How to deal with intercept and trend in practical cointegration analysis?*

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Abstract

This note gives a few practical guidelines for cointegration analysis. The focus is on testing the cointegration rank in a VAR model and on how an intercept and a trend should be incorporated in the model. Only two cases appear relevant for most economic data.

*There is no new material in this note, and all results have been derived elsewhere. The discussion of the two relevant cases should however be useful for those who use standard packages like for example EViews. I thank Marius Ooms for bringing this issue to my attention, and Peter Boswijk, Richard Paap and Dick van Dijk for helpful comments.

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1 Introduction

Cointegration analysis is an important tool when modelling economic data with trends. Ever since its formal introduction in Engle and Granger (1987), it has been popular among practitioners and theorists. The current standard for analysis is the maximum likelihood method, based on a vector autoregression [VAR], proposed in Johansen (1988, 1995). Statistical packages like EViews incorporate this method and thus allow for its wide application.

Inference in cointegration models is not easy. There are many decisions to be made, and a good summary of these is given in Doornik, Hendry and Nielsen (1998). An important decision concerns the inclusion of deterministic terms in the cointegrating VAR. Results in Banerjee, Dolado, Galbraith and Hendry (1993), Johansen (1994), and Nielsen and Rahbek (1998) show that the statistical properties of the commonly used test procedure are affected, in the sense that its size cannot be controlled in some cases, and that there is substantial power loss in other cases. Much of this literature is of a technical nature, and not easy to read for many practitioners. It is the aim of this note to collect the main results and to give a few simple practical guidelines. Note again that nothing is really new in this note, and that part of the material is included in the excellent paper by Nielsen and Rahbek (1998). It merely summarizes the current state of knowledge for those who want to use the relevant routines in, for example, EViews (version 2.0 or 3.0).

In Section 2, I give preliminaries concerning univariate and multivariate unit root analysis. In Section 3, I consider the relevant cases for economic data. The prime focus is on testing the rank of the matrix containing the cointegrating relations. In Section 4, I conclude with some remarks.

2 Some preliminaries

This section contains some preliminaries concerning unit root testing. The focus is on the model representation when an intercept and trend are included. To save notation, I only consider autoregressive models of order 1. Of course, most results carry over to higher order models, although the computations are slightly different.

2.1 Univariate autoregression

Consider a univariate time series y_t, $t = 1, 2, \ldots, n$, when it can be described by

$$y_t - \mu - \delta t = \phi_1(y_{t-1} - \mu - \delta(t-1)) + \varepsilon_t, \tag{1}$$

where ε_t is assumed to be a standard white noise process. The behavior of y_t, when generated by (1), depends on the values of ϕ_1, μ and δ. When $|\phi_1| < 1$, y_t is a trend-stationary series. When both μ and δ are unequal to zero, one might say that y_t is attracted by $\mu + \delta t$. Informally stated, when forecasting y_{n+h} at time y_n, with h large, the forecast will approximately equal $y_{n+h} = \mu + \delta(n+h)$. If $\delta = 0$, this long-run forecast is μ.

When $\phi_1 = 1$ in (1), the model reduces to

$$y_t = \delta + y_{t-1} + \varepsilon_t. \tag{2}$$

Notice that μ in (1) is not identified when $\phi_1 = 1$. Recursive substitution results in

$$y_t = y_0 + \delta t + \sum_{i=1}^{t} \varepsilon_i, \tag{3}$$

where y_0 is a pre-sample starting-value. The $\sum_{i=1}^{t} \varepsilon_i$ component is called the stochastic trend. Notice from (3) that the long-run forecast of y_t equals $y_0 + \delta t$. In other words, a nonzero drift δ in (2), implies that this forecast is a function of an intercept and a linear deterministic trend, even though there is no such deterministic trend included explicitly in (2).

An alternative way of writing (1) concerns separating the long-run forecast (for both cases $|\phi_1| < 1$ and $\phi_1 = 1$) and the drift (for $\phi_1 = 1$), which results in

$$\Delta_1 y_t = \delta + \rho(y_{t-1} - \mu - \delta(t-1)) + \varepsilon_t, \tag{4}$$

where Δ_1 is defined as $(1 - L)$, with L the usual lag operator, and where $\rho = \phi_1 - 1$. This expression immediately shows that when $\rho = 0$, y_t has a stochastic trend with drift δ. It also indicates that when $\rho < 0$, (4) is a univariate equilibrium correction equation and y_t is a stationary AR(1) series with attractor $\mu + \delta t$.

In the univariate case it appears most easy to test for $\phi_1 = 1$ in yet another version of (1), which is

$$\Delta_1 y_t = \mu^* + \delta^* t + \rho y_{t-1} + \varepsilon_t, \tag{5}$$

with

$$m^* = (1 - \phi_1)\mu + \phi_1 \delta, \tag{6}$$

$$\delta^* = (1 - \phi_1)\delta. \tag{7}$$

This representation shows that the test regression includes the deterministic trend variable, even though it disappears under the null hypothesis of $\phi_1 = 1$. However, setting δ^* equal to zero implies that one imposes, *before* any test is carried out, that $\phi_1 = 1$ (which is what one aims to test) or that $\delta = 0$ (which means that the data have no trend). If the data do have a trend, the latter assumption is not plausible. So, the practical rule is to better include the trend in (5) even though it vanishes under the null hypothesis. Alternatively, one may use a joint test for $\rho = 0$ and $\delta^* = 0$, see Dickey and Fuller (1981), and it is exactly this procedure which is to be recommended for multivariate time series below.

2.2 Multivariate autoregression

Consider the VAR(1) model

$$Y_t = \Phi_1 Y_{t-1} + e_t, \tag{8}$$

for an $(m \times 1)$ time series Y_t containing $y_{1,t}$ through $y_{m,t}$, where e_t is a $(m \times 1)$ vector white noise series. For cointegration analysis it is convenient to write (8) in equilibrium correction format, that is,

$$\Delta_1 Y_t = \Pi Y_{t-1} + e_t, \tag{9}$$

where $\Pi = \Phi_1 - I_m$. The matrix Π contains information on cointegrating relations between the m elements of Y_t. In cointegration analysis it is common to write (9) as

$$\Delta_1 Y_t = \alpha\beta' Y_{t-1} + e_t, \tag{10}$$

where α and β are $(m \times r)$ full rank matrices. When $0 < r < m$, there are r cointegrating relations between the m variables, see Engle and Granger (1987) and Johansen (1995).

The maximum likelihood cointegration test method, developed in Johansen (1988) tests the rank of the matrix Π using the reduced rank regression technique based on canonical correlations. For model (10) this amounts to calculating the canonical correlations between $\Delta_1 Y_t$ and Y_{t-1}. This gives the eigenvalues $\hat{\lambda}_1 \geq \ldots \geq \hat{\lambda}_m$ and the corresponding eigenvectors $\hat{\beta}_1, \ldots, \hat{\beta}_m$. The most reliable test for the rank of Π is the likelihood ratio [LR] test statistic Q

$$Q = -n \sum_{i=r+1}^{m} \log(1 - \hat{\lambda}_i). \tag{11}$$

The null hypothesis is that there are at most r cointegration relations. Asymptotic theory for Q is given in Johansen (1995), and the critical values for this Q for model (10) are given in Table 15.1 in Johansen (1995).

Notice that the model in (10) assumes that the m time series do not have a trend, and that the cointegrating relations $\beta' Y_t$ have zero equilibrium values. This may however not be a reasonable assumption for many economic data. In the next section, I discuss two extensions of (10), which are often more useful.

3 Two relevant cases

In this section, I expand on the contents of Section 2.2 by incorporating an intercept and trend in the VAR model. The discussion closely follows that for the univariate case in Section 2.1, most notably equation (4).

3.1 None of the m time series displays a trending pattern

The imposed restriction that the cointegrating relations $\beta' Y_t$ in (10) all have an attractor which is exactly equal to zero does not seem plausible for many economic data. Hence, it is more appropriate to extend (10) as follows

$$\Delta_1 Y_t = \alpha(\beta' Y_{t-1} - \mu_1) + e_t. \tag{12}$$

To compute the LR statistic, one should now calculate the canonical correlations between $\Delta_1 Y_t$ and $(Y_{t-1}, 1)'$. The relevant asymptotic theory is given in Johansen (1995). The critical values of the corresponding LR test appear in Table 15.2 in Johansen (1995). This case corresponds with Option 2 in the relevant routine in EViews.

3.2 Some or all of the m time series display a trending pattern

When (some or all) series display trending patterns, one should consider a multivariate version of (4), which is

$$\Delta_1 Y_t = \mu_0 + \alpha(\beta' Y_{t-1} - \mu_1 - \delta_1 t) + e_t. \tag{13}$$

In words, this model allows the individual time series to have trends by not restricting μ_0 to zero, while the cointegrating relations attain their equilibrium values at $\mu_1 + \delta_1 t$. In very special cases, all parameters in δ_1 may equal 0, but it is safe not to assume that on beforehand.

To compute the LR statistic, one should calculate the canonical correlations between demeaned first differenced series and demeaned $(Y_{t-1}, t)'$. The relevant asymptotic theory is again given in Johansen (1995). The critical values of the LR test appear in Table 15.4 in Johansen (1995). This second case corresponds with Option 4 in the relevant routine in EViews.

In case one *a priori* assumes that $\delta_1 = 0$ in (13), one implicitly assumes that there are links between the deterministic growth patterns across the m individual time series. This assumption has an impact on the value of the LR test statistic and on its asymptotic distribution. The relevant theory is given in Johansen (1995), and the critical values appear in Table 15.3 in Johansen (1995). This case corresponds with (the default) Option 3 in EViews. However, as mentioned above, the assumption that $\delta_1 = 0$ may not be a sensible assumption for many economic data. The same holds for the assumption that $\mu_1 = 0$ and $\delta_1 = 0$ in (13) (which is case II in Franses (1998, Table 10.3)).

3.3 What if one wants to allow for quadratic trends?

From the discussion above it is immediately clear which model representation is most useful when testing the rank of Π while allowing for quadratic trends. A natural extension of (13) is now given by

$$\Delta_1 Y_t = \mu_0 + \delta_0 t + \alpha(\beta' Y_{t-1} - \mu_1 - \delta_1 t - \delta_2 t^2) + e_t. \quad (14)$$

To my knowledge the relevant asymptotic theory for (14) has not been developed yet, but is should follow the basic principles outlined in Johansen (1995). A restricted version of (14), for which similar cautionary remarks should be made as above, concerns the assumption that $\delta_2 = 0$. This model is again analyzed in Johansen (1995), and the relevant critical values appear in Table 15.5 of his book. In EViews, this model (with the possibly implausible parameter restriction) appears under Option 5.

4 Concluding remarks

To summarize, there seem to be only two relevant model representations for the analysis of cointegration amongst most economic time series variables. Statistical theory for these cases has been developed in Johansen (1995). They are included in the EViews (version 2.0) statistical package, under Options 2 and 4. This conclusion should not be interpreted as that the statistical theory of other models is not relevant. Merely, for most practical purposes there seem to be only two important cases.

Once the cointegrating rank has been fixed, the next steps in empirical model building can include tests for specific values of β and tests for the statistical relevance of r sets of deterministic regressors. An excellent treatment of many of these empirical issues is given in Doornik *et al.* (1998).

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