LAG CYCLES AND LIFE CYCLES*

1. In trade cycle theories a distinction is made between exogenous and endogenous movements. An exogenous movement in economic quantities (production, prices, incomes, *etc.*) is caused by changes in the non-economic "data", the natural, technical and institutional circumstances in which economic events take place. An endogenous movement procedes whilst the "data" remain unaltered, emanating from the internal relations which exist between the economic quantities, although this movement must receive its initial impulse from a change in data. In reality smaller and greater changes in data are constantly occurring. Nevertheless most trade cycles investigators are of the opinion, and rightly so I think, that a trade cycle is an endogenous movement which is only disturbed by numerous but small changes in data. This means that the forces issuing from the mutual relationship between the economic quantities are generally stronger than the "disturbing" forces from the outside. A number of important aspects of cyclical theory and policy can accordingly be analysed, leaving the "disturbing" outside forces out of consideration. This course will be followed in this article.

Endogenous movements would not occur at all if the causal relationship between the economic quantities and the "data" were completely simultaneous, that is to say if every economic quantity were only related to the other economic quantities and the "data" of the same moment. In that case what we could call an immediate adaptation of the entire economic system to the natural, technical and institutional conditions of the moment would occur. If we assume that the latter do not change (or, which would lead to the same conclusions regarding business cycle theory, only change slowly and in one direction) there would never be an actual possibility of fluctuations occurring.

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* "Vertragingsgolven en Levensduurgolven", from *"Strijdenskracht door Wetensmacht, Opstellen aangeboden aan S. de Wolff t.g.v. zijn 60e verjaardag"*. Amsterdam 1938, p. 143–150.

For this reason it is a necessary —although not sufficient—condition that the relationships existing between the several economic quantities should be dynamic *i.e.* that the value taken by one variable at a given moment depends on the values of other variables at previous moments. For example, activity in shipbuilding depends on the profits which have been made in shipping during the previous year; or wages and prices of consumer goods depend on the production attained some time previously. It is true that owing to such a time difference between two or more phenomena which are causally connected a cycle may be caused. The examples which will be discussed later will demonstrate this point.

In the first place we may point out that such a difference in time can occur in several ways, two of which we will examine here. In the first place these time differences may be relatively slight as compared to the length of the cyclical fluctuations; in the second place they may be of the same size. The first kind finds its chief representative in all kinds of lags which occur in economic life as a result of the length of production processes and of numerous "reactions". For example the reaction of entrepreneurs to certain favourable conditions, in the shape of new investments, takes a few months during which the situation has to be watched, plans have to be made or elaborated *etc.* The reaction of wages to altered labour conditions takes some time, *i.a.* due to the contract periods and also to other reasons. The reactions of the cost of living to a changed cyclical situation depends on the period for which stocks have been laid, on the period of hire-purchase transactions and on numerous other conditions which are time-consuming.

The second category of time differences which has been mentioned above—the longer differences in time—is chiefly caused by the life of a number of durable goods. A car has an average useful life of six to seven years and when it is worn out it will be necessary to produce a new one. The latter event then is logically connected with an event pertaining to a number of years previously.

Both the shorter and longer time differences in economic relationships may lead to fluctuations of the same length as cyclical fluctuations but the two kinds of fluctuations show a few differences in character which will be scrutinized further here. For brevity's sake they will be called henceforth "lag fluctuations" and "life fluctuations".

2. An example of a mechanism with lag fluctuations is given by the following model. The following economic variables are under consideration:

L labour income
Z the income not derived from labour, to be called "profit" in short
U the value of the consumer goods sold
V the value of the investment goods sold.

The following relationships are assumed between these variables, all measured as deviations from a moving equilibrium.[1]

(1) The profit equation which shows how "profits" are calculated:

$$Z = U + V - L. \tag{1}$$

In this equation $U + V$ is the value of the products sold by all enterprises in the economy. Only wages are deducted as cost elements; the cost of the raw materials used by all enterprises is also converted to wage costs. Besides wages, depreciation should be deducted as a cost but it can be assumed that it changes so gradually that it only shows a trend with no cyclical component. Our equations, however, apply only to the latter category.

(2) It is assumed that the value of the investment goods sold fluctuates with profits. The main reasons are that, on the one hand, investments promise to pay best when profits are generally high and, on the other hand, that the financial possibilities permit investments, both out of own funds and with the aid of outside capital, most easily when profits are high. In this relation we assume a lag of one time unit, which is taken to be four months. Then we get

$$V_t = \beta Z_{t-1}, \tag{2}$$

whereby β represents a constant coefficient.

(3) Finally it is assumed that the expenditures on consumer goods are related to L and Z in the following way:

In the first place the entire amount of wages is spent on consumer goods immediately.

[1] This equilibrium may prove unstable. See: "On the theory of business-cycle control", *Econometrica*, 1938.

In the second place the consumer goods expenditure of independents is determined by both their income Z and by the rise in their capital per time unit. With regard to the latter we assume that it changes proportionally with Z. In these relationships we assume time lags of such a nature that finally we obtain the following formula:

$$U_t = L_t + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2}). \tag{3}$$

In view of the fact that our first and foremost aim is to give an example, the details of the assumption should not be taken too literally. Nevertheless they correspond fairly well to reality, *e.g.* in the United States, as an extensive statistical investigation taught me.[2]

If we choose the unit at about 34.000 million dollars, then an estimate of the average value for L and Z over the period 1919–1932, to be indicated by $\overline{L}$ and $\overline{Z}$, reaches 1; $\overline{U} = 1.8$ and $\overline{V} = 0.2$; furthermore we can put $\beta = 0.2$, $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 1$.

The three equations obtained are sufficient to determine the course of Z, V and $U - L$; if we wished to know the course of L and U separately, then a further relationship would be required. This is not essential to achieve our purpose and for this reason we limit ourselves to the above. For example we find that the following movement equation can be applied to Z.

$$Z_t - (\beta + \varepsilon_1 + \varepsilon_2) Z_{t-1} + \varepsilon_2 Z_{t-2} = 0. \tag{4}$$

The same equation can be found for both other quantities.

The exact significance of this equation is that the value taken by Z during the period $t(Z_t)$ is determined by the values Z_{t-1} and Z_{t-2} which Z has assumed during the periods $t - 1$ and $t - 2$. The distance between two time units, between the first and the last term in this equation, we will call the "interval" of this mechanism. For the values of the coefficients mentioned above we get:

$$Z_t = 1.8\, Z_{t-1} - Z_{t-2}. \tag{5}$$

The Table shown below gives an example; starting from a lowest point where $Z_1 = Z_2 = -0.5$ we find $Z_3 = -0.4$. With the aid of Z_2 and Z_3 it becomes possible to calculate Z_4 and so on. The character

[2] A few particulars in this respect are mentioned in my article in *Econometrica*, Jan. 1938 just mentioned and in my *Fondements mathématiques de la stabilisation des affaires*, Paris 1938, where somewhat different definitions were applied.

of the movement can now be clearly seen; moreover it can easily be proven.

t =	1	2	3	4	5	6	7	8	9	10
Z =	—0.5	—0.5	—0.4	—0.22	0	+0.22	+0.40	+0.50	+0.50	+0.40
t =	11	12	13	14	15	16	17	18	19	20
Z =	+0.22	0	—0.22	—0.40	—0.50	—0.50	—0.40	—0.22	0	+0.22

The period of the movements depends on the β, ε_1 and ε_2; and it can be shown that the period is increasing if $\frac{1}{4}(\beta + \varepsilon_1 + \varepsilon_2)^2$ approaches ε_2 but does not exceed this value. In our case the period is almost 14 time units or about $4\frac{2}{3}$ year; in any case it is considerably longer than the lags which "cause" it and this is significant with regard to our further argument. Apart from the way as indicated in this model lag cycles can arise in many other ways.

3. The theories which utilize the lifetime of capital goods as a main element in the explanation of the fluctuating level of economic activity, can thus be stylized into a model. We assume that increased production of capital goods leads immediately to increased purchasing power on the part of the producers and accordingly to an increased production of consumption goods; also that the price system alters immediately in conformity with production and for this reason incomes and profits do also. If we indicate the latter by Z, a parallel movement of Z_t with the production y_t of capital goods will exist. If we again measure both quantities in terms of deviations of some equilibrium development, then:

$$y_t = \eta Z_t \tag{6}$$

and all other economic quantities also will run parallel to Z_t. The production of capital goods itself can depend on two factors; these are Z_t and the quantity of capital goods which are eligible for replacement after a rigidly determined lifetime. Capital goods with widely varying lifetime T will exist; for simplicity's sake we assume that there is only one kind of production equipment but that of each two there is one with a lifetime of six and one with a lifetime of eight years. In reality the spreading is much greater and numerous intermediate stages exist but, in principle at least, this does not change our considerations in any way that we can detect. If we express the production of capital goods in the year $t - 6$ by y_{t-6} and in the year $t - 8$ by y_{t-8}, then the quantity of production units which are eligible for replacement according to their lifetime is:

$$\tfrac{1}{2}\, y_{t-6} + \tfrac{1}{2}\, y_{t-8}.$$

Our assumption concerning y_t now becomes accordingly:

$$y_t = \tfrac{1}{2}\,(y_{t-6} + y_{t-8}) + \zeta\, Z_t, \tag{7}$$

and this assumption can now be interpreted in such a way that, depending on Z_t, more or less is replaced than is necessary according to the life of the equipment and secondly that, apart from replacement, new investment still takes place also depending on Z. Combining the equation (6) and (7) we find:

$$y_t = \tfrac{1}{2}\,(y_{t-6} + y_{t-8}) + a\, y_t, \tag{8}$$

where

$$a = \frac{\zeta}{\eta}. \tag{9}$$

This therefore is the final equation of a system in which lifetime appears as the only "dynamic factor". As the course followed from year 1 to 8 is given, further developments can also be calculated. During a few of these eight years, owing to factors which we will provisionally consider as accidental, a peak has appeared in the volume of production of production equipment. There will be a propensity for this peak to occur again. This is, in a nutshell, the so-called echo-theory propounded in this country by S. DE WOLFF, not only with respect to the short (about eight-yearly) cyclical fluctuations but also with regard to the long (about forty years) cycle which we will not consider further here.

An objection which has often been made to this theory is that the damping of the cycles would be very considerable in view of the wide variation in the lifetimes of the capital goods. This is true if we leave out the term with Z_t in equation (7) which means that the term ay_t drops out of equation (8). The damping is then even underestimated by this equation since the variation of the lifetimes is greater than we have assumed for simplicity's sake.

The afore mentioned strong damping need not occur, however, if we retain the term which contains Z. For a certain value of a we may get undamped cycles or perhaps cycles which display anti-damping. Indeed it is not difficult to set up examples of this kind. With regard to $\alpha = 0.37$ equation (8) *e.g.* gives a purely periodical solution.[3] In reality, however, the values of α will have to be much greater in order to obtain undamped cycles.

4. We will now make a few comparisons between lag and life cycles. Regrettably lack of space does not allow to back all our statements with a full proof. A few results of investigations will have to suffice.

A first difference springs from the sensitivity of the period to changes in the structural coefficients (such as the figures β and ε with regard to the lag cycles and α to the life cycles). The period of the lag cycles may change very strongly, even when only small changes occur in the structural coefficients. We have to take for granted that such changes take place repeatedly; the comparative invariability in the period of actual business cycles is accordingly somewhat contradictory. The life cycles do not show such sensitivity. Comparatively large changes in α will not exert a strong influence on the period of the cycle.

A second difference is due to the sensitivity to short term disturbances. We chose as an example a country in the middle of a rising phase of the business cycle which normally lasts about five years. Let us assume that owing to an exogenous disturbance the level of economic activity movements is forced downward for one year. If the cyclical mechanism is purely a lag mechanism with an interval of less than a year, business cycles will have entirely and definitely changed their phase: because further movements are entirely determined by the "initial values" during a short period. If, on the other hand, the cyclical mechanism is a lifetime mechanism, then after this disturbance the old course can continue because, for this type of cycle, the present situation is determined by the situation prevailing for a long previous time period. In this case therefore there is a much greater inertia in the cyclical movement. In both cases the precise shape of the smaller fluctuations are determined mainly by the numerous smaller exogenous forces which often occur and which are the main basis for the short-run cyclical diagnoses which appear for example in the newspapers.

It seems to me that both these points of difference, as compared to

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[3] The function $y = C \sin 2\pi t/7$, in which C is taken at random, is a solution.

reality, must lead to the view that lag cycles which are founded on relatively short lags are not an adequate explanation of the cyclical movements. But a knowledge of the mechanism of the shorter lags and the connected relationships is vital for problems connected with business cycles. We will once more try to make this plausible by putting the question: if business cycles were caused entirely by the life mechanism of equation (8), would an anti-cyclical policy be effective and if so, what should this policy be? It is clear that an anti-cyclical policy directed at reducing the coefficients $\frac{1}{2}$ in this equation would hardly be suitable. A strong reduction in these coefficients would mean that normally only a small part of the worn production equipment would be replaced, which would be a cure worse than the complaint. A policy which influenced the coefficient α, however, would have an acceptable influence and the cyclical movement would become weaker as $1/1 - \alpha$ decreased. The final equation can be formulated as follows:

$$y_t = \frac{\frac{1}{2}(y_{t-6} + y_{t-8})}{1 - \alpha}. \tag{10}$$

This means that α itself must be reduced. However, α is only a symbol representing a number of reactions. In our example they are immediate reactions but in reality they are usually somewhat delayed reactions comprising a "lag mechanism". In order to find out by what policies α can be reduced, further study is required of the relationships characteristic of that mechanism. Although this mechanism is perhaps insufficient to explain the period of the actual movements, it is sufficient to enable us to find the suitable anti-cyclical policy measures.

As a simple example, we will mention "compensating public investments", *i.e.* public investments which move up or down contrary to the cycle of private investments and which are one of the methods of reducing α.