Testing Common Deterministic Seasonality, with an application to industrial production

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Econometric Institute Research Report 9905/A

Abstract

We propose methods to test for common deterministic seasonality, while allowing for possible seasonal unit roots. For this purpose, we consider panel methods, where we allow for individual and for common dynamics. To decide on the presence of seasonal unit roots, we introduce a decision-based approach, for which we derive the relevant critical values. We introduce an estimation method for our specific panel models. Our application concerns 16 quarterly industrial production series. One of our findings is that there is not much evidence for common deterministic seasonality.

JEL Classification: C12, C23, C44.

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1 Introduction

Many economic time series sampled at a subannual periodicity show remarkable seasonal patterns. In some main macroeconomic aggregates, seasonal variation is so strong that it dominates features at different frequencies, such as business cycles. In order to gain further insight into the causes and properties of the seasonal variation, it is of interest to compare the seasonal patterns across industries, regions, or countries and to assess to what degree they are common or idiosyncratic.

It is known that seasonality in economic time series can appear to be evolving and stochastic — sometimes clearly suggestive of non-stationary generating mechanisms — but it can also appear to be repetitive, fixed, and deterministic. Although some studies reveal interesting conclusions by only assuming deterministic seasonality (cf. Miron, 1996), many authors concur that the possibility of seasonal unit roots cannot be ignored (cf. Franses, 1996, Hylleberg, 1992, and others). The discriminatory power of existing tests that attempt to decide between stochastic and deterministic seasonal models (e.g. Hylleberg *et al.*, 1990 [HEGY], Canova and Hansen, 1995, Tam and Reinsel, 1997) is severely limited by the sample size. Therefore, additional insights are to be expected from considering a multitude of time series rather than a single one, see also Engle and Hylleberg (1996).

In this paper we focus on testing for common deterministic seasonal patterns, while allowing for the possible presence of seasonal unit roots. Hence, we build on the work in Miron (1996) and Miron and Beaulieu (1996) by considering formal statistical tests of common patterns across parameters. Also, our allowance of seasonal unit roots, while focusing on deterministic seasonality, is new. Thirdly, by considering several time series simultaneously, we aim to improve the discriminatory power of the statistical method. We illustrate our method on a set of quarterly industrial production series across sixteen countries, which are treated as a panel.

We consider two approaches that represent benchmark cases with respect to the assumptions about structural constancy of dynamics over the cross-section dimension. In the first approach, these dynamics are assumed as idiosyncratic and hence the panel decomposes into a collection of loosely connected time series. We call such models *individual-dynamics* models (ID), where the potential linkage may consist in common seasonal patterns. In the second approach, the country series are viewed as repeated measurements of the same dynamic process. Only level and seasonal constants are allowed to vary, hence this second approach resembles the traditional methods of panel analysis (see Hsiao, 1986, or Baltagi, 1995). We call such models *common-dynamics* models (CD).

In panel analysis, individuals and also time points are usually permitted to deviate from the basic relationship by unobserved level constants that may be treated as incidental parameters (fixed effects) or as random numbers drawn from a common distribution (random effects). We analyze an extension of this level-effects structure by incorporating seasonal constants. We demonstrate that the counterparts of the classical optimal estimation methods for fixed and for random effects, the least-squares dummy-variables (LSDV) and an appropriate GLS method, are the least-squares seasonal-dummies (LSSD) and a modified GLS method, respectively.

We choose to model the dynamics in the variables as low-order autoregressions, which is a natural starting point of time-series analysis. The discrimination procedure between seasonal unit roots and deterministic seasonality that is best adapted to autoregressive modeling is the one by Hylleberg *et al.* (1990), the so-called HEGY test. We follow Franses and Kunst (1999) who suggest to restrict the influence of deterministic seasonality in the presence of seasonal unit roots in order to avoid implausible expansion in the assumed data generation mechanism. We simulate critical points for HEGY–type tests, based on symmetric loss and a uniform weighting prior distribution. If we decide for seasonal unit roots, we restrict the seasonal constants. In the ID model, the resulting 16 individual autoregressions, partly with and partly without seasonal unit roots, yield a summary picture of similarities and differences in the deterministic seasonal structure across the investigated countries. In the CD model, there is only one autoregressive structure.

The outline of this paper is as follows. In Section 2, we discuss how seasonal intercepts can be interpreted in the case of seasonal unit roots in the autoregressive polynomial. Additionally, we introduce a decision-based method to decide for the presence of seasonal unit roots. In Section 3, we present the panel models, present details on the estimation routine, and outline hypotheses of interest for common seasonal deterministics. In Section 4, we apply our methods to 16 industrial production series. We find a clear rejection of common deterministic seasonality. In Section 5, we conclude with some remarks.

2 Interpreting seasonal dummies

Consider the availability of a panel of quarterly variables $y_{k,t}$ with the time index t running from 1 to T and the individual index k running from 1 to N. Here, we identify the individual index k with a particular country. Furthermore we assume that, out of the total sample of N, N_1 countries have deterministic seasonality and no seasonal unit roots and that the remaining $N - N_1 = N_2$ countries have seasonal unit roots. We introduce a modeling approach that separates deterministic seasonal cycles from seasonal unit roots. For the importance of such restrictions, see Franses and Kunst (1999) who demonstrate that the joint occurrence of deterministic and unit-root seasonality can generate expanding seasonal structures that may be implausible in the longer run.

We assume that the non-deterministic part of the process dynamics follows an autoregressive scheme of order p [AR(p)], therefore we have

$$y_{kt} = \sum_{j=1}^{p} \varphi_{kj} y_{k,t-j} + \varepsilon_{kt} \quad . \tag{1}$$

Using B to denote the lag operator, we may also write $\varphi_k(B)y_{kt} = \varepsilon_{kt}$ for (1). If $z = -1$ is a unit root of the autoregressive polynomial $\varphi_k(z) = 1 - \sum_{j=1}^{p} \varphi_{kj} z^j$, then $\sum_{j=1}^{p}(-1)^j \varphi_{kj} = 1$. If the complex pair $z = \pm i$ is a root of $\varphi_k(z)$, then formally $\sum_{j=1}^{p} i^j \varphi_{kj} = 1$ and hence, as $\varphi_k(z)$ is a real-coefficients polynomial, $\sum_{j=1}^{[p/2]}(-1)^j \varphi_{k,2j} = 1$ and $\sum_{j=1}^{[(p+1)/2]}(-1)^j \varphi_{k,2j-1} = 0$, where we use [.] to denote the largest integer function.

The data generation mechanism for the observed series is assumed to be (1) with added deterministic terms to the right-hand side, such as constants, trends, or seasonal cycles. A deterministic seasonal cycle can be represented by the equivalent parameterizations

$$\begin{aligned} d_{k1}D_{t1} + \ldots + d_{k4}D_{t4} &= g_{k0} + g_{k1}\cos(\pi t) + g_{k2}\cos(\pi t/2) \\ &\quad + g_{k3}\cos(\pi(t-1)/2), \end{aligned} \tag{2}$$

with the shorthand notation $D_{tj} = \delta^j_{t-4[(t-1)/4]}$, which are the usual seasonal dummies. Whereas the left-hand side is often easier to handle due to its symmetry over the quarters, the right-hand side gives an intuitive decomposition into the components at the spectral frequencies $\omega = 0$ (g_{k0}), $\omega = \pi$ (g_{k1}), and $\omega = \pi/2$ (g_{k2} and g_{k3}).

Franses and Kunst (1999) show that plausibility arguments typically require that $g_{k1} = 0$ if $\varphi_k(-1) = 0$ and $g_{k2} = g_{k3} = 0$ if $\varphi_k(\pm i) = 0$. These admissibility restrictions cannot be guaranteed in finite samples, as the existence of seasonal unit roots is not known exactly. This situation concerns a typical statistical decision problem. Spuriously restricting the model incurs under-specification of the deterministic part and impairs all subsequent statistical inference. Failing to impose the restrictions in the presence of seasonal unit roots leads to an efficiency loss but also to implausible longer-run predictions. The ultimate purpose of the model determines the loss involved in both events of misclassification. Because this purpose has not been stated explicitly, we opt for a simple loss function and assume that we want to maximize the frequency of correct classification.

The decision of whether a seasonal unit root is present or not is commonly based on a likelihood-ratio or similar hypothesis test. The simplest parametric test is the one suggested by Hylleberg *et al.* (1990), the so-called HEGY test. The HEGY test regresses seasonal differences ($\Delta_4 = 1 - B^4$) of the observed variables on lagged seasonal differences and on four variables that are only *partially* differenced. An example of such a partial differencing filter is $S(B) = 1 + B + B^2 + B^3 = (1 - B^4)/(1 - B)$, which is Δ_4 without the factor $\Delta = 1 - B$. The t–values of the coefficients of the partially differenced variables indicate whether the missing factor is a root of the autoregressive polynomial or not. Recently, some alternative tests for seasonal unit roots have been proposed that are based on unobserved-components models (Canova and Hansen, 1995) or on over-differencing (Tam and Reinsel, 1997). These tests are not based on a pure AR model, and therefore we will not consider them here.

The test we need here is a variant of the HEGY test, as we are not interested in testing for a unit root at $\omega = 0$. We further simplify notation by defining deterministic functions of time w_{jt},

$$w_{1t} = \cos(\pi t), \quad w_{2t} = \cos(\frac{\pi t}{2}), \quad w_{3t} = \cos(\frac{\pi(t-1)}{2}) \quad .$$

We suppress the subject index k if no confusion can arise. Presupposing a unit root at $+1$ and a lag order p in a level autoregression, the appropriate auxiliary regression for calculating the HEGY–type statistics is

$$S(B)\Delta y_t \;\; = \;\; g_0 + \sum_{j=1}^{3} g_j w_{jt} + a_1(1 + B^2)\Delta y_{t-1} + a_2(1 + B)\Delta y_{t-1}$$

$$+a_3(1+B)\Delta y_{t-2} + \sum_{j=1}^{p-3} \varphi_j S(B)\Delta y_{t-j} + \varepsilon_t \quad ,$$

$$t \quad = \quad 5 \wedge (p+1), \ldots, T \quad \text{and} \quad p \geq 3. \tag{3}$$

If $a_1 = 0$ then $\varphi(-1) = 0$ and if $a_2 = a_3 = 0$ then $\varphi(\pm i) = 0$. Critical values at some specified significance level for such a test can be obtained by simulation from the model $S(B)\Delta y_t = \varepsilon_t$. However, basing the decision on a fixed significance level results in an inconsistent classification as $T \to \infty$. Decreasing the significance level gradually would resolve this problem but nothing can be said about the optimal rate of decrease, unless an explicit decision model is specified. In other words, we impose $g_{k1} = 0$ if the observed t–value falls below a critical t–value of a_1 that increases as $T \to \infty$, and the same procedure is used for the root at $z = \pm i$. We now first turn to a discussion of how we obtain these critical values.

Using Bayes' theorem, hypothesis H_D, the deterministic seasonal model, is preferred to H_U, the unit-root model after observing X, if $P(H_D|X = x) > P(H_U|X = x)$ or

$$\frac{\int f(x|H_U, \theta_U)h_U(\theta_U)d\theta_U}{P_\pi(H_U)} < \frac{\int f(x|H_D, \theta_D)h_D(\theta_D)d\theta_D}{P_\pi(H_D)} \tag{4}$$

with P_π denoting the prior probabilities of the two hypotheses and $h_U(.)$ and $h_D(.)$ weighting priors over the corresponding parameter spaces. Fair priors imply $P_\pi(H_D) = P_\pi(H_U) = 0.5$ and yield a decision for H_U whenever $\int f(x|H_U)h_U(\theta_U)d\theta_U > \int f(x|H_D, \theta_D)h_D(\theta_D)d\theta_D$ and otherwise H_D. Notice that, in contrast to classical testing of nested hypotheses, there need not be any correspondence between θ_U and θ_D and the dimension may vary across hypotheses. In our case, some nuisance parameters can be interpreted as being common to both hypotheses, whereas the seasonal constants are specific to H_D and are absent under H_U.

Our prior setup for the determination of the relevant critical values consists of random drawings of processes with unit roots and the same number of random drawings without unit roots. The generated unit-root model is

$$\Delta_4 y_t = \mu + \varphi_1 \Delta_4 y_{t-1} + \varphi_2 \Delta_4 y_{t-2} + \varepsilon_t \tag{5}$$

for $t = 1, \ldots, 128$, including 6 starting values of 0. The sample size $T = 128$ is inspired by the empirical example (see Section 4). The pair of coefficients (φ_1, φ_2) is drawn from a uniform distribution $U(S_2)$ on the triangular stable region of second-order autoregressions. The generated deterministic-seasonality model is

$$\begin{aligned}\Delta y_t &= \mu + \varphi_1 \Delta y_{t-1} + \varphi_2 \Delta y_{t-2} + \\ & \quad \delta_1 w_{1t} + \delta_2 w_{2t} + \delta_3 w_{3t} + \varepsilon_t \quad ,\end{aligned} \tag{6}$$

with (φ_1, φ_2) drawn from the same distribution as above and $\delta_l, l = 1, 2, 3$ drawn independently from standard normal distributions. For both models, we use an $n.i.d.(0, 1)$ feeding process $\{\varepsilon_t\}$. As a sensitivity check, the models are also generated on the basis of $\varphi_2 = 0$, a $U(S_1) = U(-1, 1)$ distribution on φ_1, and an estimated first-order autoregression. The results are not very sensitive to the change and a second-order autoregression better accommodates our empirical examples, hence we do not pursue these extensions. Notice that the unit-root model has a higher lag order, which also corresponds to the empirical examples below. Like the seasonal coefficients δ_l, the unbounded nuisance parameter μ is also generated from a standard normal $N(0, 1)$ distribution. From some macroeconometric examples, see KUNST (1993), we know that the order of magnitude in the variation of μ, in the seasonal constants, and in the errors is comparable, though it seems that the variability in seasonal constants tends to be somewhat higher by a factor of around 3 to 5. Nevertheless, the decision setup appears to be justified. This defines $h(.)$.

The differences in densities $f(\tau_l|H_U) - f(\tau_l|H_D)$ are displayed in Figures 1,2,3 for the HEGY–type t–statistics τ_l at $a_l, l = 1, 2, 3$. These plots were preferred to plots of the odds ratios $f(\tau_l|H_U)/f(\tau_l|H_U)$ because of a better visual resolution. This modification does not affect the critical points. Based on 50,000 replications, the critical values are approximately at 2.9, 2.9, and 3.3. These values are determined by the points where the difference curves pass the abscissa axis.

Note the fat left tails in Figures 2 and 3, which are caused by combinations of coefficients in the deterministic model that are close to the boundary $\varphi_2 = -1$ in conjunction with negative φ_1 and relatively large seasonal constants. Such designs generate seemingly unstable behavior and negative values of τ_2 and τ_3. These in turn lead to a seeming dominance of the deterministic model for such values. In contrast to the cases of τ_1 and τ_3, the asymptotic distribution of τ_2 is symmetric under H_U and negative values occur under a stationary classical alternative (see HEGY). However, while the simulated small-sample density $f(\tau_2|H_U)$ is approximately symmetric around 0, the density $f(\tau_2|H_D)$ has most of its mass close to $\tau_2 = 6$. In our empirical example below, all values for $\tau_l, l = 1, 2, 3$, are positive. Hence, we do not view the left tails as of high empirical relevance and we consider one-sided tests only.

We close this section with a brief discussion on the representation of seasonal intercepts in the presence of seasonal unit roots. In order to simplify notation, we introduce the switching factors

$$\zeta_{k1} = I\{\varphi_k(-1) \neq 0\}, \quad \zeta_{k2} = \zeta_{k3} = I\{\varphi_k(i) \neq 0\} \quad .$$

These factors become 0 if there are seasonal unit roots at the corresponding frequencies. Hence the following model

$$\begin{aligned} \Delta y_{kt} &= g_{ko} + \sum_{j=1}^{3} g_{kj}\zeta_{kj}w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}, \\ t &= p + 1, \ldots, T, \quad k = 1, \ldots, N, \end{aligned} \tag{7}$$

is now designed to avoid seasonal cycles that expand as $T \to \infty$. The factor ζ_{k1} is estimated by $I\{\tau_{k1} > \tau^*_{1,T}\}$ and $\zeta_{k2} = \zeta_{k3}$ by $I\{\tau_{k2} > \tau^*_{2,T}\} \cup I\{\tau_{k2} > \tau^*_{2,T}\}$, where τ_{kj} are the HEGY–type t–statistics for the coefficients a_{kj} and where $\tau^*_{j,T}$ are the simulated critical values.

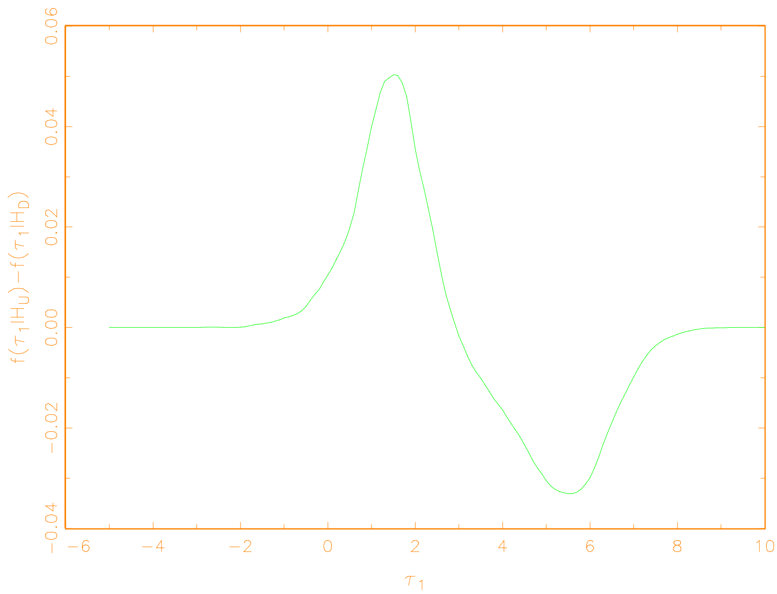

Figure 1: Difference of densities $f(\tau_1|H_U) - f(\tau_1|H_D)$ for $T = 128$. Positive values favor a unit root at -1.

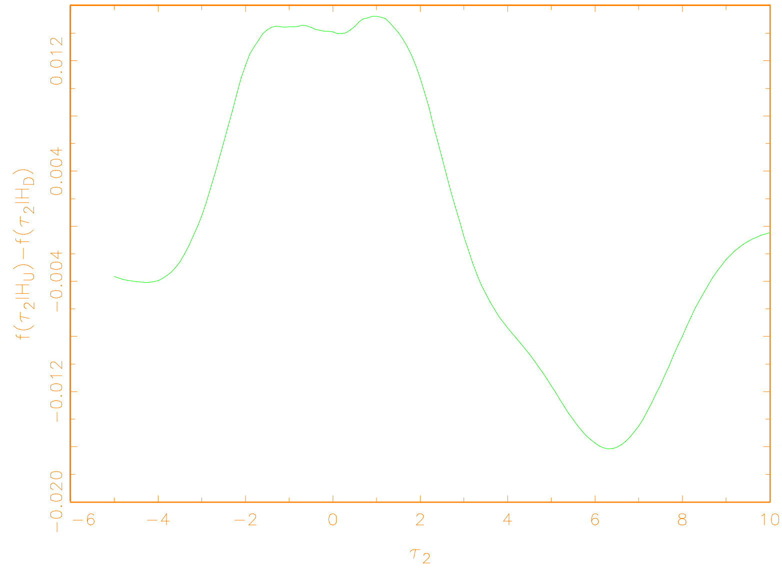

Figure 2: Difference of densities $f(\tau_2|H_U) - f(\tau_2|H_D)$ for $T = 128$. Positive values favor a unit root at $\pm i$.

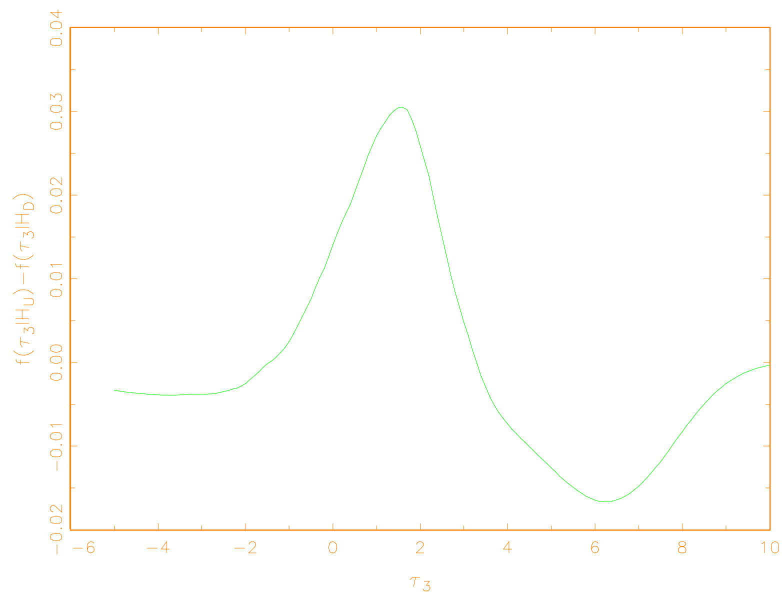

Figure 3: Difference of densities $f(\tau_3|H_U) - f(\tau_3|H_D)$ for $T = 128$. Positive values favor a unit root at $\pm i$.

3 Panel models

In our dynamic panel with *individual dynamics* (ID), we consider the following model:

$$\Delta y_{kt} = \sum_{j=1}^{4} d_{kj} D_{tj} + \sum_{j=1}^{p} \varphi_{kj} \Delta y_{k,t-j} + \mu_k + \varepsilon_{kt}, \quad t = p+2, \ldots, T.$$

The errors ε_{kt} are assumed as *i.i.d.* $N(0, \sigma_\varepsilon^2)$ noise. In order to achieve identification, one could either impose the restriction $\sum_{j=1}^{4} d_{kj} = 0$ or omit μ_k. In the following, we adopt the latter convention. One may imagine that d_{kj} are drawn independently from a joint Gaussian density such that $(d_{k1}, d_{k2}, d_{k3}, d_{k4})' \sim N(0, \sigma_d^2 \mathbf{I}_4)$. An alternative model to this *seasonal random-effects model* is the *seasonal fixed-effects model* that treats the seasonal constants $d_{kj}, j = 1, \ldots, 4$ as transient parameters. Although hybrid models may be plausible in some applications, we will assume that seasonal random effects come with random individual level constants and, similarly, seasonal fixed effects accompany fixed individual level effects.

Unless additional explanatory variables appear in the equations, the distinction between random and fixed effects is hardly relevant to the ID model, as it does not affect the efficient estimation of the coefficient parameters d_{kj}, φ_{kj}. We therefore keep the random-effects model as a reference model.

If $\varphi_k(B)$ contains seasonal unit root factors, it was discussed above that additional restrictions on $(d_{k1}, \ldots, d_{k4})$ should hold as otherwise unreasonable seasonal expansions are created by the model. If, for example, the 'full' unit-root factor $(1 + B + B^2 + B^3)$ is contained in $\varphi_k(B)$, these restrictions impose $(d_{k1}, \ldots, d_{k4}) = (0, \ldots, 0)$, while μ_k is left unrestricted.

In the classical panel model (see Hsiao, 1986, or Baltagi, 1995)

$$y_{kt} = a' Z_{kt} + \mu_k + \varepsilon_{kt},$$

it is well known that under the 'random-effects' assumption $\mu_k \sim N(\mu, \sigma_\mu^2)$ the efficient estimator for a is a GLS estimator that can be approximated by feasible GLS. This follows from the fact that the aggregate error $\mu_k + \varepsilon_{kt}$ is unobserved and that the correlation structure of $\mu_k + \varepsilon_{kt} - \mu$ is $\sigma_\mu^2 I_N \otimes E_T + \sigma_\varepsilon^2 I_{NT}$, where we use the symbol E_T to denote a $T \times T$–matrix of ones. In the 'fixed-effects' model, the μ_k are treated as incidental (or transient) parameters, not as unobserved variables. Then, the optimal method is to 'purge' all variables y and Z by subtracting means over time and estimate the resulting equation $y_{kt} - \bar{y}_{k.} = a'(Z_{kt} - \bar{Z}_{k.}) + u_{kt}$ by OLS. If the time means correspond to their population averages, it holds that $u_{kt} = \varepsilon_{kt}$.

3.1 Seasonal fixed effects

The seasonal panel can be handled in analogy with the classical panel with level effects. If the dummies are treated as incidental, quarter-by-quarter purging will result in the optimal estimation method. Quarter-by-quarter purging is achieved by pre-multiplication of all variables by $I_{NT} - I_N \otimes \bar{E}_{T/4} \otimes I_4$, where we use the notation $\bar{E}_n$ to represent $n^{-1}E_n$. The resulting estimator is called the *least-squares seasonal-dummies* (LSSD) estimator.

3.2 Seasonal random effects

If the dummy coefficients are drawn from a common distribution, we may consider the following model. For each k, there is a relationship between the $(T-p) \times 1$–vector of data without the first p observations, that is, $y_k = (y_{p+1,k}, \ldots, y_{T,k})'$, and its lags, or

$$y_k = \varphi_{k1} y_{k,-1} + \ldots + \varphi_{kp} y_{k,-p} + Dd + v_k \tag{8}$$

The symbol $y_{k,-j}$ denotes the shifted vector of observations $(y_{p+1-j,k}, \ldots, y_{T-j,k})'$. It is useful to include overall seasonal means via the $(T-p) \times 4$–matrix D and the coefficients d. These objects do not change across the N individuals. Indeed, the seasonal dummy-type influence is assumed to play different roles for different individuals, just like the lag structures. Because the seasonal constants are assumed to be drawn from a common probability distribution, the error can be decomposed as

$$v_k = s_k + u_k \quad , \tag{9}$$

where s_k is the 4–periodic vector of seasonal constants and u_k contains the remainder error which is uncorrelated over time *and* over individuals.

In more compact notation we can write $Z_k = (y_{k,-1}, \ldots, y_{k,-p})'$ for the $(T-p) \times p$ regressor matrix and $\varphi_k = (\varphi_{k1}, \ldots, \varphi_{kp})'$ for the vector of autoregressive coefficients. This yields

$$y_k = Z_k' \varphi_k + Dd + v_k \tag{10}$$

or for the whole system of N countries

$$
\begin{aligned}
y^* &= Z^*\varphi^* + D^*d + v^* \\
v^* &= s^* + u^*
\end{aligned}
\tag{11}
$$

where $y^* = (y_1^{'}, \ldots, y_N^{'})'$ is an $N(T-p)$–vector. $Z^* = \mathrm{diag}(Z_1, \ldots, Z_N)$, φ^* is a pN–vector, and D^* is a $(T-p)N \times 4$ matrix obtained by N times stacking the matrix D. This equation system could also be generalized to accommodate the case of different individual lag orders as $\sum p_k$.

The specialty of panel models is the assumption on the error variance structure which identifies the unobserved components s^* and u^*:

$$
\begin{aligned}
Es^*s^{*\prime} &= \sigma_d^2(I_N \otimes E_{T/4} \otimes I_4) = \Omega_1 \\
Eu^*u^{*\prime} &= \sigma_u^2 I_{TN} = \Omega_2
\end{aligned}
\tag{12}
$$

In analogy to the classical random-effects model, the total error variance matrix $\Omega = \Omega_1 + \Omega_2$ is inverted more easily in a different, the so-called *spectral*, decomposition $(T\sigma_d^2/4 + \sigma_u^2)\bar{\Omega}_1 + \sigma_u^2\bar{\Omega}_2$ with orthogonal component matrices. The singular $\bar{\Omega}_1$ contains the seasonal individual averaging operation, whereas the singular $\bar{\Omega}_2$ describes the purging of all observations from their individual seasonal averages, i.e.

$$
\bar{\Omega}_1 = I_N \otimes \bar{E}_{T/4} \otimes I_4, \quad \bar{\Omega}_2 = I_{TN} - I_N \otimes \bar{E}_{T/4} \otimes I_4
$$

For the GLS solution we have, using $Z_e^* = (D^*, Z^*)$,

$$
\begin{aligned}
\left(\hat{d}, \hat{\varphi}^{*\prime}\right)' &= (Z_e^{*\prime}\Omega^{-1}Z_e^*)^{-1}Z_e^{*\prime}\Omega^{-1}y^* \\
&= (Z_e^{*\prime}\left\{(T\sigma_d^2/4 + \sigma_u^2)^{-1}\bar{\Omega}_1 + \sigma_u^{-2}\bar{\Omega}_2\right\}Z_e^*)^{-1} \\
&\quad \times Z_e^{*\prime}\left\{(T\sigma_d^2/4 + \sigma_u^2)^{-1}\bar{\Omega}_1 + \sigma_u^{-2}\bar{\Omega}_2\right\}y^* \\
&= (\theta Z_e^{*\prime}\bar{\Omega}_1 Z_e^* + Z_e^{*\prime}\bar{\Omega}_2 Z_e^*)^{-1} \\
&\quad \times(\theta Z_e^{*\prime}\bar{\Omega}_1 y^* + Z_e^{*\prime}\bar{\Omega}_2 y^*)
\end{aligned}
\tag{13}
$$

which is a weighted average of the seasonal-dummies purged estimator, which we call LSSD (*least squares seasonal dummies*), and the between-seasonals estimator that reduces each individual to its four seasonal time averages. The weighting parameter is defined by $\theta = \sigma_u^2/(T\sigma_d^2/4 + \sigma_u^2)$. As in traditional panel analysis, $\theta = 1$ yields the OLS estimator and $\theta = 0$ yields the LSSD estimator.

This GLS estimator is only feasible if the variance ratio θ happens to be known. In practice, θ must be estimated. This can be done, for example, by the concentrated likelihood estimator, which requires running the two borderline regressions (dummy-purged and between-seasonals, see BREUSCH, 1987). Iterating the procedure yields the ML estimator conditional on the first p observations treated as fixed.

Let us now turn to the representation in (7). The LSSD method yields consistent estimators — in the sense of $T \to \infty$ — of the coefficients in the φ_k polynomials and switching values ζ_{kj} can be calculated from them. Imposing the identified unit roots results in new φ_k estimates. If the weights g_{kj} are treated as 'fixed effects', an OLS regression of the residuals $\hat{\varphi}_k(B)\Delta y_{kt}$ on individual constants and the cyclical functions w_{jt} yields estimates of g_{kj}.

If, on the other hand, the weights g_{kj} are treated as 'random effects', the LSSD step results in an estimate of σ_d^2 and hence of the ratio θ, which can be used in a next step to construct a weighted average of the LSSD and the between-seasonals estimators. In detail, an iterative algorithm can be set up as follows:

1. LSSD is applied and yields estimates $\hat{\varphi}_k$.

2. From the LSSD regression, switching factors $\hat{\zeta}_{kj}$ are also obtained. The countries for which $\hat{\zeta}_{kj} \neq 0$ contribute to the calculation of the $\hat{\sigma}_d^2$ weights.

3. $\hat{\theta}$ is defined by

$$\hat{\theta} = \frac{4\hat{u}'\bar{\Omega}_2\hat{u}}{(T-4)\hat{u}'(\bar{E}_{NT/4} \otimes I_4 - \bar{\Omega}_1)\hat{u}} \quad , \tag{14}$$

   where $\hat{u}$ denotes the residual vector from the previous step with respect to the non-deterministic regressors, as for example $\hat{u} = y^* - Z^*\hat{\varphi}^*$. A feasible GLS method uses this $\hat{\theta}$ and determines new $\hat{\varphi}$ estimates. Notice that the $\zeta_{kj} = 0$ are completely or partially excluded from this re-estimation.

The algorithm should be iterated to convergence. It may be complicated, however, to iterate on the switching factors $\hat{\zeta}_{kj}$, as such a step would require a re-calculation of the critical bounds in each iteration. Notice that, in (14), the part $(T-4)^{-1}N^{-1}\hat{u}'\bar{\Omega}_2\hat{u}$ constitutes an estimate of the error variance σ_u^2, whereas $(4N)^{-1}\hat{u}'(\bar{E}_{T/4}\otimes I_4 - \bar{\Omega}_1)\hat{u}$ is an estimate of the variance gain from a model with 4 common seasonal constants to a model with $4N$ country-specific seasonal constants. The formula (14) follows from an analogous concentrated maximum-likelihood approach given for the non-seasonal case by BREUSCH (1987, p.385).

The above procedure achieves consistency in the sense of $T \to \infty$ but two issues should be noted. Firstly, as in all dynamic panels, the resulting estimates are biased for finite T and they are inconsistent as $N \to \infty$. This bias is troublesome if T is small and the number of countries is potentially unlimited. In our application, however, the number of countries under investigation may be regarded as a fixed entity and T is conveniently large. Secondly, even after convergence, the estimates are not maximum-likelihood due to the interdependence of the parameters θ, $\zeta = (\zeta_{11}, \ldots, \zeta_{N3})'$, and $\varphi = (\varphi_{11}, \ldots, \varphi_{Np})'$, leading to information matrices that are not block-diagonal.

3.3 Decisions on unit roots with common dynamics

A different prior setup from the one introduced in Section 2 for an ID model corresponds to the restriction that the dynamic polynomials $\varphi_k(.)$ are identical across countries, i.e., to a *common-dynamics* (CD) panel model. If the countries are repeated measurements of the *same* autoregression, except for idiosyncratic seasonal constants, only *one* decision on unit roots has to be made. Either a unit root at -1 or at $\pm i$ is present in the joint model for all countries or not.

To obtain a decision setup, we generate a large number M of replications (here: $M = 10,000$) of the $N \times (T-6)$ unit-root model

$$(1 - \varphi_1 B - \varphi_2 B^2)\Delta_4 y_{kt} = g_{ko} + \varepsilon_{kt}, \quad t = 7, \ldots, T, \quad k = 1, \ldots, N, \qquad (15)$$

and another M replications of the $N \times (T-6)$ model with deterministic seasonality

$$\begin{aligned}(1 - \varphi_1 B - \varphi_2 B^2)\Delta y_{kt} &= g_{ko} + \sum_{j=1}^{3} g_{kj} w_{jt} + \varepsilon_{kt}, \\ & \quad t = 7, \ldots, T, \quad k = 1, \ldots, N. \qquad (16)\end{aligned}$$

Notice that in the latter case more observations could have been used at the starting point but the same sample size was kept in line with the situation

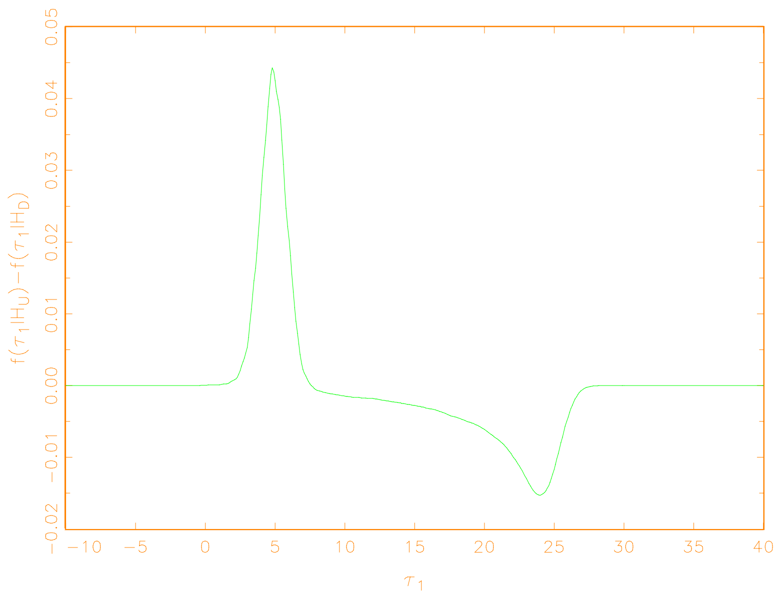

Figure 4: Difference of densities $f(\tau_1|H_U) - f(\tau_1|H_D)$ for $T = 128$ and $N = 16$. Positive values favor a unit root at -1.

of the decision maker who is confronted with the same number of data under both hypotheses. The individual constants $g_{k0}, \ldots, g_{k3}$ are drawn from a standard normal distribution, just like the disturbances ε_{kt}. Like in the setup with individual dynamics that was presented in Section 2, the autoregressive coefficients (φ_1, φ_2) are drawn from a uniform distribution on the stability region $U(S_2)$.

For the case $N = 16$ and $T = 128$, which is in line with our empirical application, Figures 4–6 display the differences in the density functions for the three HEGY–type t–statistics τ_1, τ_2, τ_3. In contrast to the odds curves in Figures 1 to 3, the estimation method for the auxiliary regression (3) is LSSD and not OLS in order to take care of the individual seasonal effects, particularly under the deterministic-dummies model. Again, the points of intersection can be used as critical values. They turn out to be at 7.6, 8.8, and 8.9, for τ_1,τ_2,τ_3, respectively.

Notice that under the unit-root model the densities for τ_2 and τ_3 are trimodal and bimodal. Further notice the large values for the critical points. Repeated measurements on the same process entail strongly inflated t–values

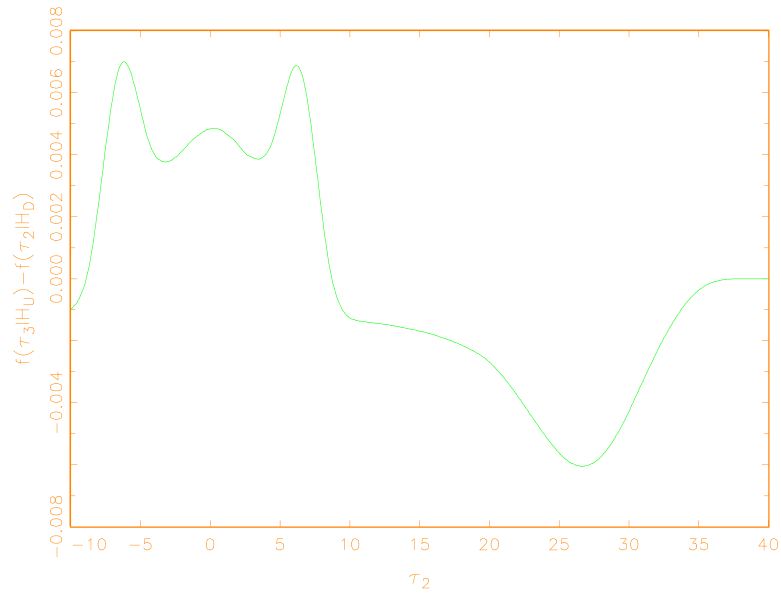

Figure 5: Difference of densities $f(\tau_2|H_U) - f(\tau_2|H_D)$ for $T = 128$ and $N = 16$. Positive values favor a unit root at $\pm i$.

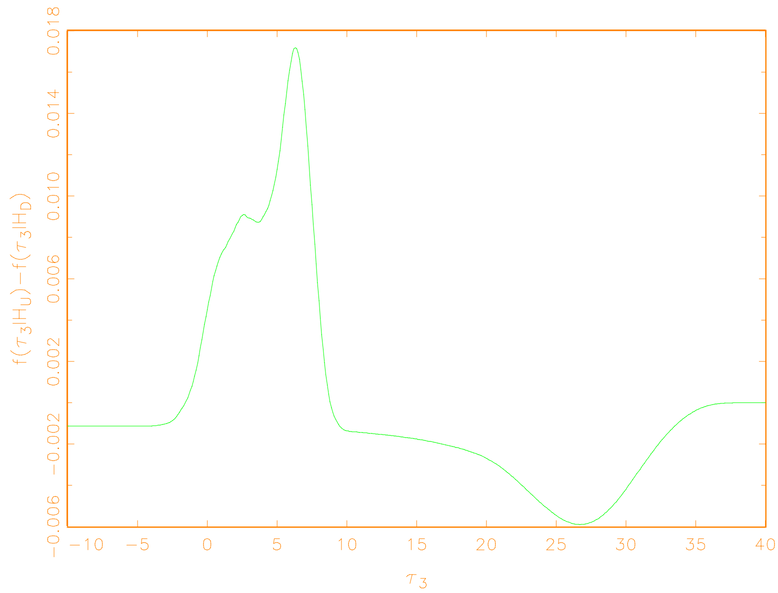

Figure 6: Difference of densities $f(\tau_3|H_U) - f(\tau_3|H_D)$ for $T = 128$ and $N = 16$. Positive values favor a unit root at $\pm i$.

for all coefficients, as the fixed starting values in the panel decrease the growth in the cross-moments matrix of the regressors that determines the standard deviation in the denominator of the t–values.

3.4 Common deterministic seasonality in the panel

In the end, we are interested in investigating whether deterministic seasonal cycles coincide across all N individuals or across a subset of the individuals. Some countries may experience similar influences from seasonality due to similar institutional reasons, for example similar holidaying behavior, or due to similar climatic cycles. Other countries may show larger differences. MIRON (1996) and MIRON AND BEAULIEU (1996) extensively discuss similarities in deterministic seasonality across countries and sectors, using more informal methods. These authors also give several economic reasons for such similarities.

An arguably extreme case of common seasonality would be that s_k (defined in (9)) is constant in the index k, that is, deterministic seasonality across the countries is the same in size and sign. In that case, the model is reduced to

$$\begin{aligned} y^* &= Z^*\varphi^* + D^*d + v^* \\ v^* &= \mu^* + u^* \end{aligned} \tag{17}$$

where D^* is the $N(T-p)\times 4$–data matrix of seasonal constants with typical element $D^*_{jl} = \delta^l_{j-4[(j-1)/4]}$ and $d = (d_1, \ldots, d_4)'$. The error component $\mu^* = (\mu_1 \mathbf{e}'_T, \ldots, \mu_N \mathbf{e}'_T)'$ contains random individual level influences such that the error variance representation simplifies to the standard form

$$Ev^*v^{*\prime} = \sigma^2_\mu (I_N \otimes E_T) + \sigma^2_u I_{TN} = \Omega_\mu + \Omega_2 \tag{18}$$

and the efficient estimator for $(d', \varphi^{*\prime})'$ is the traditional unobserved-components or random-effects estimator

$$\left(\hat{d}, \hat{\varphi}^{*\prime}\right)' = \{Z^{*\prime}_e(\Omega_\mu + \Omega_2)^{-1} Z^*_e\}^{-1} Z^{*\prime}_e (\Omega_\mu + \Omega_2)^{-1} y^* \quad ,$$

with $Z^*_e = (D^*, Z^*)$ an $N(T-p)\times(4+Np)$–matrix. Validity of this model could be interpreted as implying the presence of *common deterministic seasonality.* This is to be contrasted with the possibility that the individual lag polynomials $\varphi_k(z)$ contain seasonal unit roots such that an $r \times N$–linear combination β' across certain individuals $\beta'(\varphi_1, \ldots, \varphi_N)'$ determines r polynomials *without* seasonal unit roots. This possibility, *common stochastic seasonality*, is treated in ENGLE AND HYLLEBERG, 1996, and KUNST, 1993, and we will not deal with this issue here.

Less stringent restrictions on the deterministic seasonal components also deserve consideration. For example, it may hold that the seasonal constant in the first quarter corresponds to a seasonal constant in the second quarter, or $d_{k1} = d_{k2}$, for all countries k. Alternatively, it may be interesting to see whether $d_{k1} - g_{k0} = -d_{k2} - g_{k0}$, or whether the effects of some quarter are counterweighted by a similar effect in the next quarter, taking the longer-run growth of the variable into account. Such hypotheses would match the empirical findings in, for example, MIRON (1996). Notice that the hypothesis makes sense in this form, as g_{k0} is also determined by the remaining quarters. Such restrictions can be tested in a straightforward fashion using F–statistics. However, care has to be taken in calculating the degrees of freedom, as for some k the indicated presence of seasonal unit roots causes such restrictions to be satisfied automatically. In the next section we will focus on some tests in a more precise manner, where the hypotheses of interest will be motivated by some initial empirical results.

4 An application

In this section we apply the methods introduced in Sections 2 and 3 to (the logs of seasonally unadjusted) quarterly industrial production in 16 industrialized countries from 1962 to 1993, which thus constitute a panel with $N = 16$ and $T = 128$. Simulations and analytical derivations in KUNST AND FRANSES (1999) indicate that any bias in the LSSD estimation can be expected to be very small for this sample size. The data are displayed as Figures 7 to 9.

It is obvious that all series are trending. Notice, though, the phases of stagnation or recession in Luxemburg and the Netherlands. The intensity of seasonal patterns seems to vary widely. Some countries, among them Japan and the United States, show no obvious visual signs of seasonality at all, whereas France, Sweden, and the United Kingdom have very pronounced seasonal variation. Some countries, among them the Netherlands and Portugal, show seasonal patterns that evolve significantly over time.

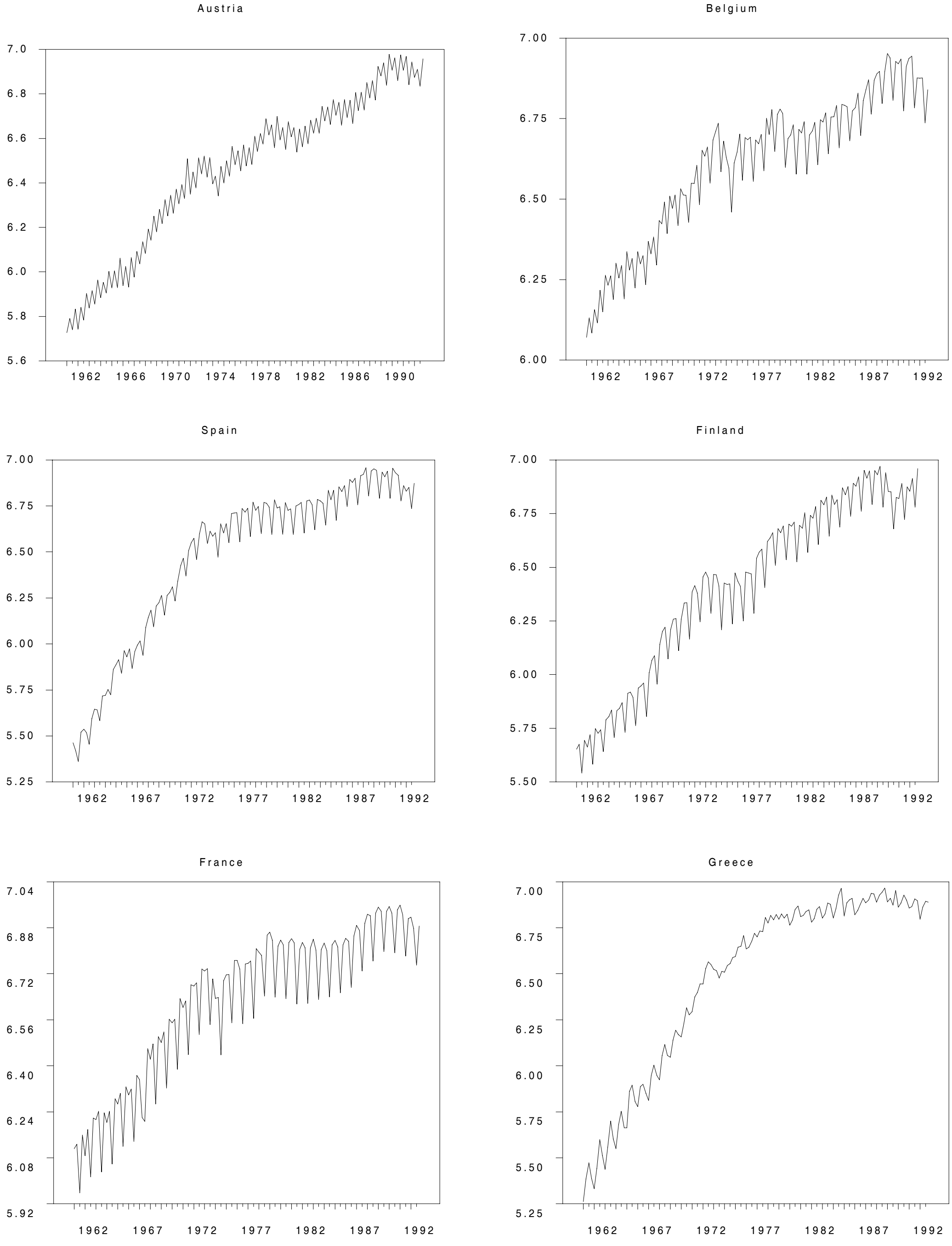

Figure 7: Industrial production, quarterly series, 1962.1–1993.4

Figure 8: Industrial production, quarterly series, 1962.1–1993.4

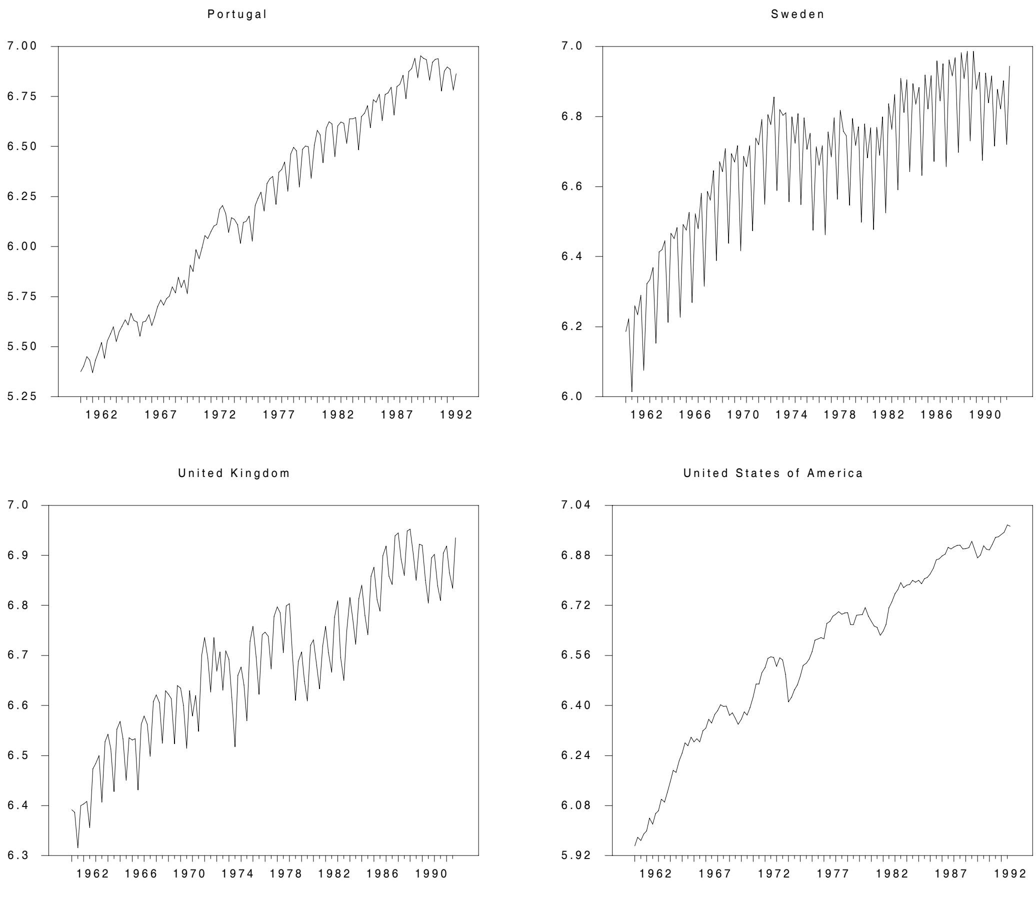

Figure 9: Industrial production, quarterly series, 1962.1–1993.4

4.1 A panel model with individual dynamics

Simple purging of all series from constant seasonal patterns by regressing them on seasonal dummy variables and entertaining low-order autoregressive models yields some variation in suggested lag orders. Utilizing AIC as the lag-order search criterion results in five first-order models, five fifth-order models, four second-order models, one third-order model, and one model of order six. If one accepts that all series are integrated at the frequency zero and hence must be differenced once to remove the singularity of the spectrum at $\omega = 0$, it appears that a lag order of $p = 4$ (after first-order differencing) captures the short-run dynamics in all series with an acceptable precision.

The critical points that were reported in Section 2 for the ID approach are now to be applied. To this aim, HEGY–type statistics are calculated for all the series from regressions of the form

$$\begin{aligned} S(B)\Delta y_{kt} &= g_{k0} + \sum_{j=1}^{3} g_{kj} w_{jt} + a_1(1+B^2)\Delta y_{k,t-1} + a_2(1+B)\Delta y_{k,t-1} \\ &\quad + a_3(1+B)\Delta y_{k,t-2} + \sum_{j=1}^{2} \varphi_j S(B)\Delta y_{k,t-j} + \varepsilon_{kt} \end{aligned}$$

for $k = 1, \ldots, 16$. The t–values for the least-squares coefficient estimates $\hat{a}_1, \hat{a}_2, \hat{a}_3$ are compared with the critical points 2.9, 2.9, 3.3. If $t(\hat{a}_1) > 2.9$, no unit root at $\omega = \pi$ is indicated. Similarly, if either $t(\hat{a}_2) > 2.9$ or $t(\hat{a}_3) > 3.3$, no unit root at $\omega = \pi/2$ is indicated. The results are summarized in Table 1. It appears that few series have no seasonal unit roots and that many countries have only one root at $\omega = \pi$.

In the next stage, in accordance with (7), some, all, or no seasonal constants are restricted at zero in the individual countries and individual autoregressions are estimated with the lag order determined by AIC. There appears to be no difference in selected lag orders between AIC and BIC, for all countries. A summary of these results is given in Table 2. Notice that the lag order now generally is higher than before, which is likely to be due to the data-admissibility constraints on the seasonal intercepts.

Excepting those cases where unit roots at both seasonal frequencies have been found, for each country one obtains estimates for deterministic dummy coefficients that are related to the four quarters. Figures 10 and 11 are scatter plots of the coefficient estimates for the first and second quarter and for the third and fourth quarter, respectively. For these two pairs we find

Table 1. Seasonal unit roots in the production series.

Country	τ_1	τ_2	τ_3	Roots
Austria	4.262	4.332	4.226	-
Belgium	3.943	4.709	3.573	-
Spain	2.769	2.321	4.789	-1
Finland	2.368	2.283	3.122	$-1, \pm i$
France	2.930	2.483	5.891	-
Greece	3.640	1.145	2.936	$\pm i$
Ireland	3.794	1.485	2.717	$\pm i$
Italy	2.788	2.438	3.976	-1
Japan	2.347	4.019	4.925	-1
Luxemburg	1.761	3.493	3.163	-1
Netherlands	3.859	0.471	3.004	$\pm i$
Norway	4.609	1.297	6.689	-
Portugal	2.695	2.006	1.990	$-1, \pm i$
Sweden	2.802	2.176	3.838	-1
United Kingdom	2.486	3.231	5.984	-1
United States	2.077	3.493	3.334	-1

Notes. The test statistics τ_1, τ_2, and τ_3 are discussed in Section 2. The critical values are 2.9, 2.9, and 3.3, respectively.

Table 2. Autoregressive models for individual countries.

Country	p	R^2	$p(Q)$
Austria	0	0.951	0.437
Belgium	0	0.911	0.000
Spain	3	0.917	0.017
Finland	1	0.681	0.643
France	1	0.948	0.000
Greece	2	0.620	0.116
Ireland	2	0.935	0.023
Italy	3	0.920	0.019
Japan	4	0.818	0.273
Luxemburg	4	0.795	0.126
Netherlands	2	0.951	0.000
Norway	2	0.716	0.329
Portugal	1	0.533	0.070
Sweden	3	0.967	0.230
United Kingdom	2	0.907	0.057
United States	4	0.698	0.347

Notes. p is the lag order selected by AIC/BIC, R^2 is the uncorrected ratio of explained variation to total variation, and $p(Q)$ is the p–value of the Ljung-Box portmanteau statistic for remainining autocorrelation after fitting the model.

the strongest correlations, where in both cases the correlation is negative.

A different representation is achieved if the estimated coefficients are sorted. We considered sorting according to the third quarter as the third quarter coefficient is the most significant one for most countries. It is typically significantly negative. The slump in the third quarter is then usually followed by a strong increase in production in the fourth quarter. Figure 12 shows, just as Figure 11, that countries with a particularly strong slump in the third quarter tend to have a particularly pronounced boom in the fourth one. The observations for the first half of the year show a less clear pattern. For most countries, the first quarter continues the growth in the fourth one whereas the second quarter is close to zero, although several cases violate this pattern. In Austria, Ireland, and the Netherlands, to a lesser degree also in Belgium, production has a strong slump in the first quarter and a second peak in spring. The amplitude of the deterministic seasonal cycle also shows strong cross-country variation. Greece, Finland, the United States, Portugal, and Japan have very weak deterministic seasonality, once we take care of seasonal unit roots.

A detailed analysis of the sources of the seasonal production cycle is provided by Miron (1996) who, however, did not separate the cases of seasonal unit roots. In line with the well-known descriptive definition of seasonality by Hylleberg (1992, p.4), the differences across countries are rooted in differences in climate and culture. Both sources may motivate secondary institutional arrangements, such as the pooling of vacations in a certain time of the year. For example, Austria may show low production activity in the first quarter due to the popularity of winter sports. In contrast, the Nordic countries, which have climatic cycles of similar amplitude, pool production in the dark half-year (first and fourth quarter) and holidaying in the light half-year (second and third quarter). An exception is Finland, where the strong seasonal variation is found to be stochastic and non-stationary and hence the deterministic cycle is absent. Note that this latter effect cannot be found by the methods of Miron (1996).

In order to see whether the strong negative correlation of the third and fourth quarters constitutes a common pattern in all or at least most countries, we employ a statistical hypothesis test. We use the Fisher-type test statistic

$$\eta = \{T(N-1) - \sum_{k=1}^{N}(p_k + \zeta_{k1} + 2\zeta_{k2})\}\frac{ESS_R - ESS_U}{ESS_U} \quad , \qquad (19)$$

where ESS_R and ESS_U denote the residual sum of squares of the restricted

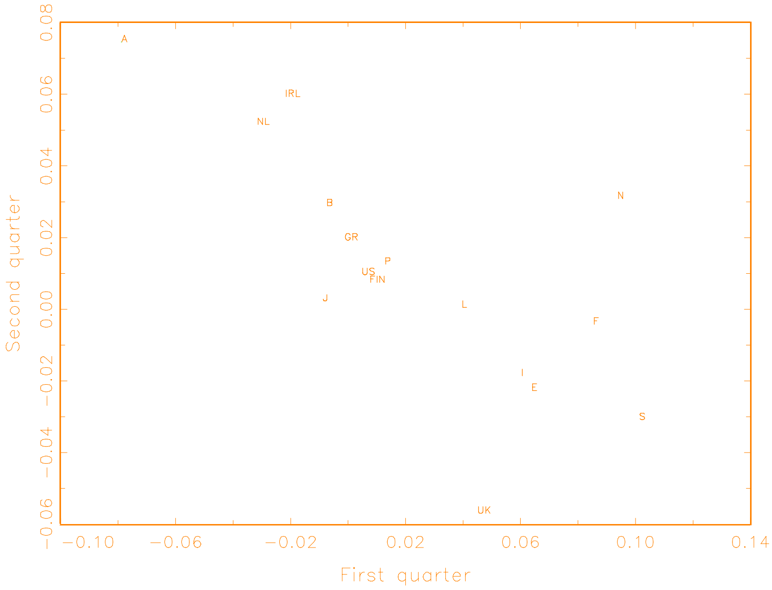

Figure 10: Scatter diagram of estimated seasonal constants in the first and second quarter.

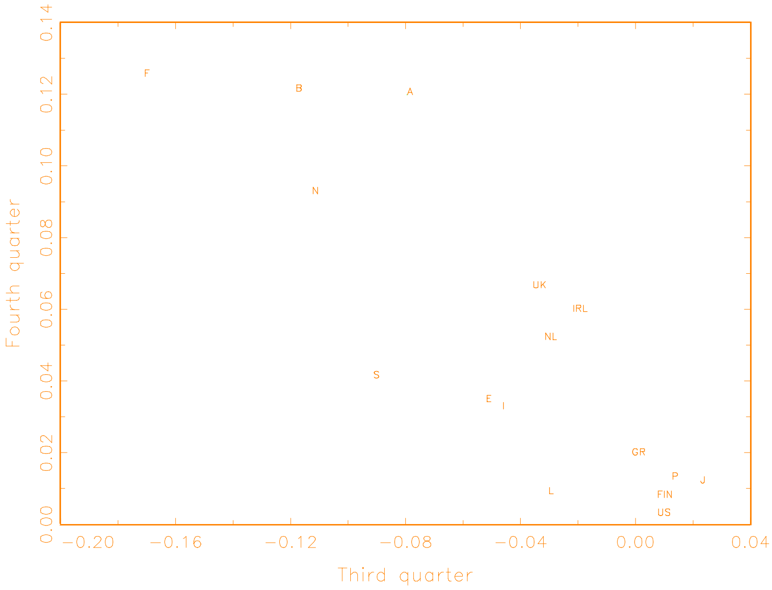

Figure 11: Scatter diagram of estimated seasonal constants in the third and fourth quarter.

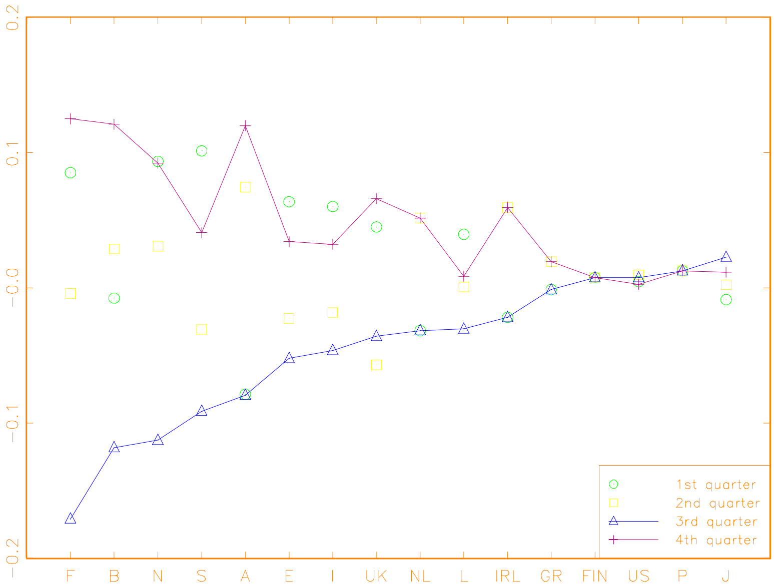

Figure 12: Estimated seasonal constants for low-order autoregressive models for the 16 countries. Countries have been sorted according to the third-quarter coefficients.

and the unrestricted model, respectively, p_k is the autoregressive lag order for country k, and ζ_{k1} and ζ_{k2} describe whether country k has seasonal unit roots at $\omega = \pi$ and $\omega = \pi/2$. The asymptotic distribution of η under the null hypothesis of the restricted model is χ^2, with the degrees of freedom determined by the number of restrictions that define the null.

Using η, a test of the hypothesis that the coefficients in the third and fourth quarter have the same absolute value but have opposed sign, i.e., $H_0 : d_{k3} - g_{k0} = -d_{k4} - g_{k0}$, yields rejection on grounds of a χ^2 distribution with 11 degrees of freedom ($\eta = 94.41$). Notice that H_0 is equivalent to the hypothesis $d_{k2} - g_{k0} = -d_{k1} - g_{k0}$ and that, therefore, the symmetry of the first and second and of the third and fourth quarter is tested jointly. Some closer inspection shows that rejection of H_0 is due to the behavior of Austria, France, Norway, Japan, and Luxemburg, whereas H_0 is accepted for the remaining cases. Although there is no obvious relationship or similarity among these five countries, Table 1 shows that the first three do not have any seasonal unit roots and therefore display the most complicated deterministic cycles, whereas Table 2 shows that Japan and Luxemburg need particularly high autoregressive lag orders. In summary, the hypothesis H_0, which may be in line with MIRON's observations, seems to hold for 11 out of the 16 countries.

Finally, any equality restrictions between two quarters, such as $d_{k3} = d_{k4}$, is rejected at extreme significance levels on grounds of a χ^2 distribution with 14 degrees of freedom. Given the visual evidence in Figure 12, this is not an unexpected finding.

4.2 A panel model with common dynamics

In a CD model, a common autoregressive structure is assumed for all of the 16 series. Information criteria and residual analysis suggest an AR lag order of 6 and hence an order of 5 in first differences. We apply the analysis of Section 3.4 to test for seasonal unit roots in the CD model on the basis of a HEGY–type test and the LSSD estimator. The HEGY–type test yields the t–values of 15.49, 8.55, and 16.65. Clearly, τ_1 and τ_3 are far beyond the critical values and, therefore, unit roots at both seasonal frequencies are rejected.

The LSSD method finds the AR(5) model

$$\Delta y_t = -0.28\Delta y_{t-1} - 0.17\Delta y_{t-2} - 0.04\Delta y_{t-3} + 0.34\Delta y_{t-4} + 0.09\Delta y_{t-5} + \varepsilon_t \quad . \tag{20}$$

The standard errors of the coefficient estimates are approximately 0.02. The characteristic polynomial has three real roots at approximately ± 1.4 and at -3.8 and a complex root pair at $\pm 1.22i$. These roots correspond to the seasonal frequencies but none of them comes very close to the unit circle. Hence, the procedure suggests to describe the seasonal features in the panel by the interaction of seasonal constants and of stationary dynamics.

An iteration of the feasible GLS algorithm outlined in Section 3.3 yields a $\hat{\theta}$ of 0.01261. Hence, the maximum-likelihood estimator is conveniently close to the LSSD solution and there are no plausible gains to be attained from further iterations.

The LSSD residuals from model (20) are regressed on individual seasonal dummies in the GLS iterations. The coefficient estimates on the seasonal dummies provide an information that is quite equivalent to the analysis in the previous section. It turns out that again the third and fourth quarter are negatively correlated with a correlation coefficient of -0.91. The negative correlation of the first and second quarter attains a value of -0.69. Figure 13 displays a scatter diagram of the third and fourth quarter values, whereas Figure 14 presents all quarters after sorting by the third one. On the whole, both figures give a visual impression that is similar to that obtained in the last section by using ID models. The main difference to the ID results is due to the fact that no seasonal unit roots are found in the CD model. Hence, for example the seasonal cycle of Finland is now viewed as mainly deterministic.

Hypothesis tests for common structures in the deterministic seasonal cycles were also conducted for the CD model. A test based on the Fisher-type test statistic (cf. (19))

$$\eta = N(T-p-4)\frac{ESS_R - ESS_U}{ESS_U} \quad , \tag{21}$$

with $p = 5$ and using the asymptotic χ^2 distribution with $2(N-1)$ and $N-1$ degrees of freedom, rejects the existence of common deterministic cycles at frequency $\pi/2$ and at frequency π. Here, ESS_U is the residual sum of squares from a CD model (20) with individual seasonal constants, and ESS_R is a CD model with the respective restrictions $(g_{k2} \equiv g_2) \wedge (g_{k3} \equiv g_3)$ and $g_{k1} \equiv g_1$. Also a test for the joint hypothesis at both frequencies rejects.

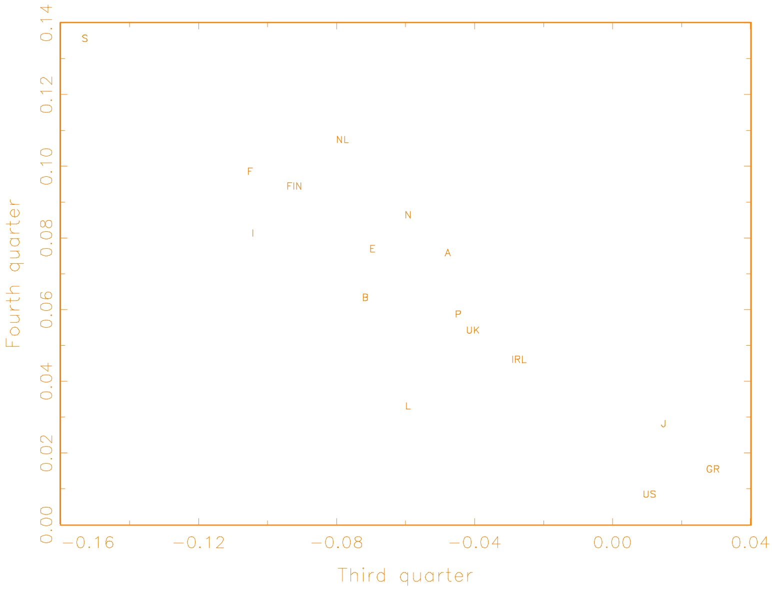

Figure 13: Scatter diagram of estimated seasonal constants in the third and fourth quarter. Estimation was based on a common-dynamics fifth-order autoregressive model for the 16 countries and on LSSD.

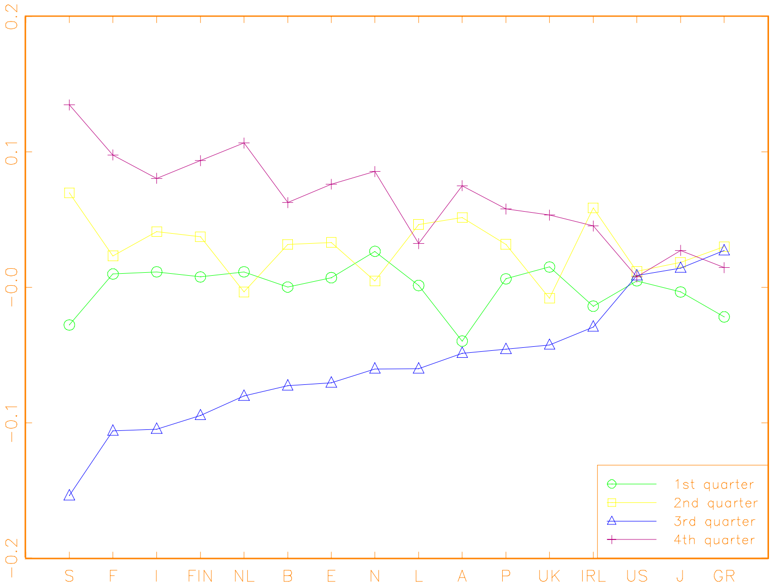

Figure 14: Estimated seasonal constants for a common-dynamics fifth-order autoregressive model for the 16 countries. Countries have been sorted according to the third-quarter coefficients.

To see whether there is some common deterministic seasonal structure among subgroups of the panel, we calculate the marginal significance levels (p–values) of restriction tests for the pairwise hypotheses $H_{0,\pi}(k,l) : g_{k1} = g_{l1}$ and $H_{0,\pi/2}(k,l) : (g_{k2} = g_{l2}) \wedge (g_{k3} = g_{l3})$. These are summarized in Tables 3 and 4. Table 3 for $\omega = \pi$ shows that the semi-annual component of the seasonal cycles in Austria, Finland, France, and Italy is nearly identical. A second group is formed by Spain, Ireland, Norway, Portugal, and the Benelux countries, though there are wider variations within this group and the hypothesis of constant g_{k1} cannot be accepted for the group as a whole. There are some similarities between Greece, Japan, and the United States, whereas Sweden and the United Kingdom remain isolated. Table 4 for $\omega = \pi/2$ suggests similarities among Finland, France, Italy, and Sweden. Another group is formed by Belgium, Spain, and Portugal. A loose link between these two groups is indicated by the fact that equality of the seasonal cycle in Spain and Finland can also not be rejected at the 10% level. A third group is formed by Japan, the United States, and Ireland. The remaining cases are rather isolated, maybe excepting Austria, where some connection to Japan and the United States is indicated.

In summary, only in some cases does the result correspond to intuition, as e.g. for the pairs Portugal/Spain or the Benelux. Furthermore, there are large differences with respect to the pooling behavior at the two seasonal frequencies. We note that the basic hypotheses of equality are also rejected for the ID model but that the significance of rejection in the CD model is much stronger.

Table 3. Marginal significance of pairwise differences between semi-annual deterministic seasonal cycles.

	A	B	E	FIN	F	GR	IRL	I	J	L	NL	N	P	S	UK
B	0.01														
E	0.02	0.74													
FIN	0.87	0.01	0.02												
F	0.91	0.01	0.02	0.96											
GR	0.00	0.00	0.00	0.00	0.00										
IRL	0.00	0.29	0.17	0.00	0.00	0.00									
I	1.00	0.01	0.02	0.87	0.91	0.00	0.00								
J	0.00	0.00	0.00	0.00	0.00	0.82	0.00	0.00							
L	0.00	0.12	0.06	0.00	0.00	0.00	0.60	0.00	0.00						
NL	0.02	0.78	0.96	0.01	0.02	0.00	0.19	0.02	0.00	0.07					
N	0.00	0.02	0.01	0.00	0.00	0.00	0.21	0.00	0.00	0.46	0.01				
P	0.00	0.04	0.02	0.00	0.00	0.00	0.33	0.00	0.00	0.64	0.02	0.78			
S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
UK	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.04	0.00	0.00	0.01	0.00	0.00	
US	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00

Table 4. Marginal significance of pairwise differences between annual deterministic seasonal cycles.

	A	B	E	FIN	F	GR	IRL	I	J	L	NL	N	P	S	UK
B	0.00														
E	0.00	0.61													
FIN	0.00	0.01	0.11												
F	0.00	0.00	0.00	0.22											
GR	0.00	0.00	0.00	0.00	0.00										
IRL	0.02	0.00	0.00	0.00	0.00	0.00									
I	0.00	0.00	0.01	0.25	0.03	0.00	0.00								
J	0.07	0.00	0.00	0.00	0.00	0.01	0.01	0.00							
L	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00						
NL	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00					
N	0.00	0.00	0.01	0.09	0.08	0.00	0.00	0.00	0.00	0.00	0.08				
P	0.01	0.27	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00			
S	0.00	0.00	0.00	0.16	0.57	0.00	0.00	0.11	0.00	0.00	0.00	0.01	0.00		
UK	0.00	0.04	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.00	
US	0.07	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.36	0.00	0.00	0.00	0.00	0.00	0.00

5 Summary and conclusion

We proposed methods to test for common deterministic seasonality, while allowing for possible seasonal unit roots, in quarterly data. For this purpose, we decided to consider panel methods, where we allowed for individual dynamics and for common dynamics. To decide on the presence of seasonal unit roots, we introduced a decision-based approach and we derived the relevant critical values. This approach can be used to the individual and common dynamics models. Our application concerned 16 quarterly industrial production series, for which at first sight common deterministic seasonal patterns would seem to exist. With the individual (or country-specific) dynamics model, we found for 11 out of the 16 countries that the production slowdowns in the first and third quarters are equal to the production booms in the second and fourth quarter, respectively. This matches with our graphical results and with those in Miron (1996). With the common dynamics models, we did not find much evidence in favor of common deterministic seasonality across countries, except for a few pairs of countries.

There are at least two directions for further research. The first amounts to applying our methods to variables other than industrial production and to time series with other sampling frequencies. For example, it may be interesting to examine of the day-of-the-week effect in stock market returns is common across countries. A second research topic concerns the use of our methods to obtain a statistical method for clustering countries, according to their time series properties.

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