of RMSE with respect to maximal loading component (RMSE/ B^{max})

| Load | RMSE | RMSE/ B^{max} |
|--------------|------|-----------------|
| F_x [N] | 0.8 | 3% |
| F_y [N] | 3.0 | 10% |
| F_z [N] | 5.2 | 1% |
| M_x [N cm] | 60.4 | 3% |
| M_y [N cm] | 25.3 | 1% |
| M_z [N cm] | 1.3 | 0% |

Table 3. Examples of test measurements and the corresponding reconstruction

| Test no. | Loading experiment (B) | | | | | | Reconstructed loads (ST) | | | | | |
|----------|------------------------|-----------|-----------|--------------|--------------|--------------|--------------------------|-----------|-----------|--------------|--------------|--------------|
| | F_x (N) | F_y (N) | F_z (N) | M_x (N cm) | M_y (N cm) | M_z (N cm) | F_x (N) | F_y (N) | F_z (N) | M_x (N cm) | M_y (N cm) | M_z (N cm) |
| 1 | 0.0 | 0.0 | 103.0 | 0.0 | 368.7 | -155.1 | 0.0 | -1.7 | 89.9 | 91.0 | 391.7 | -161.8 |
| 2 | 0.0 | 0.0 | 300.0 | -1074.0 | 0.0 | 274.2 | 0.3 | 4.1 | -318.0 | -1208.6 | -139.8 | 268.5 |
| 3 | 26.3 | 0.0 | 401.2 | 0.0 | -105.0 | 0.0 | 19.8 | -5.5 | 390.6 | 60.5 | 54.5 | 2.9 |
| 4 | 0.0 | 26.3 | 401.6 | 105 | 0.0 | 0.0 | 3.8 | 23.8 | 289.1 | 143.3 | -16.5 | -2.5 |
| 5 | 0.0 | -14.6 | 451.2 | -58.3 | 0.0 | 274.2 | -1.3 | -10.9 | 450.4 | -17.8 | 9.8 | 267.8 |
| 6 | 0.0 | 0.0 | 500.4 | 1791.4 | 0.0 | -331.2 | 1.0 | 2.5 | 485.7 | 1810.2 | 60.4 | -318.0 |

Table 4. Deviations between the applied loading components (in 45 test measurements) and the reconstructed ones from the multiplication of the measured strain and the transformation matrix (ST) given in route mean square error (RMSE), and the percentage of RMSE with respect to maximal loading component in the test series ($RMSE/B^{\max}$)

| Load | RMSE | RMSE/ B^{\max} |
|--------------|-------|------------------|
| F_x [N] | 6.3 | 24% |
| F_y [N] | 6.0 | 23% |
| F_z [N] | 12.3 | 2% |
| M_x [N cm] | 151.0 | 8% |
| M_y [N cm] | 89.6 | 5% |
| M_z [N cm] | 6.1 | 2% |

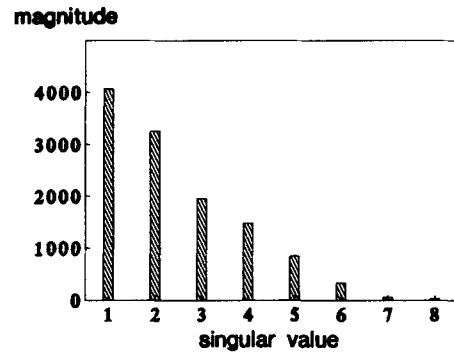


Fig. 2. The singular values as calculated in the singular value decomposition procedure in the present configuration. The last two (smaller) values were considered as zero values.

other six values. Figure 2 shows the distribution of the magnitudes of the eight singular values of strain matrix S . The two smallest values were considered as zero values. Thus, only six linear independent columns were assumed. This is indeed what was expected because there are only six independent loading components.

Finally, the loading components on the tibia could be reconstructed, using the *in vivo* strain data. An example is shown in Fig. 3. With respect to the present coordinate system the forces were predominantly axial with peaks in the range of 400 N (about body weight). The moments were in the range of 10–20 N m for bending and 3 N m for torsion. These loads were all in the range of the loads used during the *ex vivo* measurements.

DISCUSSION

This study shown that *in vivo* loading components can be reconstructed from *in vivo* strain measurements after performing a set of calibration measurements which provide the assumed linear relationship between strain and load. What is reconstructed (Fig. 3) should be interpreted as the resultant load transfer at the segment of the tibia where the strain gages were applied (Fig. 1). In fact, the reconstructed *in vivo* resultant loads create the same strain field at the segment of the strain gages as the real *in vivo* articular and nonarticular loading. In other words, the *in vivo* reconstruction is the

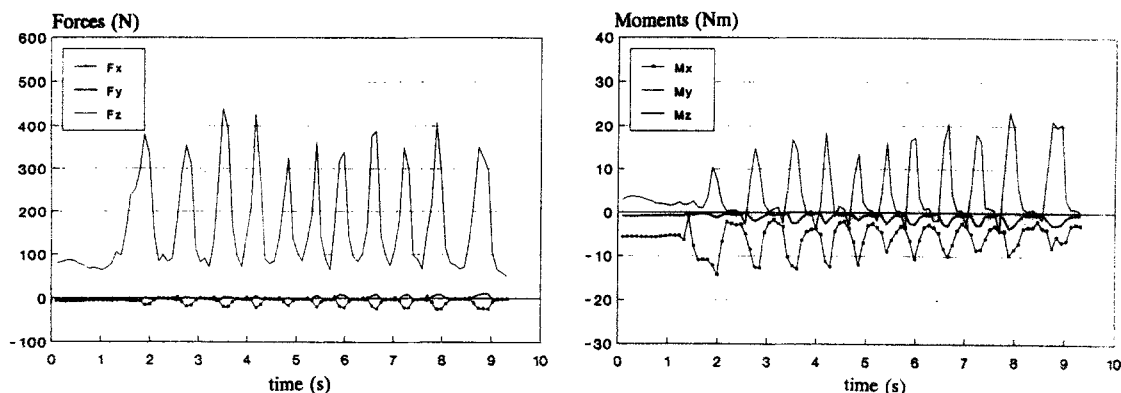


Fig. 3. The transverse forces (F_x , F_y), axial force (F_z), bending moments (M_x , M_y) and torsional moment (M_z) in the left tibia during fast walking of a goat, as reconstructed from the strain measurements using equation (1).

resultant loading transferred through the mid-section of the tibia as produced by all tendon, ligament and articular surface loading, whereby the point of application of the assumed resultant loads is represented with respect to the (arbitrarily) chosen coordinate system (Fig. 1).

To use the singular value decomposition for finding the pseudo-inverse of the strain matrix S it is required that certain conditions are met. The relationship between strains and loads must be linear and the calibration measurements must be representative for all loading cases. Furthermore, the strain gages must be located such that they are able to provide six independent signals. Hence, for six loading components at least six strain gages should be used; however, it is safer to apply more than six gages on the bone because the placement of the gages during the operative procedure is rather imprecise and it is likely that there is always some dependency within the signals. It is hard to give an accurate guideline for the optimal number and the location of the strain gages. It is clear that the gages cannot be applied at any place one likes. For instance, all the gages should not be placed along one line, since this would only provide information about the axial force and one transverse force. As a general rule for the placement of the gages one should keep in mind that for a complete reconstruction of all six loading components one needs at least two representative gages for each axis of the coordinate system, preferably as far as possible from each other. However, geometrical and clinical restrictions can make it difficult to achieve this under all circumstances.

It is obvious that small nonlinearities between load and the strain may occur. In addition, deformation due to tension or compression may not exactly be the same, small viscoelastic effects may occur and geometrical nonlinearities may occur during the loading experiments. The latter is the result of bending the tibia by an axial force or a combined effect due to an axial force and a bending moment. This will displace the tibia in the x - y plane (Fig. 1) such that the location at which the force is applied will change and an additional bending moment will thus result, which is not accounted for in the calibration process.

The reason that the accuracy of the transverse load reconstruction is relatively poor is partly because of the fact that the transverse forces themselves are relatively small. The transformation matrix is determined such that a least-squares fit, in absolute numbers, between the reconstructed and applied loads during the calibration is realized [equation (3)]. Therefore, loading components with small absolute numbers will have a large relative error in the reconstruction.

This could be prevented by introducing weight factors, such that the absolute numbers in the loading matrix B are of the same order of magnitude (Table 1, first two rows). It should be noticed that the transverse forces were small also in the reconstruction of the *in vivo* loads. The errors in the bending moments are probably due to the geometrical nonlinearities in the load application during the calibration measurements as mentioned above. This could certainly be improved by using a more accurate method of measuring the point of application of the axial force.

The fact that the bending moments and transverse forces likely produce torsion at the cross-section of the gages is no reason for a further reduction of the accuracy of the present method, since that does not affect the linear relationship between the loads and the strains. This is one of the advantages compared to the method introduced by Carter *et al.* (1981a) and Gies and Carter (1982) which will handle torsion due to an applied bending moment as adverse cross-talk: strains produced by a specific loading case are treated as if they come from another loading case. This will, of course, produce an error in the reconstruction. Furthermore, the present method is able to operate with as many strain gages and calibration measurements as one prefers, contrary to other methods described in the literature (Rybicki *et al.*, 1977; Carter *et al.* 1981a, b). It should be noticed that the pseudo-inverse method as presently performed uses as much data as is available for the reconstruction, where the other methods use a minimal amount of information.

The present method can be used for all general purposes where loads have to be reconstructed from strain measurements, without any information with respect to geometry and material properties, no matter how complicated they are, and independent from the exact location of the gages. As long as the relationship between the loads and the strains can be assumed linear and at least six gages are applied, the method will reconstruct all loading components. In fact, the method averages the stochastic errors and nonlinearities from a complex set of numerous calibration measurements, whereby the pseudo-inverse method provides the most optimal solution in the least-squares manner.

Acknowledgement—This study was sponsored in part by the Dutch Organization of Research. We thank Prof. Dr W. Hartman and Dr H. C. Schamhardt for giving us an introduction in the technique of *in vivo* bone strain measurements, and T.J.A. Mommersteeg, B. Roszek and P.J.M. can Loon for their support with the experiments.

REFERENCES

- Carter, D. R. (1978) Anisotropic analysis of strain rosette information from cortical bone. *J. Biomechanics* **11**, 199–202.
- Carter, D. R., Caler, W. E. and Harris, W. H. (1981a) Resultant loads and elastic modulus calibration of long bone cross sections. *J. Biomechanics* **14**, 739–745.
- Carter, D. R., Vasu, R., Spengler, D. M. and Dueland, R. T. (1981b) Plate and bone stresses for single- and double-plated femoral fractures. *J. Biomechanics* **14**, 55–62.
- Gies, A. A. and Carter, D. R. (1982) Experimental determination of whole long bone sectional properties. *J. Biomechanics* **15**, 297–303.
- Lanyon, L. E. (1981) The measurements and biological significance of bone strain *in-vivo*. in *Mechanical Properties of Bone* (Edited by Cowin, S. C.). Vol. **45**, pp. 93–105. ASME, New York.
- Lawson, C. L. and Hanson, R. J. (1974) *Solving Least Squares Problems*. Prentice Hall, Englewood Cliffs, NJ.
- Roszek, B., Weinans, H., van Loon, P. and Huiskes, R. (1993) *In vivo* measurements of the loading conditions on the tibia of the goat. *Acta Anat.* **146**, 188–192.
- Rybicki, E. F., Mills, E. J., Turner, A. S. and Simonen, F. A. (1977) *In vivo* and analytical studies of forces and moments in equine long bones. *J. Biomechanics* **10**, 701–705.
- Sullivan, B. J. (1984) On the use of singular value decomposition and decimation in discrete-time band-limited signal extrapolation. *IEEE Trans. Acoustic Speech Signal Processing*, **ASSP-32**, 1201–1212.