On data transformations and evidence of nonlinearity

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Abstract

In this paper we examine the interaction between data transformation and the empirical evidence obtained when testing for (non-)linearity. For this purpose we examine nonlinear features in 64 monthly and 53 quarterly US macroeconomic variables for a range of Box-Cox data transformations. Our general finding is that evidence of nonlinearity is not independent of the data transformation. Results of simulation experiments substantiate this finding.

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1 Introduction

At present there is a growing interest in considering nonlinear time series models for macroeconomic variables. Important studies that stimulated this interest are Granger and Teräsvirta (1993), Tong (1990) and Hamilton (1989), to mention just a few. Examples of empirically useful models are the Markov regime switching model (Hamilton, 1989) and the so-called Smooth Transition Autoregression [STAR] (Teräsvirta, 1994). These models allow for a description of time series that undergo various regimes, such as recessions and expansions.

Most empirical time series models are considered for data which are natural log transformed, which is one of the Box-Cox transformations, see Box and Cox (1964). Usually, this transformation is applied because it is presumed to remove nonstationarity or heteroscedasticity from the data, and that it induces symmetry and perhaps normality to the probability distribution of the time series variable in question. A casual glance at much empirical work in macroeconomics shows that the log-transformation is usually routinely applied, and that its presumed properties are seldom verified. Of course, another reason for applying the log transform is the fact that sometimes one is interested in the growth rate of a variable rather than in its level, where the growth rate roughly corresponds with first differenced logged data.

As is well known, the Box-Cox transformation is a nonlinear transformation. Therefore it is interesting to understand how this transformation influences characteristics in the data, and particularly, possible nonlinear properties. On the other hand, it may be that this transformation introduces nonlinearity in linear data. In this paper we investigate the influence of a range of Box-Cox transformations on empirical evidence of nonlinearity in many macroeconomic variables. Our analysis does not assume a correct transformation that can be found by estimating the Box-Cox parameter in a nonlinear time series model. Instead, we analyze the properties of data for a range of Box-Cox parameters in order to study possible links between their values and empirical evidence of nonlinearity. We also use artificially generated series to study whether our empirical findings can be better understood. To save space, we focus our attention on tests for STAR type nonlinearity.

The outline of our paper is as follows. In section 2 we give a description of the time series variables we consider. Section 3 deals with a description of the data transformations and the nonlinearity test used. To allow for heteroscedasticity that increases or decreases over time, we also introduce and apply a GLS-based test for nonlinearity. Section 4 gives the results of the tests applied to the empirical data. In section 5 we utilize the same procedure again, but this time for artificially generated AR and STAR time series. Our main conclusion is that evidence of nonlinearity changes over the spectrum of Box-Cox parameter values. Our simulation results help us to explain part of these findings. For some empirical time series we reject linearity for all Box-Cox values, implying that the practitioner has a free choice which data transformation to consider. In section 6 we conclude our paper with some remarks.

2 Data

In our empirical work we consider 53 quarterly time series and 64 monthly time series. These monthly series will also be analyzed in quarterly aggregated form. The monthly data sets can be divided as Monetary Issues (28), Employment (24) and Income and Expenditures (12). All quarterly data concern Production variables. All data can be obtained from the website `www.stls.frb.org/fred/data/`. Since we are investigating the interaction between nonlinearity aspects of the data and nonlinear transformations, we do not want seasonal effects influencing the outcomes. Therefore, we only use data that are already seasonally adjusted. Using seasonally adjusted data also ensures that the auxiliary test regressions for nonlinearity do not contain too many parameters. We are however familiar with the results in Ghysels, Granger and Siklos (1996) and Franses and Paap (1999), where it is documented that seasonal adjustment may introduce nonlinearity in otherwise linear time series, and that it changes the key parameters in nonlinear models, respectively.

A description of the monthly data is given in Tables 1, 2 and 3. The quarterly data are described in Table 4. The length of the monthly data sets differs from 282 to 622 months. All quarterly time series are a little over 150 data long. We assume that in all cases the size of the data sets is large enough to be able to make well-founded inference.

3 Empirical methodology

Consider a time series variable x_t, $(t = 1, \ldots, T)$, where x_t cannot have negative or zero observations. The family of Box-Cox transformations is given by:

$$y_{\lambda,t} = \begin{cases} \frac{x_t^\lambda - 1}{\lambda} & \text{for } 0 < \lambda < 1 \\ \ln(x_t) & \text{for } \lambda = 0, \end{cases} \tag{1}$$

where the transformation for $\lambda = 0$ follows from the fact that $\lim_{\lambda \to 0} \frac{x_t^\lambda - 1}{\lambda} = \ln(x_t)$. Subtracting 1 and dividing by λ does not influence the stochastic structure of x_t^λ, and hence one often considers the transformation:

$$y_{\lambda,t} = \begin{cases} x_t^\lambda & \text{for } 0 < \lambda < 1 \\ \ln(x_t) & \text{for } \lambda = 0, \end{cases} \tag{2}$$

instead of (1), without loss of generality.

Many macroeconomic time series display an upward trend. Without further testing, we assume here for all values of λ that this trend is a stochastic trend, and hence that we can induce stationarity of a time series z_t by taking first differences, defined by

$$\nabla z_t = z_t - z_{t-1}. \tag{3}$$

It appears that the decision on taking first differences can be affected by the Box-Cox data transformation, see Granger and Hallman (1991), Franses and Koop (1998) and Franses and McAleer (1998), for example. For simplicity, we abstain from a discussion on these matters, and assume that first differencing, at least approximately, removes a stochastic trend. In sum, we consider testing for nonlinearity in the time series:

$$\nabla y_{\lambda,t} = \begin{cases} x_t^\lambda - x_{t-1}^\lambda & \text{for } 0 < \lambda \leq 1 \\ \ln(x_t) - \ln(x_{t-1}) & \text{for } \lambda = 0 \end{cases} \tag{4}$$

In this paper we choose $\lambda = 0, 0.05, 0.10, \ldots, 1.00$. So, for every original time series x_t we get 21 transformed series $\nabla y_{\lambda,t}$. For practical purposes one may conjecture that only a few values of λ (like 0, 1/2 and 1) are relevant. However, we consider more such values in order to be able to better examine any systematic patterns across increasing and decreasing values of λ and evidence of nonlinearity.

To test for nonlinearity in $\nabla y_{\lambda,t}$, we consider the test for STAR advocated in Teräsvirta (1994). This test aims to discriminate between the linear autoregressive (AR) model

$$y_t = \pi_1' w_t + \eta_t, \tag{5}$$

where y_t is shorthand for $\nabla y_{\lambda,t}$, as defined in (4), and the nonlinear logistic smooth transition autoregressive (LSTAR) model

$$y_t = \pi_1' w_t + f(\tilde{w}_t; \gamma, a, c)\pi_2' w_t + \eta_t \tag{6}$$

where $f(\tilde{w}_t; \gamma, a, c) = (1 + \exp(-\gamma(a'\tilde{w}_t - c)))^{-1} - \frac{1}{2}$, $\pi_i' = (\pi_{i0}, \ldots, \pi_{ip})$ for $i = 1, 2$, $\tilde{w}_t = (y_{t-1}, \ldots, y_{t-p})'$, $w_t = (1, \tilde{w}_t)'$, $a = (a_1, \ldots, a_p)'$. In both models $\eta_t \sim N(0, \sigma_\eta^2)$. By subtracting $\frac{1}{2}$ from the logistic function we get as the null hypothesis of linearity $H_0 : \gamma = 0$ with the alternative $H_1 : \gamma > 0$. Replacing $f(\tilde{w}_t; \gamma, a, c)$ by a first order Taylor expansion, an empirically useful test procedure is as follows:

- Determine the value of p by letting it run from 1 to p_{max} and choose that value for which the Akaike or Schwarz model selection criterion (AIC or BIC) attains its minimum value. In this paper we set $p_{max} = 4$.

- Regress y_t on w_t as in (5). Compute the residuals $\hat{\eta}_t = y_t - \hat{\pi}_1' w_t$, and the sum of squares $SSR_0 = \sum \hat{\eta}_t^2$.

- Estimate the parameters in the auxiliary regression

  $$\hat{\eta}_t = \phi' w_t + \sum_{j=1}^{p} \sum_{i=1}^{j} \phi_{ij} \tilde{w}_{ti} \tilde{w}_{tj} + \nu_t,$$

  where $\tilde{w}_{ti}$ and $\tilde{w}_{tj}$ are the ith and jth component of $\tilde{w}_t$ respectively. Compute the residual sum of squares $SSR_1 = \sum \hat{\nu}_t^2$.

- Compute the test statistic

  $$LM = \frac{T(SSR_0 - SSR_1)}{SSR_0}$$

Under the null hypothesis of linearity, this LM test statistic has a $\chi^2(\frac{p(p+1)}{2})$ distribution, see Teräsvirta (1994).

The transformed series $\nabla y_{\lambda,t}$ in (4) may be heteroscedastic for some values of λ. This may disturb the outcome of the linearity test. Therefore, we propose and apply a linearity test that should be robust to some form of heteroscedasticity. This new test is based on applying GLS to the AR model for $\nabla y_{\lambda,t}$. Here, we assume that the residuals $\hat{\epsilon}_t$ are distributed as

$$\epsilon_t \sim N(0, \sigma^2_{\epsilon,t}),$$

where $\sigma^2_{\epsilon,t} = \exp(\alpha_0 + \alpha_1 t)$ for some unknown α_0 and α_1. This assumption includes the cases of homoscedasticity ($\alpha_1 = 0$), exponentially growing variance ($\alpha_1 > 0$) and exponentially decaying variance ($\alpha_1 < 0$). We consider the following procedure:

- Fit an AR(p) to $\nabla y_{\lambda,t}$. In this paper, $p\,(\leq p_{max})$ is selected by the AIC only.
- Compute the residuals $\hat{\epsilon}_t$.
- Regress $\ln(\hat{\epsilon}_t^2)$ on a constant and time t. The regression parameters are $\hat{\alpha}_0$ and $\hat{\alpha}_1$.
- Calculate $\widehat{\ln(\sigma_t^2)} = \hat{\alpha}_0 + \hat{\alpha}_1 t$, and ${\hat{\sigma}_t}^2 = \exp(\widehat{\ln(\sigma_t^2)})$ for all t.
- Compute $z_t = y_t/\hat{\sigma}_t$. If the assumption on the form of heteroscedasticity is correct, this new time series variable is homoscedastic and therefore the above test procedure for nonlinearity can be applied to z_t.

In the next two sections we apply the two LM tests to variables $\nabla y_{\lambda,t}$, where we allow λ to vary over a range of values for empirical and simulated data.

4 Results

In Tables 5 and 7 we give the results of the linearity test for the monthly data and the original quarterly data, respectively. The total number of monthly time series is 64 and we have 53 quarterly time series. The monthly data can also be aggregated to quarterly data. The results of the linearity test to these aggregated time series can be found in Table 6. The three tables have eight columns for each test for nonlinearity (OLS and GLS). For the first two columns the order p in equation (5) is determined on the basis of the AIC and the level of significance for the LM test is 5%. The first column gives the results for all available time series. The second column gives the results for those series for which linearity is rejected for $\lambda = 0$. The third and fourth column again use the AIC but this time the level of significance is 10%. Again, the third column concerns alle series, while the fourth column concerns only those series for which for $\lambda = 0$ linearity is rejected. This also applies to columns six and eight for each test. In the fifth and sixth column we select the optimal AR order by the BIC, with a test level of 5%. In the last two columns again BIC is used with the level of significance put to 10%.

The results in Tables 5, 6 and 7 lead to several conclusions. The first is that the GLS-based test finds slightly less nonlinearity, as compared with the OLS-based test. Also, when BIC is used to select p we find slightly less evidence of nonlinearity than in case we use the AIC. A cause for this may be that BIC generally finds lower values of p. As a consequence, less regressors are included in the auxiliary regression of the linearity test, and thus nonlinearity may be more difficult to find.

The second result, based on comparing the percentage of series with evidence of nonlinearity in Tables 6 and 7 versus Table 5, is that we find more nonlinearity in monthly data than in quarterly data. This substantiates earlier findings in empirical work that perhaps one may opt for fitting nonlinear time series models to monthly data instead of to quarterly data.

With respect to the Box-Cox parameter we find mixed results across Tables 5, 6 and 7. For the monthly data we find most evidence of nonlinearity (in more than 60% of the cases) for λ taking values close to 0 or 1. The rejection frequencies display a certain U-shaped pattern, where minimum evidence of nonlinearity is found for λ close to 1/2. This may be partly due to heteroscedasticity for λ close to 1. When applying GLS this U-shape disappears. In that case we find slightly less evidence of nonlinearity for larger values of λ. For the quarterly data, however, we find more evidence of nonlinearity for larger values of λ. Additionally, if we consider all even columns of Tables 5, 6 and 7, that is those columns where nonlinearity is detected for those series with nonlinearity features for $\lambda = 0$, we obtain intruiging outcomes. It appears that when $\ln(x_t) - \ln(x_{t-1})$ is nonlinear, evidence of nonlinearity for $x_t^\lambda - x_{t-1}^\lambda$ tends to decrease, but not much. Hence, in some cases, the differenced levels can perhaps be described by a linear model instead of a nonlinear model.

For these series it may hold true that the log transformation introduces nonlinearity.

In sum, we find that evidence of nonlinearity changes with the value of the Box-Cox parameter. In the next section we will use simulation experiments to examine if the patterns observed in Tables 5, 6 and 7 can be replicated.

5 Simulations

In this section we study the performance of the LM tests for linearity across various values of λ in a controlled simulation experiment. We first consider the tests in case the data y_t are linear. Next, we study the case of nonlinear data.

We act as though a time series y_t is the result of a transformation of x_t by $y_t = x_t - x_{t-1}$ (indicated by $\lambda = 1$) or a transformation $y_t = \ln(x_t) - \ln(x_{t-1})$ (indicated by $\lambda = 0$) respectively. Hence, from y_t we construct x_t by $x_t = y_t + x_{t-1}$ or $x_t = \exp(y_t) * x_{t-1}$ respectively. We need a starting value for x_0. We can choose it freely, conditional on $x_t > 0$ for all t. As soon as the artificial time series x_t is constructed, we can deal with it the same way we did with the real time series in the former section. That is, we transform it to $\nabla y_{\lambda,t} = x_t^\lambda - x_{t-1}^\lambda$ for $\lambda = 0, 0.05, \ldots, 1.00$ and apply the linearity tests of section 3 to all 21 transformed series.

The data generating process (DGP) we use in the first set of experiments is an AR model. This way we can answer the question: if the DGP is linear, can values of λ be found where the linearity test concludes nonlinearity for $y_{\lambda,t}$? We choose the AR processes as:

$$
\begin{array}{llll}
\text{Model 1:} & \lambda = 1, & y_t = y_{t-1} - 0.8y_{t-2} + \epsilon_t, & x_0 = 10 \\
\text{Model 2:} & \lambda = 1, & y_t = 0.02 + 0.9y_{t-1} - 0.795y_{t-2} + \epsilon_t, & x_0 = 10 \\
\text{Model 3:} & \lambda = 0, & y_t = y_{t-1} - 0.8y_{t-2} + \epsilon_t, & x_0 = 1 \\
\text{Model 4:} & \lambda = 0, & y_t = 0.02 + 0.9y_{t-1} - 0.795y_{t-2} + \epsilon_t, & x_0 = 0.1
\end{array}
$$

From every model we generate 1000 realisations of length 300 and count the number of times we reject linearity. The result of this exercise can be found in Tables 8 - 11. From the last rows of Tables 8 and 9, and the first rows of Tables 10 and 11, we notice that the empirical size of the test is close to the nominal size, and also that applying GLS first does not lead to a size distortion. The tables show that when $x_t - x_{t-1}$ is linear ($\lambda = 1$), for a few series the log transformation introduces nonlinearity, although this increase is not very large. On the other hand, when $\ln(x_t) - \ln(x_{t-1})$ is linear, quite some evidence for nonlinearity is found with increasing values of λ (for both the OLS and GLS tests).

Our next set of experiments concern a nonlinear STAR model:

$$y_t = 1.8y_{t-1} - 1.06y_{t-2} + (1+\exp(-\gamma(y_{t-1} - 0.02)))^{-1} * (\pi - 0.9y_{t-1} + 0.795y_{t-2}) + \epsilon_t \quad (7)$$

where $\epsilon_t \sim NID(0, .02^2)$. This DGP is also used in Teräsvirta (1994). We are free to choose different values for π and $\gamma > 0$. In this paper we choose the following parameter values for (7):

Model 5:	$\lambda = 1,$	$\gamma = 100,$	$\pi = 0.02$	$x_0 = 10$
Model 6:	$\lambda = 1,$	$\gamma = 100,$	$\pi = 0$	$x_0 = 25$
Model 7:	$\lambda = 1,$	$\gamma = 20,$	$\pi = 0.02$	$x_0 = 10$
Model 8:	$\lambda = 1,$	$\gamma = 20,$	$\pi = 0$	$x_0 = 25$
Model 9:	$\lambda = 0,$	$\gamma = 100,$	$\pi = 0.02$	$x_0 = 0.01$
Model 10:	$\lambda = 0,$	$\gamma = 100,$	$\pi = 0$	$x_0 = 100$
Model 11:	$\lambda = 0,$	$\gamma = 20,$	$\pi = 0.02$	$x_0 = 0.1$
Model 12:	$\lambda = 0,$	$\gamma = 20,$	$\pi = 0$	$x_0 = 100$

For models 5 and 6 the results are unambiguous: linearity is rejected 1000 out of 1000 times in all cases. Therefore, we did not make tables for these models. The results for models 7-12 can be found in Tables 12 - 17. Clearly, there are not many differences between the OLS-based and GLS-based test results. We can draw several conclusions from these results. First of all, and comparing the results of models 5,6,9,10 with models 7,8,11,12 we find that a smaller value of γ yields less evidence of nonlinearity. This is likely to be due to the fact that the switching function in that case takes values closer to $\frac{1}{2}$, while when $\gamma = 100$, the values of 1 and 0 are approached more frequently. Secondly, comparing the odd models with the even ones, we observe that frequently a π value of 0.02 yields more evidence of nonlinearity, although the differences with $\pi = 0$ are not very large. Thirdly, comparing models 5-8 with 9-12, we notice that if $x_t - x_{t-1}$ is nonlinear, one frequently finds that $x_t^\lambda - x_{t-1}^\lambda$ is also nonlinear, independent of the value of λ. Hence, it seems that in that case the Box-Cox transformation does not reduce evidence of nonlinearity.

On the other hand, when $\ln(x_t) - \ln(x_{t-1})$ is nonlinear, we observe that less evidence of nonlinearity is found with increasing values of λ.

For model 11 we find the U-shaped pattern, as was found for the monthly data in the former section, but for other models this pattern is not found. To investigate whether the lack of the U-Shape is perhaps due to our choice of $\lambda = 0$ or $\lambda = 1$ in the DGP, we repeat our exercise for the case when $\lambda = 0.5$. We choose the following parameter values for (7):

$$\text{Model 13:} \quad \lambda = 0.5, \quad \gamma = 20, \quad \pi = 0 \qquad x_0 = 16$$
$$\text{Model 14:} \quad \lambda = 0.5, \quad \gamma = 20, \quad \pi = 0.02 \quad x_0 = 16$$

The results are presented in Tables 18 and 19, and we observe that the U-shape is also not encountered here. For $\pi = 0$ the frequency of rejection decreases when λ is increased. For $\pi = 0.02$ this frequency is highest for the true value $\lambda = 0.5$, and lowest around $\lambda = 0$ and $\lambda = 1$.

Finally, we address the question how the results from the simulation experiments shed light on the empirical findings in Tables 5, 6 and 7. Since the monthly series seem more informative, we choose to put more weight on the results in Table 5. The rejection frequencies in this table do not display a constant pattern across values of λ, and hence we may tentatively conclude that it seems best to consider the cases where $\lambda = 0$ and for which the time series are either linear or nonlinear. Subject to the restriction that a time series is nonlinear for $\lambda = 0$, we observe that the number of series where linearity is rejected decreases with increasing values of λ. Furthermore, subject to $\ln(x_t) - \ln(x_{t-1})$ being linear, we observe an increase in evidence of nonlinearity with increasing values of λ. In sum, changing the Box-Cox parameter can lead to different conclusions regarding nonlinearity.

6 Concluding remarks

The empirical and simulation results in this paper seem to suggest that a useful starting point in practice is to take natural logs (and thus consider growth rates after first differencing). When evidence for nonlinearity (perhaps based on the value of LM test statistics) decreases (given nonlinearity of the growth rates) for other values of λ, one gains additional confidence in the necessity to use a nonlinear model for the growth rates.

On the other hand, when such evidence increases if the growth rates appear linear, one may want to stick to a linear model. We also learned that studying nonlinearity is best done for disaggregated data and that quarterly data are perhaps less useful.

For the real-life monthly time series analyzed in this paper, we can conclude that many of these require a nonlinear model for description and forecasting, independent of the value of λ. This suggests that in practice one is still free to decide on the value of the Box-Cox parameter for a given time series at hand. Whether this decision influences key parameters in the subsequent nonlinear model and its out-of-sample point and interval forecast is a topic we aim to study in our further research.

Table 1: Monthly data: Money.

Series	Description	Period	Length
1	Commercial Paper Outstanding – All Issuers	70.01 - 93.06	282
2	Currency plus Demand Deposits	59.01 - 97.09	465
3	Currency Component of Money Stock Figure	47.01 - 97.09	609
4	Debt of Domestic Nonfinancial Sectors	59.01 - 97.08	464
5	Demand Deposits at Commercial Banks	59.01 - 97.09	465
6	Institutional Money Funds	74.04 - 97.09	282
7	Large Time Deposits at Commercial Banks	59.01 - 97.09	465
8	Large Time Deposits at Thrift Institutions	70.02 - 97.09	332
9	Liquid Assets	59.01 - 97.08	464
10	Large Time Deposits – Total	59.01 - 97.09	465
11	M1 Money Stock	59.01 - 97.09	465
12	M2 Money Stock	59.01 - 97.09	465
13	M3 Money Stock	59.01 - 97.09	465
14	Commerc. Paper Outstanding, Nonfinanc. Companies	70.01 - 93.06	282
15	Other Checkable Deposits	63.01 - 97.09	417
16	Retail Money Funds	73.11 - 97.09	287
17	Savings Deposits – Total	59.01 - 97.09	465
18	Small Time Deposits at Commercial Banks	59.01 - 97.09	465
19	Small Time Deposits – Total	59.01 - 97.09	465
20	Small Time Deposits at Thrift Institutions	59.01 - 97.09	465
21	Savings Deposits at Commercial Banks	59.01 - 97.09	465
22	Savings Deposits at Thrift Institutions	59.01 - 97.09	465
23	Savings and Small Time Deposits at Commercial Banks	59.01 - 97.09	465
24	Savings and Small Time Deposits – Total	59.01 - 97.09	465
25	Total Checkable Deposits	59.01 - 97.09	465
26	Total Time Deposits at Commercial Banks	59.01 - 97.09	465
27	Total Time Deposits at all Depository Inst.	59.01 - 97.09	465
28	Travelers' Checks Outstanding	59.01 - 97.09	465

Table 2: Monthly data: Employment.

Series	Description	Period	Length
1	Aggreg. Weekly Hours Indx: Private Nonfarm Payrolls	64.01 - 97.10	406
2	Aver. Weekly Hours: Private Nonagricultural Establ.	64.01 - 97.10	406
3	Civilian Employment–16 years and older	48.01 - 97.10	598
4	Civilian Participation Rate	48.01 - 97.10	598
5	Civilian Labor Force	48.01 - 97.10	598
6	Employment Ratio	48.01 - 97.10	598
7	Index of Help Wanted Advertising in Newspapers	51.01 - 97.09	561
8	Manufacturing Employment	46.01 - 97.10	622
9	Payroll Employment of Wage and Salary Workers	46.01 - 97.10	622
10	Employment in Service Producing Industries	46.02 - 97.10	621
11	Civilian Unemployed for 15 Weeks and Over	48.01 - 97.10	598
12	Civilians Unemployed for Less Than 5 Weeks	48.01 - 97.10	598
13	Median Duration of Unemployment	67.07 - 97.10	364
14	Unemployed – All Civilian Workers	48.01 - 97.10	598
15	Unemployment Rate	48.01 - 97.10	598
16	Employment in Construction	46.02 - 97.10	621
17	Employment in Finance, Insurance, and Real Estate	46.02 - 97.10	621
18	Employment in Goods Producing Sectors	46.02 - 97.10	621
19	Employment in Government	46.02 - 97.10	621
20	Employment in Mining	46.02 - 97.10	621
21	Employment in Services	46.02 - 97.10	621
22	Employment in Transportation and Public Utilities	46.02 - 97.10	621
23	Employment in Retail Trade Industry	46.02 - 97.10	621
24	Employment in Wholesale Trade Industry	46.02 - 97.10	621

Table 3: Monthly data: Income and Expenditures.

Series	Description	Period	Length
1	Disposable Personal Income	59.01 - 97.09	465
2	Real Disposable Personal Income	59.01 - 97.09	465
3	Personal Consumption Expenditures	59.01 - 97.09	465
4	Real Personal Consumption Expenditures	59.01 - 97.09	465
5	Pers. Cons. Expend.: Durable Goods	59.01 - 97.09	465
6	Real Pers. Cons. Expend.: Durable Goods	59.01 - 97.09	465
7	Pers. Cons. Expend.: Nondurable Goods	59.01 - 97.09	465
8	Real Pers. Cons. Expend.: Nondurable Goods	59.01 - 97.09	465
9	Pers. Cons. Expend.: Services	59.01 - 97.09	465
10	Real Pers. Cons. Expend.: Services	59.01 - 97.09	465
11	Personal Income	46.01 - 97.09	621
12	Personal Savings Rate	59.01 - 97.09	465

Table 4: Quarterly data.

Series	Description	Period	Length
1	Final Sales to Domestic Purchasers	59.1 - 97.3	155
2	Real State & Local Govn. Cons. Expend. & Gross Invest.	59.3 - 97.3	153
3	Compensation of Employees	59.1 - 97.3	155
4	Consumption of Fixed Capital (GNP)	59.1 - 97.3	155
5	Corporate Profits After Tax with Iva & CCAdj	59.1 - 97.2	154
6	Corporate Profits with IVA & CCAdj	59.1 - 97.2	154
7	National Defense Gross Investment	59.1 - 97.3	155
8	Real National Defense Gross Investment	59.3 - 97.3	153
9	Disposable Personal Income	59.1 - 97.3	155
10	Real Disposable Personal Income	59.3 - 97.3	153
11	Exports of Goods And Services	59.1 - 97.3	155
12	Real Exports of Goods & Services	59.3 - 97.3	153
13	Real Federal Cons. Expend. and Gross Invest.	59.3 - 97.3	153
14	Federal Government: Current Expenditures	59.1 - 97.3	155
15	Federal Government Receipts	59.1 - 97.3	155
16	Final Sales	59.1 - 97.3	155
17	Real Final Sales	59.3 - 97.3	153
18	Fixed Private Investment	59.1 - 97.3	155
19	Real Fixed Private Investment	59.3 - 97.3	153
20	Real Govn. Cons. Expend. and Gross Invest.	59.3 - 97.3	153
21	Gross Domestic Product	59.1 - 97.3	155
22	Real Gross Domestic Product Fixed	59.1 - 97.3	155
23	Real Gross Domestic Product Chained	59.3 - 97.3	153
24	Gross Domestic Product Chain-Type Price Index	59.3 - 97.3	153
25	Gross Domestic Product Implicit Price Deflator	59.1 - 97.3	155
26	Government Current Expenditures	59.1 - 97.3	155
27	Gross National Product	59.3 - 97.3	153
28	Real Gross National Product Fixed	59.1 - 97.3	155
29	Real Gross National Product Chained	59.3 - 97.3	153
30	Gross National Product Chain-Type Price Index	59.3 - 97.3	153

Table 4: *(continued).*

Series	Description	Period	Length
31	Gross National Product Implicit Price Deflator	59.1 - 97.3	155
32	Gross Private Domestic Investment	59.1 - 97.3	155
33	Real Gross Private Domestic Investment	59.3 - 97.3	153
34	Gross Priv. Domestic Invest. Chain-Type Price Index	59.3 - 97.3	153
35	Gross Private Savings	59.1 - 97.3	155
36	Government Receipts	59.1 - 97.3	155
37	Gross Savings	59.1 - 97.3	155
38	Indirect Business Tax and Nontax Liability	59.1 - 97.3	155
39	Imports of Goods & Services	59.1 - 97.3	155
40	Real Imports of Goods & Services	59.3 - 97.3	153
41	Federal Nondefense Gross Investment	59.1 - 97.3	155
42	Real Federal Nondefense Gross Investment	59.3 - 97.3	153
43	State & Local Government Current Expenditures	59.1 - 97.3	155
44	State & Local Government Gross Investment	59.1 - 97.3	155
45	Real State & Local Government Gross Investment	59.3 - 97.3	153
46	Nonfinancial Corporate Business Profits After Tax	59.1 - 97.3	155
47	National Income	59.1 - 97.3	155
48	Real Nonresidential Invest.: Producers' Durable Equip.	59.3 - 97.3	153
49	Personal Cons. Expend. Chain-Type Price Index	59.3 - 97.3	153
50	Real Private Nonresidential Fixed Investment	59.3 - 97.3	153
51	Real Private Residential Fixed Investment	59.3 - 97.3	153
52	Proprietors Income with IVA & CCAdj	59.1 - 97.3	155
53	Personal Saving	59.1 - 97.3	155

Table 5: Monthly data: number of series (out of the 64) for which linearity is rejected

	OLS								GLS							
λ	AIC95		AIC90		BIC95		BIC90		AIC95		AIC90		BIC95		BIC90	
0	48	48	53	53	46	46	52	52	48	48	51	51	42	42	47	47
0.05	50	48	54	52	47	45	52	51	47	47	51	51	42	42	47	47
0.10	50	47	55	52	48	45	53	51	46	46	51	51	42	42	47	47
0.15	50	46	55	52	45	41	51	49	46	46	51	51	42	42	47	47
0.20	51	46	54	51	44	41	49	47	45	45	49	49	40	40	46	46
0.25	51	46	54	51	43	40	49	47	45	44	49	49	39	38	46	46
0.30	51	46	54	51	42	38	47	45	45	44	50	49	39	38	48	47
0.35	50	44	54	50	40	35	49	45	45	44	50	49	39	38	47	46
0.40	47	41	53	48	39	34	45	41	46	44	50	49	39	37	45	44
0.45	47	40	51	47	42	36	46	42	47	45	50	48	40	37	46	43
0.50	47	40	51	46	41	36	48	43	46	44	50	48	40	37	46	43
0.55	46	39	52	47	41	36	49	43	46	44	51	48	40	37	46	42
0.60	47	38	51	46	43	36	47	42	46	44	51	48	38	36	46	42
0.65	48	39	51	46	43	36	47	42	46	43	49	47	39	36	45	42
0.70	48	39	51	46	44	35	47	42	46	43	50	47	39	36	44	41
0.75	48	39	51	45	45	36	48	42	46	43	50	47	40	36	45	41
0.80	49	39	51	45	46	36	48	42	45	43	49	46	39	36	45	41
0.85	49	39	52	46	46	36	48	42	45	43	48	46	40	36	44	41
0.90	49	39	52	46	47	37	49	43	44	42	49	46	40	37	45	42
0.95	49	38	52	46	47	37	49	43	43	41	48	45	39	37	44	42
1.00	50	39	54	47	48	38	51	44	44	41	48	45	40	37	45	43

Note: λ is the Box-Cox parameter for the time series as defined in (4). AIC95, for example, means that the order p in the test regression is selected by AIC and that the confidence level for the LM test is set at 95%.

Table 6: monthly data aggregated to quarterly: number of series (out of the 64) for which linearity is rejected

λ	OLS								GLS							
	AIC95		AIC90		BIC95		BIC90		AIC95		AIC90		BIC95		BIC90	
0	27	27	30	30	22	22	25	25	21	21	26	26	18	18	20	20
0.05	25	25	29	29	18	18	21	21	20	19	26	24	16	16	22	20
0.10	25	23	28	25	18	17	20	19	20	19	26	24	16	16	24	20
0.15	24	22	27	23	18	17	20	19	22	20	26	24	19	18	22	20
0.20	24	23	28	24	19	18	21	19	23	19	26	23	20	18	23	20
0.25	25	23	28	24	19	18	21	19	25	20	31	24	20	18	24	20
0.30	25	23	29	26	21	19	24	21	23	19	29	23	20	18	24	20
0.35	26	24	28	25	23	20	25	21	22	18	28	23	19	17	23	20
0.40	26	24	29	25	23	20	26	21	24	19	28	23	20	18	24	20
0.45	26	24	31	25	23	20	27	21	24	19	29	24	20	18	25	20
0.50	26	23	30	24	22	18	25	19	24	19	29	24	22	18	25	20
0.55	27	23	32	25	22	18	25	19	23	18	28	24	21	17	25	20
0.60	27	23	33	25	22	18	26	19	23	18	28	24	21	17	25	20
0.65	27	22	34	25	22	17	27	19	22	18	29	24	22	17	26	20
0.70	29	22	32	24	23	17	26	18	24	18	29	24	22	17	26	20
0.75	29	22	32	24	23	17	25	17	23	18	30	24	22	17	26	20
0.80	29	21	32	24	25	17	27	18	24	19	30	24	23	18	26	20
0.85	29	21	33	23	25	17	29	19	24	19	30	24	23	18	26	20
0.90	30	21	33	23	27	16	30	19	24	20	29	25	22	18	25	20
0.95	30	21	34	22	28	16	32	19	25	20	31	25	22	18	27	20
1.00	30	20	36	22	29	17	33	19	25	20	31	24	22	18	28	20

Note: see Table 5.

Table 7: Quarterly data: number of series (out of the 53) for which linearity is rejected

	OLS								GLS							
λ	AIC95		AIC90		BIC95		BIC90		AIC95		AIC90		BIC95		BIC90	
0	16	16	18	18	9	9	13	13	17	17	19	19	11	11	13	13
0.05	15	15	18	18	9	9	14	13	16	16	18	17	10	10	13	12
0.10	13	13	16	16	10	9	14	12	17	16	21	18	10	10	13	12
0.15	12	12	17	16	10	9	16	12	16	16	20	18	10	10	13	12
0.20	13	11	18	15	11	9	16	12	14	14	18	15	10	10	14	12
0.25	15	11	19	15	11	8	16	12	14	14	19	15	10	10	14	12
0.30	15	9	21	15	12	7	16	11	14	14	18	14	10	10	12	10
0.35	14	7	19	13	11	5	16	11	14	14	18	14	10	10	12	10
0.40	15	7	19	12	12	5	18	10	14	12	20	14	8	7	12	10
0.45	17	7	22	13	13	4	18	10	13	12	19	14	9	7	13	9
0.50	18	8	22	13	13	4	19	10	13	12	19	14	9	7	13	9
0.55	18	7	21	12	15	4	20	10	14	12	19	14	9	7	13	9
0.60	18	7	21	12	15	4	21	9	14	12	18	13	10	8	13	9
0.65	20	7	24	12	15	4	21	9	16	12	18	13	11	8	13	9
0.70	22	8	24	11	18	5	21	8	17	11	19	13	12	7	13	9
0.75	21	7	22	10	18	5	21	8	18	11	19	13	12	7	12	8
0.80	20	6	21	9	18	4	22	8	18	11	19	13	12	7	12	8
0.85	20	6	22	10	19	4	22	8	18	11	19	13	11	6	12	8
0.90	19	6	23	10	18	4	23	8	20	12	22	14	12	7	13	9
0.95	20	6	23	9	18	4	24	8	18	10	23	14	10	5	14	9
1.00	20	7	23	10	16	4	21	7	16	8	23	13	10	5	14	9

Note: see Table 5.

Table 8: Number of series (out of 1000) for which linearity is rejected, Model 1

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	55	123	56	121	52	107	53	95
0.05	55	123	56	120	50	105	52	94
0.10	54	122	55	119	51	104	52	94
0.15	54	123	55	120	51	103	51	94
0.20	54	122	55	120	50	103	48	94
0.25	54	121	55	119	50	103	48	93
0.30	54	121	55	119	49	100	47	90
0.35	54	121	55	117	50	101	47	91
0.40	54	121	55	117	50	101	47	92
0.45	54	122	55	118	49	99	47	92
0.50	52	121	53	117	49	98	47	92
0.55	51	118	52	114	49	98	47	92
0.60	51	117	51	113	50	100	46	94
0.65	51	116	51	112	50	100	46	95
0.70	51	119	51	114	50	100	46	94
0.75	51	118	51	113	50	102	46	95
0.80	51	118	51	113	50	102	46	95
0.85	51	115	50	110	50	105	46	97
0.90	51	115	50	110	49	105	45	97
0.95	50	114	49	109	49	103	45	96
1.00	49	115	48	107	49	104	45	97

Note: The data generating process is described in section 5. Here, the DGP is a linear AR model for a series with $\lambda = 1$. The last row can thus be viewed as the empirical size of the test.

Table 9: Number of series (out of 1000) for which linearity is rejected, Model 2

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	110	187	111	188	63	122	71	128
0.05	105	175	106	179	61	118	70	124
0.10	94	163	96	168	59	119	67	123
0.15	86	159	88	158	58	119	65	124
0.20	81	153	83	151	62	131	66	133
0.25	76	145	78	142	64	125	67	127
0.30	73	134	75	136	64	128	67	129
0.35	70	126	75	129	65	125	68	126
0.40	70	121	73	124	63	130	68	131
0.45	69	117	74	120	63	127	66	129
0.50	66	109	73	114	64	122	66	121
0.55	65	103	74	107	60	126	63	127
0.60	64	101	71	106	57	127	62	128
0.65	62	101	67	107	63	129	66	128
0.70	60	103	66	108	63	128	66	129
0.75	61	108	66	111	60	126	63	129
0.80	57	107	62	109	63	124	64	128
0.85	57	109	62	110	68	122	69	125
0.90	57	110	63	112	62	124	63	125
0.95	56	115	63	116	63	122	65	123
1.00	56	111	63	114	61	123	65	124

Note: See Table 8.

Table 10: Number of series (out of 1000) for which linearity is rejected, Model 3

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	60	119	54	115	59	109	59	111
0.05	63	123	56	117	60	108	59	109
0.10	64	122	57	113	61	112	59	117
0.15	62	118	59	112	61	116	59	118
0.20	69	126	67	121	61	128	63	129
0.25	76	138	74	131	63	138	67	138
0.30	81	140	79	134	75	146	76	143
0.35	92	151	92	146	89	151	89	153
0.40	100	159	99	154	94	163	94	167
0.45	107	177	108	170	101	177	105	183
0.50	123	191	120	188	110	198	119	207
0.55	135	208	136	208	127	215	136	218
0.60	147	234	151	235	145	234	155	240
0.65	168	257	173	264	161	256	169	264
0.70	186	279	190	288	178	286	190	297
0.75	204	299	208	313	201	318	213	335
0.80	227	326	237	337	221	341	233	361
0.85	245	354	260	364	246	368	262	384
0.90	271	395	292	404	274	389	288	407
0.95	297	425	318	434	309	415	328	430
1.00	338	452	353	469	339	444	360	463

Note: The data generating process is described in section 5. Here, the DGP is a linear AR model for a series with $\lambda = 0$. The first row can thus be viewed as the empirical size of the test.

Table 11: Number of series (out of 1000) for which linearity is rejected, Model 4

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	63	107	59	104	56	116	55	115
0.05	74	139	70	130	58	113	58	111
0.10	177	281	172	266	62	118	64	117
0.15	398	515	394	511	64	124	61	121
0.20	645	756	639	741	62	130	62	136
0.25	821	882	798	864	84	156	87	162
0.30	911	955	901	945	122	204	114	201
0.35	950	977	941	971	206	306	203	299
0.40	974	987	973	985	327	433	330	419
0.45	980	992	979	991	493	610	478	585
0.50	985	995	984	995	673	755	635	733
0.55	990	996	990	996	803	854	787	847
0.60	989	997	989	996	879	912	869	905
0.65	993	998	990	997	925	955	915	943
0.70	993	997	989	996	952	970	941	959
0.75	993	997	989	994	970	976	958	968
0.80	995	999	991	996	972	982	964	977
0.85	995	999	992	996	974	988	969	983
0.90	996	999	992	996	983	991	978	987
0.95	996	999	993	997	986	993	983	990
1.00	996	999	993	997	991	994	987	993

Note: See Table 10.

Table 12: Number of series (out of 1000) for which linearity is rejected, Model 7

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	899	937	910	943	933	970	953	976
0.05	900	937	912	944	931	968	952	975
0.10	901	940	914	947	930	968	951	976
0.15	901	938	915	946	930	965	951	976
0.20	901	937	917	948	930	966	950	975
0.25	901	939	916	949	932	967	952	975
0.30	904	940	918	949	933	964	953	973
0.35	905	942	918	951	933	964	953	973
0.40	904	943	920	955	932	963	951	972
0.45	905	943	922	957	931	963	949	973
0.50	906	944	924	958	929	964	948	973
0.55	910	948	926	959	927	965	946	974
0.60	911	947	926	958	928	964	947	973
0.65	914	948	929	959	926	966	945	974
0.70	916	949	931	961	927	966	945	975
0.75	919	948	935	960	926	968	944	976
0.80	922	952	938	961	926	968	945	977
0.85	922	954	938	962	927	966	945	977
0.90	923	955	938	963	926	965	945	976
0.95	922	956	937	963	926	965	945	975
1.00	922	956	938	962	925	965	943	977

Note: The data generating process is described in section 5. Here, the DGP is a nonlinear STAR model.

Table 13: Number of series (out of 1000) for which linearity is rejected, Model 8

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	987	991	991	996	982	989	983	989
0.05	987	992	991	996	982	989	983	989
0.10	987	992	991	996	980	989	983	989
0.15	987	992	991	996	980	989	983	989
0.20	987	992	991	996	979	988	982	989
0.25	987	992	991	996	979	988	981	989
0.30	987	992	991	996	979	987	981	988
0.35	987	992	991	996	979	986	981	988
0.40	987	992	991	996	979	987	981	987
0.45	987	992	991	996	979	987	981	987
0.50	987	992	991	996	978	987	980	987
0.55	987	992	991	996	978	987	980	987
0.60	987	992	991	996	978	987	980	987
0.65	987	992	991	996	978	987	980	987
0.70	987	992	991	996	978	987	980	987
0.75	987	992	991	996	978	987	980	987
0.80	987	992	991	996	978	987	979	987
0.85	987	992	991	996	978	987	979	987
0.90	987	992	991	996	978	987	979	987
0.95	985	991	991	995	978	987	979	987
1.00	984	991	990	995	978	987	979	987

Note: See Table 12.

Table 14: Number of series (out of 1000) for which linearity is rejected, Model 9

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	1000	1000	1000	1000	1000	1000	1000	1000
0.05	1000	1000	1000	1000	1000	1000	1000	1000
0.10	1000	1000	1000	1000	1000	1000	1000	1000
0.15	1000	1000	1000	1000	1000	1000	1000	1000
0.20	997	997	997	998	1000	1000	1000	1000
0.25	993	995	990	994	1000	1000	1000	1000
0.30	989	994	985	990	999	1000	999	1000
0.35	986	993	984	992	998	999	998	999
0.40	984	989	982	987	996	999	996	998
0.45	982	987	977	983	994	998	992	996
0.50	979	984	974	982	990	996	986	994
0.55	973	982	966	975	987	993	985	990
0.60	975	983	963	976	983	991	981	990
0.65	972	982	956	971	970	982	971	984
0.70	972	981	953	964	960	977	959	976
0.75	973	982	952	966	957	970	955	966
0.80	967	979	941	962	951	968	944	961
0.85	962	973	939	958	944	968	930	955
0.90	958	970	932	949	944	965	920	951
0.95	957	968	928	943	936	964	916	948
1.00	957	968	929	943	939	964	919	947

Note: See Table 12.

Table 15: Number of series (out of 1000) for which linearity is rejected, Model 10

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	1000	1000	1000	1000	1000	1000	1000	1000
0.05	1000	1000	1000	1000	1000	1000	1000	1000
0.10	1000	1000	1000	1000	1000	1000	1000	1000
0.15	1000	1000	1000	1000	1000	1000	1000	1000
0.20	1000	1000	1000	1000	1000	1000	1000	1000
0.25	1000	1000	1000	1000	1000	1000	1000	1000
0.30	1000	1000	1000	1000	1000	1000	1000	1000
0.35	1000	1000	1000	1000	1000	1000	1000	1000
0.40	1000	1000	1000	1000	1000	1000	1000	1000
0.45	1000	1000	999	1000	999	1000	1000	1000
0.50	999	1000	999	999	999	1000	1000	1000
0.55	999	1000	999	999	999	1000	999	1000
0.60	997	1000	998	999	999	1000	999	1000
0.65	997	998	997	998	998	999	998	999
0.70	995	998	995	998	996	999	997	999
0.75	993	995	992	995	997	998	997	998
0.80	992	995	988	994	995	998	994	998
0.85	993	995	987	991	993	997	994	997
0.90	991	995	985	991	991	994	991	997
0.95	989	994	982	991	987	993	988	996
1.00	989	993	985	990	989	996	988	995

Note: See Table 12.

Table 16: Number of series (out of 1000) for which linearity is rejected, Model 11

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	947	968	956	975	945	970	950	976
0.05	922	948	938	957	938	967	945	974
0.10	872	915	884	927	930	958	941	965
0.15	837	886	840	884	917	955	931	964
0.20	824	876	805	868	902	950	920	957
0.25	816	876	792	852	886	940	905	945
0.30	826	880	794	850	865	921	880	934
0.35	839	883	798	851	831	897	841	903
0.40	848	889	797	850	790	862	798	872
0.45	856	899	800	857	762	837	763	834
0.50	870	899	810	850	754	829	742	813
0.55	872	901	815	849	755	832	711	801
0.60	880	909	821	859	739	819	698	783
0.65	883	913	823	867	740	806	691	769
0.70	891	919	828	869	752	815	689	770
0.75	899	922	839	876	759	817	698	770
0.80	901	927	841	883	766	833	699	775
0.85	907	929	852	884	791	846	724	781
0.90	911	936	858	891	809	848	745	793
0.95	916	940	863	894	819	855	757	803
1.00	924	945	870	902	834	867	766	813

Note: See Table 12.

Table 17: Number of series (out of 1000) for which linearity is rejected, Model 12

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	981	992	988	994	970	981	973	984
0.05	977	989	983	994	962	976	968	979
0.10	966	984	973	988	958	973	963	977
0.15	958	975	963	980	945	968	949	971
0.20	943	969	949	973	934	962	942	967
0.25	937	958	937	962	923	956	933	960
0.30	922	948	926	953	912	945	922	949
0.35	913	942	917	940	902	939	913	943
0.40	899	937	902	932	887	928	900	932
0.45	886	931	889	927	875	921	886	927
0.50	873	927	877	920	854	909	870	915
0.55	866	920	863	911	834	897	853	905
0.60	860	912	853	901	815	881	836	890
0.65	848	913	834	901	798	871	818	879
0.70	839	906	829	895	779	854	797	864
0.75	833	903	820	888	766	849	783	861
0.80	822	896	808	878	758	838	771	849
0.85	819	892	799	871	747	825	760	839
0.90	824	889	803	871	739	825	747	830
0.95	826	882	802	865	733	824	744	822
1.00	821	878	796	861	745	823	747	815

Note: See Table 12.

Table 18: Number of series (out of 1000) for which linearity is rejected, Model 13

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	991	996	994	997	985	987	987	988
0.05	991	996	991	996	983	987	986	988
0.10	990	995	988	991	983	987	983	989
0.15	990	994	991	994	982	986	981	988
0.20	988	994	990	993	980	985	982	989
0.25	988	996	989	994	976	982	982	988
0.30	986	995	990	996	970	980	975	988
0.35	983	993	990	996	965	979	968	985
0.40	982	990	988	996	965	978	970	983
0.45	980	988	986	992	965	978	969	984
0.50	981	986	988	992	961	975	966	981
0.55	974	988	981	992	956	974	962	980
0.60	966	983	974	988	953	972	961	977
0.65	961	975	971	982	950	970	956	973
0.70	955	972	968	979	942	965	951	969
0.75	955	970	965	978	933	955	941	962
0.80	946	967	959	976	922	954	928	961
0.85	942	961	955	972	916	949	921	955
0.90	932	957	944	966	907	944	915	953
0.95	920	950	936	960	905	933	911	943
1.00	915	942	934	956	895	930	904	937

Note: See Table 12.

Table 19: Number of series (out of 1000) for which linearity is rejected, Model 14

	OLS				GLS			
λ	AIC95	AIC90	BIC95	BIC90	AIC95	AIC90	BIC95	BIC90
0	907	940	913	946	946	967	957	969
0.05	908	947	916	951	946	967	959	969
0.10	911	951	921	956	944	966	955	969
0.15	912	953	929	961	945	965	955	969
0.20	919	950	937	961	941	966	952	969
0.25	925	956	945	969	940	966	949	970
0.30	927	959	946	974	939	965	948	970
0.35	930	962	950	975	937	963	947	968
0.40	936	964	955	977	936	959	948	967
0.45	939	962	955	973	932	958	945	966
0.50	935	961	953	972	932	956	945	965
0.55	932	961	949	969	928	953	943	962
0.60	927	958	947	966	921	952	937	964
0.65	921	957	943	966	916	953	932	964
0.70	918	955	936	963	914	951	931	962
0.75	910	952	928	962	914	951	931	963
0.80	895	941	919	955	911	947	929	960
0.85	889	932	909	950	908	945	925	959
0.90	889	925	902	943	904	945	921	959
0.95	878	920	889	937	900	944	917	956
1.00	876	919	880	931	896	942	915	954

Note: See Table 12.

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