Modeling the Conditional Covariance Between Stock and Bond Returns: A Multivariate GARCH Approach

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Modeling the Conditional Covariance Between Stock and Bond Returns: A Multivariate GARCH Approach[1]

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[1] We appreciate the comments of Guido De Bruyne, Hans Dewachter, Geert Dhaene, Ben Jacobsen, Theo Nijman, Frans Spinnewyn, Marno Verbeek and several seminar participants.

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Abstract

To analyze the intertemporal interaction between the stock and bond market returns, we allow the conditional covariance matrix to vary over time according to a multivariate GARCH model similar to Bollerslev, Engle and Wooldridge (1988). We extend the model such that it allows for asymmetric effects on conditional variances and covariances. Using weekly U.S. stock and bond market data, we find strong evidence of conditional heteroskedasticity in the covariance between stock and bond market returns. The results indicate that not only variances, but also covariances respond asymmetrically to return shocks. Regardless of the bond market shocks, bad news in the stock market is typically followed by a higher conditional covariance than good news. We find that volatility timing strategies for dynamic asset allocation significantly outperform passive strategies. Even when short-sale restrictions are present and transaction costs are high, the economic value of dynamic trading strategies is larger than that of a passive strategy. Moreover, the symmetric volatility timing strategy is outperformed by its asymmetric counterpart.

Keywords: Multivariate GARCH, Stock and Bond Market Interaction, Time-Varying Volatility, Asymmetric Effects, Impact of News.

JEL classification codes: G12, C22.

1 Introduction

The modeling of conditional volatilities of asset returns, as well as the covariances between returns, is of considerable importance for the pricing of financial securities, and (co)variances are key inputs to asset allocation and risk management in financial institutions. Consequently, accurate models and forecasts of conditional covariances are crucial. However, while there is a vast amount of literature on modeling returns and volatility, these are often restricted as they either examine the stock market or the bond market separately.[1] Little attention has been paid to the interaction between the two markets. Only since the last decade financial economists have begun to model these temporal dependencies. For example, Breen, Glosten and Jagannathan (1989) show that there is a negative relation between short term interest rates and future stock index returns, and Schwert (1989) documents that U.S. stock and bond returns and volatilities move together. A recent study by Fleming, Kirby and Ostdiek (1998) examines volatility interaction of stock, bond and money markets using a stochastic volatility model. Although they find a strong link in volatility between the three markets, they do not consider the conditional covariance between the stock and bond market returns. Studies that explicitly consider time-varying conditional covariances, using multivariate GARCH models, include Bollerslev, Engle and Wooldridge (1988), Ng (1991), Turtle, Buse and Korkie (1994), and De Santis and Gerard (1997). While these studies lay emphasis on allowing the conditional mean equation to depend on the conditional covariance terms, they do not explicitly examine the interactions between the stock and bond market.

The purpose of our study is to analyze the intertemporal interactions of stock and bond returns. To this end we allow the conditional covariance matrix of stock and bond market returns to vary over time, according to a multivariate GARCH-in-mean model similar to Bollerslev, Engle and Wooldridge (1988). We extend their model by allowing for asymmetric effects of return shocks on the conditional covariance between stock and bond returns.

[1] Some examples of stock market studies include Breen, Glosten and Jagannathan (1989), Campbell and Hentschel (1992), Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993) and Kroner and Ng (1998). Literature on the modeling of bond returns include Engle, Lilien and Roberts (1987), Engle, Ng and Rothschild (1990), Fama and French (1995) and Duffie and Singleton (1997).

Although it is often recognized that variances and covariances of returns change over time (see, e.g., French, Schwert and Stambaugh, 1987, and Schwert, 1989) their determinants are not yet well identified and documented. Among the econometric volatility models, the family of GARCH models, as introduced by Engle (1982) and generalized by Bollerslev (1986), seems to be the most fruitful. For an extensive literature overview we refer to Bollerslev, Chou and Kroner (1992) and Bollerslev, Engle and Nelson (1994). GARCH models are able to capture the phenomenon that volatilities of asset returns are clustered over time. Univariate GARCH models have appeared to be quite successful in predicting volatility. A drawback of standard GARCH models is that the arrival of "good" and "bad" news in the market (unexpected positive and negative returns, respectively) are assumed to have a symmetric impact on volatility, while typically unexpected decreases in prices tend to rise the predictable volatility more than unexpected increases of similar magnitude. This asymmetric effect of shocks in the second moment of stock returns is a well-known phenomenon in financial modeling. Recent studies have shown that more accurate volatility predictions can be obtained when asymmetric responses of volatility to news are taken into account. While many different extensions of the model have been suggested (for an excellent overview see Engle and Ng, 1993, or Bollerslev, Engle and Nelson, 1994), particularly nice extensions are the exponential GARCH, introduced by Nelson (1991), and the Glosten, Jagannathan and Runkle (1993) model. Empirical studies show that these models, which allow for the possibility that positive and negative shocks in returns affect volatility differently, work very well in practice.

While there is a large body of literature on the asymmetric volatility phenomenon in univariate ARCH models, there exists only few studies on the asymmetric effects in multivariate models, and surprisingly little attention has been paid to the asymmetric effects in the covariance between stock and bond market returns. As a portfolio manager's optimal portfolio depends on the predicted covariance between assets, relaxing the symmetric specification, may lead to superior investment choices. Other examples of applications in finance can be found in the field of risk management and derivative pricing. One of the few examples that imposes asymmetric effects in multivariate models is Kroner and Ng (1998). They use data on large and small firms to compare four popular multivariate GARCH models. Another example is Braun, Nelson and Sunier (1995), who estimate a bivariate exponential GARCH model with asymmetries in stock return betas for different sectors. However, this latter study does not explicitly consider asymmetries in covariances. Moreover, in order to examine asymmetries between different asset classes, their method is not very suitable. Because these effects on covariances between stock and bond returns in a multivariate GARCH model appear to be neglected in the literature, this paper is a first step towards filling this gap. To model the asymmetric effects on conditional covariances we develop a new approach by extending the Glosten, Jagannathan and Runkle (1993) specification to a multivariate setting. We use weekly data from 1987 to 1999 to examine the intertemporal interaction between the returns on the Standard and Poor's 500 index, and the returns on a short and long term bond.

The remainder of this paper is organized as follows. In Section 2 we describe the multivariate model which enables us to analyze time-varying covariances. In Section 3, the model is extended to capture asymmetric effects on the variances and covariances, so that we can apply it to asset market data on a short and long term bond and a stock index. Section 4 presents the data and empirical results on exploiting the model to study the time-varying covariances and their asymmetric properties. In Section 5 we evaluate the economic significance of asymmetric volatility timing using a dynamic allocation strategy. Conclusions are offered in the final section.

2 A Multivariate Approach to Modeling Time-Varying Covariances

In financial literature, the relationship between risk and expected returns on assets is well-reported. Most studies use the covariance between the return of an asset and one or more other factors as a measure of risk. A general stochastic discount factor model measures risk by the covariance between the asset's return and the stochastic discount factor. Especially in cross-sectional analysis the relationship between risk and expected return is well-documented and overwhelming empirical evidence is available (see, e.g., Fama and French, 1992, and Roll and Ross, 1994). There is a general consensus that investors require a higher expected return from holding a relatively riskier asset. Another branch of studies, that has recently received much attention in the literature, examines the *intertemporal* relationship between risk and expected return. A natural way of examining the dynamics of the conditional second moments is to consider a univariate GARCH-in-mean model (see, e.g., Engle, Lilien and Robins, 1987, French, Schwert and Stambaugh, 1987, Bollerslev, Engle and Wooldridge, 1988, and De Santis and Gerard, 1997). This way the conditional variance is allowed to influence the conditional mean, resulting in a time-varying risk premium. Empirical studies report mixed results regarding the relation between the market risk premium and time-varying market volatility.[2] To obtain a measure of risk in the *multivariate* case, we need to model the conditional covariances. As theory does not say anything about the way to model this, a natural way to fill this gap is to model the volatility by a *multivariate* GARCH process. This way we can easily examine the conditional covariance structure and interactions between the stock and bond market.

Starting from a general asset pricing model, the well-known result that expected excess return on an asset depends on the covariance between a stochastic discount factor and the return on this asset is obtained. Consider the general asset pricing model, which can be obtained by a no-arbitrage condition (see, e.g., Harisson and Kreps, 1979):

$$E_t\{m_{t+1}r_{i,t+1}\} = 1, \tag{1}$$

where $E_t\{.\}$ denotes the expectation operator, conditional on information at time t, m_{t+1} denotes the stochastic discount factor and $r_{i,t+1}$ the return on an asset i $(= 1, ..., N)$. Equation (1) holds for any asset. An expression for the conditional expected excess return on an asset at time $t + 1$ can be obtained by rewriting (1) as:

$$E_t\{r^e_{i,t+1}\} = -r_{f,t+1}Cov_t\{m_{t+1}, r_{i,t+1}\}, \tag{2}$$

where $r^e_{i,t+1}$ denotes the return on asset i in excess of the riskfree return. We hereby assume that a riskfree return, denoted by $r_{f,t+1}$, exists. Consequently, the general asset pricing model implies that the expected excess return on asset i (the risk premium) depends on the covariance between the stochastic discount factor and the return on the asset.

[2] Often a positive relation is found between the risk and the expected risk premium on an asset (see, e.g., French, Schwert and Stambaugh, 1987, and Campbell and Hentschel, 1992). However, sometimes a negative sign is found (e.g. Glosten, Jagannathan and Runkle, 1993, and Nelson, 1991).

To obtain a relationship that can be used empirically, we assume that asset prices can be described by a simple one-factor CAPM, introduced by Sharpe (1964) and Lintner (1965). Consequently, the risk will be measured by the covariance between the asset's return and the market return. This is obtained by imposing the following structure for the stochastic discount factor:

$$m_{t+1} = \eta + \psi r_{m,t+1}, \tag{3}$$

where $r_{m,t}$ is the return on the market portfolio at time t, and $\eta > 0$ and $\psi < 0$.[3] As Bollerslev, Engle and Wooldridge (1988) show, in order to test the validity of the CAPM, it is important that $r_{m,t+1}$ not only includes a stock market index. Therefore, we assume that the return on the market portfolio can be mimicked by the returns on N asset classes. Thus, the market portfolio is a weighted average of N asset classes: $r_{m,t} = \sum_{j=1}^{N} w_{j,t} r_{j,t}$, with the weights ($w_{j,t} \geq 0$) summing up to 1. A similar assumption is made by Frankel (1985) and Bollerslev, Engle and Wooldridge (1988). Under these assumptions, we can rewrite (2) such that the expected excess return on asset i is a function of its risk (cf. Merton, 1980):

$$\begin{aligned} E_t\{r^e_{i,t+1}\} &= \lambda Cov_t\{\sum_{j=1}^{N} w_{j,t+1} r_{j,t+1}, r_{i,t+1}\} \\ &= \lambda \sum_{j=1}^{N} w_{j,t+1} \sigma_{ij,t+1}, \end{aligned} \tag{4}$$

with $\sigma_{ij,t+1} = Cov_t\{r_{j,t+1}, r_{i,t+1}\}$, and $\lambda = -r_{f,t+1}\psi$ (> 0), interpreted as the market price of risk[4], which is assumed to be constant over time.[5] If $\lambda = 0$ in (4), the expected risk premium is unrelated to the predictable level of volatility of the market. We expect to find a positive relationship between the expected risk premium and the conditional volatility. Note that this framework allows for time-varying implicit beta of assets. It is easily seen from (4) that after imposing a CAPM structure, the expected excess return is proportional to the systematic

[3] As long as $E_t\{r_{m,t+1}\} > r_{f,t+1}$.

[4] Note that λ does not vary over i.

[5] It is possible to allow for time-varying market price of risk by assuming that the price of risk depends linear on a set of instruments, like a January dummy, the dividend yield, the change in the term premium and the change in a 1-month Treasury bill return (see De Santis and Gerard, 1997). We abstract from this extension.

risk of an asset, as measured by the covariance between the asset's return and the market return. Because we made the additional assumption that the market return can be mimicked by N returns, the covariance is decomposed into a sum of N covariances.

Following Bollerslev, Engle and Wooldridge (1988), we include asset-specific intercepts into the regression equation. This allows the conditional mean excess returns to be unconstrained in the estimation procedure. The regression model we take into consideration - which enables us to test the importance of the influence of past returns on current levels of returns - now becomes (for $i = 1, ..., N$):

$$r^e_{i,t+1} = \mu_i + \lambda \sum_{j=1}^{N} w_{j,t+1}\sigma_{ij,t+1} + \varepsilon_{i,t+1}, \tag{5}$$

where $\varepsilon_{i,t+1}$ represents the unexpected excess return on asset i, i.e. $r^e_{i,t+1} - E_t\{r^e_{i,t+1}\}$; it represents the "news" corresponding to asset i that is arrived in the corresponding market. Under the condition that $\sigma_{ij,t+1}$ is time-varying, (5) implies that the risk premium will be time-varying as well for $\lambda \neq 0$. The one-factor CAPM implies that risk is solely measured by the covariance between the asset's return and the return of the market portfolio. Conditional on the extra assumptions we made, we can test the CAPM relationship by testing whether $\mu_i = 0$, for each i. Next, we describe how the conditional covariances evolve over time.

The CAPM does not indicate how risk evolves over time. We model the time-varying covariances in (5) by a multivariate GARCH process. While the GARCH specification does not follow from any economic theory, it is well-known that it provides a good approximation to the heteroskedasticity typically found in financial time-series data. The most widely used volatility predicting model for financial series is the GARCH(1,1) parameterization, and we will restrict our attention to this particular specification since it has been shown to be a parsimonious representation of conditional variance that adequately fits many financial time-series. The formula for the univariate case is:

$$\sigma^2_{t+1} = \gamma + \alpha\varepsilon^2_t + \beta\sigma^2_t, \tag{6}$$

with $\sigma^2_{t+1} \equiv E_t\{\varepsilon^2_{t+1}\}$. Usually nonnegativity restrictions on the three unknown parameters are used to guarantee that the predicted variance is nonnegative.

While the majority of the GARCH literature focuses on the univariate properties, there now appears a vast amount of literature that considers multivariate extensions.[6] The GARCH(1,1) in (6) can be generalized to a multivariate setting as follows (see, e.g., Bollerslev, Engle and Wooldridge, 1988). The matrix Σ_{t+1}, containing the conditional covariances, is assumed to follow a simple multivariate GARCH(1,1) model, which can be compactly written in vector form as:

$$\text{vech}(\Sigma_{t+1}) = \mathbf{c} + A\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') + B\text{vech}(\Sigma_t), \tag{7}$$

where vech denotes the operator which stacks columns of the lower triangle (those elements on and below the main diagonal) of a $N \times N$ symmetric matrix as an $N(N+1)/2 \times 1$ vector.[7] Further, $\boldsymbol{\varepsilon}_t$ denotes the vector of error terms at time t. The vector $\mathbf{c}$ has dimension $N(N+1)/2 \times 1$, and matrices A and B have dimension $N(N+1)/2 \times N(N+1)/2$. While this model is a natural extension of the univariate GARCH model and is easy to understand, there are two major problems in estimating this model. The first problem concerns the number of parameters to be estimated and the second problem concerns the positive-definiteness constraints to be imposed on the conditional covariance matrix.

Obviously a disadvantage of the multivariate approach is that the number of parameters to be estimated in the GARCH equation increases rapidly (for example, with $N = 3$ there are 78 parameters to be estimated), which limits the number of assets that can be included. In order to reduce the number of parameters to be estimated, it is advisable to impose some restrictions on A and B, without lowering the explanatory power of the model significantly. Following Bollerslev, Engle and Wooldridge (1988), we assume that matrices A and B are diagonal. Thus, (7) can be written, after conveniently rearranging the parameter indices, as:

$$\sigma_{ij,t+1} = \gamma_{ij} + \alpha_{ij}\varepsilon_{i,t}\varepsilon_{j,t} + \beta_{ij}\sigma_{ij,t}, \tag{8}$$

$i, j = 1, ..., N$. For $N = 3$ this reduces the number of parameters to 18. Note that the univariate GARCH(1,1), defined in (6), is still nested in specification (8). Bollerslev, Engle and Wooldridge (1988) use this multivariate GARCH model to estimate the trade-off in variance among three assets: a stock index, a bond and a Treasury bill.

[6]Some examples include Harvey (1989), Bollerslev (1990), Bodurtha and Mark (1991), Ng (1991), Ng, Engle and Rothschild (1992), Braun, Nelson and Sunier (1995), Engle and Kroner (1995), Nijman and Sentana (1996), De Santis and Gerard (1997) and Kroner and Ng (1998).

[7]The multivariate model in (7) is sometimes called the VECH model.

In the univariate GARCH(1,1) the restrictions on the parameter to ensure that the variance is nonnegative are quite obvious. However, to guarantee in a multivariate setting that the conditional covariance matrix is positive definite is less straightforward. Although Bollerslev, Engle and Wooldridge (1988) do not impose any restrictions on the parameters, the covariance matrix may become nonpositive definite during the estimation procedure. If this happens, the estimated covariance matrix cannot be inverted, and the maximum likelihood method fails to compute an optimum. To circumvent this problem, Engle and Kroner (1995) suggest the use of the so-called BEKK model. The attractive feature of the BEKK specification is that it allows the conditional covariances to change sign over time while the positive definiteness of the conditional covariance matrix is guaranteed.[8] A drawback of the BEKK model is that the number of parameters increases rapidly when the number of assets increase and that the parameters cannot be easily interpreted. Moreover, the extension towards asymmetric models is not straightforward. GARCH models include the constant correlation model[9] of Bollerslev (1990) and the factor ARCH (FARCH) model of Engle, Ng and Rothschild (1990), which is a special case of the BEKK model. Compared to the VECH model, most of these models assume a very restrictive structure on the covariances, which makes them not suitable for our application. Moreover, the VECH model has the advantage that it allows cross-product terms of the negative shocks to determine the covariance. Considering all of the above, we opt to generalize the VECH model, and estimate the model using constrained maximum likelihood to ensure positive definiteness. In the next section we introduce asymmetric effects in conditional covariances. First, we briefly describe the univariate asymmetric models and then proceed by generalizing these models into a multivariate setting.

[8] More specifically, the BEKK model is given by

$$\Sigma_{t+1} = C + A_B' \varepsilon_t \varepsilon_t' A_B + B_B' \Sigma_t B_B,$$

where C is an $N \times N$ symmetric positive definite matrix, and the matrices A_B and B_B have dimension $N \times N$. Given that the matrix of constants is positive definite, the predicted conditional covariance matrix of returns will be positive definite because the second and third element in the BEKK model are expressed in quadratic terms.

[9] A major objection of this specification is that simultaneous shocks of opposite signs to two assets increase their conditional covariance.

3 Introducing Asymmetric Effects in Multivariate GARCH Models

GARCH models are quite successful in practice: they capture many stylized facts, such as volatility clustering and thick tailed returns. However, since the conditional variance is a function of the magnitudes of the lagged error terms and not their signs, GARCH models are not capable to capture the so-called *leverage effect*. This asymmetric volatility phenomenon, first noted by Black (1976), refers to the tendency that good and bad news in returns have a different impact on conditional volatility in stock markets. More specifically, bad news is followed by larger volatility than good news. The rationale of this phenomenon, according to Black (1976), is that a lower stock price increases the debt-equity ratio of a company (i.e. the financial leverage of the firm increases) and this again increases the risk of holding stocks of this company. Because firms have many fixed costs, a decrease in prices has a bigger impact on volatility than an increase in prices. It is however not likely that the large response of stock volatility can be explained by leverage alone (see Black, 1976). This is empirically supported by Christie (1982) and Schwert (1989).

Several recent papers put forward alternative explanations. Campbell and Hentschel (1992) and Bekaert and Wu (2000), e.g., use a volatility feedback approach. This implies that changes in volatility affect the level of required stock returns. Campbell and Hentschel show that volatility feedback explanation is able to explain the asymmetries in volatilities. An alternative interpretation is provided by a psychological explanation: the *following-the-herd effect*. That is, during a stock market crash, investors might pay less attention to the fundamentals, and sell their stocks when (they think that) other investors are selling stocks. This leads to a relatively high volatility when bad news arrives in the market. This idea is very similar to a result in a recent study by Veronesi (1999), who shows, using a rational equilibrium asset pricing model, where the drift of fundamentals shifts between two unobservable states, that stock prices overreact to bad news in good times and underreact to good news in bad times. Veronesi (1999) shows that this model is able to explain the asymmetric effect in stock returns.

Among financial economists there has not reached a consensus yet about the explanation of the asymmetric volatility phenomenon, and is a hot topic nowadays in financial economics. While the leverage argument can only partly explain the asymmetric nature of the volatility response to return shocks, in this paper we use the leverage effect as a synonymous for the asymmetric effect in (co)variances. We do not concentrate on the rationale behind this phenomenon. Instead we focus on estimating the importance of asymmetric effects in conditional covariances.

Numerous studies have shown that introducing a certain asymmetry in GARCH models to capture the leverage effects in conditional volatility, can substantially improve univariate models. These models are often referred to as leverage or asymmetric volatility models. One of the most successful asymmetric specification in univariate models is Nelson's (1991) EGARCH (which stands for Exponential GARCH), in which a logarithmic transformation is applied. This guarantees that variances are non-negative. A generalization of EGARCH is however inconvenient in a multivariate setting, because this would imply that all covariances between returns are positive. Nevertheless, Braun, Nelson and Sunier (1992) use a bivariate EGARCH model estimate the variances of the market portfolio and a second asset. To estimate the conditional beta between the market portfolio and a second asset they use a different specification without logarithms. This specification, which is not a very natural extension, seems less appropriate to model asymmetric covariances.

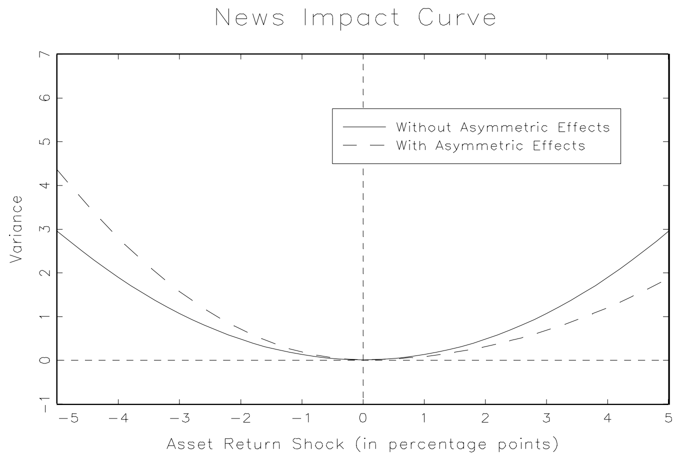
To allow for asymmetric effects in conditional covariances, a more appropriate approach is to extend the model of Glosten, Jagannathan and Runkle (1993) (GJR henceforth) to the multivariate case.[10] For the univariate case, a GARCH specification with GJR asymmetric volatility effects can be written as:

$$\sigma_{t+1}^2 = \gamma + \alpha_1 {\varepsilon_t}^2 + \alpha_2 I_{\varepsilon_t} {\varepsilon_t}^2 + \beta \sigma_t^2, \tag{9}$$

where I_{ε_t} denotes a dummy (or indicator) variable which is equal to 1 if $\varepsilon_t < 0$ (and zero otherwise), in other words the dummy is 1 if "bad" news arrives at the market, and zero in case of "good" news. Hence we expect α_2 to be positive if the leverage effect holds. Engle and Ng (1993) found that the GJR specification outperforms other asymmetric models (including EGARCH), using Japanese stock market data. Similarly, Hagerud (1997) finds empirical evidence that among several asymmetric univariate GARCH models, the GJR is one of the few specifications that is superior

[10] An additional advantage of GJR over EGARCH is that the former has an easier interpretation.

Figure 1: Typical News Impact Curve with and without Asymmetric Effects



for modeling the dynamics of the conditional variance of Nordic stock returns.

Introducing asymmetry in the *multivariate* GARCH model is not as straightforward as in the univariate case. Before considering asymmetries in the covariances, it is helpful to look at the so-called news impact curve, introduced by Engle and Ng (1993), which relates past return shocks to current volatility. This curve answers the question how past return shocks affect the conditional volatility, holding the past conditional variances constant at the unconditional sample mean. In Figure 1 we see a typical news impact curve for a symmetric and asymmetric univariate GARCH(1,1) model as in (9). From this figure one can easily see the impact of unexpected shocks on the conditional variance according to the GARCH model. While in a standard GARCH model (i.e. with $\alpha_2 = 0$) this curve is symmetric (the solid line), the curve becomes asymmetric when allowing for leverage effects (the dashed line). So while both curves are centered at $\varepsilon_t = 0$, the asymmetric news impact curve predicts that a negative return shock will lead to a higher subsequent volatility than a positive return shock of the same magnitude.

Next, we introduce a multivariate generalization of the news impact curve. Note that this implies two major changes. First, we consider covariances instead of variances. Second, the generalization of the news impact curve involves two shocks instead of only one. Instead of a *curve,* we will refer to this 3-dimensional plot as a news impact *surface.* This plot shows how past shocks in asset i and j $(i \neq j)$ affect the predicted covariance, holding past covariances constant at their unconditional sample mean levels. For illustrative purposes, Figure 2 presents a symmetric covariance surface against past return shocks on two assets, according to the Bollerslev, Engle and Wooldridge (1988) specification (formula (8)). Like in the news impact curve, we clearly see a symmetric pattern in the surface. The surface is symmetric if $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} = Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$ and $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} = Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$, where $\varepsilon^*_{i,t}$ denotes a given positive return shock in asset i. The interpretation of Figure 2 goes as follows. In the symmetric case, a return shock in asset i and j in the same direction (both positive or negative) has an identical impact on the conditional covariance. For example, a shock in the stock market return of 5% and a shock in the bond market return of 1% has exactly the same impact on the conditional covariance as the opposite shocks, i.e. -5% on the stock market return and -1% on the bond market return. Even more, the order of magnitude of the absolute value of the covariance (in excess of the shock-independent constant) for shocks of either $(5\%, 5\%)$, $(5\%, -5\%)$, $(-5\%, 5\%)$, or $(-5\%, -5\%)$, for example, are identical. However, there are good reasons to believe that there are also asymmetric effects in conditional covariances. In formula, the surface is asymmetric if $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} \neq Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$ and/or if $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} \neq Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, +\varepsilon^*_{j,t}\}$. Below we will show mathematically that if leverage effects in volatility exist, they also affect covariances.

If leverage effects exists in the variance of asset i we have that

$$Var\{r_{i,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}\} - Var\{r_{i,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}\} = \delta_{i,t} > 0, \tag{10}$$

where $\varepsilon^*_{i,t}$ denotes a given positive shock in asset i at time t, i.e. in the interval $(0, \infty)$. Using the definition of the squared correlation coefficient:

$$\rho^2_{ij,t+1} = \frac{Cov^2_t\{r_{i,t+1}, r_{j,t+1}\}}{Var_t\{r_{i,t+1}\}Var_t\{r_{j,t+1}\}}, \tag{11}$$

we can write[11]

$$Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\} - Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} =$$

[11] For simplicity, we assume symmetric time-varying correlations.

Figure 2: Typical Symmetric News Impact Surface

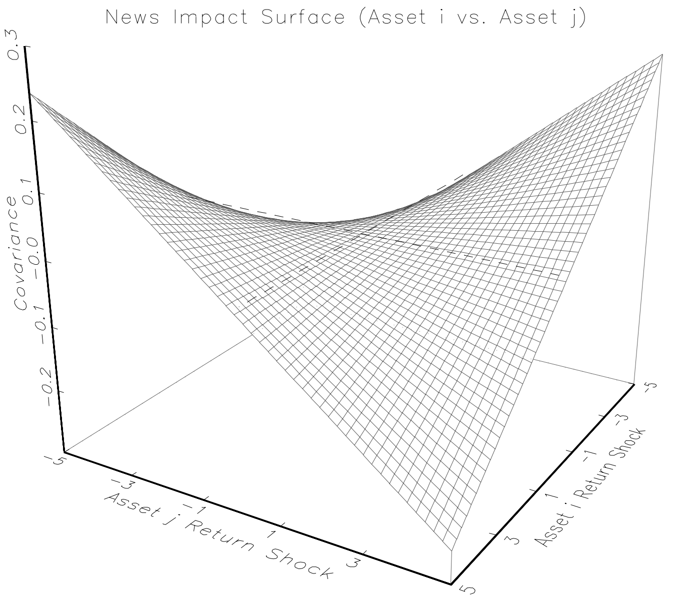
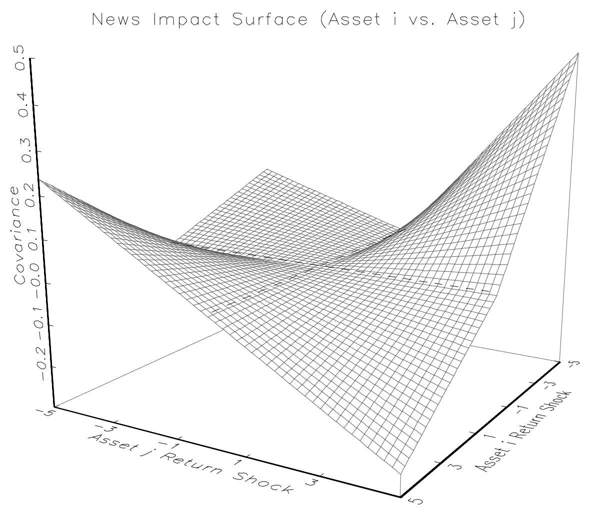


Figure 3: Typical Asymmetric News Impact Surface



$$\rho^2_{ij,t+1}Var\{r_{i,t+1}|\mathcal{I}_t;\varepsilon^*_{i,t}\}\delta_{j,t} - \rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t;\varepsilon^*_{j,t}\}\delta_{i,t} + \rho^2_{ij,t+1}\delta_{i,t}\delta_{j,t}, \quad (12)$$

where $\varepsilon^*_{i,t}, \varepsilon^*_{j,t} > 0$, and $\rho^2_{ij,t+1} > 0$. If index i denotes the stock index and index j a bond index, then $\delta_{j,t}$ corresponds to the leverage effect in bond returns. As this effect has not been documented before, we expect $\delta_{j,t}$ to be (close to) zero. It follows from (10) that, in general, the right hand side of (12) will not be equal to zero. If $\delta_{j,t}$ equals zero, the right hand side of (12) reduces to $\rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t;\varepsilon^*_{j,t}\}\delta_{i,t}$, which will be a positive number. As the conditional covariances between stock and bond returns are mostly positive numbers (see Figure 5), it follows from (12) that the conditional covariance between two assets given two negative shocks will be larger than given two positive shocks. More generally, if both stock and bond returns exhibit leverage effects, (12) implies that the conditional covariance between these assets responses asymmetrically to shocks, in such a way that the covariance will be relatively higher after two negative shocks.

Furthermore it is interesting to examine whether $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\}$ is expected to be larger than $Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$. The difference between $Cov^2\{r_{i,t+1}, r_{j,t+1}|\ \mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\}$ and $Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$ can be written as $\rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t;\varepsilon^*_{j,t}\}\delta_{i,t} - \rho^2_{ij,t+1}Var\{r_{i,t+1}|\mathcal{I}_t;\varepsilon^*_{i,t}\}\delta_{j,t}$. Let us consider the case that index i refers to a stock index (exhibiting leverage effects) and index j to a bond (without leverage effects). Then we expect that $Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} - Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\} = \rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t;\varepsilon^*_{j,t}\}\delta_{i,t} > 0$, such that the conditional covariance is larger after a negative shock in the stock market and a positive shock in the bond market than shocks of opposite signs.[12] Thus, we expect asymmetric patterns in the covariance in various dimensions. Below we will elaborate on this.

To relax the restrictive assumption of symmetric covariances, we introduce a specification which is a generalization of (9):

$$\sigma_{ij,t+1} = \gamma_{ij} + \alpha_{1ij}\varepsilon_{i,t}\varepsilon_{j,t} + \alpha_{2ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t} + \alpha_{3ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1 - I_{\varepsilon_{j,t}})\varepsilon_{j,t} + \beta_{ij}\sigma_{ij,t}, \quad (13)$$

$i, j = 1, ..., N$. Similar to the univariate case, indicator variable $I_{\varepsilon_{k,t}}$ is equal to 1 if $\varepsilon_{k,t} < 0$ (and zero otherwise), $k = i, j$, such that the space can be partitioned into four quadrants[13] in the $\{\varepsilon_i, \varepsilon_j\}$ plain. Let us partition this plane into: $Q(+,+)$,

[12] If the bond market also exhibit leverage effects, the direction of the asymmetry is an empirical issue.

[13] Strictly, we should not talk about quadrants in this setting, but octants.

$Q(+,-)$, $Q(-,+)$, and $Q(-,-)$, denoting the quadrant, corresponding to the signs of $(\varepsilon_i,\varepsilon_j)$: a "+" for a positive and a "−" for a negative shock. In (13), $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$ is nonzero for pairs of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ in $Q(-,-)$. This term assigns an asymmetric covariance effect on shocks in the same direction ($Q(+,+)$ vs. $Q(-,-)$). On the other hand, $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ is nonzero for pairs in $Q(-,+)$.[14] This term assigns an asymmetric covariance effect on shocks in the opposite direction ($Q(+,-)$ vs. $Q(-,+)$). We will refer to these latter effects as "cross effects". Counterintuitively, Kroner and Ng (1998) present an asymmetric covariance model without including this cross effect. Figure 3 represents a news impact surface for the asymmetric case. We can clearly see that the responses in the same and opposite directions do not yield identical covariances anymore. The news impact on the covariance in $Q(-,-)$ is larger than in the symmetric case, and the impact on the covariance in $Q(-,+)$ is also larger than in the symmetric case (cf. Figure 2).

The asymmetric multivariate GARCH-in-mean specification, which allows for asymmetric effects of shocks in the (co)variances, now becomes[15]

$$r^e_{i,t+1} = \mu_i + \lambda \sum_{j=1}^{N} w_{j,t+1}\sigma_{ij,t+1} + \varepsilon_{i,t+1} \tag{14}$$

$$\sigma_{ij,t+1} = \gamma_{ij} + \alpha_{1ij}\varepsilon_{i,t}\varepsilon_{j,t} + \alpha_{2ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t} + \alpha_{3ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t} + \beta_{ij}\sigma_{ij,t}, \tag{15}$$

$i,j = 1,...,N$. Joint model (14)-(15) provides an explicit model for the expected returns on assets which depend on time-varying risk premia. Equation (14) will be referred to as the mean equation and equation (15) as the covariance equation. Note that for the variance ($i = j$), (15) becomes

$$\sigma^2_{i,t+1} = \gamma_{ij} + \alpha_{1ii}\varepsilon^2_{i,t} + \alpha_{2ii}I_{\varepsilon_{i,t}}\varepsilon^2_{i,t} + \beta_{ii}\sigma^2_{i,t}, \tag{16}$$

which is equivalent to the formula for the GJR specification for a single asset given in (9). Thus, our model provides a generalization of the asymmetric GJR model by allowing explicitly for asymmetric conditional covariance terms. On the other hand, the joint model (14)-(15) is a natural extension of the GARCH-in-mean model of Bollerslev, Engle and Wooldridge (1988). In the next section we

[14] Note that this quadrant is situated on the right side of each plot.

[15] This multivariate GARCH specification can be easily extended by adding exogenous variables to the equation.

will empirically examine whether the joint model (14)-(15) is able to explain the return, volatility and covariance between a short term bond as implied by the 6-month LIBOR rate, the long term bond (10 year), implied by swap rates, and the return on the Standard and Poor's 500 index.

4 Empirical Results

4.1 Data

In order to examine the asymmetric volatility in the stock and bond market, our asset return data include the weekly excess returns on a stock index, a short and a long term zero-coupon bond. These U.S. asset market data cover the period April 8, 1987 - March 24, 1999 (624 observations), such that we can examine some volatile periods (1987-1988, 1990 and 1998) and less volatile periods (1991-1995). We assume that the return on the market portfolio can be mimicked by returns on three assets: the return on a short term zero-coupon bond as implied by the 6-month LIBOR (London Interbank Offer Rate) (denoted by $r_{1,t}$), the return on a long term (10 year) zero-coupon bond (denoted by $r_{2,t}$), implied by swap rates, and the return on the Standard and Poor's 500 index (denoted by $r_{3,t}$). For reasons of convenience, we will refer to these asset returns as the short (term) bond returns, the long (term) bond return, and the stock index return. All returns were converted to excess returns (denoted by $r^e_{1,t}$, $r^e_{2,t}$, and $r^e_{3,t}$ respectively) using the riskfree rate approximated by the 3-month LIBOR rates.

Using bond returns implied by LIBOR and swap rates, which are also used in some recent papers; see, e.g., Duffie and Singleton (1997), Dai and Singleton (1999) and Piazzesi (2000), may be more relevant than using government bond returns, because most of the interest rate derivatives are priced using LIBOR and swap rates. Moreover, these rates are minimally affected by credit risk because of their special contractual netting features (see Duffie and Huang, 1996). To calculate the one-period holding returns on the 6 month zero-coupon bond and the return on the 10 year zero-coupon bond, we use (see, e.g. Campbell, Lo and MacKinlay, 1997, p.398):

$$r^b_{n,t+1} = \frac{p^b_{n-1,t+1} - p^b_{n,t}}{p^b_{n,t}}, \tag{17}$$

where $r^b_{n,t+1}$ denotes the one-period holding return on an n-period bond at time

$t+1$, and $p^b_{n,t}$ denotes the price of a zero-coupon bond at time t, with time-to-maturity n (weeks). Because we do not observe $p^b_{n-1,t+1}$ we follow, e.g., Campbell and Shiller (1991) and approximate it by $p^b_{n,t+1}$.

Following Frankel (1985), we keep the market weights ($w_{j,t}$) constant over time, and employ values that roughly correspond to the ones in Bollerslev, Engle and Wooldridge (1988), namely 0.1, 0.1 and 0.8 respectively. A disadvantage of this approach is that measurement errors are likely to be introduced into the model. However, we expect these to be of minor importance, because the market portfolio weights are relatively constant over time (see Bollerslev, Engle and Wooldridge, 1988). Moreover, using market capitalization data to approximate the weights is not free of measurement errors either.

4.2 Estimation Results

In this section the estimation results of the temporal interaction between U.S. stock and bond markets are presented. The relation between expected return and risk is examined, and, in particular, we study the empirical significance of asymmetric responses of conditional covariances to return shocks.

The mean and covariance equations are estimated by maximum likelihood, which enables us to estimate the two non-linear processes jointly. In order to use maximum likelihood we need to make distributional assumptions about the error terms. If we assume that $\boldsymbol{\varepsilon}_{t+1} \sim N(\mathbf{0}, \Sigma_{t+1})$, the loglikelihood function (for the sample $1, ..., T$) is given by

$$\ell(\boldsymbol{\theta}) = -\frac{1}{2}TN\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\det\Sigma_t(\boldsymbol{\theta}) - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t'(\boldsymbol{\theta})\Sigma_t^{-1}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t(\boldsymbol{\theta}), \qquad (18)$$

where $\boldsymbol{\theta}$ denotes the vector of unknown parameters, the $N \times 1$ vector $\boldsymbol{\varepsilon}_t(\boldsymbol{\theta})$ contains the error elements $\varepsilon_{i,t}(\boldsymbol{\theta}) = r^e_{i,t} - \mu_i - \lambda\sum_{j=1}^{N} w_{j,t}\sigma_{ij,t}$, $i = 1, ..., N$, and $\Sigma_t(\boldsymbol{\theta})$ contains the covariance terms $\sigma_{ij,t}(\boldsymbol{\theta})$, as defined in (15). The conditions under which the maximum likelihood is consistent and asymptotically normal are derived by Bollerslev and Wooldridge (1988).

The estimates are obtained by numerical methods using the Berndt, Hall, Hall and Haussman (1974) (BHHH) optimization algorithm. Without any restrictions, the multivariate VECH model is likely to produce nonpositive definite matrices, so that the maximum likelihood method fails to compute an optimum. To guarantee positive definiteness of the conditional covariance matrix, we use the constrained maximum likelihood optimization procedure of GAUSS and impose that the smallest eigenvalue of each covariance matrix has to be positive during estimation. The large number of parameters to be estimated combined with the fact that the model is highly nonlinear, complicates estimation of the system. In order to improve convergence, starting values that are sufficiently close to the optimum are crucial. We use starting values based on unconditional sample statistics and preliminary estimates of univariate GARCH models.

The estimation results of joint model (14)-(15) are given in Table 1. The first column in this table (Model 1) refers to the Bollerslev, Engle and Wooldridge (1988) model (BEW model in shorthand notation), i.e. model (14)-(15) without asymmetric terms in the (co)variance equations. The second column of Table 1 presents the results of the asymmetric covariance model.

As Model 1 is nested in Model 2, we can easily test one against the other using the likelihood ratio test. The results clearly suggest that asymmetric effects are important in modeling the conditional covariances between stock and bond market returns. The likelihood ratio test statistic is 147.68, and with the degrees of freedom being equal to 9, the null hypothesis is soundly rejected at the conventional significance levels. This suggests that the model specification with asymmetric effects in covariances is superior to the BEW model. Consequently, economic interpretations are mainly concentrated upon this specification.

There are a number of compelling observations to be made concerning these estimation results, and subsequently we schedule our comments in the following order: first, the mean equation (Subsection 4.2.1), second, the dynamics in the covariance structure (Subsection 4.2.2), the asymmetric effects in the variances (Subsection 4.2.3), and finally, the asymmetric effects in the covariances (Subsection 4.2.4).

4.2.1 Mean Equation

While Bollerslev, Engle and Wooldridge (1988) conclude that the market price of risk is high and significant (0.499 with a standard error of 0.160), we do not find such a high value. An important difference between Bollerslev, Engle and Wooldridge (1988) and the approach in this paper is that they use *quarterly* data from 1959 to 1984, whereas we use (more recent) *weekly* data. While there is a

Table 1: **Estimates of Multivariate GARCH Model**

This table reports the maximum likelihood estimation results of the model

$$r^e_{i,t+1} = \mu_i + \lambda \sum_{j=1}^{N} w_{j,t+1}\sigma_{ij,t+1} + \varepsilon_{i,t+1}$$

$$\sigma_{ij,t+1} = \gamma_{ij} + \alpha_{1ij}\varepsilon_{i,t}\varepsilon_{j,t} + \alpha_{2ij} I_{\varepsilon_{i,t}}\varepsilon_{i,t} I_{\varepsilon_{j,t}}\varepsilon_{j,t} + \alpha_{3ij} I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1 - I_{\varepsilon_{j,t}})\varepsilon_{j,t} + \beta_{ij}\sigma_{ij,t},$$

where indicator variable $I_{\varepsilon_{k,t}}$ equals 1 if $\varepsilon_{k,t} < 0$ (and zero otherwise), $k = i, j$. The index $i = 1$ refers to the short term zero-coupon bond, $i = 2$ to the long term zero-coupon bond, and $i = 3$ to the stock index. The estimated asymptotic standard errors are reported in parentheses. All statistics presented refer to the period April 8, 1987 - March 24, 1999 ($T = 624$).

Explanatory	**(Model 1)**		**(Model 2)**	
Variables	Estimate	Std. Error	Estimate	Std. Error
	Mean Equations			
$Const_1$	-0.1047*	(0.0017)	-0.1045*	(0.0015)
$Const_2$	0.0000	(0.0005)	-0.0154*	(0.0015)
$Const_3$	0.1262	(0.0742)	0.1224	(0.1000)
$\sum_{j=1}^{N} w_{j,t+1}\sigma_{ij,t+1}$	0.0036	(0.0425)	0.0063	(0.0979)
	Covariance Equations			
$Const_{11}(\times 100)$	0.0101	(0.0153)	0.0074*	(0.0016)
$Const_{21}$	0.0045*	(0.0015)	0.0035*	(0.0011)
$Const_{22}$	1.7062*	(0.2750)	1.8820*	(0.1490)
$Const_{31}$	0.0106	(0.0093)	0.0016	(0.0013)
$Const_{32}$	0.5716	(0.4580)	0.5718*	(0.2037)
$Const_{33}$	0.0602*	(0.0236)	0.3636*	(0.0948)
$\varepsilon^2_{1,t}$	0.1406*	(0.0218)	0.1111*	(0.0266)
$\varepsilon_{1,t}\varepsilon_{2,t}$	0.0448*	(0.0132)	0.0294*	(0.0096)
$\varepsilon^2_{2,t}$	0.1342*	(0.0240)	0.0748*	(0.0307)
$\varepsilon_{3,t}\varepsilon_{1,t}$	-0.0463	(0.0407)	-0.0798*	(0.0360)
$\varepsilon_{3,t}\varepsilon_{2,t}$	-0.0339	(0.0228)	0.0832*	(0.0402)
$\varepsilon^2_{3,t}$	0.0645*	(0.0151)	0.0319	(0.0389)
$I_{\varepsilon_{1,t}}\varepsilon^2_{1,t}$	-	-	0.0565	(0.0349)
$I_{\varepsilon_{2,t}}\varepsilon_{2,t}I_{\varepsilon_{1,t}}\varepsilon_{1,t}$	-	-	0.0325*	(0.0171)
$I_{\varepsilon_{2,t}}\varepsilon^2_{2,t}$	-	-	0.1959*	(0.0555)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}I_{\varepsilon_{1,t}}\varepsilon_{1,t}$	-	-	0.1021*	(0.0409)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}I_{\varepsilon_{2,t}}\varepsilon_{2,t}$	-	-	-0.1178	(0.0775)
$I_{\varepsilon_{3,t}}\varepsilon^2_{3,t}$	-	-	0.1697*	(0.0530)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t}(1 - I_{\varepsilon_{2,t}})\varepsilon_{2,t}$	-	-	0.1003	(0.0577)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}(1 - I_{\varepsilon_{1,t}})\varepsilon_{1,t}$	-	-	0.2581*	(0.0651)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}(1 - I_{\varepsilon_{2,t}})\varepsilon_{2,t}$	-	-	0.4045*	(0.0933)

*denotes significance at the 5% level. (continued on next page)

Table 1 (Continued): **Estimates of Multivariate GARCH Model**

Explanatory	(Model 1)		(Model 2)	
Variables	Estimate	Std. Error	Estimate	Std. Error
$\sigma^2_{1,t}$	0.8434*	(0.0213)	0.8536*	(0.0184)
$\sigma_{21,t}$	0.8649*	(0.0414)	0.8814*	(0.0327)
$\sigma^2_{2,t}$	0.1016	(0.1226)	0.0160	(0.0347)
$\sigma_{31,t}$	0.3286	(0.5646)	0.7176*	(0.0769)
$\sigma_{32,t}$	0.2941	(0.5830)	0.2854	(0.2106)
$\sigma^2_{3,t}$	0.9205*	(0.0189)	0.7946*	(0.0375)
Log-likelihood	1449.94		1523.78	

* denotes significance at the 5% level.

positive relation between the expected market risk premium and the conditional market covariance, this relationship is not statistically significant for both specifications. Thus, our results suggest that the expected returns are independent of the time-varying reward to risk. A possible explanation is that investors typically hold undiversified portfolios (see, e.g. Barber and Odean, 2000), so that a risk measure including idiosyncratic risk might be more appropriate to measure the intertemporal risk return relationship (see Goyal and Santa-Clara, 2001). While our theoretical model says that the expected returns only depend on the risk term, some of the estimated coefficients of the constant terms in the mean equation are significantly different from zero. These results do not support the simple one-factor CAPM.

4.2.2 Dynamics in Volatility

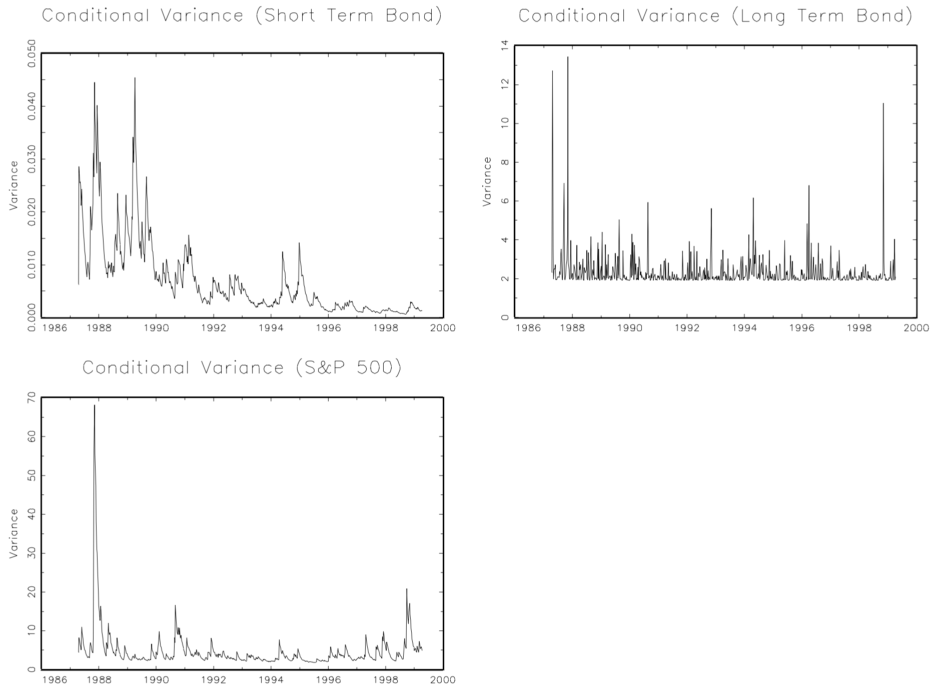
Next, we consider the estimation results of the parameters that govern the dynamics in the variances and covariances. It appears that covariances change substantially over time, as most of the corresponding estimated parameters are statistically significant at the 5 percent level. Hence, the constant covariance hypothesis can be rejected. This result is consistent with the findings of Bollerslev, Engle and Wooldridge (1988), Harvey (1989) and Bodurtha and Mark (1991), who also document strong evidence in favor of heteroskedastic covariances. The estimates for the coefficients on the product of the return shocks (i.e. the $\varepsilon_i\varepsilon_j$'s) in Model 2 range from 0.032 to 0.111 for the variances, and from -0.080 to 0.083 for the covariances. A negative estimate for the ARCH term in the covariance equation means that two shocks of the same sign affects the conditional covariance between the corresponding assets negatively, while two shocks of opposite signs have a positive effect on the forecasted covariance. Apparently, negative (or positive) shocks in both the S&P 500 and the short term bond leads to a significant decrease in their next period's covariance, while the opposite holds for the covariance between short and long bond returns and between long bond and S&P 500 returns. However, this interpretation only holds if we neglect the asymmetries in covariance. We will see below that the introduction of these asymmetric effects lead to more complex relationships.

Finally, the estimates for the coefficients on lagged volatility (i.e. the $\sigma_{ij,t}$'s) are statistically significant and range from 0.016 to 0.854 for the variances and from 0.285 to 0.881 for the lagged conditional covariances. Obviously, not only variances, but also covariances tend to cluster over time. Striking is the low estimate for the parameter corresponding to the lagged variance in the long term bond return. This, combined with the fact that the constant term in the volatility equation for this asset is relatively high, indicates that the volatility of the long term bond return is harder to predict than volatility in stock returns and short bond returns. The low value of the estimated lagged volatility coefficient is also found using a univariate analysis (not reported). The technical explanation for this low value is probably the influence of the outlier in the long bond return in 1987. An outlier correction, by removing the outlier observation, results in a substantial higher estimate. The estimated coefficient of lagged volatility becomes 0.677. Obviously, the outlier has a big impact, and causes the predictable component to decrease.

Figure 4 and 5 present the plots of the conditional variance and covariance forecasts over time, based on the estimation results of Model 2. The figures show that the conditional variances and covariances are not constant over time and are especially volatile during the periods 1987-1988 (the October 1987 crash), 1990-1991 (recession and Gulf war), and 1998-1999 (the Millennium crash). Like Schwert (1989) we find that U.S. stock and bond volatilities tend to move together.[16] Furthermore, the figures suggest that in general covariances between assets are higher (lower) in times of high (low) volatility. Looking at Figure 5, we see that the conditional covariance between short and long bond returns, as well as between

[16] Moreover, Engle, Ng and Rothschild (1990) uncover that changes in U.S. bond volatility are closely linked across maturities.

Figure 4: The Estimated Conditional Variances



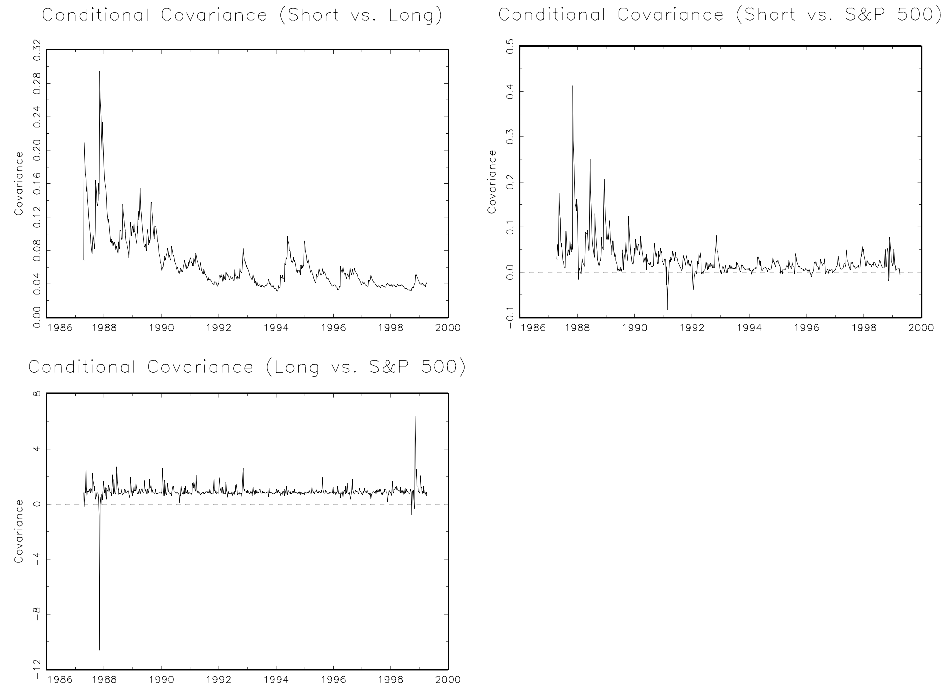
short bond returns and the stock index returns are highly clustered over time (the corresponding estimated influence of lagged covariances are 0.881 and 0.718 respectively). We further see that the degree of clustering between long bond returns and stock index returns is much lower (the corresponding estimated influence of lagged covariances is 0.211).

To examine whether the time-variability in covariances is solely due to the variation in variances, we consider the conditional correlation coefficients. Let $\rho_{ij,t+1}$ denote the conditional correlation coefficient between return i and j at time $t+1$:

$$\rho_{ij,t+1} = \frac{Cov_t\{r_{i,t+1}, r_{j,t+1}\}}{\sqrt{Var_t\{r_{i,t+1}\}}\sqrt{Var_t\{r_{j,t+1}\}}}, \tag{19}$$

If $\rho_{ij,t+1}$ is constant over time, the variability in covariances is solely due to variation in variances. In that case, modeling of time-varying covariances is not very interesting, as all the dynamics are captured in variances. Figure 6 presents the estimated correlation coefficients, and shows that correlation coefficients vary con-

Figure 5: The Estimated Conditional Covariances



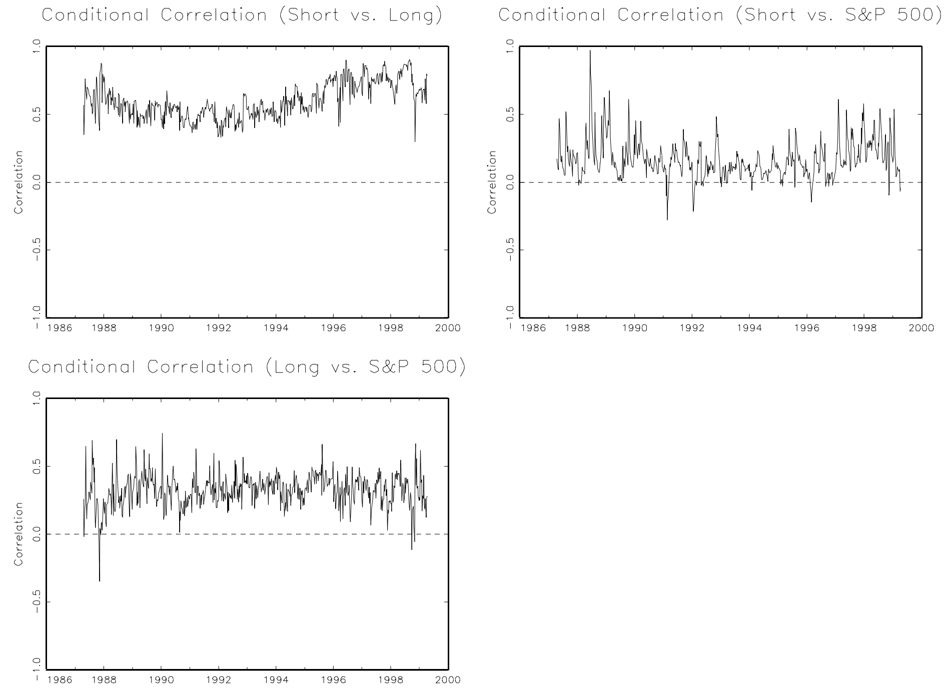
siderably over time. This is in line with Tse (2000), who rejects for different countries that conditional correlations are constant over time. Tests of constancy of these correlation coefficients (not reported), by performing regression of the correlation coefficients on a constant and lagged correlation coefficients, clearly show that the correlation coefficients are not constant over time. Consequently, the variability in covariances is not solely due to time-varying variances, and modeling time-varying covariances is important.[17]

4.2.3 Asymmetric Effects in Variances

In this subsection, we address the degree of importance of the asymmetric effects in the variances (i.e. $I_{\varepsilon_{i,t}}\varepsilon_{i,t}^2$, $i = 1, 2, 3$). The results in Table 1 indicate that this effect is especially pronounced in the variance of long term bond returns and stock

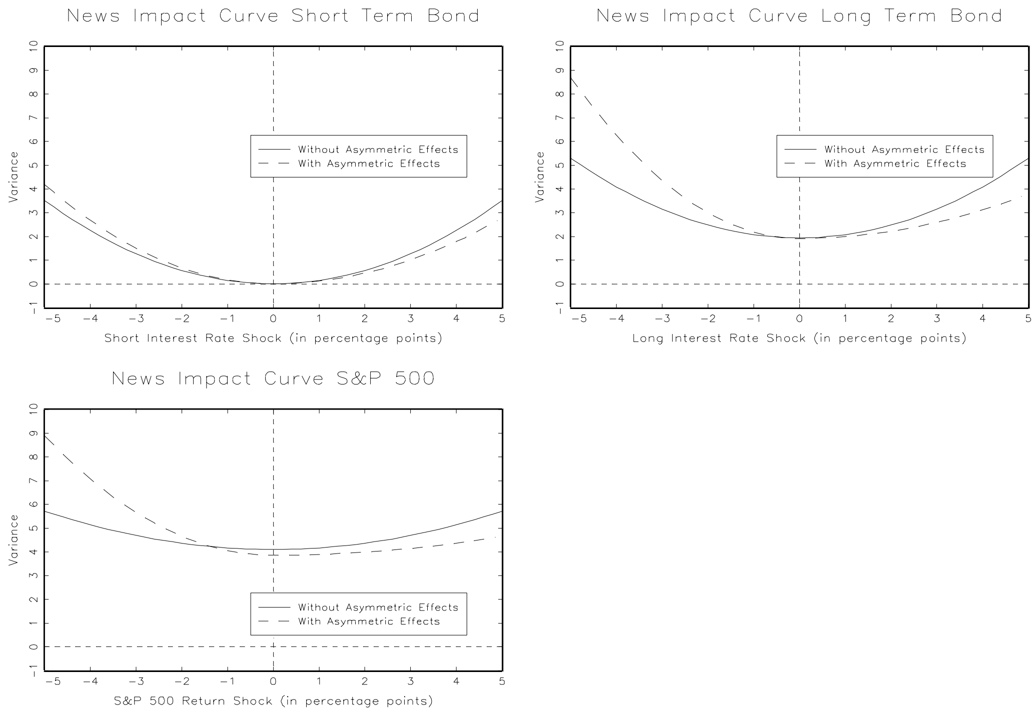
[17]If correlation coefficients were constant over time, we could simply calculate conditional covariances via the estimated variances (see (19)).

Figure 6: The Estimated Conditional Correlation Coefficients



index returns. For example, the estimated coefficient of the variable that captures the negative shocks in the S&P 500 return is equal to 0.170, which means that negative return shocks in the S&P 500 are followed by a relatively high conditional variance. Given existing results in the literature (see, e.g., Glosten, Jagannathan and Runkle, 1993, and Engle and Ng, 1993), it is not surprisingly that we find this asymmetric effect in the variance of the stock index. However, for bond returns this effect is not reported before in the literature. The presence of asymmetric effects in the variance of Treasury bond returns means that the leverage explanation of Black (1976) cannot be the (only) valid argument of this effect, as his explanation, based on the debt-equity ratio, only holds for stocks. The news impact curves, discussed in Section 3, for the three assets using the estimates from Table 1 are given in Figure 7. The solid lines represent the symmetric impacts on volatility of shocks in the asset returns, calculated using Model 1. The dashed lines represent the asymmetric impact on volatility, which are calculated using the estimates of

Figure 7: Estimated News Impact Curves with and without Imposing Symmetry

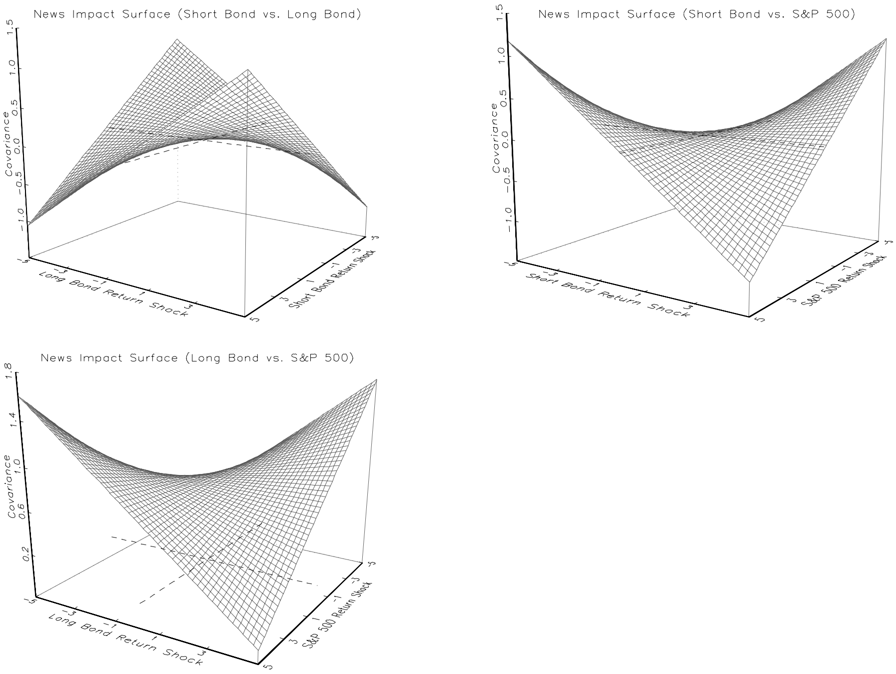


Model 2. Figure 7 illustrates that the models predict that a negative return shock is followed by a higher subsequent volatility than a positive return shock of the same magnitude. While this effect is small for the short term bond, it is substantial for the long term bond and the S&P 500 index.

4.2.4 Asymmetric Effects in Covariances

Next, we focus on the asymmetries in *covariances*. The results in Table 1 show that not only variances, but also covariances exhibit significant leverage effects. The asymmetric effects for shocks with the same sign (i.e. $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$, $i \neq j$) seem to be important, as the corresponding estimated coefficients are statistically significant for two out of three cases. While the asymmetric effects in the covariance between the long bond return and the S&P 500 return is statistically negligible, the leverage effect in the covariance between the other assets are statistically sig-

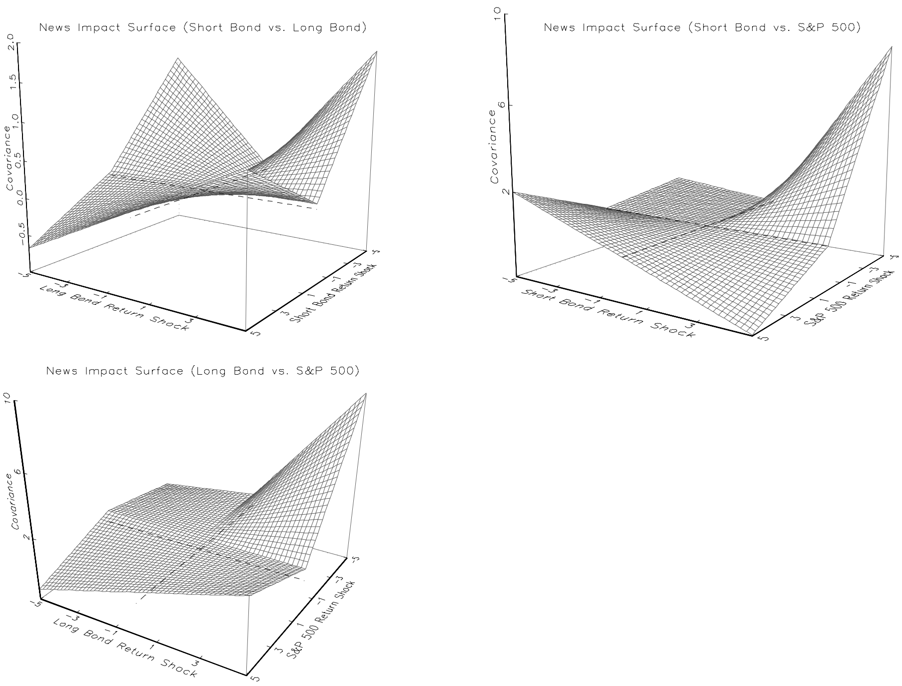
Figure 8: Estimated News Impact Surfaces Imposing Symmetry



nificant. A positive sign of the coefficients indicates that next week's conditional covariance between returns is higher when there are two negative shocks rather than two positive shocks. Below, interpretations will be given using estimated news impact curves and surfaces. The cross effects in the asymmetry, i.e. when shocks in the two assets are of opposite signs (i.e. $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1 - I_{\varepsilon_{j,t}})\varepsilon_{j,t}$, $i \neq j$), seem also to be important. An estimated positive sign of the corresponding parameter indicates that the conditional covariance between returns is higher when there is a negative shock in i and a positive shock in j rather than a positive shock in i and a negative shock in j of the same magnitude.

The estimated news impact surfaces imposing symmetry, based on results from Model 1, are shown in Figure 8, while Figure 9 presents estimated news impact surfaces which allow for asymmetries, obtained from Model 2. The interpretation of these surfaces is more difficult than the news impact curves, as there are two shocks instead of one. The symmetric news impact surface for short and long bonds in Figure 8 shows that the conditional covariance is high after return shocks of the

Figure 9: Estimated News Impact Surfaces Allowing for Asymmetry



same sign, while shocks in opposite direction lower the conditional covariances. This is because bond returns are (highly) positively correlated (see Figure 6). As these assets move together, shocks in the same direction involves a higher forecasted risk than shocks in opposite direction. This makes sense, as it is riskier to invest in two assets that are highly positively correlated than to invest in two assets that are less correlated.

The remaining two plots in Figure 8 show the opposite: a shock in both asset returns of the same sign decreases the covariance between the assets. A possible explanation is that a large negative stock return shock typically leads to a higher demand for bonds and thus raises bond prices, and vice versa.

Figure 8 shows very clearly that, when one uses a symmetric model, shocks of the same magnitude (in absolute value) in both assets, e.g. 5% or -5%, imply an identical impact on the conditional covariance. Figure 9 presents the news impact surfaces, allowing for asymmetries. The first plot contains the asymmetric news impact surface for the short en long bond returns. The slope of the covariances in

$Q(-,+)$ is not downward anymore. A negative shock to the short bond return, combined with a positive shock in the long bond return, results in a relatively high conditional covariance. Apparently, a negative shock in the short bond is followed by a relatively high degree of risk in the bond market. This is a novel result which could not be found using standard symmetric covariance models.

We further see that there is a significant asymmetry in positive and negative shocks in the stock index returns. Bad news in stock index returns is followed by a large increase in the covariance between the bond return and the stock index return. This holds irrespectively whether the news in the short bond market is good or bad. This is what we expected ex-ante (see Section 3). Figure 9 further uncovers that the conditional covariance is especially high after a positive shock in the (short or long) bond return and a negative shock in the stock index return. Thus, the cross effects in asymmetries, described in Section 3, seem to be important. Bear markets (and stock market crashes) are typically followed by more risky periods than bull markets. A possible economic arguments for this effect is Black's leverage argument, that bad news in the stock market leads to higher debt-equity ratios of companies, and this again increases the risk of holding stocks. However, this argument can not explain the asymmetries in the bond markets. An alternative explanation is the volatility feedback effect, examined by, e.g., French, Schwert and Stambaugh (1987) and Bekaert and Wu (2001). If volatility is priced, unanticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline. Another possible explanation is the following-the-herd effect. Investors tend to pay less attention to the fundamentals during a stock market crash, and follow the herd by selling (a part of) their stocks if they observe that other investors are selling. This leads to relatively high volatility when bad news arrives in the market. However, it should be stressed that this interpretation is subjective, and further research on the asymmetric effects in variances and covariances is needed to underpin the interpretations.

4.3 Specification Tests

When modeling the conditional covariance, it is important whether the specification is a statistically adequate representation of the data. In particularly, it must be the case that the standardized residuals, $\breve{\boldsymbol{\varepsilon}} = \hat{\Sigma}_t^{-1/2}\hat{\boldsymbol{\varepsilon}}_t \sim$ i.i.d.$(0, I)$. In Table 2 we present the test statistics for the (normalized) covariance for the three assets

Table 2: **Diagnostic Tests for Covariance Specification**

	$\breve{\varepsilon}_1$	$\breve{\varepsilon}_2$	$\breve{\varepsilon}_3$	$\breve{\varepsilon}_1^2$	$\breve{\varepsilon}_2^2$	$\breve{\varepsilon}_3^2$	$\breve{\varepsilon}_1\breve{\varepsilon}_2$	$\breve{\varepsilon}_1\breve{\varepsilon}_3$	$\breve{\varepsilon}_2\breve{\varepsilon}_2$
Mean	-0.0486	0.2282	-0.0135	0.9750	0.9940	1.0023	0.9946	-39.2854	-34.2143
Std. Dev.	0.9880	0.9713	1.0018	1.0836	0.5162	1.9247	24.8338	884.4523	703.9095
Skewness	0.1635	-0.4468	-0.7299	4.1854	2.0636	5.3763	-14.7819	-21.6252	-18.1501
Kurtosis	2.2451	1.4990	4.6424	30.0875	15.0728	50.6090	375.0372	510.0952	363.9575
Ljung-Box Statistics									
Q(6)	5.4864	10.5162	7.5515	4.5182	8.6452	10.1651	3.2165	4.8425	2.5405
Q(12)	7.1508	14.5426	10.2948	8.5161	10.512	15.2115	6.6843	6.8422	5.2012
Q(18)	11.589	17.84533	15.1872	14.6548	16.256	17.2565	8.6468	9.4156	9.3188
Q(24)	16.168	20.4563	22.2565	19.6654	29.5152	19.5666	12.1635	12.5464	13.5118

Notes: This table reports summary statistics and Ljung-Box statistics for standardized residuals and standardized products of residuals. $Q(r)$ denotes the Ljung-Box test statistic for rth order serial correlation in the standardized cross-product of residuals. The 95% critical values for $Q(6)$, $Q(12)$, $Q(18)$ and $Q(24)$ are 12.6, 21.0, 28.9 and 36.4, respectively.

combinations. The tests to evaluate the adequacy of the model are based on the standardized residuals and the standardized products of residuals from the asymmetric covariance model. We consider the mean, standard deviation, skewness and kurtosis.. In addition we present the Ljung-Box tests for serial correlation in the normalized cross-product of residuals. As can be seen from the table, the sample skewness and the sample kurtosis for the standardized residuals should not show any significant skewness and leptokurtosis. The sample means do not significantly differ from zero (the t-statistics are obtained by the mean divided by the standard deviation times $\sqrt{T}$) and the squared residuals do not differ significantly from one. Further, the Ljung-Box tests do not reject the models. Consequently, the tests reveal no significant departure from the null hypothesis of temporal independence.

5 The Economic Value of the Volatility Timing

In this section we examine the economic value of exploiting the multivariate GARCH model. The appealing estimation results do not necessarily imply economically useful implications for forecasting volatility. Hence, the performance of the model is further evaluated through some measures of economic value. One of the most important applications of (asymmetric) volatility prediction is portfolio selection. To examine the economic gains of using the asymmetric model, we compare the performance of this model to allocate wealth with the restricted (symmetric) one and with a passive portfolio.

There has been only few studies that addresses the economic value of volatility timing. Without doubt, the most important contribution being Fleming, Kirby and Ostdiek (2001). We follow their strategy by evaluating the impact of volatility timing on the short-term allocation strategy performance. To easily compare different volatility timing strategies, we consider an investor who minimizes his portfolio variance subject to a particular target rate of return. In formula:

$$\min_{\mathbf{w}_{t+1}} \ \mathbf{w}'_{t+1}\Sigma^{-1}_{t+1}\mathbf{w}_{t+1}, \tag{20}$$

$$\text{s.t. } \mathbf{w}'_{t+1}\boldsymbol{\mu} + (1 - \mathbf{w}'_{t+1}\boldsymbol{\iota})r_{f,t+1} = \mu_p,$$

where $\boldsymbol{\mu} = E\{\mathbf{r}_{t+1}\}$, $\mathbf{w}_{t+1}$ is the vector of portfolio weights on the risky assets, and μ_p is the target expected return. The proportion invested in the riskfree asset is $w_{0,t+1} = 1 - \boldsymbol{\iota}'\mathbf{w}_{t+1}$. Solving (20) for $\mathbf{w}_{t+1}$ gives us the optimal weights:

$$\mathbf{w}^*_{t+1} = \frac{(\mu_p - r_{f,t+1})\Sigma^{-1}_{t+1}(\boldsymbol{\mu} - r_{f,t+1}\boldsymbol{\iota})}{(\boldsymbol{\mu} - r_{f,t+1}\boldsymbol{\iota})'\Sigma^{-1}_{t+1}(\boldsymbol{\mu} - r_{f,t+1}\boldsymbol{\iota})}. \tag{21}$$

To calculate the optimal portfolio, we need the conditional forecasts of the covariance matrix. We employ three types of conditional covariances: a constant, a *symmetric* time-varying, and an *asymmetric* time-varying covariance matrix. In every case we assume that the expected return is constant over time. The reason for this is threefold. First, we want to concentrate on only the volatility timing. Second, there is little evidence that (economically significant) predictable patterns in returns exist at the weekly level. Third, a long sample period is needed to produce reliable estimates in a forecast regression for first moments (see Merton, 1980). Ideally, out-of-sample forecasts, generated by the model, are used to evaluate the performance. However, this would mean that for each observation the model has to be re-estimated, which is unfortunately a too time consuming operation.[18] Therefore we employ in-sample forecasts.

[18] Out-of-sample forecasts are easy to obtain using Foster and Nelson's (1996) rolling estimator. However, to extend this estimator to capture asymmetric effects is less straightforward.

We will compare the performance of the dynamic strategies with the static one (i.e. the constant covariance matrix), and we compare the dynamic strategy which entails the asymmetric effects with the dynamic strategy that only considers the symmetric covariances. If the asymmetric extension has no economic value, then the ex post performance of the two strategies should be statistically indistinguishable. A suitable performance measure, which captures the trade-off between risk and return, is:

$$\hat{U}_p(\gamma) = \frac{1}{T}\sum_{t=0}^{T-1}\left[r_{p,t+1} - \frac{1}{2}\gamma r_{p,t+1}^2\right], \tag{22}$$

where $r_{p,t+1}$ denotes the portfolio return. Unfortunately, due to data limitations we cannot use high frequency data to determine the ex post (co)variances like in Marquering and Verbeek (2000). Instead, we follow Fleming, Kirby and Ostdiek (2001) by employing the squared portfolio returns.

The above approach enables us to compare alternative investment strategies by calculating the associated average utility levels. We can determine the economic value of volatility timing by calculating the maximum fee per week, an investor would be willing to pay for holding the dynamic portfolio rather than a passive one. To find the maximum fee for holding portfolio a rather than portfolio b, Δ_{ab}, we solve

$$\frac{1}{T}\sum_{t=0}^{T-1}\left[(r_{a,t+1} - \Delta_{ab}) - \frac{1}{2}\gamma(r_{a,t+1} - \Delta_{ab})^2\right] = \frac{1}{T}\sum_{t=0}^{T-1}\left[r_{b,t+1} - \frac{1}{2}\gamma r_{b,t+1}^2\right], \tag{23}$$

where the indices a and b refer to the active and passive strategies, respectively. The Δ_{ab} will be reported using different values of γ.

Table 3 presents summary statistics and performance measures on several dynamic and passive portfolios. The unconditionally mean-variance efficient passive portfolio, constructed using the unconditional means, variances, and covariances, has a lower mean than the market portfolio, which is a weighted portfolio of the short bond, the long bond and the S&P 500 index. However, the unconditionally mean-variance efficient passive portfolio with a target return of 0.20, has a lower standard deviation, such that the Sharpe ratio for this portfolio is slightly higher than the Sharpe ratio of the market portfolio. More interestingly, the volatility timing strategies, both with target return of 0.20, generate a higher Sharpe ratio. The highest Sharpe ratio is obtained for the asymmetric volatility timing model.

Table 3: **Evaluation of Various Strategies**

Mean and Std. Dev. denote the mean return and the standard deviation of the return on the corresponding strategy in %, respectively. The Sharpe ratio equals the average excess return of the strategy divided by the sample standard deviation. The maximum fee an investor is willing to pay for holding one of the dynamic portfolios rather than the passive portfolio is denoted by Δ_{ab}.

	Mean	**Std. Dev.**	**Sharpe**	Δ_{ab} $(\gamma = 3)$	Δ_{ab} $(\gamma = 6)$	Δ_{ab} $(\gamma = 9)$
Market portfolio	0.2095	1.7151	0.0543	–	–	–
Passive portfolio	0.1984	1.4861	0.0552	–	–	–
Vol. timing	0.1934	1.2460	0.0618	0.1874	0.2812	0.3722
Asym. vol. timing	0.1947	1.2284	0.0638	0.2214	0.3281	0.4092

Next, we compare the performance of the dynamic portfolios to the performance of the unconditionally mean-variance efficient passive portfolio. Using performance measures based on the ex post utility levels, enables us to obtain an economic value of volatility timing. For different values of γ, Table 3 presents Δ_{ab} in percentages per week. For example, an investor with $\gamma = 6$ who currently holds his wealth in the passive portfolio, is willing to pay a fee of 0.28% to switch to the volatility timing strategy and 0.33% to switch to the asymmetric volatility timing strategy. These numbers indicates sizeable gains due to volatility timing, and increase with higher risk-averse levels. Moreover, the strategy using the asymmetric model outperforms its symmetric counterpart.

Table 4 presents the same statistics in the presence of transaction costs and short-sale constraints. We assume that the transaction costs are equal to τ percentage points of the value traded on the stock market, such that the transaction costs equal: $\tau W_t|\Delta w_{3,t+1}|$, where W_t denotes the wealth at time t, and $\Delta w_{3,t+1} = w_{3,t+1} - w_{3,t}$, where index 3 refers to the stock market weight. Consequently, the return after transaction costs is equal to $r_{p,t+1} - \tau|\Delta w_{3,t+1}|$. Panel A presents the results for $\tau = 0.5\%$ and Panel B for $\tau = 1\%$.

Although the economic gain of volatility timing after transaction costs and short-sale constraints is less pronounced, the predictability captured by volatility modeling is still economically significant. We find that volatility timing strategies for dynamic asset allocation matrix significantly outperform passive strategies. Moreover, the symmetric volatility timing strategy is outperformed by its asymmetric counterpart. Thus, we do not only find evidence the asymmetric volatility models are superior statistically, but we also find that the predictability captured

Table 4: **Evaluation of Various Strategies in Presence of Transaction Costs and Short-Sale Constraints**

Mean and Std. Dev. denote the mean return and the standard deviation of the return on the corresponding strategy in %, respectively. The Sharpe ratio equals the average excess return of the strategy divided by the sample standard deviation. The maximum fee an investor is willing to pay for holding one of the dynamic portfolios rather than the passive portfolio is denoted by Δ_{ab}.

Panel A: 0.5% transaction costs						
	Mean	**Std. Dev.**	**Sharpe**	Δ_{ab} ($\gamma = 3$)	Δ_{ab} ($\gamma = 6$)	Δ_{ab} ($\gamma = 9$)
Market portfolio	0.2095	1.7151	0.0543	–	–	–
Passive portfolio	0.1984	1.4861	0.0552	–	–	–
Vol. timing	0.1922	1.3270	0.0581	0.0949	0.1288	0.2073
Asym. vol. timing	0.1936	1.2994	0.0603	0.1394	0.2605	0.2989
Panel B: 1% transaction costs						
	Mean	**Std. Dev.**	**Sharpe**	Δ_{ab} ($\gamma = 3$)	Δ_{ab} ($\gamma = 6$)	Δ_{ab} ($\gamma = 9$)
Market portfolio	0.2095	1.7151	0.0543	–	–	–
Passive portfolio	0.1984	1.4861	0.0552	–	–	–
Vol. timing	0.1904	1.3204	0.0561	0.0084	0.0754	0.1540
Asym. vol. timing	0.1929	1.2988	0.0589	0.0544	0.1819	0.2064

by the asymmetric volatility model is economically significant.

6 Conclusions

In this paper we analyzed the bond and stock market interactions by modeling the time-varying covariances between stock and bond market returns. A multivariate GARCH-in-mean parametrization is employed, which nests the Bollerslev, Engle and Wooldridge (1988) model. The main contribution of this paper is that it extends the multivariate model by allowing for asymmetric effects in covariances between stock and bond returns. We have shown that if asymmetric effects exist in the variance of stock returns, then it is likely that asymmetric effects are also present in the covariances between stock returns and returns on a second asset as well. To model the asymmetric effects on conditional covariances we have developed a novel approach by generalizing the Glosten, Jagannathan and Runkle (1993) specification towards a multivariate setting. The model is estimated using weekly U.S. asset market data on the S&P 500 index and a short and long zero-coupon bond.

The main empirical findings can be summarized as follows. As the conditional covariances change substantially over time, the constant covariance hypothesis should be rejected. While theory says that high returns should be associated with high conditional covariances, our results suggest that this relationship is not statistically significant. With respect to asymmetric effects in the variances, we find that weekly returns on the S&P 500 index, as well as the long bond returns exhibit significant leverage effects. The finding of leverage effects in the stock index returns is to be expected and already well-documented, but the strong evidence of such effects in Treasury bond returns is a novel result. This implies that Black's (1976) explanation (using the debt-equity ratio) cannot be the (only) explanation of the asymmetric volatility phenomenon. The rationale behind these effects remains an important area of future research.

Not only variances, but also covariances between stock and bond returns exhibit significant asymmetric effects. Overall, our findings imply that a symmetric specification is too restrictive to model the conditional covariances. Especially bad news in the stock market is followed by a much higher conditional covariance than good news in the stock market. This holds irrespectively the sign of the bond market shock. Taking into account the predictable component in the covariances turns out to be economically profitable. An investor who uses a dynamic allocation strategy by employing volatility timing outperforms an investor who holds a passive portfolio. Even when short-sale restrictions are present and transaction costs are high, the economic value of dynamic trading strategies is larger than that of a passive strategy. Finally, the symmetric volatility timing strategy is outperformed by its asymmetric counterpart.

A challenge for future research is to refine the economic rationale behind the asymmetric effects in variances and covariances. As we do not find any support for the CAPM, multi-factor or intertemporal asset pricing models may be more appropriate. Further, the multivariate asymmetric model could, for example, be used to analyze asymmetries in asset returns between different countries and to analyze the interactions with announcement effects.

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