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ABSTRACT. A model based on differences between workers regarding their preferences for wage and leisure drives the heterogeneity of firms result. The more industrious workers are driven to small firms due to free riding in large firms. An industry consisting of small and large firms turns out to produce more output than an industry consisting of only large firms. Some comparative statics results are derived with respect to the size of large firms, the productivity difference between firms, and monitoring capabilities.

#### I. Introduction

Casual empiricism indicates that large firms have difficulties organizing their research departments. The New York Times (1984) reported that "For the first time since the early days of the auto industry, the General Motors Corporation has bought a minority interest in a small company rather than swallowing it whole". The reasons given for GM's partial acquisition were:

- preserving incentives in the small high tech firm:
- hiring difficulties of experienced people;
- access to potential breakthroughs and guiding research.

A similar example is Westinghouse and its experience with the high tech firm Unimation. The Wall Street Journal (1984) reported about Westinghouse: "They learned that their financial incentives would be lower under Westinghouse and they felt the time it took to make business decisions would lengthen". Williamson (1985) has reported about similar cases and concludes that "large companies

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Tilburg University B601 P.O. Box 90153 5000 LE Tilburg The Netherlands are becoming increasingly aware that the bureaucratic apparatus they use to manage mature products is less well-suited to supporting early stage entrepreneurial activity".

There are several interesting observations to be made with respect to the above phenomena. The first one concerns the employment of labour across firms. The question is whether or not the more productive workers are employed by large firms. The study by Garen (1985) predicts that individuals who acquire more schooling will be employed by larger firms and that larger firms will pay higher wages. Monitoring/evaluation costs rise with firm size in his model. Large firms rely in their wage compensation scheme therefore less on their own evaluation of workers than small firms do and more on other indicators such as schooling. Only workers with higher activity are willing to put forth the effort of getting a higher education in order to enter a large firm and obtain a higher wage. Farrell and Scotchmer (1988) derive a similar result for partnerships. Several studies support this relationship empirically (Brown et al., 1990).

These analyses are not saying anything with respect to workers within a certain ability class (i.e., a certain level of schooling) due to unobservable characteristics of workers. Ability classes are usually broad and encompass many different characteristics. These differences within a certain class will influence the choice of a small or large firm by a particular worker. Our model addresses this question. It supports a claim by Stigler (1962) that: "Men should, in general, enter smaller companies, the greater their ability". The explanation in our model for the hypothesis of Stigler is that free riding in large firms drives the more industrious workers away from large firms, into small firms. They are employed by a less productive technology in order to prevent having free riding colleagues. The hypotheses of Garen and Stigler therefore both hold because one has to distinguish between observable and non-observable characteristics of workers, given a certain level of schooling. Garen deals with observable characteristics whereas Stigler's hypothesis and our model are about unobservable characteristics.

A second observation is that neoclassical theory predicts that all firms are identical in long run equilibrium with perfect information and free entry. The inefficient firms are driven out of the industry when profits go to zero for the most efficient firms. However, empirical evidence suggests that small and large firms coexist in many industries. The large firms are usually considered more productive, ceteris paribus. If the larger firms are more productive in fact, then the existence of small firms remains a puzzle. Explanations have to deal somehow with the ceteris paribus condition regarding the internal functioning of firms.

Firms are viewed as entities employing workers in a production technique. The output of the firm is assumed to be divided between the workers of the firm. A firm in which the shares of all workers are the same is called a partnership (Farrell and Scotchmer, 1988). Our reward schedule for workers in large firms corresponds to those of partnerships in section two, but workers differ with respect to their preference of providing effort. (Another difference is that incumbent workers can keep new workers out in the model of Farrell and Lander, whereas they cannot in our model.) Small firms employ only one worker in our set-up. The assumption of equal shares is relaxed in section three. The share in the output of the firm of a worker will be based upon a combination of equal share and an imperfect observation of the effort level provided. A group of people with possibly unequal shares is called a team (Farrell and Lander, 1989). We continue to use the word firm for each production function, regardless of the number of workers employed and the renumeration scheme used.

We assume that the output of each firm depends on the effort level provided by the worker(s). The production of the large firm is influenced by two opposing forces. On the one hand, the technology of the large firm is more productive than those of small firms, ceteris

paribus. This might induce workers to supply higher levels of effort. On the other hand, the individual effort levels cannot be costlessly observed by the firm. We assume that individual remunerations have to be based on joint performance, which creates a moral hazard problem. Workers may take advantage of this by supplying less effort. This moral hazard problem may be dealt with by hiring a monitor. However, there are problems associated with the use of a monitor, such as deciding which disciplinary powers are available to the monitor and the possibility that the monitor shirks. These considerations make us pursue a different approach.

One of the resons for the neo-classical result of identical firms in equilibrium is that labour is homogeneous. We will relax this assumption and employ a distribution of workers which will drive the heterogeneity of firms result. Workers are assumed to derive utility from wages and leisure. Some of them care mainly about wages, whereas others put more weight on leisure. Workers sort themselves across production techniques in order to maximize utility. This will result in an industry equilibrium consisting of small and large firms. Notice that the equilibrium size distribution of firms in this paper is not the result of a stochastic process, which underlies Gibrat's law (see, e.g., Simon and Bonini, 1958; Klepper and Graddy, 1990), but is generated by an economic process (i.e., the sorting of workers across production techniques). The objective of this paper is not to derive Gibrat's law for certain types of industries, but to develop a model showing firm heterogeneity in equilibrium due to endogenous economic forces.

This article is organized as follows. Section two develops the model and delineates some of the forces at work by an example. Section three analyses monitoring. Finally, a summary and avenues for further research are provided.

### II. The model

# II.1. Firms

A firm is viewed as a production function employing a set of workers. The small firm is characterized by production technique  $f_1(\cdot)$  and is able to employ only one worker. The large firm has

production technique  $f_2(\cdot)$ . It is more productive than the small firm, ceteris paribus. We formulate this as  $f_2(e_1, \ldots, e_N) > f_1(e_1) + \cdots + f_1(e_N)$ , where N > 1,  $e_i$  is the effort of worker i and  $e_i \in [0, 1]$ . If N = 1, then  $f_2(e_1)$  is defined to be zero, regardless of the effort level provided. We assume that both  $f_1(\cdot)$  and  $f_2(\cdot)$  are strictly monotonic, continuous and concave.

The fruits of production of the small firm go completely to the worker. (The worker is self-employed.) Workers in a large firm split the gain of their productive efforts equally.

#### II.2. Workers

Each worker is characterized by an utility function. The utility of each worker depends on the effort level, *e*, provided on the job and the wage, *w*, that will be paid for services rendered. Workers have to make two choices. They have to decide what effort levels they are going to provide and by which firm they want to be employed. Workers are assumed to maximize expected utility and are risk neutral.

There are two types of workers in the economy. Type I (industrious) workers have utility function  $U^1(w, e) = w$ . Type II (lazy) workers are characterized by utility function  $U^2(w, e)$ , where  $U^2_w > 0$ ,  $U^2_e < 0$  and is continuous and strictly quasi concave.

A partition  $\pi = \{S_0, \ldots, S_K\}$  of the set of workers S satisfies

$$\bigcup_{k=0}^{K} S_k = S$$

$$S_k \neq 0, k \in \{0, \dots, k\}$$

$$S_k \cap S_m = 0, \text{ with } k \neq m, k, m \in \{0, \dots, k\},$$

where  $S_0$  is defined as the set of workers employed by small firms and  $S_k$  in the set of workers employed by large firm k, k = 1, ..., K. Define M as the set of workers of type I and N as the set of workers of type II, so that  $S = M \cup N$ .

Given  $\pi$ , let  $e_{ik}$  denote the effort level of worker i in  $S_k$  and the vector  $e_k = (e_{jk})$  for all  $j \in S_k$ . If  $i \in S_k$ , then define  $e'_k = (e_{jk})$  for all  $j \in S_k \setminus \{i\}$ . An effort allocation is the (K + 1)-tuple  $E = (e_0, e_1, \ldots, e'_k)$ . Define  $E' = (e_0, e_1, \ldots, e'_k, \ldots, e'_k)$ .

The problem facing worker i, employed by firm k, is deciding what effort level to supply, given the

effort levels of the other workers employed by firm k. For a given partition  $\pi$  and effort allocation E, define  $e_{ik}^*$  as the solution value of  $e_{ik}$  for the problem

$$U'(e_{1k}^*) = \max_{e_{ik}} U'(w_k, e_{ik})$$
s.t.  $w_k = \begin{cases} f_1(e_{ik}) &, k = 0 \\ f_2(e_k, e_{ik}) / |S_k|, k = 1, \dots, K, \end{cases}$ 

where  $|S_k|$  is the number of workers employed in firm k. Notice that each worker in a large firm receives a wage which is an equal share of the output of the firm. The optimal firm for worker i is  $k^*$ , given  $\pi$  and E, which is the solution to the problem

$$U^{i}(e_{ik\bullet}^{*}) = \max_{k} U^{i}(w_{k}, e_{ik}^{*}).$$

# II.3. Equilibrium

The economic process is modelled as a two stage game. In the first stage, workers decide by which firm they want to be employed; in the second stage, workers decide on an optimal effort level, taking first-stage decisions as given. The game will be solved for its subgame perfect Nash equilibrium. This is done by using the method of backward induction. So we find an optimal effort level for each worker, given a certain coalition structure. Next, workers choose by which firm they want to be employed, taking into account the effort levels that will be chosen in the second stage. Formally, an equilibrium coalition structure is a partition and an effort allocation  $\{\pi, E\} = \{(S_0, \ldots, S_k), (e_0, \ldots, e_k)\}$  such that

$$\forall_{i \in S_k} \not\exists_{\hat{e}_{ik} \neq e_{ik}} \{ U^i(\hat{e}_{ik}) > U(w_k^i, e_{ik}) \}$$

$$\forall_{i \in S_k} \not\exists_{k \neq k} \{ U^i(e_{ik}) > U^i(w_k, e_{ik}) \},$$
where  $k = 0, \dots, K$ .

# II.4. Example

We will formulate an example showing that an industry consisting of small and large firms outperforms an industry composed of only large firms. It turns out that even for large productivity differences the free riding in large firms is responsible for industrious workers choosing small firms.

Suppose that there is one industrious worker with utility function  $U^1(w, e) = w$ . There are two type II workers and the utility function of such a worker is  $U^2(w, e) = w - c(e)$ , where  $c(\cdot)$  is a convex function. We will take  $c(e) = e^2$ . The production technology of a small firm is  $f_1(e) = e$  and the technology of the large firm is

$$f_2(e_{1k},\ldots,e_{|S_k|k})=\alpha\sum e_{ik},$$

where the productivity parameter  $\alpha$  is larger than one. This choice reflects the assumption that large firms are more productive than small firms, *ceteris* paribus. The second stage problem of worker i is

$$U'(e_{ik}^*) = \max_{e_{ik}} U'(w_k, e_{ik})$$
s.t.  $w_k = \begin{cases} f_1(e_{ik}) & , k = 0 \\ f_2(e_{1k}, \dots, e_{|S_k||k})/|S_k|, k = 1, \dots, K. \end{cases}$ 

The first stage problem of worker i is

$$U^{i}(e_{ik^{*}}^{*}) = \max_{k} U^{i}(w_{k}, e_{ik}^{*}).$$

Two conditions will now be formulated in order to establish that an industry consisting of small and large firms is an equilibrium coalition structure. Suppose that the industrious worker is employed by a small firm and the two lazy workers by a large firm. The effort level provided by the industrious worker is 1, whereas the lazy workers supply an effort level equal to  $\alpha/4$ , which is the solution of the symmetric Nash-equilibrium. The utility levels are 1 and  $3\alpha^2/16$ , respectively. If a type II worker is employed by a small firm, then his utility maximizing effort level is 1/2 and the corresponding utility level is 1/4. A lazy worker will therefore not move to a small firm when

$$3\alpha^2 \ge 4$$
.

It has also to be ensured that an industrious worker does not move to a large firm. Suppose that there is just one large firm. The industrious worker will supply an effort level of one and the lazy workers supply  $\alpha/18$ . The wage received by every worker is  $\alpha(3 + \alpha)/9$ . The industrious worker will not stay in this large firm when

$$\alpha(3+\alpha) < 9$$
.

The industry structure consisting of one small firm

employing the industrious worker and one large firm employing the lazy workers is an equilibrium industry structure when the above inequalities are satisfied. This is achieved when

$$2/\sqrt{3} < \alpha < \sqrt{11.25} - 1.5$$
.

So, there is a range of intermediate values of the productivity parameter  $\alpha$  for which small and large firms exist together. If the large firm is much more productive than the small firm, then the small firm will disappear and the industrious worker will be employed by a large firm. On the other hand, a small productivity difference between small and large firms will eliminate the large firm because the higher productivity of the large firm will not compensate the lazy worker enough for the free riding of his colleague.

The industry structure of one small and one large firm is not only an equilibrium for intermediate values of the productivity parameter but it is also attractive from an output point of view. The industry output when there is one large and one small firm is

$$1 + \alpha^2/2$$
.

The industry consisting of one large firm produces

$$\alpha(9+\alpha)/9$$
.

It is easy to show that the industry output of one small and one large firm is always higher than the output of an industry consisting of just one large firm.

Notice that the above example is not sensitive to the number of industrious and lazy workers. A large firm will in equilibrium consist of a limited number of lazy workers, because utility levels will decrease beyond a certain firm size. (Utility levels decrease already beyond two workers in the above example with  $c(e) = e^2$ .) This is due to the reduction in disutility of effort not being large enough to compensate for the lower wage. The industry output result in the above example is also not affected by the number of lazy workers, because the production of a large firm consisting of only lazy workers is always  $\alpha^2/2$ . Finally, observe that both types of workers are needed in order to get the heterogeneity of firms result. Only one type of worker will result in all firms being identical, except for some cases with integer problems.

# III. Monitoring

This section will investigate the effect of monitoring on the productivity of the large firm and the coexistence of small and large firms. The monitoring technology will be modelled by adopting the specification of Farrell and Lander (1989). They distinguish between two types of effort. A selfish effort level represents the activities spend on promoting private returns, whereas a team effort level contributes to the team's goal, i.e., production. The costs of these two effort levels are born by the worker and unobservable. In return, he gets a share of the team's benefit. This share consists of a fixed component and some imperfect indicator of the team effort level provided by the worker. The fixed component is determined by the number of workers in the firm, whereas the imperfectness of the indicator represents how well the team effort level is measured. This is modelled by a weighted sum of team and selfish effort level. Each worker decides which selfish and team effort level to provide.

#### III.1. Model

Each individual chooses a selfish effort level  $s_i$  and a team effort level  $e_i$ . We will assume that these effort levels are in the unit interval. A type II worker bears an unobservable direct cost  $c(s_i + e_i)$ , which we assume is differentiable, increasing and convex. A type I worker does not incur cost of supplying effort. Workers get a share of the output of the firm. The fractional share,  $\tau_i$ , of the output in firm k that worker i receives is

$$\sigma_i = \delta \tau_i / \Sigma \tau_i + (1 - \delta) / |S_k|,$$

where

$$\tau_i = \gamma e_i + (1 - \gamma) s_i$$

is observable. The share  $\sigma_i$  depends on  $\tau_i$  (an imperfect indicator of  $e_i$ ) in order to encourage team effort. The parameter  $\gamma$  describes how well  $\tau_i$  measures  $e_i$ , and  $\delta$  represents the weight given to this imperfect observation in setting members' rewards. It is assumed that  $\gamma$  is smaller than 0.5.

A lazy worker i employed by firm k chooses  $e_{ik}$  and  $s_{ik}$  to maximize

$$U_i(e_{ik}, s_{ik}) = \sigma_i \cdot f(e_{ik}, e_k^i) - c(e_{ik} + s_{ik}).$$

Two first-order conditions regarding type I workers are obtained by calculating the symmetric Nashequilibrium in the second stage. The comparative statics results are those of Farrell and Lander (1989). They show that the team effort is negatively related to the number of workers employed by the firm. This is the free riding effect. The second comparative statics result is that improved monitoring capabilities  $(\gamma)$  of a large firm increases the attractiveness of greater team effort levels, i.e., less imperfect devices to measure  $e_i$  will result in higher levels of  $e_i$ . More weight given to  $e_i$ in the determination of  $\tau_i$  implies that less weight is given to the selfish effort level  $s_i$ . However, the cost of supplying effort remains the same, which induces a worker to reduce his selfish effort level. Improved monitoring capabilities are therefore positively related to team effort levels and negatively related to selfish effort levels.

The above result regarding the monitoring capabilities suggests that the range of parmeter-values for which small firms exist is negatively related to  $\gamma$ . Large firms are faced in the above model with the difficulty of assessing individual contributions to output. Once these problems are reduced by increasing monitoring capabilities, then their superior productivity features more prominently. Greater team effort levels in large firms will increase payments in large firms. This might be sufficient to attract industrious workers, away from small firms.

The allocation of workers across firms is given when the choices in the second stage of the game are made. These second stage decisions are taken into account when the choice of the firm by a worker is considered in the first stage. It is therefore not obvious how the size of the large firm will change due to a change in the monitoring capabilities. First, improved monitoring capabilities will result in a higher effort level by a lazy worker. This will increase the wage received, but also the cost of providing the higher level of effort. This last effect may dominate and actually reduce the size of the firm. The next subsection will show this with an example. Second, an increase in these capabilities will increase the team effort level provided by a type two worker. This makes it more attractive for an additional worker to move to a large firm. However, this move increases the free riding problem and may prevent the move.

### III.2. Example

The example of subsection II.4 is appropriately adapted in order to take the monitoring technology into account. Suppose that  $c(e + s) = (e + s)^2$ . If a large firm is employing n type II workers, then the utility maximizing selfish and team effort level are

$$e = \alpha (1 - \gamma)^2 / 2n(2\delta + 1) (1 - 2\gamma)^2$$
  
and

$$s = (2\delta - \gamma(4\delta + 1)) \times$$

$$\times \alpha(1 - \gamma)/2n(2\delta + 1)(1 - 2\gamma)^2,$$

when there is an interior solution. Two effects are clearly illustrated by the above expressions. First, improved monitoring capabilities  $(\gamma)$  will induce higher team effort levels by the individual workers. This will attract more workers to a firm. Second, more workers (n) in a firm exacerbates the free riding problem. This might prevent an increase in the size of the firm.

Improved monitoring capabilities may actually reduce the size of the firm. This depends on the utility of higher wages versus the disutility of providing higher effort levels. Only type II workers are needed to illustrate this claim.

It can be shown that for  $\alpha=2$ ,  $\delta=0.8$  and  $\gamma=0.2$  a large firm consisting of two workers can not exist in equilibrium. Both workers are rather self-employed. However, a large firm consisting of three workers exists in equilibrium for  $\gamma=0.2$ . The lower effort level due to more free riding and the moreless low level of the monitoring capabilities results in a lower wage received by each worker. However, this is more than compensated for by the reduction in the cost of providing effort. Improved monitoring capabilities, e.g.,  $\gamma=0.22$ , will result in utility levels for labor in a large firm consisting of two workers which are higher than the self-employment utility level.

The comparative statics results regarding the weight given to the imperfect observation in setting members' rewards are counterintuitive. An increase in  $\delta$  will reduce the effort level provided for the team (e), whereas the selfish effort level (s) will increase. The increase in  $\delta$  puts more weight on the imperfect indicator of the effort level supplied  $(\tau)$ . Recall that this indicator is a linear

combination of the team and selfish effort levels. The weight put on the team effort level  $(\gamma)$  is by assumption smaller than 0.5. The most attractive way of increasing  $\tau$  for the worker is therefore to increase s and lower e.

### IV. Summary and further research

We have developed a model in which workers first choose a firm and next decide upon an effort level. A large firm is assumed to be more productive than a smaller firm, ceteris paribus. However, small firms exist for intermediate values of the productivity diference parameter. The free riding in large firms is responsible for this result because it drives some workers into small firms. Workers with a strong preference for high wages are driven to small firms, whereas those having a strong preference for leisure are employed by large firms. It turns out that an industry structure consisting of small and large firms may produce more than an industry consisting of only large firms because the small firms provide a lower bound on the free riding in large firms.

The introduction of a monitoring technology shows that the relationship between the monitoring capability and the attractiveness of large firms for industrious workers depends on several endogenous vraiables. Improved monitoring capabilities induce higher team effort levels in the second stage, but this attracts more workers and therefore worsens the free riding problem.

An avenue for future research might be to look at optimal reward schedules. We have not explored the possible benefits and costs of contracting. Various parameters which might be part of a contract were treated as exogenous variables in the above model. Some of the comparative statics results regarding these parameters might be qualified when they are determined together.

We have only distinguished small and large firms. It might be desirable to split the category of small firms into two categories. Workers seeking employment by small firms might choose to work in a small firm as an employee or work for themselves. A force guiding the selection of workers in such an environment might be differences between risk postures in the population of workers. Firm heterogeneity was driven in our model by economic forces. A richer model explaining the size

distribution of firms has to include other aspects like the size and composition of demand, financial constraints and a richer description of the motivation of entrepreneurs (e.g., Kandel and Lazear, 1990; Brown *et al.*, 1990).

The research and development aspects of the cases described in the introduction were not treated. Geroski (1990) summarized his empirical findings regarding the relationship between innovation and industry structure by "... our data suggests that the price which has to be paid for high levels of innovation may not include tolerating the growth of highly concentrated, imperfectly competitive market structures". Our model might address some of the issues involved. The analysis suggests that an industry consisting of small and large firms might achieve an innovation earlier than an industry consisting of just large firms, due to increased competition more than offsetting the free riding in large firms with the superior technology.

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