# A SHIPBUILDING CYCLE?*

The basic problem of any theory on endogenous trade cycles may be expressed in the following question: how can an economic system show fluctuations which are not the effect of exogenous, oscillating forces, that is to say fluctuations due to some "inner" cause? Or if we take the simplest case, that of a single, isolated market, how can price and turnover fluctuate in this manner in a market which is considered isolated?

We are greatly indebted to MOORE[1] for his example of an "elementary cycle", and to HANAU[2] for his statistical elaboration of such an "elementary cycle". I refer to the so-called "pork cycle". The very interesting examples quoted by MOORE and others where this "elementary cycle" plays a part are much more complicated insofar as they show disturbances of the cycle which stem from exogenous influences and therefore interrupt the cycle, in most cases even before one period has come to an end. The most important cause for disturbance from the outside is the different yield per acre of the agricultural products in question.[3]

In this article I should like to demonstrate a relationship between shipbuilding and freight rates which will bring us to another type of "elementary fluctuation", a type which, it seems to me, displays interesting characteristics of consequence to the theory of economic fluctuations in general, and also are not without meaning for the field of shipbuilding in particular.

---

* Ein Schiffbauzyklus? Weltwirtschaftliches Archiv, 34. Band (1931 II), p. 152–164.

1 H. L. MOORE, *Synthetic Economics*, New York 1929.

2 A. HANAU, "Die Prognose der Schweinepreise", *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 18; 3., vollst. neu bearb. Aufl. des Sonderheftes 2, Berlin 1930.

3 With most agricultural products, the acreage is connected with the prices of the previous year. If the yields per acre were constant, an entirely analogous cycle of prices and production would be the result. Now and then parts of such cycles can be observed, *e.g.* for cotton about the year 1906.

In addition to a thorough treatment of this type of fluctuations I will deal briefly with the relationship between the "shipbuilding cycle" and the "pork cycle" and in conclusion add a few related remarks on the latter cycle.

## 1. SOME STATISTICS ON THE SHIPBUILDING CYCLE

Fluctuations in shipbuilding determine to a large extent the fluctuations in the increase in total tonnage of a country's merchant marine, since the number of sunk and scrapped ships is relatively small. This fact is illustrated by the lower portion of Fig. 1 in which curve C represents the annual increase in the total tonnage of Great Britain, the United States and Germany (deviation from trend) and curve D the total world launchings (idem, reduced). In much the same way the volume of shipbuilding is largely dependent on the level of freight rates. These in turn are clearly correlated with total available tonnage. It will be understood that both relationships are subject to a certain lag so that, for example, freight rates will be high if total tonnage was low shortly before. An increase in tonnage will appear about one year after the occurrence of increased rates because one year is approximately the construction time of a new ship Such retarded relations (lag correlations) are shown in the two top portions of Fig. 1. Line A shows the fluctuations in freight rates, derived from the index of homeward freights published by "Fairplay",[4] in deviations from a parabolic trend of the second degree; while curve B represents the fluctuations in the total tonnage of the three big sea-faring countries (Great Britain, United States, Germany) in terms of percentages of deviation from a parabolic trend of the second degree.

The two relationships described here are the basis for a third which is the most significant to us: the relationship between the increase in total tonnage and the volume of the total tonnage of about two years earlier. The two curves are remarkably similar, as will be seen from the centre portion of Fig. 1.

Before we consider the consequence of this relationship I should like to state briefly its social and economic significance.[5] This is, obviously,

[4] *Fairplay*, London, Vol. 67 (1916 II), No. 1754, p. 946

[5] See also: J. TINBERGEN, "Scheepsbouw en conjunctuurverloop", *De Nederlandsche Conjunctuur*, 's-Gravenhage, Jg. 1931, Afl. 1, p. 14.

that the volume of shipbuilding is primarily a function of the shipowners' demand for freight capacity—costs seem to be less variable —and furthermore, that the level of freight rates, at least during the period under review, depends mainly on the shipowners' supply of freight capacity. The demand of importers and exporters considered as an aggregate appears to fluctuate much less. We do by no means want to deny the influence of these other factors, but their effect is probably secondary.

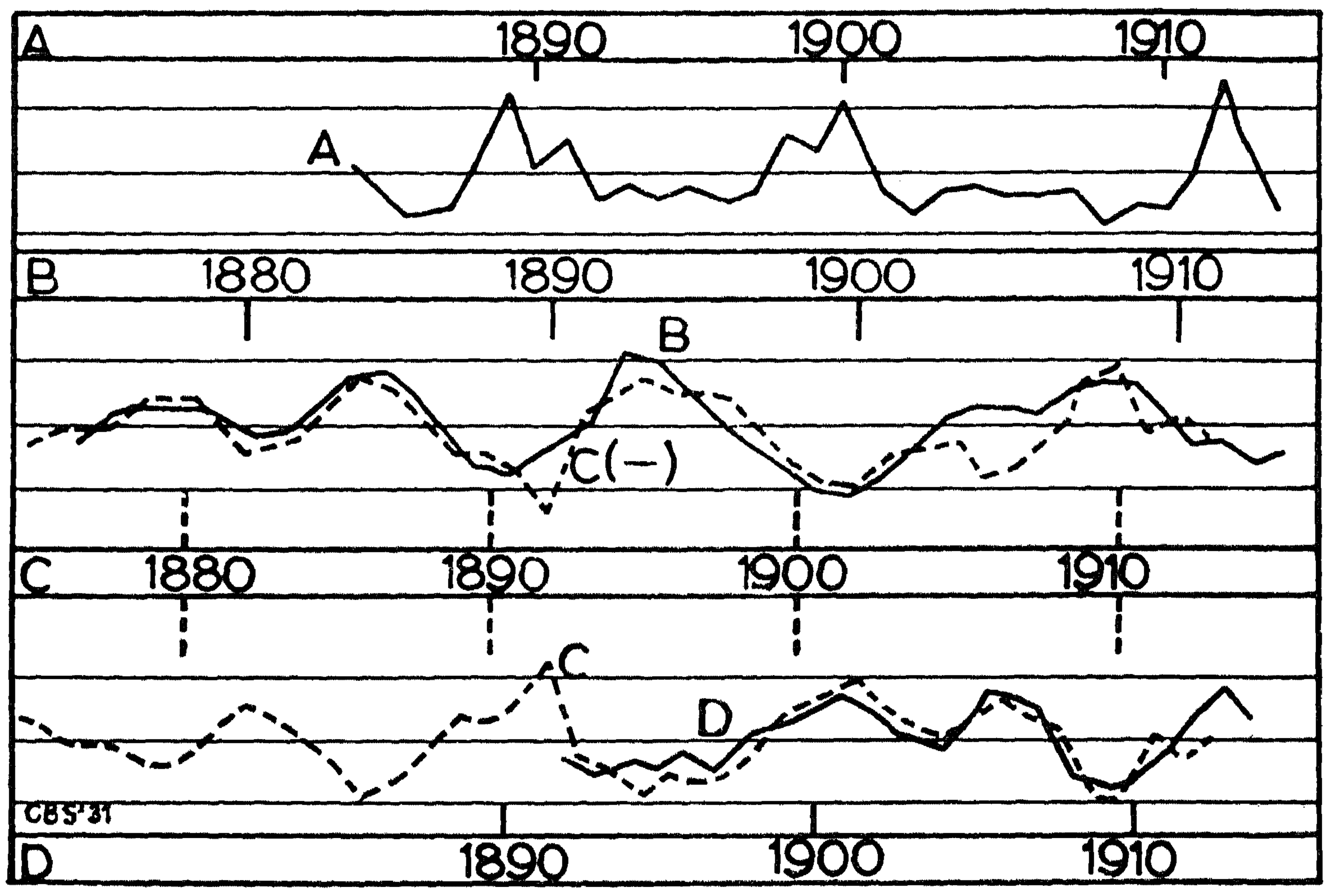


Fig. 1

Index of freight rates (A), total tonnage of British, American and German merchant marines (B), increase in tonnage (C), world launchings (D) prior to 1913 (deviations from trend).[6]

We have seen that an increase in the total tonnage is in the first place dependent upon the volume of tonnage of about two years earlier. At this point emerges an interesting theoretical problem: how great would the total tonnage and its rate of increase have been if the relationship described above had had no exceptions? This relationship,

[6] For the figures compare: *De Nederlandsche Conjunctuur*, Jg. 1931, Afl. 1, p. 22.

this "reaction mechanism", will in most cases and at any rate in the present one, prove to result in a cyclical movement which I have briefly called the "shipbuilding cycle". The following paragraphs reveal the characteristics of this shipbuilding cycle. The only successful method in my opinion is a strictly mathematical approach to the problem because this is the only way to arrive at a determinate solution.[7]

## 2. DETERMINING THE PROBLEM

We want to obtain the development over time of total tonnage; time will be expressed as $t$, tonnage as $f(t)$, so that the function $f$ is our unknown quantity.

The data given are in the first place the abovementioned relationship between "increase" and "tonnage some time ago". The rate of increase will be expressed here as $f'(t)$, the tonnage $\theta$ years ago—we are stating the problem generally at first—as $f(t-\theta)$. The intensity of the reaction, *i.e.* the volume of the increase which corresponds to a tonnage level of one unit above the trend, shall be expressed as $a$. Since a high level corresponds with a low increase, the relationship discovered may be expressed in the following equation

$$(1) \qquad f'(t) = -af(t-\theta) \qquad (a > 0).$$

Apart from this equation which defines the law according to which the subsequent conditions develop from the present condition, something should also be indicated about the initial conditions, since naturally the shape of the development is influenced by them. It will be understood that not only should the total tonnage at some initial moment be given but also the development during the total initial period of length $\theta$. It is only then that the further development will be defined since the increase will in each case be determined by the conditions given $\theta$ years earlier. Another given quantity, therefore, must be the development in an interval of, shall we say $0 \leqq t < \theta$, which will be expressed here by the equation:

$$(2) \qquad f(t) = g(t) \qquad 0 \leqq t < \theta,$$

---

[7] Readers who are not versed in mathematics can omit paragraph 3 without losing the line of reasoning.

Finally the solution should meet another requirement in order to have economic significance: it must be real and finite.

In the following paragraph we will first try to find the solution to the general equation (1) and subsequently work out the particular form of the result which corresponds to the value of the constant in our particular case. These values are as follows: in the shipbuilding cycle, the value of $\theta$ is 2 (in years), the value of $a$ in the period under review lies between 1 and 1/2.[8] The development in the course of two years may generally be assumed to be a simple curve (that is it may be illustrated as either a parabola or a sinusoid or an exponential curve etc. with only minor deviations).

## 3. THE MATHEMATICAL SOLUTION OF THE PROBLEM[9]

The solution of equations of this kind (functional equations) in analytical mathematics is, as will be known, usually not found "methodically" but experimentally. If a solution has been found and we can prove that only one solution is possible, then we may conclude that we have found the correct solution.

In the case of equation (1) the most obvious way to obtain a solution is in the following manner: assume

$$f(t) = e^{\alpha t+\beta} = Ce^{\alpha t}, \tag{3}$$

where the two constants are still to be defined and must be assumed *a priori* in as general a form as possible, in other words we assume that they are complex quantities. Substituting (3) into (1) we get:

$$\alpha Ce^{\alpha t} = - a\, Ce^{\alpha(t-\theta)}, \tag{4}$$

which after dividing by $Ce^{\alpha t}$ reads:

$$\alpha = - ae^{-\alpha\theta}. \tag{5}$$

From (4) and (5) it follows that $C$ can be chosen arbitrarily, whereas

[8] In fact, that the value decreases slowly in the period under review. Strictly speaking, $a$ is therefore also a function of $t$. Because of its slow variability, however, the best way is to assume $a$ to be constant for the time being and substitute it as variable in the solution found. In physics this is known as the method of adiabatic variables. (*Cf.* P. EHRENFEST, *Ann. der Physik*, Bd. 51, 1916, p. 327.

[9] The author is greatly indebted to Dr. J. DROSTE, Professor of Mathematics at the University of Leiden, for some critical remarks on this paragraph.

$a$ depends on equation (5). To solve this last equation we substitute thus

(6) $$-a\theta = z = x + iy,$$

or

(7) $$a = -\frac{x+iy}{\theta},$$

and then write for (5)

$$\frac{z}{a\theta} = e^z,$$

which for $a\theta = b$ turns out to be

(8) $$z = be^z.$$

Splitting this complex equation into its real and its imaginary components we get:

(9A,B) $$x = be^x \cos y \qquad y = be^x \sin y,$$

Eliminating $x$ we get:

(10) $$x = \frac{y}{\operatorname{tg} y}$$

(11) $$\frac{y}{\sin y} = be^{\frac{y}{\operatorname{tg} y}}$$

or

(11') $$b\frac{\sin y}{y} = e^{-\frac{y}{\operatorname{tg} y}}$$

A graphical solution of equation (11') is made possible by the graph in Fig. 2. The drawn curves represent the right-hand and the dotted curves the left-hand side of different values for $b$, namely $b = 0.25$; 1; 2.

The following conclusions may be drawn from the figure:

1. The intersections for $y = 2\pi$, $4\pi$ *etc.* cannot be used as solutions because they lead to infinite values for $a$.
2. The other intersections are acceptable solutions which shall be expressed as $y_k$ ($k = 1$ in the first interval of length $2\pi$, $k = 2$ in the second interval *etc.*).
3. The solution for $y_1$ is missing if $eb < 1$, *i.e.* if

(12) $$b(\equiv a\theta) < \frac{1}{e} \sim 0.37.$$

4. All $y_k$ increase if $b$ increases.

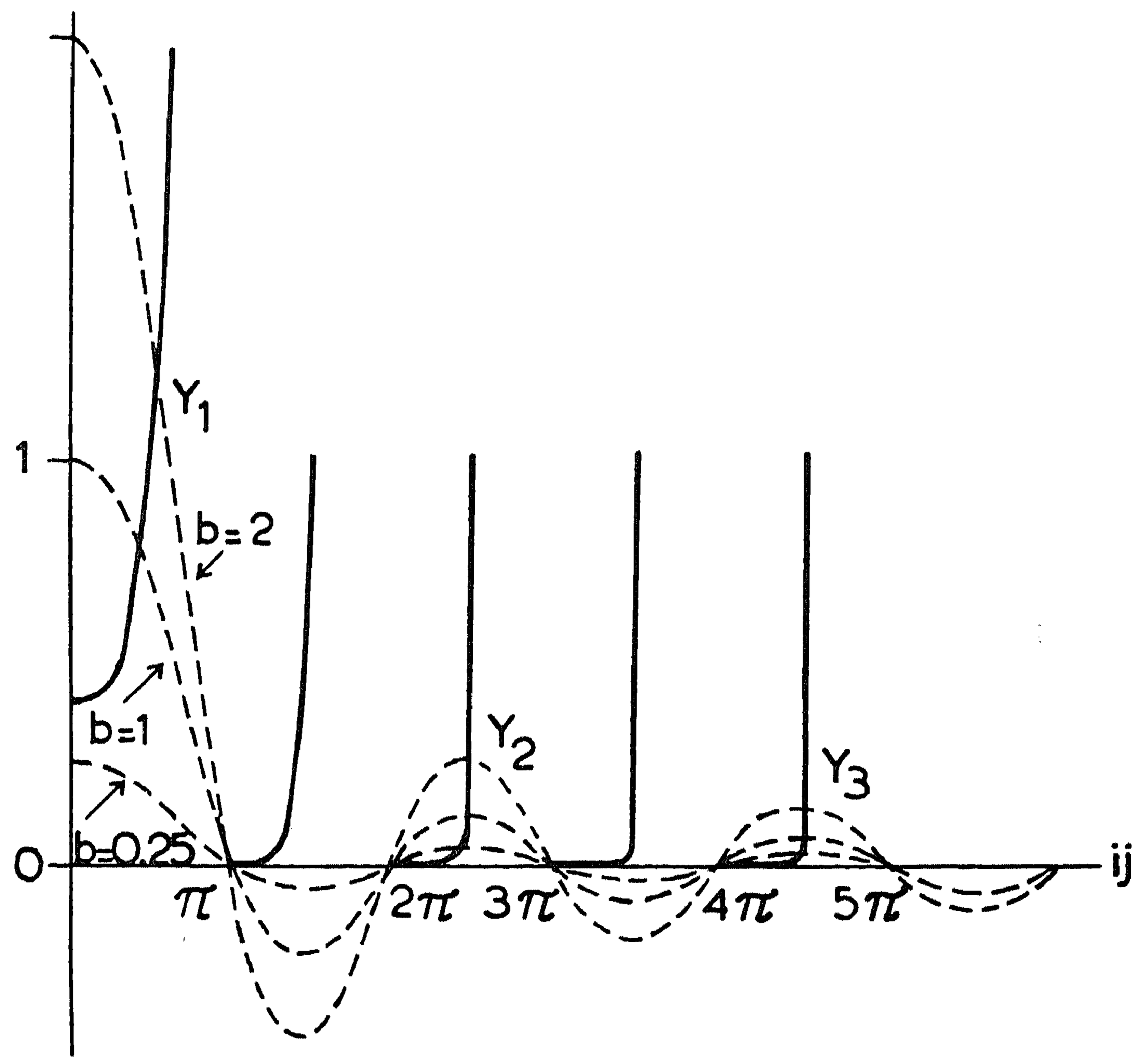


Fig. 2.

It will be seen that we have found a number of particular solutions which take the form of:

$$f = C_k e^{+a_k t},$$

in which $a_k$ is given through equation (7) if $x$ and $y$ in this equation have the subscript $k$. From equations (10) and (11′) it follows furthermore that also $-y_k$ and $x_k$ form a pair of roots. The corresponding $a_{-k}$ is conjugate to $a_k$.

Since the original equation is linear, each sum of two or more solutions is again a solution so that the general form of the solutions found now reads:

$$f(t) = \sum_{-\infty}^{+\infty} {}^{k} C_k e^{\alpha_k t}, \quad \text{where } C_0 = 0. \tag{13}$$

Only those solutions that are real have an economic significance, from which it may be concluded that also $C_{-k}$ is conjugate to $C_k$. Each pair of terms from (13):

$$C_k e^{\alpha_k t} + C_{-k} e^{\alpha_{-k} t}$$

represents a sine wave with an arbitrary phase and initial amplitude, but with a fixed period and a fixed damping degree, since the last two quantities are defined by $\alpha_k$.

If it were possible to represent any development of function $f(t)$ for the interval $0 \leqq t < \theta$ by the proper selection of $C_k$, then (13) would be the general solution of the equation quoted above. In the following we shall proceed as if this assumption were proven already. For those cases which are of interest to us any possible errors will be small. However, the proofs which should be made at this point will be omitted because of their length. We will proceed as if they have been included. This means that equation (13) presents a solution to the problem only when $b \geqq 1/e$ (compare (12)).

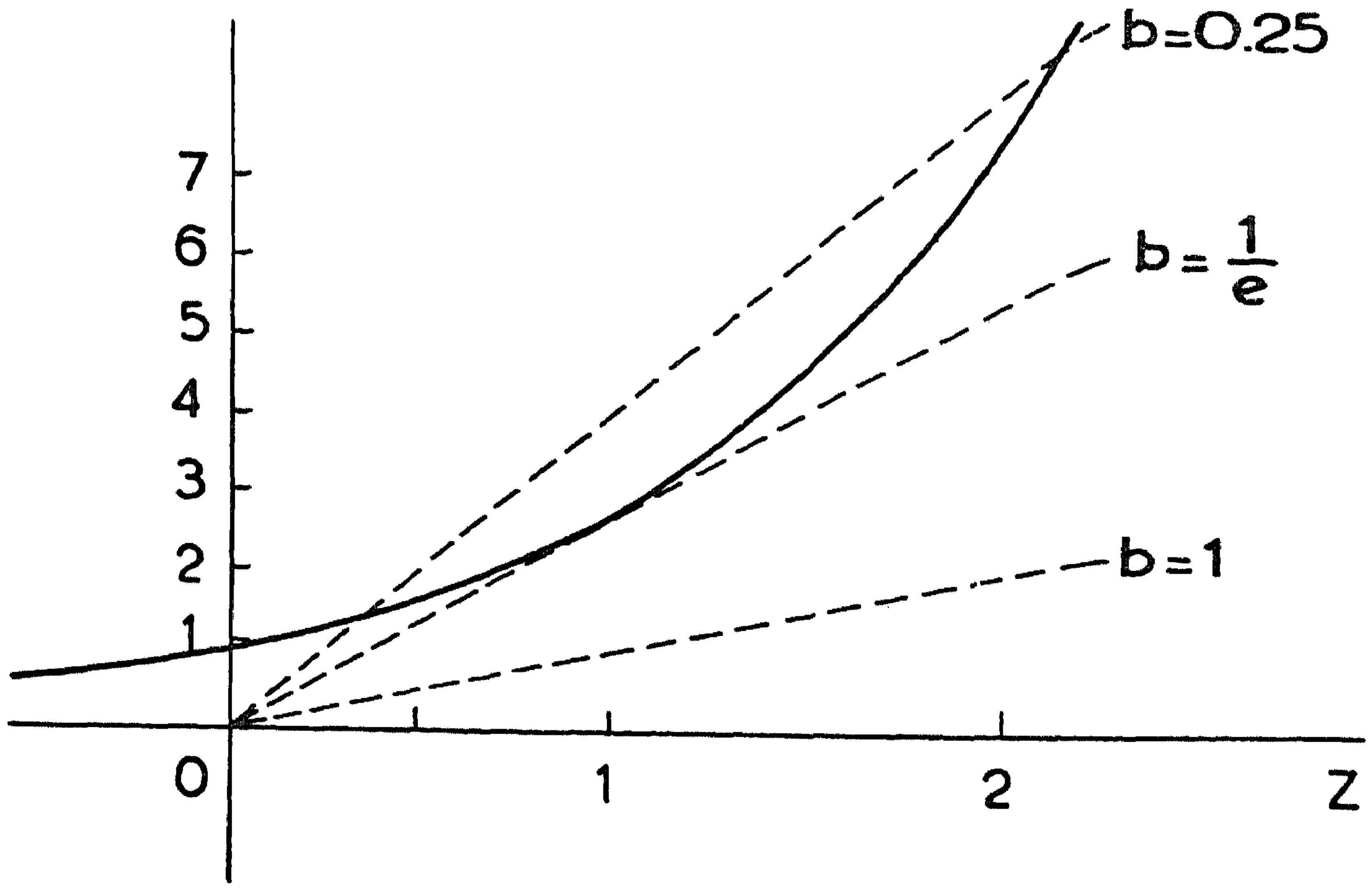


Fig. 3.

We must not forget that the solution of equation (8) which has been attempted with the help of equations (10) and (11′), is correct only if $y \neq 0$, *i.e.* if $z$ is actually a complex figure.

If $y = 0$ and hence $z$ is real, the division of equation 9B by 9A to get equation (10) is not permissible and the solution of (8) must be made directly. In this case it can be done quite simply (see Fig. 3).

If we write (8) thus

$$\frac{z}{b} = e^z, \tag{8'}$$

then the right-hand side in Fig. 3 is again represented by the drawn line and the left-hand side by the several dotted curves where $b$ takes the values $1$, $1/e \sim 0.37$ and $0.25$. The figure shows that (8) has a real solution in cases where $b < 1/e$ applies, in other words where the complex solution does not exist. In general the equation actually has two roots: $z'$ and $z''$, which coincide only for $b = 1/e$ in $z = 1$. In the above cases the following two terms take the place of the solution with $y < 2\pi$:

$$C'_1 e^{-\frac{z_1'}{\theta}t} + C''_1 e^{-\frac{z_1''}{\theta}t}, \tag{14}$$

except for the border-line case of $b = 1/e$ where, according to a familiar theorem of linear differential equations, $a$ in (14) reads instead:

$$(C'_1 + C''_1 t) e^{-\frac{t}{\theta}}. \tag{15}$$

With the same reservation made for equation (13) we will further assume that for cases where $b < 1/e$ or $b = 1/e$ the general solution is given through (13) except that instead of the terms for $k = 1$ we must read the terms under (14) and (15) respectively.

## 4. THE ECONOMIC SIGNIFICANCE OF THE SOLUTION

In the following we shall try to give an explanation of the most important features of equations (13), (14), (15) for economic theory and, in particular, the theory of trade cycles in nonmathematical terms. The fluctuations following the law of the "shipbuilding mechanism" are determined by:

1. the lag $\theta$,
2. the intensity of reaction $a$,
3. the movement during an "initial period" from which point onwar(
   the mechanism has been undisturbed.

The movement is actually a combination of several elementar) fluctuations from which it results by superposition. These elementar) fluctuations are partly of a cyclical nature and partly consist of a unilateral approximation to a state of equilibrium. If the product ($b$ of lag period and reaction intensity exceeds 0.37, all components ar( cyclical.

The period of the cyclical components, *i.e.* the interval between tw( consecutive zero levels with equidirectional movement of the com-

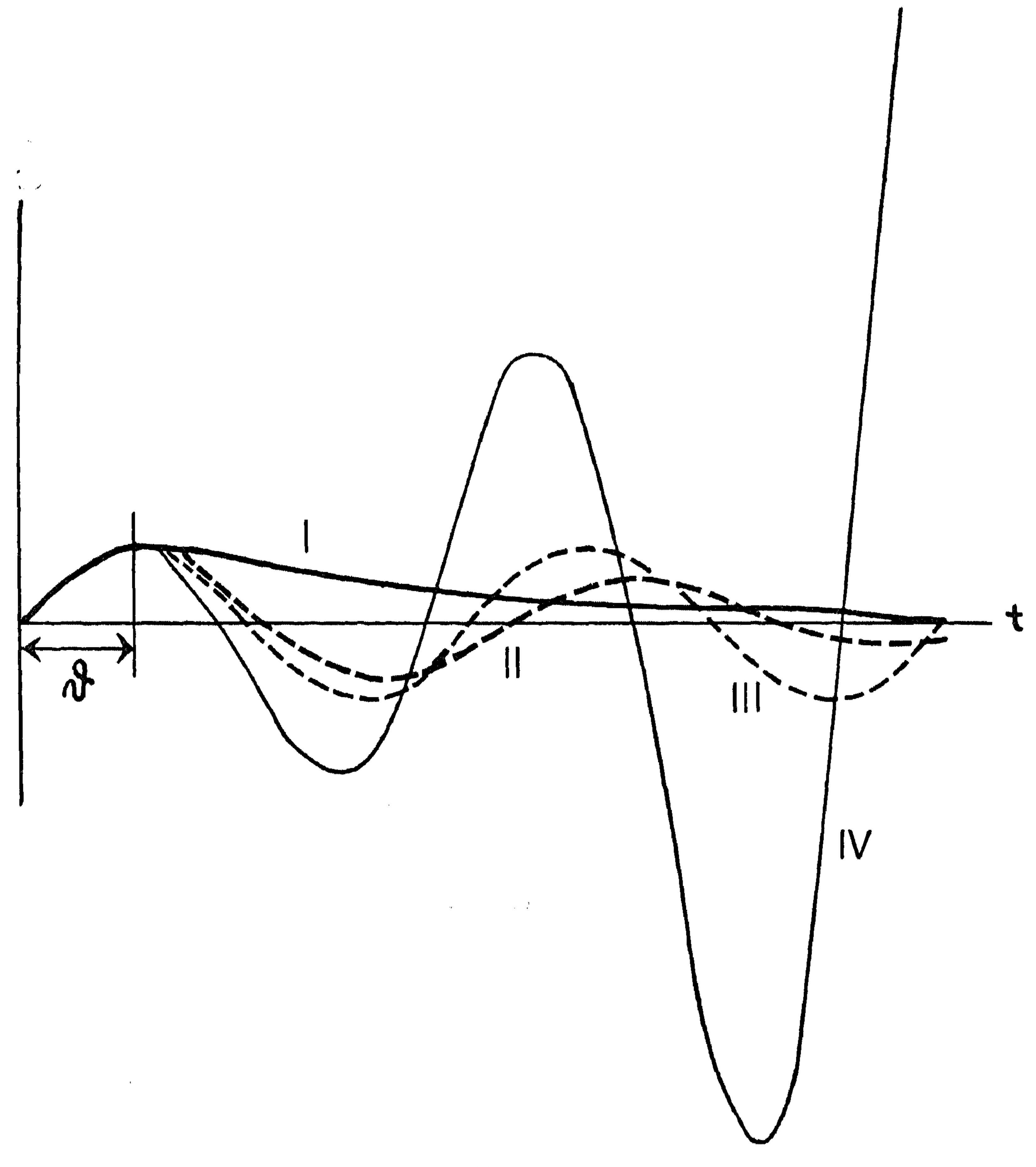


Fig. 4.

ponent in question, is dependent on only $\theta$ and $a$. One may say that it is inherent in the mechanism. There are a whole number of cycles beginning with the maximum period, whereas all further components show steadily decreasing periods. The length of the maximum period is at least equal to $2\theta$ for cases where $b > 1/e$. The second lies always between $\theta$ and $\frac{2}{3}\theta$, the third between $\frac{1}{2}\theta$ and $\frac{2}{5}\theta$, the one in the order of $k + 1$ between $\theta/k$ and $2\theta/2k + 1$. For $b > 1/e$ the maximum period already lies between $\theta$ and $\frac{2}{3}\theta$, the one in the order of $k$ between $\theta/k$ and $2\theta/2k + 1$. The "initial development" (see under 2 above) determines the relative importance of the components. The importance of the bigger cycles will be greater if the smaller ones in the initial development are not recognizable. In the present case of shipbuilding the periods which are shorter than $\theta$ (= 2 years) in connection with the "mechanism" are probably insignificant.

Since, in general, the periods which are shorter than the lag period are of little interest we shall leave them out of our discussion.

The movement therefore presents itself as follows: if (1) $b$ is smaller than $1/e$ (0.37) or equal to $1/e$ (0.37), in other words in the case of short lags and/or small intensity of reaction there is no cyclical motion, just a unilateral adaptation to the state of equilibrium $f(t) = 0$, *i.e.* to the trend. If (2) $b$ lies between 0.37 and $\pi/2 \simeq 1.57$, then we get a damped sine wave, *i.e.* a gradual approximation to the state of equilibrium by the steadily decreasing amplitude of the fluctuations.

If (3) $b = \pi/2$, we get a pure sine wave, that is to say, a cyclic motion with constant amplitude. Finally, if (4) $b > \pi/2$, in other words exceeds 1.57, we get sine waves with amplitudes increasing in time. In the latter two cases therefore there is no approximation to a state of equilibrium.[10] Fig. 4 is a diagrammatical representation of a number of possible cases which are all based on the same initial development. The length of the period is for:

$$b < \frac{1}{e}: \infty,$$

$$\frac{1}{e} < b < \frac{\pi}{2}: > 4\theta,$$

[10] See also: U. Ricci, "Die synthetische Ökonomie von Henry Ludwell Moore", *Zeitschrift für Nationalökonomie*, Vienna, I (1929/30), p. 649 *sq.*

$$b = \frac{\pi}{2}: 4\theta,$$

$$\frac{\pi}{2} < b \quad : < 4\theta, \text{ but } > 2\theta,$$

that is four times the lag period in the case where we have pure sine waves, larger than four times the lag period in case of damped sine waves, and smaller (but not smaller than twice $\theta$) in case of steadily increasing amplitudes. It seems to me that this result is of vital importance to the theory of economic dynamics. It permits remarkable conclusions about the relationships between the constants quoted above, particularly the significance of the intensity of reaction for the type and length of the waves. Moreover, it gives us a clue to a method of judging the stability of an economic system in general.

Its importance in the field of shipbuilding is in my opinion expressed in the statement that in shipbuilding an "endogenous" cycle of about eight years (since $b$ is a value of between 2 and 1 which results in cycles of between 7.5 and 8.7 years) exists and that development showed a tendency to quieten down during the pre-war period (about 1900 when $b < \pi/2$, that is the second example given above).[11]

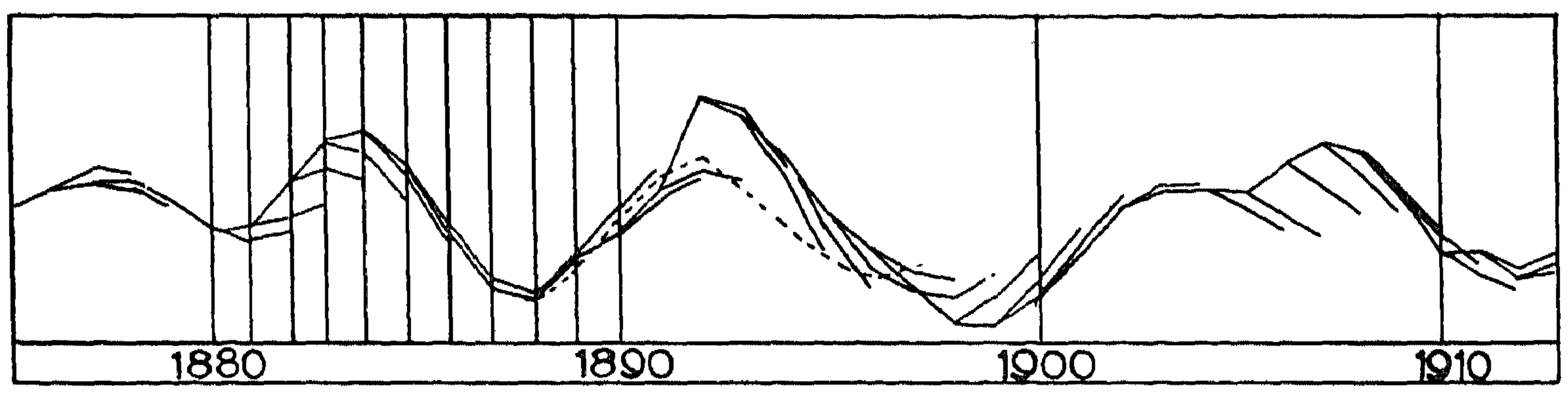


Fig. 5.

The reason why in the course of history this fact has not been recognized at all or at least only vaguely is explained on the one hand by the slow rate of decrease in amplitude and on the other hand by the fact that exogenous disturbances appear which change the total tonnage level. This is made clear in Fig. 5, where the thin curve represents

[11] This also means that at the same time the period of the fluctuation has a tendency to increase.

total tonnage during the years 1875 to 1913, and the thick curves indicate individual developments during a period of two years, which would for any given year be the combined result of the past two years' development and of the lag mechanism. The dotted curve illustrates the development over a period of 12 years which would have resulted from developments between 1885 and 1887 if during this entire period the mechanism had had an undisturbed effect.

As one can see from this figure, developments dependent on the mechanism and real developments coincide satisfactorily in most years, whereas in some years, *e.g.* in 1882, 1892 and 1905, there are distinct outside disturbances which can be attributed chiefly to general trade fluctuations. At the same time the graph permits an estimation of the relative importance of the "proper" shipbuilding cycle as opposed to general business cycles which are exogenous for shipbuilding.[12]

### 5. COMPARISON BETWEEN THE "SHIPBUILDING CYCLE" AND THE "PORK CYCLE"

After the discussion of the characteristics of the "shipbuilding cycle" it is profitable to draw a comparison between this cycle and the "pork cycle". The common feature of these two cycles is an endogenous movement which is caused by supply lagging behind price and influenced by the "reaction intensity" whereby supply responds to deviatons from the normal price.

The first difference lies in the relation between lag and period. In the pork cycle the period is exactly twice, in the shipbuilding cycle it is always more than twice, and in our example almost four times the lag. As can easily be seen, this difference is largely due to the fact that in the shipbuilding cycle it is the increase in tonnage which is significant. In the end the basic reason is that a ship is a durable good while a pig is more of a non-durable. Generally speaking we should therefore differentiate between durable good cycles

[12] The operation of the mechanism can also be detected fairly well in the postwar period (see other publications of the author) although not as distinctly as before the war. However, the time that elapsed since the war is a little too short to allow us to make a conclusive judgment.

and non-durable good cycles although the boundary is not clearly defined.[13]

Conceivably a second difference might be seen in the fact that in shipbuilding cycles the waves can be both undamped and damped. This difference, however, is not real; nor is another difference which could be mentioned, namely the existence of several periods in the shipbuilding industry compared to only one period in pork cycles. But if we analyse the pork cycle in a manner similar to the shipbuilding cycle, we will easily reach the conclusion that the "pork mechanism" corresponds to a damping or anti-damping as soon as the reaction intensity is smaller or larger respectively than the one causing the waves of constant amplitude. We also find that fluctuations with smaller periods can be generated by the mechanism, namely those with periods of 1/3, 1/5, 1/7 *etc.* of the period of the "main cycle". These fluctuations, it seems to me, are of slight significance, just as in the case of the shipbuilding cycle. On the other hand, the interdependence between the rate of damping and the reaction intensity is of importance because it demonstrates that when the reaction intensity diminishes the amplitude can also diminish. This is already attempted in practice. Moreover, it can easily be proven that, contrary to what was demonstrated with regard to shipbuilding, the length of the lag is of no consequence to the degree of damping in the pork cycle.

---

[13] After writing the present article I also found that housing, *e.g.* in Hamburg, showed a similar durable good cycle. After 1900 the regular trade cycle is predominant. (*Cf.* K. HUNSCHA, "Die Dynamik des Baumarkts", *Vierteljahreshefte zur Konjunkturforschung*, Sonderh. 17, Berlin 1930).