CHAPTER VI

COMPLICATIONS CREATED BY BOUNDARY CONDITIONS

1. In chapter III, § 3 the rôle of boundary conditions was already indicated in a general way. We have to deal with this matter in some more detail now. Suppose we have to do with what we called, in chapter IV, a normal problem. The system of structural relations will yield us a set of values z_l^0 for the political parameters. The question now arises whether or not these values satisfy the boundary conditions. If they do not, the solution cannot be accepted for the very reasons that led us to formulate the boundary conditions: as for e.g. physical, social or political reasons, In other words: there does not exist a solution to the broader problem of finding values which obey at the same time the structural relations and the boundary conditions. We shall have to reformulate our problem and content ourselves with a more modest task-setting of the economic policy under discussion.

This comes to giving up one or more of the targets originally set. To start with we shall, of course, only give up conditional targets. Instead we shall add one or more boundary conditions and it seems the most natural procedure to add those particular boundary conditions that were, with the previous set-up of the problem, violated. There is a great deal of chance that we shall obtain an acceptable solution then. There is, however, no guarantee that the new solution will be acceptable. If the number of violated boundary conditions surpasses the number of conditional targets, it may also be impossible to reformulate the problem beforehand, i.e. to formulate an alternative where the number of equations is equal to the number of unknows. The case may therefore present itself that no solution to our policy problem, however restricted by the elimination of conditional targets, exists.

Apart from these possibilities there are other potential complications. If originally one boundary condition was violated and hence one conditional target replaced by that boundary condition, it may happen that another boundary condition will be violated in turn. But it may as well happen that, with originally two boundary conditions violated, the introduction of one of them (with the necessary omission of one conditional target) is already sufficient to make the problem soluble. Generally speaking the number of possibilities is large and it is, as a rule, difficult if not impossible to study them or detect them in a systematic way. It is by trial and error, albeit perhaps by a somewhat systematicised trial and error method, that we have to proceed. This is one reason why in this chapter more particularly than in the previous ones examples are a useful tool in the study of these problems.

2. Starting again with *example* (1), where no data are included, we may impose the boundary condition that no wage reduction is accepted, i.e. $l' \geqq 0$. This condition is violated by the solution found in chapter IV, § 3 where $l' = -0.11$. Considering the equilibrium in the balance of payments as the unconditional target, i.e. $D = -0.04$, we have to give up the other target $y = 0$ and now have the following equations:

$$\begin{aligned} \xi_2 p - \xi_0 \qquad\quad &= -\xi_1 y + 0.04 \\ -\delta p \qquad\qquad\quad &= \mu y + 0.04 \\ p \qquad - \pi_0 &= \pi_2 y \\ p \qquad\qquad\quad &= 0 \end{aligned}$$

It is easily seen that the solution now runs:

$$y = -\frac{0.04}{\mu} = -0.09;\ \pi_0 = 0.01;\ \xi_0 = -0.05.$$

It is evidently only at the expense of a rather large amount of unemployment that the stability of wages is to be attained, whereas the decrease in public expenditure must also be larger than before. If we had also imposed a boundary condition saying e.g. that $\xi_0 \geqq -0.04$, that second condition would now be violated, making the new solution also inacceptable. In that case therefore the conclusion would be that no acceptable solution to the balance of payments problem would be possible.

3. For our next example we take *example* (2) but, to begin with, assume *no change in the data*. Let us consider as unconditional targets the reduction to one-half of the balance of payments deficit ($D = -0.02$) and the maintenance of real national expenditure ($x = 0$), representing e.g. the necessity of an increased defence programme. As conditional targets, on the other hand, we consider $a = l^R = 0$.

Imposing no boundary conditions we find the solutions given in chapter IV, § 4, i.e. the first line of table VI 1 (p. 49). It appears that the targets set can only be reached by a fall in nominal wages and in nominal profit margins by 5 % and an increase in productivity by 4 %. Supposing now that it is considered psychologically *impossible to impose a decrease in nominal wages* — say, because workers do not believe that a fall in prices will be realised —; then there is a boundary condition $l \geqq 0$. Adding this condition to the equations, we may omit either of the conditional targets. It seems natural, in this case, to drop the condition that $l^R = 0$ (real wage unchanged) since the constancy of the nominal wage rate presumably will even lead to an increase in real wages. Our new problem therefore is: given $D = -0.02$; $x = 0$; $a = 0$ and $l = 0$; unknown are now only three instruments h, π_0 and τ and one variable l^R that, in this connection, might be considered as an irrelevant variable. The results are given as case 2 in the table below. It appears that, in fact, $l^R > 0$

now, but that a stronger decrease in profit margins and a stronger increase in indirect taxes are required. In a sense social equilibrium has been disrupted (unless it is believed that in the initial situation non-workers incomes were quite out of proportion to workers incomes).

Another problem with one boundary condition arises if we assume that an (*extra*) *increase in productivity will not be possible* at short notice e.g. since in the planned figures from which we start a certain increase was already taken account of and since efficiency increase is a slow process. This is equivalent to imposing the condition $h = 0$ meaning in fact that one of the instruments originally recognised is now dropped. (An alternative treatment, algebraically almost identical, would be to impose a condition $h \leqq 0.02$ e.g.). For the moment we assume that nominal wage decreases are, on the other hand, considered possible. The problem is now as indicated in line 3 of the table, if we drop the target $a = 0$, or as in line 4 if we drop $l^R = 0$. Case 3 is very much like case 1, but there is a higher rate of employment, profit margins are lower and taxes higher.

Case 4 is a very strange case and very instructive for the theory of economic policy. It appears that equations (2 II) and (2 IV) now become incompatible; they are

$$- 0.02 - \delta (l - l^R - \tau) = 0$$
$$- 0.03 + \eta (l - l^R - \tau) = 0$$

Going back to their original form in order to find the economic interpretation of this situation we may state the problem in the following way. Since $h = a = 0$ also $y = 0$; in addition $x = 0$. In the balance of payments equation (2 II′) this means (since no changes in data are assumed to exist either) that the only regulative variable for closing the balance of payments remains the variable p; in fact this equation now runs:

$$D - \delta p = 0$$