Testing for Converging Deterministic Seasonal Variation in European Industrial Production

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Abstract

In this paper we consider deterministic seasonal variation in quarterly industrial production for several European countries, and we address the question whether this variation has become more similar across countries over time. Due to economic and institutional factors, one may expect convergence across business cycles. When these have similar characteristics as seasonal cycles, one may perhaps also find convergence in seasonality. To this aim, we propose a new method, which is based on treating the set of production series as a panel. By testing for the relevant parameter restrictions for moving window samples, we examine the hypothesis of convergence in deterministic seasonality while allowing for seasonal unit roots. We derive the estimation bias, and show that it is very small for samples of more than three years of quarterly observations. Our main empirical finding is that there is almost no evidence for convergence in seasonality.

Key words: Convergence; Deterministic seasonal variation; European industrial production.

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1 Introduction

The last few years have witnessed an obvious interest in examining whether there is convergence amongst major economic variables, measured for European countries. Examples of such variables are interest rates, stock market returns, industrial production, and determinants of labor markets. There are several reasons why such convergence may exist. These include institutional factors like the establishment of the European Monetary Union. Also, the fact that many European countries have small (relative to, for example, the US) open economies suggests the plausibility that these economies respond similarly to exogenous shocks such as, for example, sudden oil price increases and the Asian stock market crisis.

Because industrial production is a key macroeconomic variable, it is relevant to study convergence of its main features amongst countries. Indeed, given the fact that many European countries are closely related, one may expect that a common European business cycle may have emerged over time. In this paper, we also consider this variable, where we now focus on its country-specific seasonal variation. Recent evidence in MIRON (1996) (and several other studies of the same author and his associates) indicates that business cycle variation and seasonal variation are closely related. Hence, where there would be some form of convergence amongst the growth patterns of industrial production in European countries, one may hypothesize that there is a converging tendency for seasonal variation in these growth rates as well.

The empirical results in MIRON (1996, table 3.28) suggest that seasonal growth rates for European countries in particular seasons have the same sign, and also that often growth rates are large in the last quarter while there is a substantial decrease in the first quarter. In this paper we investigate if these seemingly common patterns also indicate that the size of seasonal growth has become more similar across countries. For this purpose we propose a new method which is based on treating the set of industrial production series as a panel. By testing for the relevant parameter restrictions for moving window samples, we examine the hypothesis of converging seasonal variation. Similar to MIRON (1996), we assume that seasonal variation can sufficiently be described by seasonal dummy variables. However, in contrast to the approach taken in MIRON (1996), we allow for the potential presence of seasonal unit roots.

The outline of this paper is as follows. In Section 2, we discuss two panel models which are relevant for our purposes. Because we aim to use moving window samples and these samples may contain a small number of observations, we investigate in Section 3 the small-sample behavior of our estimator of interest. The reader who is not interested in these results can simply skip this section. Briefly, we find the bias to be very small in samples containing only 12 quarterly observations, which is the smallest sample we use for our empirical work. In Section 4, we apply our method to several European countries, for which we have over three decades of quarterly observations. In Section 5, we present the overall conclusion, which is that we find virtually no evidence for converging deterministic seasonal variation, and we discuss a few potential explanations.

2 The models

In this section, we provide details on the representation of the panel data models we use in our empirical work below, and on the relevant estimators and tests. First, in Section 2.1, we consider the individual-dynamics model, while in Section 2.2 we consider the common-dynamics model, which turns out to be very useful for our purposes.

2.1 An individual-dynamics model

If all countries are assumed to follow their own idiosyncratic dynamic patterns, or, in other words, business cycles are seen as country-specific, one has the individual-dynamics (ID) model. Additionally, we assume one unit root at $\omega = 0$ (the zero frequency), fixed seasonal effects, and autoregressive individual dynamics of order p_k. We use [.] for the largest integer function and $\delta(i, j)$ for Kronecker's δ function. Thus, we define $D_{tj} = \delta(j, t - 4[(t - 1)/4])$ as the usual quarterly dummy variables. Then, the ID model reads as

$$\begin{aligned} \Delta y_{kt} &= \sum_{j=1}^{4} d_{kj} D_{tj} + \sum_{j=1}^{p_k} \varphi_{kj} \Delta y_{k,t-j} + \varepsilon_{kt}, \\ & \quad t = p_k + 2, \ldots, T, \quad k = 1, \ldots, N, \end{aligned} \tag{1}$$

where N denotes the number of countries, and Δ denotes first differencing. The polynomials $\varphi_k(z) = 1 - \sum_{j=1}^{p_k} \varphi_{kj}(z)$ may have seasonal unit roots at $\omega = \pi, \pi/2$. Then, in order to avoid diverging seasonal trends, it appears plausible to impose the restrictions suggested by FRANSES AND KUNST (1999a). To this aim, it is convenient to use a different representation for the deterministic seasonal part , that is, to use

$$d_{k1}D_{t1} + \ldots + d_{k4}D_{t4} = g_{k0} + g_{k1}w_{1t} + g_{k2}w_{2t} + g_{k3}w_{3t}, \tag{2}$$

where we use the abbreviations $w_{1t} = \cos(\pi t)$, $w_{2t} = \cos(\pi t/2)$, $w_{3t} = \cos(\pi(t-1)/2)$ for the cycles at the seasonal frequencies. This allows to separate the individual long-term growth rates g_{k0} from the quarterly growth rates. We will not attempt to test restrictions on these growth rates, as it is, for example, done in the literature on 'convergence' (cf. ANDRES *et al.*, 1996, among others). Instead, we view them as freely varying and we constrain focus on the coefficients $g_{kj}, j = 1, 2, 3$, which describe the shape and amplitude of the deterministic seasonal cycle.

The analysis of FRANSES AND KUNST (1999a) suggests to impose $g_{k1} = 0$ if $\varphi(-1) = 0$ and $g_{k2} = g_{k3} = 0$ if $\varphi(i) = \varphi(-i) = 0$. In order to simplify notation, we define the switching factors

$$\zeta_{k1} = I\{\varphi_k(-1) \neq 0\}, \quad \zeta_{k2} = \zeta_{k3} = I\{\varphi_k(\pm i) \neq 0\}.$$

Hence the following model (B is the backward shift operator)

$$\begin{aligned} \Delta y_{kt} &= g_{ko} + \sum_{j=1}^{3} g_{kj}\zeta_{kj}w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}, \\ & \quad t = p_k + 2, \ldots, T, \quad k = 1, \ldots, N, \end{aligned} \tag{3}$$

is designed to avoid seasonal cycles that expand as $T \to \infty$. The coefficients g_{kj} are not identified for $\zeta_{kj} = 0$ but they are kept in (3) for the ease of notation.

It may be interesting to see whether the deterministic seasonal cycles show similarities across the panel, even if the business-cycle dynamics and the seasonal unit-root properties are allowed to vary among countries. To this aim, one may consider the restricted model

$$\Delta y_{kt} = g_{ko} + h_k \sum_{j=1}^{3} g_j \zeta_{kj} w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}. \tag{4}$$

This model is identified if, for example, $\|(g_1, g_2, g_3)\| = 1$ is imposed for a normalization, and if $\zeta_{kj} = 1$ for some k. Note that this model allows the nonseasonal dynamics φ_k, the cases of seasonal unit roots ζ_{kj}, the growth rates

g_{k0}, and the amplitude of the seasonal cycle h_k to be country-specific, whereas the shape of the deterministic seasonal cycle is restricted to be common to all countries.

It turns out, however, that this restricted model does not properly reflect similarities in deterministic cycles across countries. For example, if $\varphi_k(\pm i) = 0$, then $\zeta_{k2} = \zeta_{k3} = 0$ and no restriction is imposed on the deterministic seasonal sine wave $h_k w_{1t}$ in country k. On the other hand, restricting attention to the cases without seasonal unit roots may (and actually does so in our empirical application) eliminate most countries from the analysis. We decided to adopt a compromise solution and to exclude only those countries where seasonal unit roots at *both* seasonal frequencies have been detected. In the remaining cases, seasonal unit roots are not imposed during estimation and deterministic cycles are allowed to consist of both the $\cos(\pi t)$ and the $\cos(\pi t/2)$ component. This model reads

$$\begin{aligned} \Delta y_{kt} &= g_{ko} + \{1 - (1-\zeta_{k1})(1-\zeta_{k2})\}\, h_k \sum_{j=1}^{3} g_j w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}, \\ & \quad t = p_k + 2, \dots, T, \quad k = 1, \dots, N. \end{aligned} \tag{5}$$

Noting that the restricted model has

$$\begin{aligned} n_R &= \sum_{k=1}^{N} p_k + N + 2 + \sum_{k=1}^{N} \{1 - (1-\zeta_{k1})(1-\zeta_{k2})\} \\ &= \sum_{k=1}^{N} p_k + N + 2 + \sum_{k=1}^{N} \zeta_{k1} + \sum_{k=1}^{N} \zeta_{k2} - \sum_{k=1}^{N} \zeta_{k1}\zeta_{k2} \end{aligned} \tag{6}$$

free coefficient parameters and the unrestricted ID model (3) has

$$n_U = \sum_{k=1}^{N} p_k + N + \sum_{k=1}^{N} \zeta_{k1} + 2\sum_{k=1}^{N} \zeta_{k2} \tag{7}$$

such coefficients, the restriction of a common shape in the deterministic seasonal cycles can be tested by a Fisher test. The Fisher statistic is defined as

$$F = \frac{ESS_R - ESS_U}{ESS_U/DF_U} \quad , \tag{8}$$

where $DF_U = N(T-1) - \sum_{k=1}^{N} p_k - \sum_{k=1}^{N} \zeta_{k1} - 2\sum_{k=1}^{N} \zeta_{k2}$ is the degrees of freedom of the unrestricted model, i.e., the difference of the NT observations and the n_U coefficient parameters to be estimated. The abbreviations ESS_U and ESS_R denote the residual sums of squares in the unrestricted and the restricted model, respectively. Under the restricted model, F is asymptotically distributed as χ^2 with $\sum_{k=1}^{N} \zeta_{k2}(1+\zeta_{k1}) - 2$ degrees of freedom.

To obtain the sums of squares ESS_R and ESS_U, the parameters in (3) and (5) must be estimated. We first choose the switch factors ζ_{kj} from a novel Bayesian decision procedure that is based on the seasonal unit root statistics developed by Hylleberg *et al.* (1990) [HEGY] and lag orders p_k selected by AIC. The details of this decision procedure are given in Franses and Kunst (1999b). We use these simulated critical values instead of the significance points given by HEGY in order to achieve the consistency of the other parameter estimates in the second step, which is conducted by least squares.

A convenient way to estimate the restricted model is to conduct the following steps, after eliminating those $N - N^*$ countries where seasonal unit roots at both seasonal frequencies have been found:

1. Purge the deterministic seasonal cycle from each country k by regressing $\Delta y_{kt}, t = 1, \ldots, T$ on a constant and on $w_{jt}, j = 1, 2, 3$. Call the residuals Δy^*_{kt}.

2. Estimate the individual autoregressive dynamics from the purged series by N^* least-squares regressions.

3. Filter the original Δy_{kt} by the estimated $\hat{\varphi}_k$, yielding filtered values f_{kt}.

4. Run a canonical analysis between the N^*–dimensional vector of the filtered f_{kt} and the 3–dimensional vector w_{jt}. This is equivalent to a reduced-rank regression of f_{kt} on w_{jt} with the rank of the coefficient matrix fixed at one. Estimates for the loading coefficients $h_k, k = 1, \ldots, N^*$, and for the shape coefficients $g_j, j = 1, 2, 3$, are obtained.

For the technique of canonical analysis in reduced-rank regression problems, see Tso (1981). For the dimensionality of the application that we report in Section 4, only the shape coefficients g_j are obtained as canonical vectors, whereas the loading coefficients h_k are obtained by simple linear regression.

2.2 A common-dynamics model

Panel models differ mainly with respect to how much common structure is assumed across the N individuals. In the ID model, no common structure is assumed in the general model (3) although we suggested testing for a common shape of the seasonal cycle in (5). Imposing more common structure can increase the power of restriction tests.

Alternatively, the common-dynamics (CD) model assumes that the N countries are subject to the same low-order autoregressive dynamics. In other words, the CD model assumes that $\varphi_k \equiv \varphi$. This simplifies the structure (1) to

$$\begin{aligned} \Delta y_{kt} &= g_{k0} + \sum_{j=1}^{3} g_{kj} w_{tj} + \sum_{j=1}^{p} \varphi_j \Delta y_{k,t-j} + \varepsilon_{kt}, \\ & t = p+2, \ldots, T, \quad k = 1, \ldots, N. \end{aligned} \tag{9}$$

The common polynomial $\varphi(z)$ may contain seasonal unit roots at -1 or at $\pm i$. Then, the restrictions suggested by FRANSES AND KUNST (1999a) imply that the deterministic cycle is restricted for *all* individuals. For example, if $\varphi(-1) = 0$, then $g_{k1} \equiv 0$ for $k = 1, \ldots, N$ and the regressor w_{t1} is excluded from all individual relationships.

For a decision on the presence of seasonal unit roots, seasonal unit-root tests such as the test by HEGY can be applied. The HEGY statistics are the t–values on the coefficient $b_j, j = 2, 3, 4$ in the regression

$$\begin{aligned} \Delta_4 y_{kt} &= g_{k0} + b_2 y^{(2)}_{k,t-1} + b_3 \Delta_2 y_{k,t-2} + b_4 \Delta_2 y_{k,t-1} + \sum_{j=1}^{p^*} \gamma_j \Delta_4 y_{k,t-j} + \varepsilon_{kt}, \\ & t = p^* + 5, \ldots, T, \quad k = 1, \ldots, N, \end{aligned}$$

where $\Delta_4 = 1 - B^4$, $\Delta_2 = 1 - B^2$, and $y^{(2)}_{k,t} = (1 - B + B^2 - B^3) y_{k,t}$. Note that the usually included extra regressor $(1 + B + B^2 + B^3) y_{k,t-1}$ is absent, as we do not test for a unit root at +1. Omitting this regressor increases the test power at the other frequencies. For the case $N = 1$, simulated significance points for finite T and for $T \to \infty$ are given by HEGY. For the case $N > 1$, these are no longer valid.

For $N = 1$, the HEGY statistic τ_2 can be shown to converge, under the null hypothesis $\varphi(-1) = 0$, for $T \to \infty$, and for the homogeneous regression with $b_0 = 0$, to the Dickey-Fuller distribution, which is formally written as

$\int BdB(\int B^2)^{-1/2}$ in stochastic integrals over the Brownian mo

–81.