ON THE THEORY OF TREND MOVEMENTS*

CONTENTS

1. INTRODUCTION

The purpose of the present paper is to make a contribution to the theory of trend movements or, to state it differently, to the dynamics of long-term economic movements. The object of this particular field of economics may first be characterized by clearly defining its position among related fields. These are statics on the one hand, and the dynamics of short-term movements on the other. The object of statics is to find the equilibrium position of an economy whose population, stock of capital goods, state of technology *etc.* are given and in most cases are assumed to be constant. The dynamics of short-term movements deals with the explanation of "fast" movements, such as trade cycles, or else of speculative special movements such as those

* "Zur Theorie der langfristigen Wirtschaftsentwicklung", *Weltwirtschaftliches Archiv*, 55 Band (1942 I), p. 511–549.

caused by fluctuations in crops, *etc*. In these dynamic models also, the assumption of a constant population, stock of capital goods, and state of technology is frequently utilized.

The purpose of a theory of trend movements is to investigate economic movements extending over decades or even centuries. Such a theory must not overlook the development of population, capital *etc*., in fact it must make these the special subject of its analysis. Its central problem may be briefly expressed in the following question: how do production, employment, living standards and other factors change under the influence of population growth, technical development and capital formation? A more exact definition of this problem will be given later, yet this question is sufficient to suggest the scope of the subject.

It must be borne in mind that the theory of trend movements does not concern itself with the short-term fluctuations which show the actual movements. The study of the components of such movements is left to the analysis of the dynamics of these short-term movements. This limitation in scope must be made in order that the analysis does not become too complex and difficult. Whether or not such dissociation from the actual movements is generally permissible will be discussed later (see section 4).

In this paper even more restrictions will be made. Such restrictions, however, are not of vital importance for the contents of our theory and could be eliminated in a complete elaboration. Furthermore we will not take the trouble to explain the so-called long cycles. Their effect appears primarily in price movements; their influence on production, employment, and capital formation as well as the fluctuations of real wages is secondary. This does not mean, however, that we shall consider them to be irrelevant. The basic problem of trend movements, above all that of the deflated variables of the economy, is of a different nature and consists in discovering the basic causes of continuous increases in these series. Our analysis will be aimed particulary at the decisive determinants of economic growth and the extent to which it can be influenced.

This approach is perhaps even more justified with regard to the third restriction to be introduced, that of an elimination of monetary phenomena from our model of economic development. Monetary aspects are especially important for short-term movements and most of all for explaining fluctuations in prices, wages and rates of interest. In the case of long-term movements the determination of the fluctuations of these latter variables may be separated almost entirely from those of the real quantities in the economy, as will be shown later. We will therefore omit them for the most part and perhaps publish in another article an investigation into these problems.

Finally—and this is the end of our definition of the subject—we will not discuss individual goods and markets but will always consider the economy as a whole.

2. FIRST STEPS IN PUBLISHED LITERATURE

As is often the case in economics, the literature on long-term trends is classified into two distinctly separate fields between which there exists a vacuum. On the one hand, we have the statistical literature describing the outward appearances of the trend movement; this literature gives rather detailed numerical illustrations of the phenomena but states the economic interrelations only in a more or less superficial manner, as we shall show later. On the other hand, there is the theoretical literature which, apart from a very few exceptions, presents a qualitative analysis and for this reason excludes *a priori* the possibility of formulating numerical conclusions. This is why econometrics in this field as well appears to be the most adequate method of investigation. Econometrics is defined as the combination of a mathematical-economic treatment with a statistical analysis.

The greater part of the statistical literature has been published by Harvard University. Harvard's research in the field of trade cycles has led to the publication of a wealth of statistical data on which most of the published series of trend components are based. It will be remembered that there are two methods of defining trends: the non-mathematical and the mathematical. The non-mathematical method uses either free-hand curves—which can hardly be called scientific—or moving averages which cannot have any significance for the theory of trends. The mathematical method first assumes a definite type of curve—a straight line, a parabola, or an exponential curve—and defines the constants of this curve by the method of least squares. This method, it must definitely be granted, has significance for the theory of trends. This is because a connection with this theory is possible if the choice of the type of curve is based on a theory. Here we should like to add immediately that only in exceptional cases has this ever happened in an entirely satisfactory manner. But there are a few such cases.

The most common types of curves are those mentioned above. The use of the straight line may generally be explained by the fact that, for short intervals, it can be considered to be the first approximation to any other curve; the parabolas of several powers can be explained as approximations of higher order. The exponential curve (whose points describe a geometrical series of ordinate values for equidistant abscissæ) may be visualized in the first place as a linear development of logarithms; or, in the second place,—and this is theoretically clearer—as the result of a simple law of growth, whereby the annual increase in the variables under review is proportional to the magnitude of these variables already reached. Here we recognize the first signs of a theory, of a certain interlinkage of causes. It is true that a linear equation for development may be just as easily constructed, yet for most economic processes the latter will be less natural than the formula for the exponential curve. The simple formula mentioned above can sometimes be applied to the growth of population and to the formation of capital. However, this is true only for especially uncomplicated conditions which hardly ever occur in reality.

Others may be added to the three simplest types of curves we discussed above. First to be mentioned is the logistic curve expressed in the equation

$$x = \frac{a}{1 + e^{-bt+c}},$$

where t is time and a, b, and c are the constants. The corresponding growth curve is obtained if the increase of one variable is proportional:

a. to the quantity x already reached of this variable and
b. to the remaining space of development $a - x$; therefore a plays the part of an absolute limit which is drawn for variable x.

The validity of this law of growth has been established to a degree for certain kinds of bacteria colonies which were supplied with a given quantity of food per unit of time. There have also been attempts to explain the growth rate of human populations by this formula. Under certain circumstances reliable approximations are obtained.

In some respects the investigation of long-term trends at Harvard culminates in the work of KUZNETS and SNYDER. In his "Secular Movements in Production and Prices"[1], KUZNETS investigated a large number of production and price series, whereby he defined, among other things, the "primary trend".

In doing so he made use of two types of curves, namely the logistic curve mentioned above and the Gompertz-curve expressed in the formula:

$$x = ae^{-bc^t}$$

Both have a common characteristic: growth saturation. In his book KUZNETS tries first of all to explain these saturation phenomena. These are correlated chiefly with the "penetration" of certain new products into the economy and are therefore more characteristic of individual markets than of an entire economy. On the other hand, KUZNETS uses qualitative methods and what he gives—although very interesting—is more an enumeration of the relevant symptoms than a formulation of an economic theory. Since he dealt only with data for individual markets, this would indeed have been a difficult task.

In his small German volume, "Wesen und Bedeutung des Trends"[2] he gives a brief summary of the results he obtained. In the appendix he promises to present "a brief historical survey of the treatment of the problem in economics",[3] yet for the most part he gives only a very interesting discussion of the statistical treatment of the problem. An economic theory of the trend, however, is nowhere to be found.

The works of SNYDER[4] are more important for our purpose in as far as they deal with the economy as a whole. As a trend curve SNYDER frequently uses an exponential curve or at least suggests it by implying

[1] S. S. KUZNETS, *Secular movements in production and prices. Their nature and their bearing upon cyclical fluctuations.* (Hart, Schaffner & Marx Price Essays, 46). Boston and New York 1930.

[2] S. S. KUZNETS, Wesen und Bedeutung des Trends. Zur Theorie der säkularen Bewegung. *Veröffentlichungen der Frankfurter Gesellschaft für Konjunkturforschung,* H.7., Bonn 1930.

[3] *Idem,* note from the editor.

[4] C. SNYDER, *Business cycles and business measurements. Studies in quantitative economics,* New York 1927.

logarithmic methods. Unfortunately, one looks in vain for a quantitative theory of the trend.

In Germany, WAGEMANN published important material containing extremely interesting ideas about long waves of various kinds.[5] Other material has been published by a number of other authors, especially HOFFMANN.[6] However, a theory as we conceive it is not offered.

To my recollection, the beginnings of such a theory are found only in CASSEL's "Theoretische Sozialökonomie".[7] In paragraph 6 of chapter I he deals with the "uniformly progressing economy". With a truly exact theory he demonstrates that the magnitudes assumed by its variables correspond to exponential curves. All the same, such a uniformly progressing economy postulates a constant technology and is also in other respects very simplified in nature.

As far as I can see, all the other theoretical literature is purely qualitative. All one has to do is to open a few textbooks on economics or a few manuals of business cycle theory to confirm this opinion. As must be expected with a qualitative analysis, there is considerable difference of opinion about the significance of the individual factors. Some authors exaggerate the importance of population growth,[8] others overemphasize technical development, and still others the formation of capital.[9] An added difficulty is the diverging conceptions of the term technical development. We shall come back to this later (see section 5).

3. THE PURPOSE OF A TREND THEORY

It may be useful before tackling the subject itself to ask ourselves what is the purpose of trying to find a theory of long-term movements.

[5] E. WAGEMANN, *Struktur und Rhythmus der Weltwirtschaft. Grundlagen einer weltwirtschaftlichen Konjunkturlehre*, Berlin 1931. To my knowledge, WAGEMANN has so far given his views on long waves only in lectures.

[6] W. HOFFMANN, "Wachstum und Wachstumsformen der englischen Industriewirtschaft von 1700 bis zur Gegenwart", *Probleme der Weltwirtschaft*, 63, Jena 1940.

[7] G. CASSEL, "Theoretische Sozialökonomie", *Lehrbuch der Allgemeinen Volkswirtschaftslehre*, Abt. 2, Leipzig 1918.

[8] So H. VON BECKERATH, "Der moderne Industrialismus. Gewerbepolitik". *Grundrisse zum Studium der Nationalökonomie*, Bd. 11, 1, Jena 1930, p. 13.

[9] To an extent, this is the case with P. H. DOUGLAS, *The Theory of Wages* (New York 1934), insofar as he attributes technical development to an increase in capital intensity in production.

One can differentiate between direct and indirect purposes. The direct purpose is first to explain the rising movement of the abovementioned economic variables; second to find a mathematical formula for the curves in question, and finally, to determine the influence of such factors as population growth, technical development, wage demands, capital formation and the rate of growth. An explanation of the growth curve has sometimes been attempted by previous authors with a more or less pronounced secondary objective of propaganda. The intention was either to demonstrate the advantages of a liberal economic policy (*e.g.* for raising national prosperity) or else its disadvantages (the development of certain disproportions). The purpose we have in mind, however, can only be to become more familiar with the process of development and hence to ascertain its degree of susceptibility to control.

The indirect purpose of our attempt is found in the following:

a. A theory of trends makes it possible to give a better foundation to the treatment of the problems of statics. It will be agreed that in some cases it is unsatisfactory to try to solve important problems of economic analysis and policy under the assumption of a constant capital quantity or a constant population. In particular, the investigation of measures which have different effects over short and long periods of time is rendered more difficult by this assumption. In statics there is only one level of equilibrium of the economic variables; this level is either raised or lowered by the measure concerned. In reality, however, there is a continuous movement which may, for instance, have a downward direction in the beginning and move upward with time, in contrast to the uni-directional development which would have occurred had no such measure been taken. Differentiations of this kind, however, are obscured by the static analysis of the problem.

b. A theory of trends also creates a better foundation for the analysis of the dynamics of short-term movements. Here the same applies *mutatis mutandis* as for statics. A particularly important problem is illustrated by the question of the influence of certain trade policies on the long-term movement. The treatment of such questions requires a theory which represents both long-term and short-term movements simultaneously. It is true that this makes it basically very complicated. The construction of such a theory, however, is facilitated by a preceding theory regarding the long-term movement itself.

4. DOES A THEORY OF LONG-TERM MOVEMENTS DISREGARDING SHORT-TERM MOVEMENTS HAVE ANY SIGNIFICANCE?

This question can be discussed only briefly within the scope of this article. Since it is of elementary importance, however, we cannot omit it entirely. In order to give an answer let us assume that we already have a complete theory explaining both long-term and short-term movements. In accordance with such a theory the economic system under review will follow different movements which can be classified into periodic components and aperiodic components.[10] The sum of the aperiodic components determines the trend movement of the system. The periodic components show different degrees of damping. Thus there are two types of economic systems, *viz.* such where all periodic components are damped, *i.e.* where the amplitudes of oscillation are decreasing and such where one or more of these components show ever-increasing amplitudes. The movements of the first type of system will deviate only temporarily from the trend movement, whereas the movement of the second type of system will generally show considerable fluctuations along the trend. For such systems a study of the trend has no point because it does not form the "center of gravity" of the short-term components. In the case of systems of the first type the trend actually constitutes the "general tendency" (la tendence générale) of the total movement. In our eyes this is the justification of a theory of the long-term movements. Therefore, such a theory exists only for systems of the first type. This justification applies at the same time to statics. For a certain number of economic systems, but only for them, statics has a meaning. This group, like the "first type of systems" is characterized by the damping of its periodic movements. It differs from the latter systems insofar that its trend runs only horizontally so that there is no progressive movement or in other words no trend component.

[10] The assumptions on which these conclusions are based and their proofs can be found in my paper: "Einige Grundfragen der mathematischen Konjunkturtheorie", *Archiv für mathematische Wirtschafts- und Sozialforschung*, Leipzig, Bd. 3 (1937), p. 1 *sq.*, 83 *sq.*

The approximation of the actual movement by the trend depends upon the amplitude of short-term movements (which depend in turn upon the accidental disturbances to which the system is exposed). Under certain circumstances it can be a very close approximation. However, a theory of trends as we give it will probably be too inaccurate for questions of economic policy, *e.g.* the problem of full employment. We shall come back to this subject later.

Now we should like to discuss shortly a mathematical question closely connected to the previous one. All movements of a dynamic system, as we have shown[11] can be perceived as solutions of the so-called final equation of the system, that is, each solution of this equation corresponds to a component of the total movement. The question now arises whether —with a given final equation—we can find the trend components without first solving them completely. Only then we can speak of a special theory of the trend movement. Under certain circumstances, the detailed discussion of which is outside the scope of this article, we can show that we can reach this aim if we neglect the small lags in the final equation. This simplifies the equation; it has fewer solutions, is easier to solve and the remaining solutions yield exactly the same trend components. This procedure, which is also valid for static analysis, might be justified as follows (although we do not know the exact conditions to which it is subject): a trend movement is always a slow movement. Therefore, the difference between two subsequent values of a variable is very small. We may therefore treat it as of no importance.

5. THE ELEMENTS OF THE THEORY

5.1. THE PRODUCTION FUNCTION

It was already pointed out that for the time being we shall discuss the real variables of the economy, relating to goods and services, but not deal with individual markets. We shall therefore deal with only one type of goods (which can be used for both consumption and investment purposes) and call its volume of production u. Apart from that we shall introduce two factors of production, labour and capital. The third factor, land, will not be considered here because its volume

[11] TINBERGEN, *loc. cit.*

is assumed to be constant. For most countries this will be correct. The volume of labour applied will be expressed as a, the volume of capital applied, in which we will assume land to be included, as K.[12] The quantity for any given moment may be measured either by the volume of goods incorporated therein or by the market value of the productive property corrected for price fluctuations. We understand by this the market values divided by a general price index such as the cost of living or the index of capital good prices. Actually this involves a number of other problems which we do not intend to discuss here.

Between the volume of production u and the volumes of applied production factors a and K there exists a technical relation which changes in the course of time and which we call the production function or production equation; that is, u is a function of a and K:

$$u = \varphi\ (a,K).$$

There is little knowledge of the exact shape of the production function. As far as we know, only DOUGLAS[13] tried to examine the function statistically. DOUGLAS is occupied with the production of the entire industry which he considers as a whole. He does not go as far as we do, since we consider the total production of all industries and trades of a country as a whole. He forms the following function:

$$u = ca^{\lambda}\ K^{\mu},$$

[12] As far as possible, our symbols are the same as in other publications.

[13] DOUGLAS gives a summary of the first series of investigations on this point in his book *The Theory of Wages*. Later the following articles appeared: M. L. HANDSAKER AND P. H. DOUGLAS, "The theory of marginal productivity tested by data for manufacturing in Victoria", *The Quarterly Journal of Economics*, Cambridge, Mass., Vol. 52 (1937/38), p. 1 *sq.*, p. 215 *sq.* – G.T. GUNN AND P. H. DOUGLAS, "Further measurements of marginal productivity", *Ibidem*, Vol. 54 (1939/40), p. 399 *sq.* – M. BRONFENBRENNER AND P. H. DOUGLAS, "Cross-section studies in the Cobb-Douglas function", *The Journal of Political Economy*, Chicago, Ill., Vol. 47 (1939), p. 761 *sq.* – G. T. GUNN AND P. H. DOUGLAS, "The production function for American manufacturing in 1919, *The American Economic Review*, Evanston, Ill., Vol. 31 (1941), p. 67 *sq.* – For critical remarks on the works of DOUGLAS see a.o.: J. M. CLARK, "Inductive evidence on marginal productivity, *Idem*, Vol. 18 (1928), p. 449 *sq.* – D. DURAND, "Some thoughts on marginal productivity, with special reference to Professor Douglas' analysis", *The Journal of Political Economy*, Vol. 55 (1937), p. 740 *sq.* – H. MENDERSHAUSEN, "On the significance of Professor Douglas' production function", *Econometrica*, Menasha, Wis., Vol. 6 (1938), p. 143 *sq.*

where c, λ and μ are constants. By suitable selection of the units for u, a, and K we can make $c = 1$. We shall proceed in the same manner. Douglas' function is therefore an exponential formula of the simplest imaginable type. Still, it should be preferred to the even more simple additive linear formula $u = c_1a + c_2K$ because the latter shows differential quotients which are independent of a and K. This would mean that the optimal productivity of labour and capital respectively is independent of the volume of the applied factors of production. Such an assumption is clearly too special. Douglas' selection of the former formula therefore is understandable.

DOUGLAS is furthermore of the opinion that the values λ and μ must satisfy the relation

$$\lambda + \mu = 1$$

that is, that the formula must be linear homogenous. This means that a proportional increase of a and K, *i.e.* of the applied volumes of labour and capital, must result in a proportional increase of the volume of production. In general this applies only within the limits of constant returns. This condition exists if all enterprises have their optimal size and if their number is large. In such a case a small change of production can take place by increasing the number of such optimal enterprises. For an analysis of slow movements which do not deviate much from the state of equilibrium, and for a whole country in which the individual enterprises are small, this assumption seems justified.

This formula has been generalized by EDELBERG[14] for the case where three production factors are used. If the volume of land used is n, the formula reads

$$U = a^{\lambda} K^{\mu} n^{\nu},$$

where

$$\lambda + \mu + \nu = 1.$$

As mentioned before, we shall combine capital and land in the following and express the total volume of the two production factors as K. Moreover, we shall use DOUGLAS' formula of the linear homogenous

[14] V. EDELBERG, "An econometric model of production and distribution", *Econometrica*, Vol. 4 (1936), p. 210 *sq.*

type. In one respect we will generalize it by attaching a factor increasing with time ε^t; we therefore write:

$$u = \varepsilon^t a^\lambda K^{1-\lambda}. \tag{1.1}$$

Thus we take into account the possibility of an increasing (perhaps also decreasing) effectiveness of the production process in time and we have the possibility of including the element of technical development in our model.

The term "technical development" warrants some explanation. It is used by several authors with contradictory meanings. Those who think along statistical lines generally conceive it as the increase in the (statistical) labour productivity, *i.e.* the relation between the volume of production and the volume of labour, probably because it is the easiest to be determined. It is obvious, however, that every reduction of the real costs, whether through reduction of the capital costs per production unit or labour costs per unit, must also be considered as a technical development. It is also possible that labour costs will rise if capital costs decrease more than proportionally.

An increase in labour productivity may be obtained by two different means. Firstly, the production function can remain unchanged and the capital intensity of production can be increased. Secondly, we can also change the production function, in other words obtain a higher volume of production with the same volume of labour and capital. DOUGLAS assumed that in the periods he studied the production function of the industry remained unchanged and that the increase in labour productivity is due exclusively to the increased capital intensity of industrial production. He points out that this process also deserves the name of technical development, in any case in outward appearance, insofar as it is linked with the introduction of new capital goods and working methods. It must not be forgotten in this connection that these methods of higher capital intensity — although belonging to the same production function — may have been previously unknown. This is true particularly for the country with the highest capital intensity in the world.

Such an increase in labour productivity will, however, always be accompanied by a corresponding decrease of capital productivity; we have here the process of mechanization, the replacement of labour by capital. In addition we have the simultaneous increase of both labour productivity and capital productivity, which may be called rise in efficiency. The latter will occur if ε^t in our formula (1.1) increases. We should like to emphasize once more that we understand and define the terms labour productivity and capital productivity in their strictly statistical meaning, *viz.* as simple quotients; hence they have nothing in common with the economic terms marginal productivity of labour and capital respectively.

For the logic of our analysis another problem is of interest, namely whether we may consider labour productivity as a datum or not. This obviously depends on the production function. If we use DOUGLAS' production function as a basis for our analysis, labour productivity is apparently no datum but a function of some variables of the problem, the quantities a and K. We shall soon come across another production function where labour productivity constitutes an independent datum.

DOUGLAS has also tried his best to determine the value of the exponent λ statistically. Not all the methods of determination he quotes are convincing, as I pointed out in another publication[15]; nevertheless his results should be fairly close to the real values. What he found was $\lambda = 3/4$; λ specifying, as we shall see later, what fraction of the national income goes to labour as income, and for most countries this fraction is actually around 3/4. In the following we shall therefore use DOUGLAS' linear-homogenous formula, and take $\lambda = 3/4$.

The main point we should remember is that DOUGLAS' function is based on the hypothesis that labour and capital are completely substitutable. We are not certain that this complies with the facts. In order to get an idea of the consequences of such a hypothesis we have made some computations on the basis of a different production function,[16] a function which presupposes that labour and capital are entirely complementary, *i.e.* exactly the contrary. Obviously this assumption goes much too far. A certain degree of substitutability no doubt exists. The second production function may best be expressed

---

[15] J. TINBERGEN, "Professor DOUGLAS' Production Function", *Revue de l'Institut international de statistique*, 1942, 1/2.

[16] In: "Het streven naar efficiency en de werkgelegenheid", *Nederlandsch Instituut voor Efficiency, Publicatie Nr. 199*, Purmerend 1941.

in a formula which differs slightly from the above. If the volume of production is stated as u and the two production factors are entirely complementary, this means that both a and K are strictly defined by u; we therefore write

$$a = gu \qquad K = hu, \tag{1.2, 2.2}$$

where g represents the quantity of labour per production unit and h the quantity of capital per production unit. The values g and h need not be constants; in any case g will show a decreasing trend in the course of time. As can easily be seen, g is the reciprocal of labour productivity; in this production function, labour productivity is therefore a datum, *i.e.* a non-economic, pre-determined quantity.

5.2. Demand and Supply of Labour and Capital

We shall further assume that production is in the hands of enterprises competing with one another, paying both workers and capital owners for their cooperation according to the prevailing market prices. These market prices—the real wage rate l' and the real interest rate m'—are therefore considered by the individual enterprises as fixed quantities. For every entrepreneur it is therefore a question of achieving the maximum profit according to the formula

$$u_i - l'a_i - m'K_i,$$

where u_i, a_i and K_i represent the production obtained by an entrepreneur and the volumes of labour and capital respectively applied. Hence it follows that

$$\frac{\partial u_i}{\partial a_i} = l' \quad \text{and} \quad \frac{\partial u_i}{\partial K_i} = m',$$

i.e. every entrepreneur will expand production to a point where the value of the output of the last worker is just equal to his wages, in other words where the law of marginal productivity of wages is applicable. Much the same is true for the interest rate: the employment of capital is expanded to a point where marginal productivity is equal to the interest rate. Substituting the expression of the DOUGLAS function for u_i we get:

$$\tfrac{3}{4}\left(\frac{K_i}{a_i}\right)^{\frac{1}{4}} \varepsilon^t = l' \qquad \tfrac{1}{4}\left(\frac{a_i}{K_i}\right)^{\frac{3}{4}} \varepsilon^t = m'.$$

From this we may conclude that the same formulæ also apply[17] for the quantities a and K (total volumes of labour and capital):

$$\frac{3}{4}\left(\frac{K}{a}\right)^{\frac{1}{4}} \varepsilon^t = l' \quad \frac{1}{4}\left(\frac{a}{K}\right)^{\frac{3}{4}} \varepsilon^t = m'. \tag{2.1; 3.1}$$

These equations could be called the demand equations of labour and capital. In case labour and capital are assumed to be complementary, a mutually independent variation of labour and capital volumes it not possible. For a given wage and interest rate, production will take place as long as a profit is left, and it will be terminated if no profit can be squeezed out. However, this is independent of the volume of production. The latter will be expanded—provided it is profitable—to a point where either the capital is fully employed or the demand for labour causes wages to rise to such an extent that no more profit is left.[18] The latter case we call underemployment of capital; we get

$$al' + km' = u. \tag{3.2}$$

where k is the quantity of capital employed which need not be consistent with the quantity of total capital.

Let us now consider supply. The supply of production factors is —generally speaking—dependent on prices. It may therefore show a certain elasticity. Accordingly, supply can be bigger at one time and smaller at another. If supply were completely inelastic there would be no underemployment of capital or labour. In many of the older theories this underemployment was excluded beforehand. However, since we are interested in unemployment we shall in any case assume a certain elasticity of labour supply.[19] For simplicity's sake we shall

---

[17] For every i it is $\frac{K_i}{a_i} = \left(\frac{4\,l'}{3\,\varepsilon^t}\right)^4$; therefore also $\frac{\Sigma K_i}{\Sigma a_i} = \frac{K}{a} = \left(\frac{4\,l'}{3\,\varepsilon^t}\right)^4$ or $\frac{3}{4}\left(\frac{K}{a}\right) \varepsilon^t = l'$. This would not apply if DOUGLAS' function were not linear.

[18] The asymmetrical treatment of labour and capital is not necessary but it is a consequence of an asymmetry in the treatment of the supply functions which has been adopted for simplicity's sake and which will be discussed immediately.

[19] This means that we consider the unemployment which will then remain, as voluntary unemployment. Involuntary unemployment will occur only if for any reasons the wage does not adapt itself immediately. This may be the consequence of either a temporary inertia of the market or a permanent

assume an entirely inelastic supply on the capital market. This is by the way not too far from the results found by DOUGLAS.[20]

With regard to labour supply we shall therefore assume that the percentage of the population available for employment depends on the wage rate, that is on the relation between wages and a rate which is normally considered adequate. If we indicate the flexibility of wages by λ', the normal wage rate by l^0 and the population by b, the supply equation may be written as follows:

$$\frac{l'}{l^0} = \left(\frac{a}{b}\right)^{\lambda'}. \tag{4}$$

The reason for introducing the normal wage rate is to remind us that wage demands may change in the course of time, in the first place through improvement of the technical possibilities of production and in the second, because of a change in social conceptions. It will be difficult to determine this normal wage statistically; at any rate we shall not be able to do this directly. We shall be able to give instead, with the help of our formulæ, an idea of the influence of any possible changes in these demands on the development of economic variables.

In order to express the possible movement of the normal wage rate we write

$$\overset{0}{l} = \overset{00}{l} \lambda_0^t$$

where l^{00} is a constant and $\lambda_0 - 1$ the annual rate of increase of the normal wage.

There is considerable divergence in estimates of the numerical value of wage flexibility or, inversely, elasticity of labour supply. As DOUGLAS points out in his book, cited before, there are economists who attribute to flexibility the value zero and others who think it is infinite and assert that even negative values are possible, except between 0 and —1. This is the well-known example of certain tropical races which actually illustrate such conditions. With an increasing wage rate these people become less inclined to work because they can make a living in a shorter time. The same inclination is also found in other races. DOUGLAS makes extended investigations of geographical comparisons, that is

phenomenon, for example if workers are afraid that an "adaptation" of wages would lead to inacceptable wages. From the point of view of the working class as a whole, unemployment is "put up with" and it might for this reason be considered as voluntary even if for the individual worker it is indeed an involuntary fact.

[20] DOUGLAS, *The theory of wages*, other pages, Chapt. XVIII.

comparisons between different towns in the United States and finds that a negative supply elasticity exists there, too. He states that for certain age brackets and for most women, it is rather high, while for men between the ages of 20 and 60 it is almost zero; which implies that λ' would be very high there.

Another possibility for determining the wage flexibility is a comparison between real wage fluctuations and fluctuations of the employment level. These investigations are based on the entirely plausible assumption that population and normal wages change slowly. Strange to say that here we find a positive correlation, *i.e.* a positive flexibility: rising employment generally occurs with rising real wages. The correlations in general are not very high—as also with DOUGLAS—and for this reason the values of flexibility are very unreliable. This is shown in Table I where the results for four different countries have been combined. The remarkable thing about these results is that the flexibilities determined according to the diagonal regression are lower where the living standard is higher. The lowest living standard is found in France, next comes Germany, then Great Britain, the highest

TABLE I

WAGE FLEXIBILITY IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES, DETERMINED FROM FLUCTUATIONS IN REAL WAGE RATES AND EMPLOYMENT LEVEL*

Country	Flexibility according to 1st Regression	2nd Regression	Diagonal Regression	Coefficient of Correlation	Period
Germany	1.25	3.6	2.1	0.59	1887–1913
Great Britain	0.47	3.4	1.3	0.37	1870–1910
France	1.22	20.0	5.0	0.26	1895–1913
United States	0.97	1.2	1.1	0.91	1897–1916

* Sources: For England: Wages: See appendix to this article. Unemployment: Trade union figures from: "Seventeenth abstract of labour statistics of the United Kingdom", *Abstract of Labour statistics, Board of Trade* (Department of Labour), (Cd. 7733) London 1915. – Other countries: J. KUCZYNSKI, "Die Entwicklung der Lage der Arbeiterschaft in Europa und Amerika, 1870–1933", *Statistische Studien zur Entwicklung der Reallöhne und Relativlöhne in England, Deutschland, USA, Frankreich und Belgien,* Basel 1934.

standard being that of the United States. The figures therefore seem to have importance. For the time being, the question of the value of flexibility and whether it is positive or negative remains open.

The last determinant in formula (4) is the size of the population. This we shall consider to be a datum, in other words we neglect the —in my opinion—weak effects of the economic situation on the size of population. We shall assume that it develops according to the exponential law, that is

$$b = b_0 \beta^t,$$

where β is the proportion of population between any given and the previous year and b_0 the size of population for $t = 0$. For the greater part of the nineteenth century the exponential law gives us a good approximation. For the end of the century as well as for the twentieth century a logistic curve would be better. In case of short intervals (10–20 years) the exponential law—which if β is small hardly differs from a straight line—may still be applied as a first approximation. Because of the complexity of the formulæ we shall not operate with the hypothesis of a logistic population development.

In the case of capital supply we shall assume that the interest rate exercises no influence. At this point, however, another much more important fact demands our attention. In principle, the quantity of capital available at a given moment is a product of the accumulation of newly formed capital during all preceding years. Mathematically speaking, this means that it is an integral of the volume of capital formation over time. Thus a logical connection with the past is formed. With regard to new capital formation in any individual period we shall assume it to be proportional to income during such a period, *i.e.* we assume the savings rate to be constant during this time. As a first step this appears justified. For a closer study one might operate with a somewhat more complex relation, although this is not at all necessary. Indicating the savings rate by $\varkappa$, we shall assume that

$$\dot{K} = \frac{dK}{dt} = \varkappa u. \tag{5}$$

5.3. Demand and Supply of Products

The demand equations given in the previous paragraph for labour and capital are at the same time supply equations for finished products.

It must be remembered that we assumed every entrepreneur will expand his production to the point where he receives no additional profit. Evidently, the marketability of the products is of no importance here. This theory is based on the implicit assumption that for every unit produced, there is a corresponding unit of income to buy the product. Production, we must not forget, simultaneously creates income equal to the value of the product. We have here the old view that "les produits s'achètent avec des produits", the "Theorie der Absatzwege" (Say's law). We can also put it in this way: the supply of goods determines the volume of production. It is assumed that K is fully employed, which in turn determines u and a. In general this will be the case when demand equals production capacity or even exceeds it. In times of great enterpreneurial initiative and vast expansion, such as the nineteenth century, this probably applies. In the twenties and thirties of the twentieth century, however, there is more reason to believe that demand lags behind the production capacity. In this case we may not assume that K is completely employed in production. A distinction will have to be made between the quantity of capital available and the quantity of capital employed. The latter shall be written k. The above equations will be supplemented by a demand equation for products. In times of structural depression, therefore, the volume of production is partly determined by demand.

Since classical theory did not deal with these problems, we have here an entirely new field of economics. It has above all been KEYNES who investigated the question of the demand function by, for instance, coining the term "propensity to consume". However, we have to do with demand for both consumer and investment goods. Since we are primarily occupied with economic development between 1870 and 1914 we shall dismiss the details of the demand function. To demonstrate that our theory can also be used for an analysis of the trend movement in the twentieth century, we shall just mention the most important factors controlling the demand for all kinds of goods.

In the first place, income is an important factor in determining the demand for a thing, as we know from both the general theory and from Keynes' arguments as well as from household budgets. Apart from income, the size of the family, *i.e.* population for the economy as a whole, will have a certain influence. The whole income is more likely to be consumed if many persons have to live on it than only a few. This is also what the statistics of household budgets teach us. If we refer to income in this connection it is always in the sense of real income, *i.e.* income corrected for price level fluctuations. For this reason we do not yet have to mention the price level as another factor. Mathematically, the income would simply be stated as u. It is probable, however, that the influence of income on the demand for consumer goods shows a lag, so that u_{t-1} would be decisive for demand in the year t. In addition to the factors mentioned above it is a safe assumption that expenditure is also influenced by habit; such habits will frequently be based on the average prosperity during a somewhat longer and more extended period. This might mean that the average income of the previous ten or fifteen years is also a contributory determinant. The main task of an economic analysis would be to find the exact influence of each individual factor on demand. The first steps in this direction have hardly been taken.[21]

In addition we have the demand for investment goods. Seen over an extended period of time—neglecting fluctuations in demand due to inventory speculation—two components are of significance: the demand for replacement purposes and that for new investments. The first may develop—according to the so-called echo principle—in a wavelike manner, whereby the period is determined by the mean life of the capital goods to be replaced.[22] The latter will in all probability form the most variable component of the total demand, which is decisive for the entire rhythm of the economy. Periods of high investment such as large railway investments alternate with periods of reduced

[21] I should like to refer to my attempts to give an explanation of the short-term fluctuations in the total volume of demand in my book: *Les cycles économiques aux Etats-Unis d'Amérique de 1919 à 1932.* (Vérification statistique des théories des cycles économiques, 2. – Série de publication de la Société des Nations, II. Questions économiques et financières, 1939. II. A. 16.) Genève 1939.

[22] Compare J. EINARSEN, *Reinvestment cycles and their manifestation in the Norwegian shipping industry.* (Publication No. 14 from the University Institute of Economics, Oslo), Oslo 1938. – J. MEULDIJK Jr., "Der englische Schiffbau während der Periode 1870–1912 und das Problem des Ersatzbaues", *Weltwirtschaftliches Archiv*, Bd. 52 (1940 II), p. 524 *sq.* – S. DE WOLFF, *Het economisch getij* (Bijdrage tot de verklaring van het conjunctuurverschijnsel, Amsterdam 1929) even tries to explain long waves in trade cycles in this manner.

willingness to make investments. Inventions as well as the opening up of new countries and continents are among the most important determinants. Apart from these, numerous other factors may be of contributory influence. In this field, as in others, research is still in its rudimentary stage.

We shall leave it at that, however, for, as we have seen, the intensity of this demand determines the production volume only when demand decreases. As soon as it exceeds a certain limit, the production increase is stopped by capacity limits, so that the latter is actually decisive. We must make only one qualification: demand for labour in this case, where u is predetermined, must be replaced by another function. This function exhibits a much smaller elasticity than the one derived under section 5, Item 2 above.[23]

5.4. Summary

In the previous paragraphs the formulæ which define the development of production, employment, capital formation, real wages and interest have been stated. Here we should like to briefly repeat and summarize the results.

A. The supply of goods is decisive (periods of expansion)

A 1 *The production process follows the* DOUGLAS *function (capital and labour substitutable)*

A 1 a. Population grows according to the exponential law.

Production equation:

$$u = \varepsilon^t a^{\frac{3}{4}} K^{\frac{1}{4}} \tag{1.1}$$

Demand equations: Labour:

$$\tfrac{3}{4}\left(\frac{K}{a}\right)^{\frac{1}{4}} \varepsilon^t = l' \tag{2.1}$$

Capital:

$$\tfrac{1}{4}\left(\frac{a}{K}\right)^{\frac{3}{4}} \varepsilon^t = m' \tag{3.1}$$

[23] See also our publication in the Dutch language on this subject: *Het streven naar efficiency en de werkgelegenheid, loc. cit.*

Supply equations: Labour:

(4) $$l' = \left(\frac{a}{b}\right)^{\lambda'} l^{00} \lambda_0^t$$

Capital:

(5) $$\dot{K} = \varkappa u$$

Development of population:

(4') $$b = b_0 \beta^t$$

A 1 b. Population develops according to a logistic curve. In place of (4') we write now:

(4'') $$b = \frac{B}{1 + e^{-b_1 t + b_2}}$$

A 2. *Labour and capital are completely complementary in the production process*

A 2 a. Full employment of capital

Production equations:

(1.2) $$a = gu$$

(2.2) $$K = hu$$

Supply equations: Labour:

(4) $$l' = \left(\frac{a}{b}\right)^{\lambda'} l^{00} \lambda_0^t$$

Capital:

(5) $$\dot{K} = ku$$

A 2 b. Underemployment of capital

Production equations:

(1.2) $$a = gu$$

(2.2) $$k = hu$$

Demand equation: Capital:

(3.2) $$al' + km' = u$$

Supply equations: Labour:

(4) $$l' = \left(\frac{a}{b}\right)^{\lambda'} l^{00} \lambda_0^t$$

Capital:

(5.2) $$m' = \left(\frac{k}{K}\right)^{\mu'}$$

B. The demand for goods is decisive. This case will be considered only for the DOUGLAS function.

Production equation: $u = \varepsilon^t a^{\frac{3}{4}} k^{\frac{1}{4}}$

Demand equations: comp. section 5.3.

Supply equations: same as under A.

Demand equation for goods: comp. section 5.3.

In the following we shall deal mainly with case A 1 a, the most simple case which represents a usable approximation for the period 1870–1914. The data of the development therefore are: the growth rates of population, efficiency and "normal" wages, the savings rate $\varkappa$ and wage flexibility. To facilitate the following computations we should like to add a few words on the selection of the units. We shall use the statistical data of some countries for certain periods (in general 1870–1914). The most important simplification is to substitute the number 1 for the values of most of the variables in the middle of the period under review; t being assumed for this middle to be equal to zero. The form of the equations, however, does not permit this procedure to be used for every variable. For $t = 0$ we shall therefore make the values for a, u, K and b equal to 1. Hence if follows that $t = 0$, $l' = 3/4$, $m' = 1/4$, whereas $l_{00} = 3/4$. One consequence of our selection is that a and b are not expressed in comparable units; this however is no disadvantage—we are studying only the relative movements. It is clear that $\dot{K}$ cannot be considered as an independent variable.

To end our summary, we should like to point out that the only thing new in our set of formulæ is that K (in DOUGLAS' function) is not assumed to be given, but is the product of previous accumulation. In this way we have obtained a system of development.

6. THE PROBLEM OF INTRODUCING MONETARY VARIABLES INTO THE MODEL

Model A 1 a, which was explained above, can easily be developed into a model including also the most important monetary phenomena. Whereas we do not intend to proceed on the basis of such a model, we should still like to discuss it briefly in order to demonstrate that no serious difficulties arise from the inclusion of monetary variables. If we take the wage rate as l, the interest rate as m, and the price level as p, we get first of all:

$$l' = \frac{l}{p} \qquad m' = \frac{m}{p}. \tag{6;7}$$

The price level may be understood to depend on the quantity of money in circulation M, which in turn may either be assumed to be given (according to WALRAS) or depend on gold reserves (according to CASSEL). If we assume furthermore the speed of circulation to be a technically determined quantity γ—chiefly through terms of payment and the division of labour between the individual enterprises—we get:

$$up = \gamma M. \tag{8}$$

This is the simplest way to include monetary aspects in the model. The last equation may be formulated more generally, *e.g.* if we want to consider the demand for money for hoarding. We can also give more consideration to the determinants of banking policy. It would lead too far to particularize, the more so because relations in real terms are not changed as long as we do not alter our equations (1) through (5). In other words: such subtilities influence merely the monetary sphere and not the exchange of goods. Naturally this does not mean that the same is also true for short-term movements or for model B. In these cases the consequences of monetary policy will be much more significant.

7. THE MATHEMATICAL SHAPE OF THE TREND MOVEMENT

Since the given formulæ determine the movements of the economic system, various conclusions can be drawn from them which shall now be discussed. In order not too much to complicate our formulæ, we have substituted numerical values for λ', *i.e.* the values 0, $\frac{1}{2}$, ∞, and —1. Although we shall have to make four different computations, the formulæ remain simple and there is moreover a method by which they can be checked. The meaning of the four individual values is the following: for $\lambda' = 0$ the wage rates are absolutely rigid, they are, as it were, dictated by the workers' organizations in an economy which is not planned centrally. For $\lambda' = \infty$, on the other hand, they are "absolutely flexible", *i.e.* supply is entirely inelastic and independent of wage rates, the same number of workers is available at all wages. The values $\frac{1}{2}$ and —1 are, as far as their numerical value is concerned, arbitrary and have been chosen for simplicity's sake.

Qualitatively, $\lambda' = \frac{1}{2}$ gives the situation in case of positive supply elasticity—as we have derived it from the time series—and $\lambda' = -1$ gives the situation in the case of a negative supply elasticity—as DOUGLAS found it in his geographical comparisons. The numerous computations we have made shall not be quoted here in full because they are mathematically simple. Instead, we should like to offer a few examples and merely state the results for the others.

One of the first conclusions to be drawn refers to the mathematical shape of the curve over a period of time. From it we can learn which methods of trend computation are justified by our theory. For this our equations must be solved. Taking $\lambda' = 0$ we get from (1.1) through (5):

(9) $$u = \varepsilon^t a^{\frac{3}{4}} K^{\frac{1}{4}}$$

(10) $$\tfrac{3}{4}\left(\frac{K}{a}\right)^{\frac{1}{4}} \varepsilon^t = l' = \tfrac{3}{4}\lambda_0^t$$

(11) $$\dot{K} = \varkappa u.$$

Equation (3.1) may be eliminated because it is useful only for determining m'. From (9) and (11) u can easily be eliminated thus:

(12) $$\dot{K} = \varkappa\, \varepsilon^t a^{\frac{3}{4}} K^{\frac{1}{4}}.$$

In order to eliminate a, too, we have to make use of (10):

(10′) $$\frac{K}{a}\varepsilon^{4t} = \lambda_0^{4t}$$

or

(10″) $$a = K\left(\frac{\varepsilon}{\lambda_0}\right)^{4t},$$

so that we get:

(13) $$a^{\frac{3}{4}} = K^{\frac{3}{4}}\left(\frac{\varepsilon}{\lambda_0}\right)^{3t}$$

(12) and (13) now lead us to the differential equation for K:

(14) $$\dot{K} = \varkappa \frac{\varepsilon^{4t}}{\lambda_0^{3t}} K.$$

The solution is as follows:

(15) $$\frac{\dot{K}}{K} = \frac{d \log K}{dt} = \varkappa\left(\frac{\varepsilon^4}{\lambda_0^3}\right)^t$$

$$\log K = \frac{\varkappa}{\log\left(\frac{\varepsilon^4}{\lambda_0^3}\right)}\left(\frac{\varepsilon^4}{\lambda_0^3}\right)^t + C. \tag{16}$$

All the logarithms are natural, and C is an arbitrary constant. The value of C is defined by the condition that for $t = 0$ $K = 1$, *i.e.* $\log K = 0$, or

$$0 = \frac{\varkappa}{\log\left(\frac{\varepsilon^4}{\lambda_0^3}\right)} + C. \tag{17}$$

This equation can be expressed a little more elegantly. The figures ε and λ_0 deviate only slightly from 1 and can be written thus:

$$\varepsilon = 1 + \varepsilon'$$
$$\lambda_0 = 1 + \lambda_0'$$

where ε' and λ_0' are small figures. For these we have:

$$\log \varepsilon = \log(1 + \varepsilon') \simeq \varepsilon' \tag{18}$$

$$\log \lambda_0 = \log(1 + \lambda_0') \simeq \lambda_0' \tag{19}$$

A similar expression can later be used for β, *viz.*:

$$\log \beta = \log(1 + \beta') \simeq \beta'. \tag{20}$$

Since

$$\frac{\varepsilon^4}{\lambda_0^3} = 4 \log \varepsilon - 3 \log \lambda_0, \text{ we get:}$$

$$C = -\frac{\varkappa}{4\,\varepsilon' - 3\lambda_0'} \tag{21}$$

Thus we have:

$$\log K = \frac{\varkappa}{4\,\varepsilon' - 3\lambda_0'}\left\{\left(\frac{\varepsilon^4}{\lambda_0^3}\right)^t - 1\right\}. \tag{22}$$

It will be seen that in this simple case already a much more complex time dependency is apparent than in the usual statistical trend determinations. Not only is equation (22) a function which is more complex than the exponential law, it also refers only to $\log K$. In some instances we get the Gompertz-curve. For K the formula is even more complicated. Much the same applies for the other variables.

From (10″) follows:

(23) $$\log a = \log K + 4t(\log \varepsilon - \log \lambda_0).$$

Finally, we get from (9):

$$\log u = t \log \varepsilon + \tfrac{3}{4} \log a + \tfrac{1}{4} \log K$$

(24) $$= t \log \varepsilon + \log K + 3\, t\, (\log \varepsilon - \log \lambda_0)$$

Equations (23 and (24) can be simplified by using ε' and $-\lambda_0'$

(23′) $$\log a = \log K + 4\, t\, (\varepsilon' - \lambda_0')$$

(24′) $$\log u = \log K + 4 t \varepsilon' - 3\, t\, \lambda_0'.$$

Similar computations can be made for those cases where λ' has another value. For $\lambda' = \frac{1}{2}$ our equations (1.1) through (5) read as follows:

(25) $$u = \varepsilon^t\, a^{\frac{1}{2}}\, K^{\frac{1}{2}}$$

(26) $$\left(\frac{K}{a}\right)^{\frac{1}{4}} \varepsilon^t = \left(\frac{a}{b}\right)^{\frac{1}{2}} \lambda_0^t$$

$$\dot{K} = \varkappa u$$

The differential equation for K can now be somewhat modified. From (26) it follows that:

(28) $$a^{\frac{3}{4}} = \frac{K^{\frac{1}{4}}}{b^{\frac{1}{2}}} \frac{\varepsilon^t}{\lambda_0^t} = K^{\frac{1}{4}} \left(\frac{\varepsilon}{\beta^{\frac{1}{2}}\, \lambda_0}\right)^t$$

Thus we get

$$\dot{K} = \varkappa u = \varkappa \varepsilon^t\, K^{\frac{1}{2}} \left(\frac{\varepsilon}{\beta^{\frac{1}{2}}\, \lambda_0}\right)^t$$

or:

(29) $$\frac{\dot{K}}{K^{\frac{1}{2}}} = K \left(\frac{\varepsilon^2}{\beta^{\frac{1}{2}}\, \lambda_0}\right)^t.$$

The solution of this equation reads:

(30) $$K^{\frac{1}{2}} = \frac{\varkappa}{2} \frac{\left(\frac{\varepsilon^2}{\beta^{\frac{1}{2}}\, \lambda_0}\right)^t}{\log \left(\frac{\varepsilon^2}{\beta^{\frac{1}{2}}\, \lambda_0}\right)} + C.$$

The arbitrary constant C must again meet the condition that for $t = 0$ $K = 1$; that is:

$$1 = \frac{\varkappa}{2 \log \dfrac{\varepsilon^2}{\beta^{\frac{1}{2}} \lambda_0}} + C. \tag{31}$$

Here again it is advisable to make use of the symbols ε' and λ_0', as well as of β', so that we get:

$$K^{\frac{1}{2}} = 1 + \frac{\varkappa}{2} \frac{\left(\dfrac{\varepsilon^2}{\beta^{\frac{1}{2}} \lambda_0}\right)^t - 1}{2\, \varepsilon' - \frac{1}{2} \beta' - \lambda_0'}. \tag{32}$$

The formula for K which would result if we were to raise it to the second power would again be very complicated: it results in a sum of various exponential terms.

Further computations will not be given here, only a few results. For $\lambda' = -1$ we get:

$$K = 1 + \frac{\varkappa}{\lambda_0' + \beta'} \{(\lambda_0 \beta)^t - 1\}, \tag{33}$$

and for $\lambda' = \infty$:

$$K^{\frac{3}{4}} = 1 + \frac{k}{\beta' + \frac{4}{3} \varepsilon'} \{(\beta^{\frac{3}{4}} \varepsilon)^t - 1\}. \tag{34}$$

All these formulæ have one disadvantage in common: we cannot readily determine the influence of the individual data on the development of the quantities K, a and u. In order to get a reliable impression of the relations, it is necessary to make use of other methods. The details of the mathematical form of the trend development are lost this way, just as if we replaced a curve by a straight line. Such details, therefore, can be found only with the help of the exact formulæ (22), (33) and (34).

8. THE GROWTH RATE OF CAPITAL, EMPLOYMENT AND PRODUCTION AND ITS EXPLANATION

One of the first methods of approximation consists in the computation of the growth rates of our three main variables K, a and u for the middle of the period under review, *i.e.* for $t = 0$. We should like to quote a few examples to illustrate the computation and for the rest give results only. For $\lambda' = 0$, for instance, the starting point for K is equation (22). Since the growth rate is equal to $\dot{K}/K = d \log K/dt$, all we have to do is differentiate (22) with respect to time.

$$\frac{d \log K}{dt} = \frac{\varkappa}{4\,\varepsilon' - 3\,\lambda_0'} \left(\frac{\varepsilon^4}{\lambda_0^3}\right)^t \log \frac{\varepsilon^4}{\lambda_0^3} = \varkappa \left(\frac{\varepsilon^4}{\lambda_0^3}\right)^t. \tag{35}$$

For $t = 0$ we get from the above: $d \log K/dt = \varkappa$. A simpler way to do this would have been just to divide equation (11) by K:

$$\frac{\dot{K}}{K} = \frac{\varkappa u}{K}.$$

For $t = 0$, $u = K = 1$, therefore $\dot{K}/K = \varkappa$. This method cannot be applied, however, to the other variables, and moreover it does not give us a chance to work out second approximations (*i.e.* find the results for $t \neq 0$).

For $\lambda' = \frac{1}{2}$ we use formula (32) and remember that

$$\frac{\dot{K}}{K} = \frac{d \log K}{dt} = \frac{d\,\frac{1}{2} \log K}{\frac{1}{2}\,dt} = \frac{d \log K^{\frac{1}{2}}}{\frac{1}{2}\,dt} = \frac{1}{\frac{1}{2} K^{\frac{1}{2}}} \quad \frac{dK^{\frac{1}{2}}}{dt}$$

We finally get the following results:

TABLE II

GROWTH RATES FOR CAPITAL QUANTITY, EMPLOYMENT, AND PRODUCTION FOR THE MIDDLE OF THE PERIOD AND FOR DIFFERENT VALUES OF λ'.

$\lambda' =$	0	$\frac{1}{2}$	∞	-1
$\dot{K} =$	$\varkappa$	$\varkappa$	$\varkappa$	$\varkappa$
$\dot{u} =$	$\varkappa + 4\varepsilon' - 3\,\lambda_0'$	$\frac{\varkappa}{2} + \frac{\beta'}{2} + 2\,\varepsilon' - \lambda_0'$	$\frac{1}{4}\varkappa + \frac{3}{4}\beta' + \varepsilon'$	$\beta' + \lambda_0'$
$\dot{a} =$	$\varkappa + 4\varepsilon' - 4\,\lambda_0'$	$\frac{\varkappa}{3} + \frac{2}{3}\beta' + \frac{4}{3}\varepsilon' - \frac{4}{3}\lambda_0'$	β'	$-\frac{\varkappa}{3} + \frac{4}{3}\beta' - \frac{4}{3}\varepsilon' + \frac{4}{3}\lambda_0'$

The table shows primarily the influence of the data on the growth of u and a. It also appears, however, that this influence is strongly dependent on λ'. There are even changes from positive to negative signs and *vice versa:* the influence of the normal wage growth rate λ_0' on the growth rate of both production and employment is negative for positive values of λ' and positive for negative values of λ'. This means that a rapid increase in wage demands with negative supply elasticity leads to an even more rapid increase of production compared to a slow increase in wage demands. At first sight this may appear strange. The explanation lies in the fact, that if wage demands rise rapidly, a given wage is more readily felt to be inadequate than if they rise slowly. In case of a negative elasticity of labour supply, the compensation is sought in a bigger labour supply. This leads to higher production. At the same time it leads to a lower (or more slowly rising) real wage level.

A second change of signs is recognized in the influence of ε' on employment: this influence is positive for positive values of λ', negative for negative values of λ'. The explanation is obviously found in the fact that an increase in the efficiency of production, under otherwise identical circumstances, causes a rise in wages; if the supply elasticity of labour is negative, this leads to a reduction in supply and employment. The table shows that production is not affected thereby: the increased production per workman manifestly compensates exactly the reduced number of workers.

A third change of signs shows the influence of $\varkappa$ on employment. It appears that a more rapid capital formation, and consequently the increase in labour productivity, has similar effects as in the former case. Production is not affected here either. All this shows that an exact knowledge of the supply elasticity of labour is very important and interesting.

Quite apart from the change of signs in the cases discussed above, the value of λ' shows a considerable influence on the results of Table II. For example for $\lambda' = 0$—*i.e.* in the case of rigid wages—the increase in population has no effect on the growth rate of production or employment. Rising values of λ' cause a corresponding rise of the influence of population growth; for $\lambda' = \infty$—*i.e.* rigid labour supply—employment naturally rises hand in hand with population. Not so production: it only rises at 3/4 of the population growth. The per capita production therefore drops, *i.e.* the living standard diminishes. For $\lambda' = -1$, employment rises even more briskly than population, precisely by 1/3. The influence of efficiency on production and employment, by contrast, becomes stronger as we move toward the left in our table. For $\lambda' = 0$ (rigid wages) the growth rate of production and employment is even four times the speed of efficiency growth. (It should be remembered that the meaning of the word efficiency is not absolutely identical here with increase in labour productivity. It comprises a simultaneous and equally strong increase in capital productivity (see section 5)). For $\lambda' = \infty$, *i.e.* for rigid labour supply, efficiency as we take it has no effect on employment, and for $\lambda' = -1$ even a negative one.

Finally, we should like to ascertain in which respect our table leads to unambiguous conclusions. There are several, of which we will single out a few. As far as the growth rate of production is concerned, the signs of $\varkappa$, β' and ε' are all alike, that is to say positive. This means that an increased rate of capital formation, population growth and efficiency result in a rising rate of production. To arrive at this conclusion, it is true, it would not have been necessary to establish all these formulæ. However, there is more to be inferred from this. The coefficients of $\varkappa$ and β never exceed 1; and they reach these values only in extreme cases. This means that an increase of the growth rate of capital formation and population does not lead to a proportional increase of the growth rate of production. An increase in the rate of population growth, therefore, does not enable us to increase per capita production, but in general only to lower it. Capital formation must proceed at a quicker pace than population growth if the living standard is to be raised. As far as capital formation goes, the same conclusion can be drawn for employment.

We should like to add a few remarks on the question: what are the consequences of technical development for the level of production and employment? As we have already seen, the term "technical development" is interpreted in different ways. It is often used synonimously with the term "increase in labour productivity"; however, there it is more inclusive. The following is of importance for the present problem. The question of the influence of a certain phenomenon on the development of certain economic variables is truly significant only if such a phenomenon is a datum in the economic sense of the word. However, we have already seen that it depends on the production function whether labour productivity is a datum or not. We might add that it is moreover dependent upon the supply functions of the production factors. Thus, the number of workers forms a datum if supply is assumed to be absolutely inelastic, and does not if supply is assumed to be elastic. In the latter case only the number of population—as it were an even more "distant" phenomenon—forms a datum *etc.*

In our model A 1, in which the DOUGLAS production function is used, labour productivity forms no datum. The question of its influence on the development of production and employment would therefore have no definite meaning. Only in the case of a rigid wage rate, that is for $\lambda' = 0$, can the question have meaning. It must be understood that the wage is always equal to the marginal productivity of labour, the latter being—if we are using the DOUGLAS function—always proportional to the mean labour productivity in the statistical sense. Therefore, variations in labour productivity can only occur provided that equal variations in the wage rate take place. The latter is, as must be borne in mind, a datum for $\lambda' = 0$ and thus can be independently varied. It is in this case equal to the normal wage (*i.e.* the wage demand); as may be seen from Table II, the influence of the wage rate on the development of production as well as of employment is negative. The influence of an increase of labour productivity on the development of production and employment is therefore also negative. In the present case this is explained by the fact that such an increase in productivity can only be brought about by an increase in the capital intensity of production and that in addition the capital quantity only be sufficient to produce a slow rise in production. It may be shown, however, that a negative effect on employment may also occur in the case of an underemployment of capital.

At first sight it may be hard to believe that raising the labour productivity has an depressing effect on the trend movement of employment. It is frequently argued that the development in the nineteenth and the twentieth century denied this. It must not be forgotten, however, that the development was not characterized by a rising labour productivity alone. Both capital goods and arable acreage rose materially. Another argument is that technical development is not equivalent to an increasing labour productivity, as we have stressed several times. We believe, therefore, that our conclusion is justified and should like, at the end, to formulate the result a little differently: for employment, an increase in capital productivity and capital quantity has a favourable effect, is not in all cases true for an increase in labour productivity.[24]

9. STATISTICAL INVESTIGATIONS FOR GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES BETWEEN THE YEARS 1870–1914

After all these theoretical considerations we should like to come to some statistical investigations in order to convey a general idea of the magnitude of the variables studied above. For this purpose we have compiled statistics for Germany, Great Britain, France and United States – for the period 1870 to 1914.

For research in trade cycles this period might be called classic in a double sense. In the first place, it has been the subject of several well-known works, and in the second, economic development during this period shows a certain smoothness. At the same time one gets the impression that many of the difficulties encountered during the years 1919–1939 can only be understood against the background of trends during the period from 1870–1914.

We have now compiled the following data for each of these countries during the said period: the growth rates of total population (β'), working population ($\dot{a}$), real wage ($\dot{l}'/l'$), total production volume ($\dot{u}$), and volume of capital ($\dot{K}$) in the sense that we understand it, *i.e.* including land. There is probably no need to emphasize that with regard to the last two quantities this procedure is rather hazardous. Some of the material that had to be used was very incomplete and its reliability could not be checked in all cases. We therefore recommend that the reader regard our attempt as preliminary. Perhaps the statistical agencies in the individual countries—but only perhaps—can furnish better estimations. The computation of the first three quantities will need no explanation; the method will be clear and the sources are listed in the appendix. When determining the total production volume, an effort has been made to give a weighted index for the production of agriculture, industry, transportation and services. In

[24] See also our publications in the Dutch language on this subject: "Technische ontwikkeling en werkgelegenheid", *Uit leven en wetenschap*, Amsterdam 1940. – *Het streven naar efficiency en de werkgelegenheid*, other pages.

principle, the weights have been based on the size of the working population or its contribution to national income. The estimation of the capital quantity is based—unless other estimations were available—on the same conception, *i.e.* we have made use of weighted indices of the quantitative development of the individual components of the capital goods stock. These components are: the number of live stock, transportation (length of the railway network, number of engines, passenger and freight cars, tonnage of ships), the industrial apparatus (horsepowers), available housing and acreage. The weights are proportional to the value of the components in the base year.

The annual growth rate has been computed from the available figures in the following way: if data were available only from enterprises' censuses, such data were used, and if annual data were available, from figures with, for example, ten year intervals. Since the trade cycle movement during this period is rather moderate, it causes only a very small element of error in such computations. The results have been combined in Table III. In this table we recognize first a few familiar facts, especially with regard to population growth. In the second place we find that in all four countries the working population grew more rapidly than population as a whole. In Great Britain, where the process of industrialization was nearly at an end, the difference between the two growth rates amounts to very little. In Germany the gap is not great either, probably because the process had only been under way for a short time. The greatest difference is found in France and the United States where the process had reached its climax.

TABLE III

ANNUAL GROWTH RATES OF SOME IMPORTANT ECONOMIC VARIABLES IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES 1870–1914 (%)

Symbol	Variable	Germany	Great Britain	France	US
β'	Total population	1.1	0.9	0.1	2.1
$\dot{a}$	Working population	1.6	1.1	0.8	3.0
$\dot{l}'/l'$	Real wages	0.9	0.8	1.2	0.8
$\dot{u}$	Production	3.4	1.6	1.9	4.1
$\dot{K} = \varkappa$	Capital quantity *	2.6	1.8	0.8	2.9

* (including land)

It is remarkable that the figures for the real wage increase differ only slightly; the difference is less substantial than with the figures for per capita production of working population as demonstrated by this comparison:

TABLE IV

GROWTH RATES OF REAL WAGE AND PRODUCTIVITY IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES

Growth rate (%)	Germany	Great Britain	France	US
Real wage	0.9	0.8	1.2	0.8
Per capita production of working population	1.8	0.5	1.1	1.1

The reason for the real wage increases being more or less uniform in the four countries is perhaps that during this period there was still a certain freedom of movement on the part of the workers, manifest chiefly in the opportunity to emigrate to America. This might also be the explanation for the fact that the increases in Great Britain and the United States—the countries with the highest real wages—is the lowest, whereas in France where real wages were lower, it is the highest.

When making an analysis of the growth rates of the amount of capital we must not forget that these refer to the capital inside a country. In the European countries they are therefore exclusive of capital exports, in America inclusive of capital imports. The growth rates say little about the savings quota because they bear no reference to income but only to the existing quantities of capital. What we can show with these statistics is the speed of growth of efficiency. Computed according to our formula (1.1) this quantity turns out to be:

$$\varepsilon' = \dot{u} - \tfrac{3}{4}\,\dot{a} - \tfrac{1}{4}\dot{K}.$$

This computation has been tabularized in Table V.

The sequence of these magnitudes is the same as with the increase in the rate of per capita production and is probably correlated with the corresponding phase of the industrialization process.

TABLE V

COMPUTATION OF THE RATE OF GROWTH OF EFFICIENCY ε' IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES DURING THE PERIOD 1870–1914

No.	Symbol	Germany	Great Britain	France	US
1	$\frac{3}{4}\dot{a}$	1.2	0.8	0.6	2.3
2	$\frac{1}{4}\dot{K}$	0.7	0.5	0.2	0.7
3	Sum (1) + (2)	1.9	1.3	0.8	3.0
4	$\dot{u}$	3.4	1.6	1.9	4.1
5	$\varepsilon' = (4) - (3)$	1.5	0.3	1.1	1.1

In conclusion we should like to take another look at Table II. As we have already seen, the production increase $\dot{u}$ can be explained by capital formation for at most an amount $\varkappa$ and by population increase for at most β'. Since the coefficients together are 1, the sum of these two factors together will never explain a value of $\dot{u}$ exceeding the larger of the two quantities $\varkappa$ and β'. Since $\varkappa$ is always the larger of the two figures, Table III shows also that a maximum of 2.6% out of 3.4% can be thus explained in the case of Germany, at the most 1.1% out of 1.9% in the case of France, and in the case of the United States a maximum of 2.9% out of 4.1%. Only in the case of Great Britain is it possible to explain the total production increase by capital formation and population increase.

Were we to adopt DOUGLAS' view that λ' is negative, the influence of capital and population increases would not surpass $\frac{1}{4}\varkappa + \frac{3}{4}\beta'$,

TABLE VI

EXPLANATION OF PRODUCTION INCREASE IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES IF WAGE FLEXIBILITY IS ASSUMED TO BE NEGATIVE

	Germany	Great Britain	France	US
Production increase	3.4	1.6	1.9	4.1
explained from:				
Capital & population increase	1.5	1.2	0.3	2.3
Efficiency increase	1.5	0.3	1.1	1.1

i.e. 1.5%, 1.2%, 0.3% and 2.3% for the respective countries. Hence it follows that the effect of the efficiency increase ε' on production increase $\dot{u}$ would be limited to not more than $1 \times \varepsilon'$, *i.e.* 1.5% for Germany, not more than 0.3% for Great Britain, 1.1% for France and 1.1% for the United States. The results have been combined in Table VI.

Thus we have made an attempt, at least for these cases, to estimate the relative importance of the components assumed in our theory. There is surely no need to stress that our attempt can only claim to be accurate provided that 1) our theory is accepted, 2) our statistical information is assumed to be correct and 3) DOUGLAS' view concerning the sign of λ' is agreed to.

Up to this point we have always interpreted the statistical results in the sense of our model A1a, in other words we have assumed that the DOUGLAS function is valid, that population growth follows an exponential law, that capital supply is entirely inelastic. It is not our intention to go into the details of another interpretation, because we are of the opinion that for the period under review our interpretation is a usable first approximation. However, we will mention other possibilities of interpretation such as the one incorporated in our model A 2 a. In this model a different production function was assumed in which labour and capital were thought of as being entirely complementary, and the (statistical) labour productivity and capital productivity u/a and u/K, or, if so preferred, labour productivity and capital intensity u/a and K/a, figure as independent data. Their annual growth rates may be easily computed from Table III:

TABLE VII

ANNUAL GROWTH RATES OF LABOUR PRODUCTIVITY, CAPITAL PRODUCTIVITY AND CAPITAL INTENSITY IN GERMANY, GREAT BRITAIN, FRANCE AND THE UNITED STATES 1870–1914 (%)

Quantity	Germany	Great Britain	France	US
Labour productivity	1.8	0.5	1.1	1.1
Capital productivity *	0.8	—0.2	1.1	1.2
Capital intensity *	1.0	0.7	0.0	—0.1

* including land

An interpretation of our figures within the definition of our models A 2 b and B is pointless for the period under review because these models refer to periods of structural underemployment of capital.

10. APPENDIX

Sources and computational bases for the statistical investigation: In this appendix some detailed information concerning the sources and computational bases for the statistical analysis are given. A complete survey would be much too voluminous for practical purposes.

Total population and working population:

These figures are taken from the censuses of occupational surveys of the various countries.

Real wages:

Germany and France: C. VON TYSZKA, "Löhne und Lebenskosten in Westeuropa im 19. Jahrhundert (Frankreich, England, Spanien, Belgien). With annex: "Lebenskosten deutscher und westeuropäischer Arbeiter früher und jetzt, *Schriften des Vereins für Sozialpolitik*, Bd. 145, T.3) München u. Leipzig 1914. (Für Deutschland: "Löhne in Preussen", *Idem* p. 289).

Great Britain: The wage rate is that of Bowley and Wood, taken from: W. L. LAYTON, *An introduction to the study of prices. With special reference to the history of the nineteenth century*, London 1920, p. 184. The index of living costs is that of C. CLARK, *National income and outlay*, London 1937, p. 231. By way of comparison the figures of A. H. HANSEN, "Factors affecting the trend of real wages", *The American Economic Review*, Vol. 15 (1925), p. 27 *sq.* have been used; they showed a growth rate of 0.81% per annum as against 0.83% in our computations.

United States: Wage rate from: W. I. KING, "The wealth and income of the people of the United States", *The Citizen's Library of Economics, Politics and Sociology, N.S.*, New York 1923, p. 168. Living costs: own computations from data of C. SNYDER and R. S. TUCKER. By way of comparison figures from HANSEN (*loc. cit.*) were again used showing a growth rate of 0.85% as against our 0.69%.

Production:

Germany: The growth rate is an estimated average of the rates for the index of goods (industry and agriculture) according to R. WAGENFÜHR, "Die Industriewirtschaft, Entwicklungstendenzen der deutschen und internationalen Industrieproduktion 1860 bis 1932", *Vierteljahreshefte zur Konjunkturforschung*, Sonderh. 31, Berlin 1933) and for an index of transportation services. The weights are 18.6 and 3.8 according to the contributions to the national income in the year 1895 according to Helfferich. The transportation index was combined from the figures for railway and mail traffic.

Great Britain: Own computation using: W. HOFFMANN, "Ein Index der industriellen Produktion für Grossbritannien seit dem 18. Jahrhundert", *Weltwirtschaftliches Archiv*, Bd. 40 (1934, II), p. 383 *sq.*, L. DRESCHER, "Die Entwicklung der Agrarproduktion Grossbritanniens und Irlands seit Beginn des 19. Jahrhunderts", *Bemerkungen zum Index der Agrarproduktion, idem* Bd. 41 (1935 I), p. 270 *sq.* and from official data on transportation (railways, tramways and mail) and services (housing and domestic services). The weights are based on the data of FLUX for the national income in the year 1907.

France: The growth rate is a weighted average of the rates of industrial production, agricultural production and transportation. Index of industrial production according to WAGENFÜHR (*loc. cit.*); agricultural production computed from data in: "L'évolution de l'économie française, 1910–1937" *Tableaux, statistiques, publ. par l'Institut scientifique de recherches économiques et sociales* sous la direction de Ch. Rist, Paris 1937) for all data on agricultural products and estimations of the meat production; transportation index from official data on railway traffic, inland navigation and mail traffic. Weights according to the number of employees in the year 1911 (industry 7, agriculture 8.5 and transportation including mail 1.2 million).

United States: The production figures were obtained from data by KING (*loc. cit.*) by dividing them by the index of living costs (see above under real wages) or an index of investment goods prices (own computation from data in: *Wholesale prices for 213 years, 1720 to 1932*, P.I.: G. F. WARREN AND F. A. PEARSON, *Wholesale prices in the United States for 135 years, 1917 to 1932*, Cornell University, Agricultural Experiment Station, Memoir 142, Ithaca, New York, 1932, p. 98, 100, and KING, (*loc. cit.*).

Capital quantity:

Germany: Quantities used: Length of railways, number of engines, passenger and freight cars, number of horses, cattle, pigs, sheep and goats, freight capacity of inland craft, net tonnage of sea craft, number of dwellings, horsepowers of machines in industrial enterprises, arable acreage. Weights: Railways 2, ships 1, live stock 1, dwellings 6, industrial apparatus 9, acreage 10. These figures correspond, in billion Marks, to the national wealth in the year 1909 according to: A. STEINMANN-BUCHER, *350 Milliarden deutsches Volksvermögen (Das Volksvermögen Deutschlands, Frankreichs, Grossbritanniens und der Vereinigten Staaten von Amerika, Neue Massstäbe und Wege für deutsche Politik und Finanzwirtschaft*, Berlin 1909) and according to estimates from other sources.

Great Britain: According to: P. H. DOUGLAS, "An estimate of the growth of capital in the United Kingdom 1865–1909", *Journal of Economic and Business History*", Cambridge, Mass., Vol. 2 (1929/30), p. 675. To the "active capital" according to DOUGLAS, other quantities pertaining to land and residential buildings have been added.

France: Quantities used: Same as for Germany except those for inland navigation. Weights: Railways 2, cattle 1, buildings 4, ships 0.3, industrial apparatus 4, land 5. These figures correspond in billion ffrs.) to the national wealth in the year 1912 according to E. THÉRY. Compare *Volksvermögen* (W. WINKLER), Handwörterbuch der Staatswissenschaften, 4, gänzlich umgearb. Aufl., Jena, Bd. 8 (1928), p. 780.

United States: All figures according to KING (loc. cit.); monetary data have been corrected with the help of computed price indices (*cf.* under production).