Forecasting volatility with switching persistence GARCH models

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Abstract

In this paper we examine the forecasting performance of five nonlinear GARCH(1,1) models. Four of these have recently been proposed in the literature, while the fifth model is a new one. All five models allow for switching persistence of shocks, depending on the value and/or sign of recent returns. We consider the models for weekly data on 5 major stock markets. Our results indicate that all models improve upon the linear GARCH(1,1) model and that our new model sometimes yields favorable forecasting results.

Keywords: Generalized AutoRegressive Conditional Heteroskedasticity, Forecasting, Stock market indices.

JEL classification: C22, C53, G15

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1 Introduction

A well-known characteristic of financial time series is that volatility changes over time. These changes tend to appear in clusters, which means that there are periods displaying high volatility and other periods with low volatility. Another characteristic is that financial series tend to be leptokurtic, i.e. its kurtosis exceeds the kurtosis of a standard normal distribution. In order to model these features Engle (1982) developed the AutoRegressive Conditional Heteroscedasticity model [ARCH] in which the variance of a series of financial returns is modeled in terms of past disturbances. Bollerslev (1986) introduced a generalization of this model, named GARCH, which additionally contains lagged conditional variances.

Both models, however, fail to cope with a further stylized fact, namely that returns are often skewed to the left. Negative outlying observations are larger than positive ones and are more likely to occur. Additionally, there is the leverage effect, which means that volatility is higher after negative shocks than after positive shocks of equal size. This observed asymmetry has led to the proposal of several modifications of the linear GARCH model. Because these models allow positive and negative shocks to have a different impact on future volatility, we will call them switching persistence GARCH models. Two of these models are discussed in the survey of Bollerslev, Engle and Nelson (1994).

In this paper we examine the forecasting performance of five switching persistence models. The first is the Quadratic GARCH model, proposed by Engle and Ng (1993) and Sentana (1995). The next model is that suggested by Glosten, Jagannathan and Runkle (1993), thus named GJR. This is a type of threshold model, where in the case of negative returns a term is added to the conditional variance. The third asymmetric model is introduced in Fornari and Mele (1996). It is called AARCH(1,1) and consists of a mixture of two nonlinear GARCH models. The fourth model is the so-called Logistic Smooth Transition ARCH developed by Hagerud (1996). This model uses a logistic function to take account of the asymmetry of the returns. Finally, we propose a new model. Our model extends the model of Hagerud (1996) by incorporating two transition functions. The first function concerns the asymmetry and the second function concerns switching persistence in the volatility. A motivation for our novel model is the fact that an estimated linear GARCH model often indicates high persistence of shocks, while a closer look at the data sometimes suggests that highly volatile periods do not last that long. We calculate volatility forecasts from these five models and compare these with a measure of volatility recently proposed in Andersen and Bollerslev (1998). Our application concerns weekly stock market data for five countries for a sample running from 1984 to 1996.

The outline of the paper is as follows. The next section reviews the four models proposed elsewhere in the literature, and our newly proposed model. Section 3 outlines the data used and discusses the empirical method. Section 4 contains the main results on forecasting, where we use four different criteria to evaluate the volatility forecasts. Section 5 concludes this paper with some remarks.

2 Switching persistence GARCH models

This section presents the nonlinear GARCH models we aim to consider in our forecasting experiment. The models have a similar structure. They consist of two equations, that is a conditional mean equation and a conditional variance equation. For all models the conditional mean equation for a return series y_t is

$$\begin{aligned} y_t &= \mu + \varepsilon_t, &(1)\\ \varepsilon_t &\sim N(0, h_t). &(2) \end{aligned}$$

The conditional variance equation for the GARCH (1,1) model is given by

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

see Bollerslev (1986). The GARCH(1,1) model is almost invariably found to be useful to describe financial time series, see Bera and Higgins (1993) and Bollerslev *et al.* (1994), among others. Estimated parameter values are typically in the range [0.1,0.2] for α and [0.8,0.9] for β, respectively. Also, $\alpha + \beta$ is often found to be very close to unity. Together, this implies that shocks can have high persistence. Notice that the GARCH(1,1) model is symmetric in the sense that positive and negative shocks have the same effect.

The first nonlinear variant of (3) we consider in this paper is proposed in Glosten *et al.* (1993), see also Rabemananjara and Zakoïan (1993). In this GJR model the conditional variance is modelled as

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \delta D_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{4}$$

where the dummy variable D_t is defined by

$$D_t = \begin{cases} 1 & \text{if } \varepsilon_t < 0, \\ 0 & \text{if } \varepsilon_t \geq 0. \end{cases} \tag{5}$$

The introduction of this dummy variable implies that negative returns have a different impact on volatility than positive returns, depending on the value of δ. When δ is positive, negative returns have more impact than positive returns. The Quadratic GARCH model is originally suggested by Engle and Ng (1993) and is also studied in Sentana (1995). This model assumes the following conditional variance equation:

$$h_t = \omega + \alpha(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1}. \tag{6}$$

A negative value of the parameter γ ensures that negative returns have a larger impact on volatility than a positive return of similar size. Franses and van Dijk (1996) compared the performance of the GJR model and the QGARCH model using a different measure of the volatility than the one we will apply below. These authors found that the QGARCH model outperforms both the GJR model and the GARCH model quite frequently. In the present study we will extend their work by investigating whether three other nonlinear models yield even further improvement.

The next model we consider is the so-called Augmented ARCH [AARCH] introduced by Fornari and Mele (1996). This model can be viewed as an extension of the

GJR model since its conditional variance equation is given by

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + (\kappa + \phi\varepsilon_{t-1}^2 + \psi\frac{\varepsilon_{t-1}^2}{h_{t-1}})S_{t-1}, \tag{7}$$

where the dummy variable S_t is defined by

$$S_t = \begin{cases} 1 & \text{if } \varepsilon_t > 0, \\ -1 & \text{if } \varepsilon_t \leq 0, \end{cases} \tag{8}$$

Fornari and Mele (1996) derive the stability regions of the various parameters in (7). The fourth model we examine is introduced in Hagerud (1996). In this model the conditional variance is assumed to be generated by

$$h_t = \omega + [\alpha_1 + \alpha_2 F_1(\varepsilon_{t-1})]\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{9}$$

The function $F_1(\cdot)$ is the logistic growth function, given by

$$F_1(\varepsilon_t) = (1 + \exp\{-\theta_1\varepsilon_t\})^{-1} - \frac{1}{2}, \qquad \theta_1 > 0. \tag{10}$$

When ε_t is close to zero, the transition function takes on a value close to zero and one approximates the standard linear GARCH model. In case ε_t is large and positive, the function $F_1(\cdot)$ approaches 1/2, and when ε_t is large and negative, the function $F_1(\cdot)$ approaches -1/2. Depending on the value and sign of α_2, this smooth transition ARCH [STARCH] can allow for asymmetric returns. In contrast to the GJR model, this STARCH model assumes smooth changes between the regimes.

The fifth and final model we consider here is a new one. This model extends the STARCH model by introducing a second switching function, that is, the conditional variance equation is now modelled as

$$h_t = \omega + [\alpha_1 + \alpha_2 F_1(\varepsilon_{t-1})]\varepsilon_{t-1}^2 + [\beta_1 - \beta_2 F_2(\varepsilon_{t-1}]h_{t-1}, \tag{11}$$

where $F_1(\cdot)$ is as in (10), but we assume that $F_2(\cdot)$ is defined by

$$F_2(\varepsilon_t) = (1 + \exp\{-\theta_2\varepsilon_t^2\})^{-1}, \qquad \theta_2 > 0. \tag{12}$$

Furthermore, we impose the parameter restrictions $\beta_1 - \beta_2 > 0$ and $\beta_1 - \beta_2/2 < 1$. From (11) it is clear that a large positive or negative shock leads to a smaller coefficient for the lagged variance ($\beta_1 - \beta_2$) than when ε_{t-1} is small ($\beta_1 - \beta_2/2$). The motivation for this is that conventional models have strong persistence. With a large shock the persistence in the GARCH(1,1) model, where $\alpha + \beta$ normally is in the range [0.75,0.95], leads to forecasting high volatility when in fact the volatile period is already over. Hence, our model allows for a substantial reduction in persistence as compared to the linear GARCH(1,1) model. In the sequel of our paper we will evaluate the out-of-sample forecasting performance of the above five models for the conditional variance.

3 Data and research method

In this section we give the data and the method we will use to evaluate the forecasting abilities of the linear and nonlinear GARCH models. The data, supplied by Datastream, cover 3393 daily observations from 31/12/83 to 31/12/96. The observations concern five stock market indices, namely the AEX (Amsterdam), DAX (Frankfurt), DJI (New York), FTSE (London) and the NIKKEI (Tokyo). To reduce the impact of outliers in the daily data, and also to remove holiday and weekend effects, we transform our data to weekly returns (Wednesdays). This amounts to 678 weekly returns, which are expressed as percentages.

In Table 1 we present some key statistics on these data. From the skewness and kurtosis it can easily be seen that these data (except perhaps for the DAX) contain outlying observations, as for example, the stock market crash of October 1987. To investigate these outliers more closely, we calculate the Median Absolute Deviation (MAD), instead of the standard deviation, in order to determine whether an observation can be classified as an outlier. The MAD is defined by

$$\text{MAD} = c \cdot \text{med}(|y_t - \text{med}(y_t)|), \tag{13}$$

where c is a constant equal to 1.4826.

An observation is an outlier when it exceeds the median plus or minus three times the MAD. We find 3 outliers for the AEX, none for the DAX, 2 for the DJI and NIKKEI and just one in case of the FTSE. After removal of these outliers, the resulting third and fourth moments improve, as can be seen from the second panel of Table 1. In our further analysis we will use the outlier corrected series.

We have 13 years of weekly returns data. To evaluate the out-of-sample forecasting preformance, we estimate the parameters of the six GARCH models (one linear, five nonlinear) for a sample containing six years of weekly returns. Next, we generate 52 one-step-ahead predictions for the subsequent year. Then, we move our sample one year ahead, and we repeat this exercise 6 times. In sum, this gives seven times 52 one-step-ahead forecasts for each model for each stock market.

Our volatility forecasts are to be compared with a measure of true volatility. Franses and van Dijk (1996) used a, by then fashionable, measure for true volatility (that is, squared returns) which, as Andersen and Bollerslev (1998) convincingly argue, is not a proper one. Therefore, in our present study, we use their alternative measure, that is,

$$\sigma_t^2 = \sum_{m=1}^{5} y_{(m),t}^2, \tag{14}$$

where $y_{(m),t}$ is the return on day m in week t; see Andersen and Bollerslev (1998). Notice that the intended use of this accumulated squared daily returns measure is another reason to consider weekly returns instead of daily returns.

To evaluate the differences between predicted volatility and true volatility as in (14), we use the following criteria:

- Median Squared Error (MedSE);
- Mean Squared Error (MSE);
- Mean Mixed Error for overprediction (MME(o));
- Mean Mixed Error for underprediction (MME(u)).

The first two criteria are well-known, so we only give the definitions for the mean mixed errors, which are proposed in Brailsford and Faff (1996). The MME(u) penalizes underpredictions more than overpredictions. The formula for the MME(u) is:

$$\text{MME(u)} = \frac{1}{T}\left(\sum_{t=1}^{T}\sqrt{\left|\hat{h}_t - \sigma_t^2\right| \cdot (1-U_t)} + \sum_{t=1}^{T}\left|\hat{h}_t - \sigma_t^2\right| \cdot U_t\right), \tag{15}$$

where T is the number of predictions, U_t is an indicator function such that $U_t = 1$ if $\hat{h}_t < \sigma_t^2$ (underprediction) and $U_t = 0$ otherwise (overprediction), and σ_t^2 are the volatilities as defined in (14). The formula for the MME(o) is analogous, and reverses the square root operator.

4 Forecasting results

The results of forecasting weekly volatility with various models for various years and four criteria are given in Tables 2 to 7. In Tables 2, 3 and 4 we present the results for the MedSE and MSE. In Tables 5, 6 and 7 we present those for the MME(o) and MME(u). Tables 2 and 5 contain the values of the criteria for all six models. Tables 3 and 6 give the values of the five nonlinear models relative to the linear GARCH model (a value below one indicates that a particular model is better). Table 4 and 7 give the rank of the six models (when evaluated against each other) and the total rank. All results are presented for each year, 1990 to 1996, and for each stock market.

A first major conclusion from all tables is that there is no single model that completely dominates the other models. Secondly, for some years (for example, 1990 and 1992), the differences between the forecasting abilities of the models differ to a large extent, while for other years (for example, 1994) the models almost perform equally well. Thirdly, the results in Table 4 and 7 suggest that there are not many differences between the various stock markets. A general finding is that the GJR and QGARCH model are the best performers, that the linear GARCH model is the worst (or second to worst), and that our newly proposed model does not beat the QGARCH or GJR model. Only for FTSE, there is some indication of success. Finally, it can be observed that these findings also hold across the evaluation criteria.

5 Conclusion

In this paper we evaluated the relative forecasting performance of the linear GARCH model, two well-known and established nonlinear GARCH models and three recently proposed nonlinear GARCH models (of which one was advocated in the present paper). Our results show that the linear model clearly gets rejected against the nonlinear models, and that the newly proposed models loose from the established ones. These findings hold across stock markets and evaluation criteria.

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Table 1: Descriptive statistics before and after the removal of outliers

	Mean	Median	Minimum	Maximum	St. dev.	Skewness	Kurtosis
Raw series							
AEX	0.20	0.39	−17.40	8.10	2.23	−1.27	11.08
DAX	0.19	0.31	−12.10	8.85	2.46	−0.50	4.67
DJI	0.24	0.41	−14.10	7.55	2.03	−0.94	8.19
FTSE	0.21	0.33	−24.90	7.95	2.21	−2.15	27.26
NIKKEI	0.10	0.32	−12.80	11.00	2.58	−0.32	6.04
Outliers removed[1]							
AEX	0.27	0.39	−9.30	8.10	2.04	−0.30	4.90
DAX	0.19	0.31	−12.10	8.85	2.46	−0.50	4.67
DJI	0.28	0.42	−8.08	7.55	1.91	−0.33	4.51
FTSE	0.24	0.33	−7.66	7.95	1.99	−0.03	4.01
NIKKEI	0.12	0.32	−11.20	11.00	2.53	−0.12	5.28

[1] In case of the AEX 3 outliers were removed, the DJI and NIKKEI had 2 outliers removed and the FTSE had only 1 outlier. For the DAX no outliers were found.

Table 2: MedSE and MSE for the predicted volatilities for the different models[1]

	MedSE						MSE					
AEX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	9.04	9.01	7.69	11.14	9.05	10.43	25.09	23.77	21.05	28.16	25.66	27.45
1991	6.14	5.79	6.04	6.09	6.83	6.08	41.08	40.53	40.65	40.93	40.00	42.28
1992	3.65	2.97	2.88	4.02	3.21	3.11	12.38	11.61	11.27	12.34	11.72	12.07
1993	1.18	0.65	0.83	1.00	0.51	0.57	2.77	3.25	2.89	2.81	2.93	3.20
1994	1.55	1.38	1.67	1.33	1.89	1.30	6.69	6.62	6.38	8.31	5.92	6.61
1995	1.25	1.43	1.34	1.65	1.60	1.47	2.15	2.21	2.14	2.33	2.32	2.19
1996	0.91	0.64	0.50	1.17	0.60	0.62	6.25	5.75	6.42	6.13	5.88	6.01
DAX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	15.53	19.87	22.01	18.53	18.58	19.18	119.22	102.09	105.84	131.28	100.23	112.27
1991	13.12	16.71	15.48	20.91	15.55	14.67	357.36	357.48	363.88	392.06	355.85	355.12
1992	8.56	8.84	8.58	11.03	8.25	8.61	20.07	18.01	18.61	21.92	17.68	18.60
1993	5.30	4.43	4.11	6.31	4.25	4.15	11.33	9.17	9.86	11.50	8.97	9.28
1994	3.68	3.81	3.57	4.21	4.61	3.95	15.16	14.33	14.36	16.00	14.54	14.41
1995	6.11	5.80	5.35	6.88	4.90	5.36	10.94	11.40	10.46	11.13	10.67	11.88
1996	3.03	3.05	2.30	4.33	2.86	3.32	13.62	13.79	14.02	14.53	14.07	13.58
DJI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	6.81	6.72	5.09	4.02	5.67	5.54	20.44	22.06	22.67	21.42	22.30	24.09
1991	7.91	7.17	7.01	9.57	7.71	5.66	19.75	19.33	18.56	20.71	22.41	22.92
1992	6.87	5.92	5.15	5.40	6.21	7.26	7.68	7.13	6.30	6.78	7.13	11.84
1993	1.59	2.10	2.87	1.41	2.60	4.81	2.34	2.66	3.68	2.08	3.15	8.87
1994	1.88	1.83	1.84	2.12	1.84	1.88	5.06	5.14	5.08	4.82	5.15	6.62
1995	1.75	1.68	1.80	2.18	1.69	1.32	2.55	2.61	2.74	2.53	2.62	2.55
1996	1.71	1.31	1.29	1.43	0.92	1.19	6.01	6.24	6.27	6.24	5.48	6.33
FTSE	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	4.30	3.67	5.53	1.61	3.20	3.70	14.24	12.30	12.62	14.40	12.39	12.68
1991	3.23	3.02	3.40	3.25	2.89	3.55	6.78	6.78	6.86	6.68	7.14	7.48
1992	6.92	4.44	6.23	9.56	4.73	3.75	56.04	50.46	50.57	55.62	56.26	50.23
1993	4.31	2.27	1.87	2.15	2.98	1.93	4.69	2.67	3.02	2.57	3.59	2.52
1994	1.79	2.75	2.89	2.03	2.37	2.94	4.37	6.71	4.92	5.92	6.47	9.02
1995	4.20	1.43	1.43	10.53	3.10	2.62	4.55	2.51	2.67	13.49	3.46	3.37
1996	1.02	0.86	0.75	0.98	1.55	0.52	1.69	1.62	1.51	1.64	2.08	1.50
NIKKEI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	31.18	21.71	26.39	29.41	29.71	24.83	643.52	477.36	604.18	712.36	431.75	432.13
1991	5.30	5.49	4.20	5.68	5.83	5.49	83.06	76.77	77.41	87.49	80.96	77.54
1992	44.66	31.94	27.89	37.17	37.36	40.66	336.27	307.60	305.94	354.05	303.62	305.97
1993	15.71	11.64	12.11	15.06	11.29	15.83	52.90	45.78	47.77	53.32	47.11	48.63
1994	7.64	8.88	9.10	9.35	11.62	14.98	73.72	73.50	80.32	72.07	70.18	86.02
1995	21.39	13.31	11.66	36.75	14.32	13.99	128.32	84.52	90.77	125.42	84.07	84.38
1996	9.09	4.78	3.60	7.61	2.76	3.17	17.04	14.52	13.72	15.67	15.71	14.63

[1] G11 denotes the linear GARCH(1,1) model, GJR the model in (4), Q11 that in (6), AAR is as in (7), LSM is (9) and NEW is our newly proposed model in (11).

Table 3: MedSE and MSE relative to the GARCH(1,1) model[1]

	MedSE					MSE				
AEX	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.997	0.851	1.232	1.001	1.154	0.947	0.839	1.122	1.023	1.094
1991	0.943	0.984	0.992	1.112	0.990	0.987	0.990	0.996	0.974	1.029
1992	0.814	0.789	1.101	0.879	0.852	0.938	0.910	0.997	0.947	0.975
1993	0.551	0.703	0.847	0.432	0.483	1.173	1.043	1.014	1.058	1.155
1994	0.890	1.077	0.858	1.219	0.839	0.990	0.954	1.242	0.885	0.988
1995	1.144	1.072	1.320	1.280	1.176	1.028	0.995	1.084	1.079	1.019
1996	0.703	0.549	1.286	0.659	0.681	0.920	1.027	0.981	0.941	0.962
DAX	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	1.279	1.417	1.193	1.196	1.235	0.856	0.888	1.101	0.841	0.942
1991	1.274	1.180	1.594	1.185	1.118	1.000	1.018	1.097	0.996	0.994
1992	1.033	1.002	1.289	0.964	1.006	0.897	0.927	1.092	0.881	0.927
1993	0.836	0.775	1.191	0.802	0.783	0.809	0.870	1.015	0.792	0.819
1994	1.035	0.970	1.144	1.253	1.073	0.945	0.947	1.055	0.959	0.951
1995	0.949	0.876	1.126	0.802	0.877	1.042	0.956	1.017	0.975	1.086
1996	1.007	0.759	1.429	0.944	1.096	1.012	1.029	1.067	1.033	0.997
DJI	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.987	0.747	0.590	0.833	0.814	1.079	1.109	1.048	1.091	1.179
1991	0.906	0.886	1.210	0.975	0.716	0.979	0.940	1.049	1.135	1.161
1992	0.862	0.750	0.786	0.904	1.057	0.928	0.820	0.883	0.928	1.542
1993	1.321	1.805	0.887	1.635	3.025	1.137	1.573	0.889	1.346	3.791
1994	0.973	0.979	1.128	0.979	1.000	1.016	1.004	0.953	1.018	1.308
1995	0.960	1.029	1.246	0.966	0.754	1.024	1.075	0.992	1.027	1.000
1996	0.766	0.754	0.836	0.538	0.696	1.038	1.043	1.038	0.912	1.053
FTSE	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.853	1.286	0.374	0.744	0.860	0.864	0.886	1.011	0.870	0.890
1991	0.935	1.053	1.006	0.895	1.099	1.000	1.012	0.985	1.053	1.103
1992	0.642	0.900	1.382	0.684	0.542	0.900	0.902	0.993	1.004	0.896
1993	0.527	0.434	0.499	0.691	0.448	0.569	0.644	0.548	0.765	0.537
1994	1.536	1.615	1.134	1.324	1.642	1.535	1.126	1.355	1.481	2.064
1995	0.340	0.340	2.507	0.738	0.624	0.552	0.587	2.965	0.760	0.741
1996	0.843	0.735	0.961	1.520	0.510	0.959	0.893	0.970	1.231	0.888
NIKKEI	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.696	0.846	0.943	0.953	0.796	0.742	0.939	1.107	0.671	0.672
1991	1.036	0.792	1.072	1.100	1.036	0.924	0.932	1.053	0.975	0.934
1992	0.715	0.624	0.832	0.837	0.910	0.915	0.910	1.053	0.903	0.910
1993	0.741	0.771	0.959	0.719	1.008	0.865	0.903	1.008	0.891	0.919
1994	1.162	1.191	1.224	1.521	1.961	0.997	1.090	0.978	0.952	1.167
1995	0.622	0.545	1.718	0.669	0.654	0.659	0.707	0.977	0.655	0.658
1996	0.526	0.396	0.837	0.304	0.349	0.852	0.805	0.920	0.922	0.859

[1] See Table 2.

Table 4: The rank of the models by MedSE and MSE[1]

	MedSE						MSE					
AEX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	3	2	1	6	4	5	3	2	1	6	4	5
1991	5	1	2	4	6	3	5	2	3	4	1	6
1992	5	2	1	6	4	3	6	2	1	5	3	4
1993	6	3	4	5	1	2	1	6	3	2	4	5
1994	4	3	5	2	6	1	5	4	2	6	1	3
1995	1	3	2	6	5	4	2	4	1	6	5	3
1996	5	4	1	6	2	3	5	1	6	4	2	3
Total	29	18	16	35	28	21	27	21	17	33	20	29
DAX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	1	5	6	2	3	4	5	2	3	6	1	4
1991	1	5	3	6	4	2	3	4	5	6	2	1
1992	2	5	3	6	1	4	5	2	4	6	1	3
1993	5	4	1	6	3	2	5	2	4	6	1	3
1994	2	3	1	5	6	4	5	1	2	6	4	3
1995	5	4	2	6	1	3	3	5	1	4	2	6
1996	3	4	1	6	2	5	2	3	4	6	5	1
Total	19	30	17	37	20	24	28	19	23	40	16	21
DJI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	6	5	2	1	4	3	1	3	5	2	4	6
1991	5	3	2	6	4	1	3	2	1	4	5	6
1992	5	3	1	2	4	6	5	3	1	2	3	6
1993	2	3	5	1	4	6	2	3	5	1	4	6
1994	4	1	2	6	2	4	2	4	3	1	5	6
1995	4	2	5	6	3	1	2	4	6	1	5	2
1996	6	4	3	5	1	2	2	3	5	3	1	6
Total	32	21	20	27	22	23	17	22	26	14	27	38
FTSE	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	5	3	6	1	2	4	5	1	3	6	2	4
1991	3	2	5	4	1	6	2	2	4	1	5	6
1992	5	2	4	6	3	1	5	2	3	4	6	1
1993	6	4	1	3	5	2	6	3	4	2	5	1
1994	1	4	5	2	3	6	1	5	2	3	4	6
1995	5	1	1	6	4	3	5	1	2	6	4	3
1996	5	3	2	4	6	1	5	3	2	4	6	1
Total	30	19	24	26	24	23	29	17	20	26	32	22
NIKKEI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	6	1	3	4	5	2	5	3	4	6	1	2
1991	2	3	1	5	6	3	5	1	2	6	4	3
1992	6	2	1	3	4	5	5	4	2	6	1	3
1993	5	2	3	4	1	6	5	1	3	6	2	4
1994	1	2	3	4	5	6	4	3	5	2	1	6
1995	5	2	1	6	4	3	6	3	4	5	1	2
1996	6	4	3	5	1	2	6	2	1	4	5	3
Total	31	16	15	31	26	27	36	17	21	35	15	23
Overall	141	104	92	156	120	118	137	96	107	148	110	133

[1] See Table 2

Table 5: MME (o) and MME(u) for the predicted volatilities for the different models[1]

	MME(o)						MME(u)					
AEX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	3.04	2.96	2.76	3.22	3.05	3.19	2.32	2.29	2.27	2.47	2.32	2.57
1991	2.74	2.72	2.75	2.73	2.70	2.80	2.25	2.17	2.23	2.19	2.18	2.36
1992	1.94	1.89	1.85	1.95	1.89	1.91	1.83	1.71	1.73	1.82	1.73	1.79
1993	1.22	1.28	1.22	1.22	1.21	1.26	1.18	0.99	1.04	1.07	0.92	0.97
1994	1.63	1.63	1.59	1.85	1.44	1.63	1.28	1.22	1.24	1.19	1.37	1.20
1995	1.12	1.13	1.12	1.13	1.13	1.13	1.15	1.22	1.20	1.24	1.22	1.19
1996	1.57	1.52	1.55	1.57	1.55	1.53	1.21	1.01	0.97	1.21	1.00	1.04
DAX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	5.29	5.03	5.26	5.69	4.70	5.02	4.19	3.89	3.87	3.85	4.04	4.15
1991	5.36	5.61	5.62	5.78	5.55	5.61	3.57	3.87	3.86	5.39	4.72	3.74
1992	2.36	2.23	2.22	2.44	2.23	2.24	3.06	3.03	3.14	3.22	2.95	3.02
1993	1.90	1.84	1.89	1.91	1.86	1.83	2.27	1.79	1.82	2.40	1.66	1.80
1994	2.20	2.25	2.17	2.39	2.20	2.26	2.01	2.33	1.96	1.99	2.29	2.04
1995	1.82	1.80	1.73	1.83	1.76	1.83	2.35	2.39	2.20	2.42	2.17	2.50
1996	2.05	2.00	2.00	2.08	2.07	1.99	1.81	1.78	1.63	2.14	1.77	1.78
DJI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	2.70	2.79	2.82	2.75	2.84	2.81	2.31	2.35	2.14	2.16	2.37	2.23
1991	2.38	2.37	2.39	2.45	2.52	2.40	2.50	2.43	2.31	2.68	2.51	2.59
1992	1.58	1.55	1.48	1.51	1.54	1.63	2.44	2.35	2.16	2.25	2.32	2.79
1993	1.15	1.20	1.29	1.15	1.25	1.54	1.33	1.43	1.74	1.22	1.57	2.31
1994	1.41	1.41	1.41	1.40	1.41	1.43	1.65	1.66	1.66	1.39	1.66	1.82
1995	1.13	1.13	1.15	1.15	1.13	1.11	1.33	1.32	1.41	1.39	1.33	1.27
1996	1.58	1.61	1.65	1.65	1.46	1.61	1.29	1.14	1.17	1.25	1.07	1.12
FTSE	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	2.07	1.97	2.03	2.08	2.02	2.02	2.23	1.79	2.10	1.48	1.68	2.00
1991	1.59	1.64	1.60	1.59	1.68	2.67	1.81	1.76	1.81	1.55	1.76	1.91
1992	3.22	2.95	3.04	3.22	3.07	2.92	2.75	1.93	2.28	2.86	2.51	1.85
1993	1.39	1.16	1.20	1.17	1.24	1.12	1.95	1.38	1.47	1.39	1.62	1.33
1994	1.41	1.49	1.42	1.76	1.45	1.57	1.47	1.95	1.59	1.23	1.84	2.16
1995	1.43	1.16	1.26	1.83	1.29	1.31	1.86	1.17	1.22	3.28	1.56	1.59
1996	1.03	0.99	0.96	1.02	1.09	0.92	1.01	0.99	0.93	0.98	1.20	0.86
NIKKEI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	11.47	9.89	11.07	12.38	9.38	9.37	5.84	4.71	5.10	3.84	4.96	4.78
1991	4.59	4.47	4.45	4.86	4.66	4.51	2.24	2.10	2.17	2.20	2.04	2.09
1992	9.22	9.36	8.83	9.84	9.08	9.21	4.76	3.85	3.71	4.11	4.14	4.09
1993	4.10	3.92	4.14	4.16	4.08	3.88	3.09	2.93	2.89	3.01	2.75	3.37
1994	3.16	3.30	3.42	3.15	3.24	3.60	3.21	3.07	3.06	3.39	3.64	3.69
1995	5.21	4.59	4.51	5.22	4.59	4.60	5.16	3.77	3.95	5.67	3.83	3.80
1996	2.43	2.23	2.16	2.38	2.27	2.22	2.58	2.14	1.98	2.24	2.01	2.01

[1] See Table 2.

Table 6: MME(o) and MME(u) relative to the GARCH(1,1) model[1]

	MME(o)					MME(u)				
AEX	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.974	0.908	1.059	1.003	1.049	0.987	0.978	1.065	1.000	1.108
1991	0.993	1.004	0.996	0.985	1.022	0.964	0.991	0.973	0.969	1.049
1992	0.974	0.954	1.005	0.974	0.985	0.934	0.945	0.995	0.945	0.978
1993	1.049	1.000	1.000	0.992	1.033	0.839	0.881	0.907	0.780	0.822
1994	1.000	0.975	1.135	0.883	1.000	0.953	0.969	0.930	1.070	0.938
1995	1.009	1.000	1.009	1.009	1.009	1.061	1.043	1.078	1.061	1.035
1996	0.968	0.987	1.000	0.987	0.975	0.835	0.802	1.000	0.826	0.860
DAX	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.951	0.994	1.076	0.888	0.949	0.928	0.924	0.919	0.964	0.990
1991	1.047	1.049	1.078	1.035	1.047	1.084	1.081	1.510	1.322	1.048
1992	0.945	0.941	1.034	0.945	0.949	0.990	1.026	1.052	0.964	0.987
1993	0.968	0.995	1.005	0.979	0.963	0.789	0.802	1.057	0.731	0.793
1994	1.023	0.986	1.086	1.000	1.027	1.159	0.975	0.990	1.139	1.015
1995	0.989	0.951	1.005	0.967	1.005	1.017	0.936	1.030	0.923	1.064
1996	0.976	0.976	1.015	1.010	0.971	0.983	0.901	1.182	0.978	0.983
DJI	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	1.033	1.044	1.019	1.052	1.041	1.017	0.926	0.935	1.026	0.965
1991	0.996	1.004	1.029	1.059	1.008	0.972	0.924	1.072	1.004	1.036
1992	0.981	0.937	0.956	0.975	1.032	0.963	0.885	0.922	0.951	1.143
1993	1.043	1.122	1.000	1.087	1.339	1.075	1.308	0.917	1.180	1.737
1994	1.000	1.000	0.993	1.000	1.014	1.006	1.006	0.842	1.006	1.103
1995	1.000	1.018	1.018	1.000	0.982	0.992	1.060	1.045	1.000	0.955
1996	1.019	1.044	1.044	0.924	1.019	0.884	0.907	0.969	0.829	0.868
FTSE	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.952	0.981	1.005	0.976	0.976	0.803	0.942	0.664	0.753	0.897
1991	1.031	1.006	1.000	1.057	1.050	0.972	1.000	0.856	0.972	1.055
1992	0.916	0.944	1.000	0.953	0.907	0.702	0.829	1.040	0.913	0.673
1993	0.835	0.863	0.842	0.892	0.806	0.708	0.754	0.713	0.831	0.682
1994	1.057	1.007	1.248	1.028	1.113	1.327	1.082	0.837	1.252	1.469
1995	0.811	0.881	1.280	0.902	0.916	0.629	0.656	1.763	0.839	0.855
1996	0.961	0.932	0.990	1.058	0.893	0.980	0.921	0.970	1.188	0.851
NIKKEI	GJR	Q11	AAR	LSM	NEW	GJR	Q11	AAR	LSM	NEW
1990	0.862	0.965	1.079	0.818	0.817	0.807	0.873	0.658	0.849	0.818
1991	0.974	0.969	1.059	1.015	0.983	0.938	0.969	0.982	0.911	0.933
1992	1.015	0.958	1.067	0.985	0.999	0.809	0.779	0.863	0.870	0.859
1993	0.956	1.010	1.015	0.995	0.946	0.948	0.935	0.974	0.890	1.091
1994	1.044	1.082	0.997	1.025	1.139	0.956	0.953	1.056	1.134	1.150
1995	0.881	0.866	1.002	0.881	0.883	0.731	0.766	1.099	0.742	0.736
1996	0.918	0.889	0.979	0.934	0.914	0.829	0.767	0.868	0.779	0.779

[1] See Table 2.

Table 7: The rank of the models by MME(o) and MME(u)[1]

	MME(o)						MME(u)					
AEX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	3	2	1	6	4	5	3	2	1	5	3	6
1991	4	2	5	3	1	6	5	1	4	3	2	6
1992	5	2	1	6	2	4	6	1	2	5	2	4
1993	2	6	2	2	1	5	6	3	4	5	1	2
1994	3	3	2	6	1	3	5	3	4	1	6	2
1995	1	3	1	3	3	3	1	4	3	6	4	2
1996	5	1	3	5	3	2	5	3	1	5	2	4
Total	23	19	15	31	15	28	31	17	19	30	20	26
DAX	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	5	3	4	6	1	2	6	3	2	1	4	5
1991	1	3	5	6	2	3	1	4	3	6	5	2
1992	5	2	1	6	2	4	4	3	5	6	1	2
1993	5	2	4	6	3	1	5	2	4	6	1	3
1994	2	4	1	6	2	5	3	6	1	2	5	4
1995	4	3	1	5	2	5	3	4	2	5	1	6
1996	4	2	2	6	5	1	5	3	1	6	2	3
Total	26	19	18	41	17	21	27	25	18	32	19	25
DJI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	1	3	5	2	6	4	4	5	1	2	6	3
1991	2	1	3	5	6	4	3	2	1	6	4	5
1992	5	4	1	2	3	6	5	4	1	2	3	6
1993	1	3	5	1	4	6	2	3	5	1	4	6
1994	2	2	2	1	2	6	2	3	3	1	3	6
1995	2	2	5	5	2	1	3	2	6	5	3	1
1996	2	3	5	5	1	3	6	3	4	5	1	2
Total	15	18	26	21	24	30	25	22	21	22	24	29
FTSE	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	5	1	4	6	2	2	6	3	5	1	2	4
1991	1	4	3	1	6	5	4	2	4	1	2	6
1992	5	2	3	5	4	1	5	2	3	6	4	1
1993	6	2	4	3	5	1	6	2	4	3	5	1
1994	1	4	2	6	3	5	2	5	3	1	4	6
1995	5	1	2	6	3	4	5	1	2	6	3	4
1996	5	3	2	4	6	1	5	4	2	3	6	1
Total	28	17	20	31	29	19	33	19	23	21	26	23
NIKKEI	G11	GJR	Q11	AAR	LSM	NEW	G11	GJR	Q11	AAR	LSM	NEW
1990	5	3	4	6	2	1	6	2	5	1	4	3
1991	4	2	1	6	5	3	6	3	4	5	1	2
1992	4	5	1	6	2	3	6	2	1	4	5	3
1993	4	2	5	6	3	1	5	3	2	4	1	6
1994	2	4	5	1	3	6	3	2	1	4	5	6
1995	5	2	1	6	2	4	5	1	4	6	3	2
1996	6	3	1	5	4	2	6	4	1	5	2	2
Total	30	21	18	36	21	20	37	17	18	29	21	24
Overall	122	94	97	160	106	118	153	100	99	134	110	127

[1] See Table 2