Chapter 5

SEVERAL SECTORS; FIXED PRICES; SUBSTITUTION

5.1. Model with Substitution between Factors

5.11 In Sec. 1.56, we discussed briefly various types of substitution playing a role in modern economies. Of those, intertemporal substitution is so much the core of the development problem that it finds treatment in every chapter of this book. Substitution between factors was dealt with in Chap. 3, without distinguishing between products. We shall take up this subject again in this chapter, in a model with several sectors (see Secs. 5.12ff.).

A third form of substitution is the one possible through international trade. This will be the subject of a few models for *open* countries, to be taken up in Secs. 5.2 and 5.3.

A fourth type of substitution is the substitution open to the consumer. Although this may be effectuated by such quantitative devices as rationing, it is more customary and more attractive to let prices play a role. This is why the subject will be taken up in Chap. 6, where prices are introduced as variables.

Although the models of Secs. 5.1 to 5.3 use the methods of input-output analysis, there is scope for using the more complicated method of linear programming for a number of the problems under discussion. This will be undertaken in Chap. 6. In this chapter, we shall treat finally some nonlinear relationships between inputs and outputs which can be considered to constitute examples of substitution also (Secs. 5.5 and 5.6).

Introducing the element of substitution in one way or another means increasing the degrees of freedom of the policy maker. While the only degree of freedom in the models of Chap. 4 was the rate of saving, which we assumed to be determined already, these models actually had no degree of freedom except the *additional instruments of economic policy* introduced in Sec. 4.6. In the present chapter, there are more degrees of freedom. This means that planning can be determinate only after further aims of economic policy have been defined. The problems to be presented as examples of the use of the models will be based on the further aim of maximizing national income to be obtained each year with the aid of the capital available.

5.12 In this subsection we discuss first a two-sector model, developed by Professor P. C. Mahalanobis.[1] The model considers a closed economy and distinguishes two sectors, sector 1 producing investment goods, sector 2 consumer goods only. The increase in production is dependent on the investments made in each sector. The policy problem discussed with the help of this model is how to allocate between the two sectors the investment goods produced.

5.13 The *variables* of the model are

- c volume of consumption goods produced (by sector 2)
- j volume of investment goods produced (by sector 1)
- w^{11} volume of investment goods used to increase the production capacity in sector 1
- w^{12} volume of investment goods delivered to sector 2
- y national income

5.14 The *equations* of the model are the following:

$$\Delta c = \zeta^{12} w^{12} \tag{5.14.1}$$

This is an alternative formulation of similar equations discussed in Sec. 4.6, the coefficient ζ^{12} being equal to $1/\kappa^{12}$ and the investment lag being equal to one time unit. Thus

$$w_t^{12} = \kappa^{12}(c_{t+1} - c_t) \tag{5.14.1$'$}$$

$$\Delta j = \zeta^{11} w^{11} \tag{5.14.2}$$

Similarly, this equation expresses the increase in the production of investment goods that is made possible by an increase in the production capacity of the investment-goods industry.

$$w^{12} = \Lambda^{12} j \tag{5.14.3}$$

$$w^{11} = \Lambda^{11} j \tag{5.14.4}$$

These equations express the allocation of total investments between the two sectors. The coefficients Λ^{11} and Λ^{12} are instruments of economic policy and add up to 1: $\Lambda^{11} + \Lambda^{12} = 1$.

[1] Some Observations on the Process of Growth of National Income, *Sankhyā*, vol. 12, p. 307, 1953; and The Approach of Operational Research to Planning in India, *Sankhyā*, vol. 16, p. 3, 1955.

$$y = c + j \tag{5.14.5}$$

Income equals the sum of the production of consumption and investment goods.

5.15 The five equations determine the five variables c, j, w^{11}, w^{12}, and y for given values of the coefficients Λ^{11} or Λ^{12}.

The solution for j is obtained from the difference equation, which we find by substituting (5.14.4) in (5.14.2). The solution is

$$j_t = (1 + \Lambda^{11}\zeta^{11})^t j_0 \tag{5.15.1}$$

where j_0 is the investment volume in the initial period 0.

Similarly we find the solution for c by substituting (5.14.3) in (5.14.1), by using the solution for j_t and by solving the difference equation we find. The solution is

$$c_t = c_0 + \frac{\Lambda^{12}\zeta^{12}}{\Lambda^{11}\zeta^{11}}[(1 + \Lambda^{11}\zeta^{11})^t - 1]j_0 \tag{5.15.2}$$

By adding up the solutions for c and j, we find the solution for y:

$$y_t = y_0 + \left(1 + \frac{\Lambda^{12}\zeta^{12}}{\Lambda^{11}\zeta^{11}}\right)[(1 + \Lambda^{11}\zeta^{11})^t - 1]j_0 \tag{5.15.3}$$

5.16 The model does not introduce explicitly the rate of savings. However, this rate can be derived from the solutions for j and y. We find

$$\sigma_t = \frac{j_t}{y_t} = \frac{(1 + \Lambda^{11}\zeta^{11})^t j_0}{y_0 + (1 + \Lambda^{12}\zeta^{12}/\Lambda^{11}\zeta^{11})[(1 + \Lambda^{11}\zeta^{11})^t - 1]j_0} \tag{5.16.1}$$

This expression shows that σ is not constant, but changes over time with constant values of Λ^{11} and Λ^{12}. Thus, it appears that what seemed to be an additional degree of freedom—the possibility of substitution between the two capital goods—proves to be not a real one. We cannot choose independently the rate of savings and the allocation of investments between the two sectors at the same time (see Sec. 4.34).

Further analysis of Eq. (5.16.1) shows that, for constant values of Λ^{11} and Λ^{12}, the rate of savings increases and approaches asymptotically $\Lambda^{11}\zeta^{11}/(\Lambda^{11}\zeta^{11} + \Lambda^{12}\zeta^{12})$ for $t \to \infty$, if $\Lambda^{12}/\Lambda^{11} < w_0^{12}/w_0^{11}$ or $\Lambda^{12} < w_0^{12}/j_0$. Verbally, if the share of investment goods going into the consumer-goods industry is lower than this share in the initial period, the rate of savings increases. Similarly, we find that the rate of savings decreases and approaches the same asymptotical value for $t \to \infty$, if $\Lambda^{12}/\Lambda^{11} > w_0^{12}/w_0^{11}$ or $\Lambda^{12} > w_0^{12}/j_0$. The rate of savings remains constant only if the allocation of the investments between the two sectors is the same as in the initial period.

5.17 The use that can be made of this model for planning purposes is to choose the value of Λ^{12} (or Λ^{11}) so as to maximize the target variable of economic policy. Different targets can be considered, each leading to a different choice. The following cases may be mentioned.

1. To maximize y at a certain point of time T. As the solution we find that if T is below a certain value T_0, depending on the coefficients ζ, Λ^{12} should be equal to 1. If T is higher than another value T_1 (with $T_1 > T_0$), then Λ^{12} should be equal to zero. Only if T is in the range between the critical values T_0 and T_1, values of Λ^{12} (or Λ^{11}) between 1 and 0 are optimal. This range may be very small depending on the values for the ζ's.

This result can be understood if we deduce from Eq. (5.15.3) the value of the increase in y per time period at the time t:

$$\Delta y_t = (\Lambda^{11}\zeta^{11} + \Lambda^{12}\zeta^{12})(1 + \Lambda^{11}\zeta^{11})^t j_0$$

The second expression at the right-hand side increases with increasing values for Λ^{11}, while the first expression decreases, if $\zeta^{12} > \zeta^{11}$, which we normally can expect to be the case. For small values of t, the increase in the second term will be smaller than the decrease in the first term and, therefore, $\Lambda^{11} = 0$ (or $\Lambda^{12} = 1$) will lead to the highest increase.

2. To maximize c at a certain point of time T. Here also there exists a critical value T_0' below which $\Lambda^{12} = 1$ leads to a maximum value of c. However, there does not exist a second critical value T_1' above which Λ^{12} should be 0, for with $\Lambda^{12} = 0$, consumption remains constant.

5.18 Professor Mahalanobis has elaborated the two-sector model into a four-sector model by subdividing the consumer-goods industry in the following subsectors: factory production of consumer goods (sector 2), production of consumer goods (including agricultural products) in small and household industries (sector 3), and services, such as health and education (sector 4). Sector 1 indicates the investment-goods sector. In addition, the model introduces the labor requirements for each sector. The problem to be solved with the help of the model is again to determine the allocation of the investments over the four sectors.

The equations of the model are

$$\Delta a = \Delta a^1 + \Delta a^2 + \Delta a^3 + \Delta a^4 \qquad (5.18.1)$$

Total newly created employment (Δa) equals the sum of the new employment created in each sector separately (Δa^i).

$$\Lambda^i j = \Xi^i \, \Delta a^i \qquad i = 1, \ldots, 4 \qquad (5.18.2)$$

These equations introduce for each sector i a constant capital-labor ratio Ξ^i, the amount of net investment required for the employment of an additional worker. The equations relate the new investments in each sector to the new employment created. The Λ^i's have to fulfill the condition $\Sigma^i \Lambda^i = 1$.

$$\Delta y = j \cdot \sum_{1}^{4}{}^{i} \zeta^i \Lambda^i \qquad i = 1, \ldots, 4 \tag{5.18.3}$$

In this equation ζ^i is the output-capital ratio of sector i, and, consequently, $\Sigma \zeta^i \Lambda^i$ represents the weighted average of the output-capital ratios with the shares of each sector in total investments as weights.

The use which Mahalanobis has made of this model is, in our opinion, not the most appropriate. He deduces the value of Λ^1 from the two-sector model and fixes on a priori grounds the values for Δy, Δa, and j. In this case no real policy problem remains.

A more suitable use would be, for example, to give certain weights (ω_i) to an increase in employment and in income and to derive from the model the values for the Λ's which maximize the weighted sum $\omega_1 \Delta a + \omega_2 \Delta y$ over a given period. Alternative approaches are also possible.

5.2. Open-economy Models

5.21 We shall now take up again some open-economy models but, contrary to what was done in Sec. 4.4, introduce into them the element of substitution through international trade. We do this by making the choice of industries a major instrument of development policy. In the train of thought of the models here considered, the choice will depend above all on the comparative advantages a country may have in some industries and the comparative disadvantages in others. Maintaining the assumption of fixed and given prices of all products, we find that a comparative advantage will show itself in lower costs per unit of product. Costs in these examples are all inputs excluding inputs of factors of production, since their values are part of the national income, which we want to maximize.

5.22 The variables to be included in the models are

- v^h production of good h
- $v^{hh'}$ input of commodity h into current production of h'
- $w^{hh'}$ input of commodity h into investments in sector h'
- c^h consumption of good h
- e^h export of good h (negative sign stands for imports)
- y national income
- s savings

5.23 The relations of the models are

$$w^{hh'} = \frac{\kappa^{hh'}}{\theta^{h'}} (v^{h'}_{t+\theta^{h'}} - v_t^{h'}) \tag{5.23.1}$$

$$s = \sum_h \sum_{h'} w^{hh'} \tag{5.23.2}$$

$$s = \sigma y \tag{5.23.3}$$

$$y = \sum_h \varphi^{0h} v^h \tag{5.23.4}$$

$$v^h = c^h + e^h + \sum_{h'} v^{hh'} + \sum_{h'} w^{hh'} \tag{5.23.5}$$

$$c^h = \gamma^h(y - s) + \bar{c}^h \tag{5.23.6}$$

$$v^{hh'} = \varphi^{hh'} v^{h'} \tag{5.23.7}$$

These relations do not need explanation, since they are identical with those of Sec. 4.6, except for the terms e^h, which, however, we met in Sec. 4.4.

5.24 As mentioned earlier, we are going to demonstrate the use of the model by solving the *second-stage planning problem of the choice of industries.* It is assumed again that a provisional choice of σ has been made and that at time $t = 0$ all v_0^h are given: the economy simply uses to the full all production capacities it has inherited from the past. This means that y_0 as well as s_0 is also given.

Confining ourselves first to the case where again all $\theta^{h'}$ are equal and choosing $\theta^{h'} = \theta$, the solution of our problem consists in maximizing $y_{t+\theta}$ under the side condition that a given amount of investment is available, that is,

$$\sum_h \sum_{h'} w_0^{hh'} = \sum_h \sum_{h'} \frac{\kappa^{hh'}}{\theta} (v_\theta^{h'} - v_0^{h'}) = s_0 \tag{5.24.1}$$

Since the $v_0^{h'}$ are known, this side condition is a linear condition upon the v_θ^h, whereas the maximand

$$y_\theta = \sum_h \varphi^{0h} v_\theta^h \tag{5.24.2}$$

is also linear in v_θ^h. We must add, strictly speaking, some boundary conditions; to begin with

$$v_\theta^h > v_0^h \tag{5.24.3}$$

or, if capital goods are assumed to have a finite life, $v^h > v_0^h - d^h$, where d^h represents depreciation allowances in sector h; we shall neglect this detail, however.

There will be further boundary conditions which we will discuss later.

As the problem is now stated, each additional unit of s_0 devoted to investment into sector h' yields an addition to production $v^{h'}$ equal to

$$\frac{1}{\sum_h \kappa^{hh'}} = \frac{1}{\kappa^{h'}} \tag{5.24.4}$$

as can be read from Eq. (5.24.1); here $\kappa^{h'}$ is the capital-output ratio of sector h'. The contribution to national income in year θ is, according to (5.24.2),

$$\frac{\varphi^{0h'}}{\kappa^{h'}} \tag{5.24.5}$$

As a rule, these figures will be different for the various sectors. In order to get the maximum value of y_θ, all investment should therefore be directed to sector h^m, showing the maximum value of (5.24.5). If, by coincidence, two sectors had the same, highest, figure, it would not matter which sector was chosen. After the $v_\theta{}^h$ had been chosen, the equations of Sec. 5.23 would enable the planning authority to determine the unknowns for year 0, that is, $w_0{}^{hh'}$, $e_0{}^h$, which can be found from Eqs. (5.23.1) and (5.23.5).

5.25 Some comments on the economic significance of the procedure just described are called for.

1. The assumption that σ has already been determined on the basis of a macromodel implies some general *knowledge about export possibilities*, for example, in the form suggested in Sec. 4.4. If this knowledge were exact, the values found for $e_0{}^h$ should be in conformity with it. In the absence of such conformity, there may be scope for revising the original assumptions, which may influence the value to be given to σ.

2. The device obtained from the maximization of y_θ is, in the language of the theory of international trade, one of *complete specialization*. Had such a device been followed in the past also, the country would produce one commodity only and import all others. The device is clearly unrealistic. It is based on the same two assumptions on which the simpler theories of international trade are based: (a) absence of transportation costs and (b) smallness of the country's supply in relation to the world market; otherwise the price cannot be considered fixed and given. A more realistic approach can be obtained by the following additional assumptions.

a. Some sectors may show very high transportation costs, making it impossible to have any international trade, whether exports or imports, meaning that $e_t{}^h = 0$ for $t = \theta$ as well as for $t = 0$.[1]

[1] This assumption has an implication for the choice of industries which deserves some further elaboration. It implies that there is no point in isolated investment

b. There may be boundary conditions set to the quantities v_θ^h which can be sold at the given price levels. This implies that the sector showing maximum (5.24.5) must have a v^h equal to this boundary value; if not all savings are needed for the corresponding investment, the next highest value of (5.24.5) will indicate a second sector to invest in, and so on.

These boundary conditions, though they make the model much more realistic, are, at the same time, its weakness: they have to be arbitrary. It makes much more sense here to introduce either nonlinear cost functions or a quantity-price relationship implying that larger quantities can be sold only at a lower price. Nonlinear cost functions will be taken up in Secs. 5.4 to 5.6. Quantity-price relationships will be taken up in Chap. 6.

5.26 A few remarks should be added about the case of *nonuniform gestation periods*. As in Sec. 4.7, we shall treat this case by considering a simple concrete model with two sectors, showing two different *lags* taken here to be $\theta^1 = 2$ and $\theta^2 = 3$ years. For practical purposes the lags can always be assumed to be commensurate. Considering the expressions, in terms of v's, for income in years 2 and 3, we have

$$y_2 = \underline{\varphi^{01}v_2{}^1} + \varphi^{02}v_2{}^2 \tag{5.26.1}$$
$$y_3 = \varphi^{01}v_3{}^1 + \underline{\varphi^{02}v_3{}^2} \tag{5.26.2}$$

With the aid of our savings s_0 we can contribute to $v_2{}^1$ and $v_3{}^2$, the underlined terms; decisions on $v_2{}^2$ have already been taken before (at time -1, since $\theta^2 = 3$), whereas decisions with regard to $v_3{}^1$ have to be

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projects in any single sector; a project should always include a number of complementary investments in the sectors with very high transportation costs (the so-called national sectors; see Sec. 1.52). The size of the necessary complementary investments may be estimated with the aid of our models, that is, with input-output models, by the following set of equations:

$\Delta v^h = \Delta e^h + \sum_{h'} \varphi^{hh'} \Delta v^{h'}$	for all sectors
$\Delta v^h = 1/\kappa^h$	for original sector of project
$\Delta e^h = 0$	for complementary (national) sectors
$\Delta v^h = 0$	for international sectors other than original project sector

Terms Δc^h and $\sum_{h'} \Delta w^{hh'}$ have been omitted from Eq. (5.23.5) since no change in consumption or investment is assumed for the purpose of the present problem. The only question concerning us here is the necessary additional production in the national sectors needed in order that all interindustry deliveries are available. This is a first approximation. A more elaborate system may be set up in order to take into account the increase in consumption made possible by the increase in national income as a consequence of the project.

taken later (at time +1). In these circumstances it seems reasonable to *maximize* $\varphi^{01}v_2^1 + \varphi^{02}v_3^2$, that is, *the contributions to future income which can be made out of savings* s_0. This principle accepted, the procedure is practically identical to the one outlined in Sec. 5.24.

5.27 The reader should be reminded of what was said in Chap. 1 and in Sec. 3.5 about the role of aims in the design of economic policy. Clearly it is possible to use our models for the solution of planning problems with aims different from those assumed in the preceding sections.

5.3. Models with a Structural Break

5.31 It is essential for the process of industrialization that, from time to time, new sectors be added to the economy. In the language used in this book it is better to say that an "empty sector" becomes a normal sector. In fact, even if in the economy studied there is no production of a certain commodity, the sector has to be included in our model, showing consumption, interindustry deliveries, and imports of that commodity. What changes therefore is only the side condition $v^h = 0$ characteristic of the sector's "emptiness." This change constitutes, however, a change in the logical structure of a model, to which it is useful to give some attention.

5.32 The reasons for such a break to occur may be, on the one hand, changes in the price or cost data or in the boundaries introduced in Sec. 5.25 or, on the other hand, changes in the size of demand inside the country. As we saw in our models of Chaps. 4 and 5, changes in prices or costs will translate themselves into changes in φ^{0h}, and these can influence the choice of industries. The same is true of changes in boundary conditions. If the break occurs for one of these reasons, the procedure discussed will automatically result in a change in the production pattern.

A change in the size of home demand may be another reason, in practice, for starting home production. Economies of scale will often be involved. The models so far presented do not show this feature, since, mathematically speaking, it introduces nonlinear relationships which it is much more difficult to handle, especially if a large number of variables are involved. Again it may be helpful to proceed in stages and to use "partial models" in order to decide whether production of a certain commodity should be started. This decision taken, our linear models may again be used to study the consequences for the economy as a whole. We shall deal with some examples of this partial nonlinear research in Sec. 5.6.

5.33 The development of an economy introducing a new item to its production program passes through *three stages.* In the first stage, production is zero and investment in the sector is zero too. In the second stage, preparations for future production must be taken in the form of investments, which are positive now, while production is still zero. In the third stage, finally, production will be positive. In our symbols, we have

Stage I:	$v_t^h = v_{t+\theta}^h = 0$	
Stage II:	$v_t^h = 0$	$v_{t+\theta}^h > 0$
Stage III:	$v_t^h > 0$	$v_{t+\theta}^h > 0$

Clearly the length of stage II is θ, or rather θ^h.

5.34 In principle, the general mathematical solutions presented in Secs. 4.2, 4.4, 4.6, etc., are valid only for periods without a structural break. From the moment of a structural break, the path of development may change and has to be calculated anew.

5.4. Models with Increasing Marginal Costs

5.41 So far it has been assumed that production has taken place under conditions of constant returns, implying constant marginal costs equal to average costs. This assumption, usual in input-output analysis, has been responsible for the result indicating that complete specialization should be aimed at in order to maximize national product. In various situations this is an unrealistic assumption. We shall now consider alternative assumptions. They can be labeled by the well-known concepts of increasing marginal costs (or diminishing returns) and decreasing marginal costs (or increasing returns). The former phenomenon is characteristic of any given enterprise when capacity limits are approached and of some industries as a whole, especially agriculture and mining. The latter phenomenon is characteristic of enterprises or industries using *indivisible factors of production* and showing overcapacity. The phenomenon of increasing marginal costs is compatible with the assumption of free competition and can be dealt with analytically in a rather simple way. The phenomenon of decreasing marginal costs, or of indivisibilities, is incompatible with free competition in that it creates, at least if flat-rate pricing is applied, permanent losses to the competitors. There are therefore complications of a particular type; we shall deal with some of these later (see Secs. 5.5 and 5.6).

5.42 If in some sectors increasing marginal costs prevail, relations (5.23.7), for these sectors, will have to be replaced by nonlinear relationships. A simple example is a quadratic relationship.

$$v^{hh'} = \varphi_1^{hh'}v^{h'} + \tfrac{1}{2}\varphi_2^{hh'}(v^{h'})^2 \tag{5.42.1}$$

Such relationships may facilitate the solution of our maximum problem. Income y is no longer a linear function of all v^h, but a quadratic function of some v^h. This means that the expression $\varphi^{0h'}/\kappa^{h'}$ has to be replaced by a more complicated one, depending on $v^{h'}$, and that a maximum y may be found for finite values of the $v^{h'}$, characterized by equal marginal income-capital ratios. In the case where all sectors showed rather rapidly increasing marginal costs, this maximum would be represented by

$$\frac{\partial y_\theta}{\partial v_\theta{}^h} + \Psi \frac{\partial s_0}{\partial v_\theta{}^h} = 0 \qquad h = 1, \ldots, H \tag{5.42.2}$$

where Ψ is a Lagrangian multiplier and where

$$\frac{\partial y_\theta}{\partial v_\theta{}^h} = \varphi^{0h} - v_\theta{}^h \sum_{h'} \varphi_2{}^{h'h} \qquad \varphi^{0h} = 1 - \sum_{h'} \varphi_1{}^{h'h} \tag{5.42.3}$$

whereas, as before, $\partial s_0/\partial v_\theta{}^h = \kappa^h/\theta$, leading to

$$\frac{\varphi^{01} - v_\theta{}^1 \sum_{h'} \varphi_2{}^{h'1}}{\kappa^1} = \frac{\varphi^{02} - v_\theta{}^2 \sum_{h'} \varphi_2{}^{h'2}}{\kappa^2} = \cdots = \frac{\varphi^{0H} - v_\theta{}^H \sum_{h'} \varphi_2{}^{h'H}}{\kappa^H} \tag{5.42.4}$$

This solution will only apply, however, if all sectors but one show increasing costs, and if the values found satisfy the boundary conditions $v_\theta{}^h \geq v_0{}^h$. Otherwise, a number of $v_\theta{}^h$ will have to be put equal to $v_0{}^h$, while the others satisfy conditions of the nature of (5.42.4).

5.5. Indivisibilities

5.51 It is well known that indivisibilities play a role in the process of development, that is, the fact that some investment projects make sense only when they have a certain minimum size. This is true for railways, because at least one track has to be constructed over the whole distance between two centers before transportation can take place at all. It is true for almost any project in some sense. In order to be an economic proposition a factory has to have a certain minimum size before all types of overhead expenses are justified. A university with a few hundreds of students only would not provide a full-time job to most of the teaching staff, and so on.

When it comes to expressing this well-known truth in terms of development models, it must be stated that very little is known quantitatively about the precise role indivisibilities play. There are no very clear macroeconomic observations which can be explained only by the phenomenon of indivisibilities. Although one can easily imagine all sorts of impacts of them on macroeconomic development, there are all sorts of "escapes" to indivisibilities too. It is true that any single railway constitutes an example, but the number of railway connections in most countries is so large that the construction of the railway network can be and actually has been spread over a long period in most countries, and the investment involved in each single year need not be a very large portion of total investment. Even if the construction of an irrigation dam for a small region is very much an indivisibility, for a country as a whole such projects need not coincide and do not therefore take such a large portion of investment resources. Often also the long construction period helps to spread the effects. Although it cannot be denied that the Aswan High Dam in Egypt will absorb a considerable portion of the country's investment resources, to quote a recent example, it is hardly possible to say that indivisibilities play a standard role in the development of every country which can be represented by some standard feature in models for development planning. The most natural way to deal with the problem is to admit that among the projects which together constitute the investment program of a country there may be some whose size is considerable, and that in appraising them the consequences for the economy as a whole should be carefully considered. In the subsections of this section, a model for the appraisal of projects will be shown in which the size of the projects is considered given and may be large. The choice of the size of some types of projects in which indivisibilities play a role is treated in Sec. 5.6.

5.52 The model for the *appraisal of investment projects*, which is explained in the following subsections, assumes that a number of big and indivisible projects is given from which a selection has to be made. The model helps to estimate the impact of a given investment program on the national product and its development over time. The model assumes an open economy and two scarce factors of production: capital and foreign exchange.

Two groups of economic variables are distinguished: those which apply to the projects and those which do not. The project variables are indicated by an upper index h, where $h = 1, 2, \ldots, H$, if H is the total number of available projects from which the selection has to be made. Each index corresponds to an investment project. A combination of projects is called an investment program, and the total of all h-variables the program sector. Variables with an upper index 0 apply to variables outside the program sector, to be called the rest of the economy. Variables which are a combination of 0- and h-variables do not carry an index.

5.53 The following variables are used in the model, leaving out the upper indices.

Y	net national product (or income)
v	volume of gross product
S	value of savings
c	volume of consumption expenditure
k	real capital stock in use
e	volume of exports
i	volume of imports
F	balance-of-payments deficit
M^h	foreign debt incurred for the execution of project h
m^i	interest rate for foreign debts
$\bar{m}$	rate of discount
p	price level of national product
p^i	price level of imports

5.54 The equations of the model are

$$Y^0 = v^0p - i^0p^i - \delta k^0p \tag{5.54.1}$$
$$Y^h = v^hp - i^hp^i - \delta k^hp - m^iM^h \tag{5.54.2}$$

These equations define in the usual way the contribution of the rest of the economy and of each project to national income. Vertical integration is assumed within the rest of the economy and within each project. The term m^iM^h in (.2) represents the interest paid on the foreign debt incurred for project h. The coefficient δ represents the rate of depreciation on the capital stock in use. This coefficient is assumed to be the same in both sectors.

$$Y^0 = c^0p + \dot{k}^0p + e^0p - i^0p^i \tag{5.54.3}$$
$$Y^h = c^hp + \dot{k}^hp + e^hp - i^hp^i - m^iM^h \tag{5.54.4}$$

These equations define the contribution to national income from the expenditure side. They have the function to determine, in combination with other equations, the export volumes. The terms $\dot{k}^0p$ and $\dot{k}^hp$ represent the net investment in the rest of the economy and in project h, respectively.

$$c^0p = \gamma^0(Y^0 + \Sigma Y^h) + \bar{c}^0p \tag{5.54.5}$$
$$c^hp = \gamma^h(Y^0 + \Sigma Y^h) + \bar{c}^hp \tag{5.54.6}$$
$$i^0 = \iota^0v^0 \tag{5.54.7}$$
$$i^h = \iota^hv^h \tag{5.54.8}$$
$$S^0 = \sigma^0Y^0 \tag{5.54.9}$$
$$S^h = \sigma^hY^h \tag{5.54.10}$$
$$S = S^0 + \Sigma S^h \tag{5.54.11}$$

The equations (.9) and (.10) formulate an essential feature of the model. They express the assumption that each project has its own rate of savings. Differences in the rate of savings between projects may be the consequence of differences in the distribution of the income generated by each project. Especially the distribution in labor and nonlabor income will be relevant. Roughly speaking, this distribution is correlated to the labor or capital intensity of the project.

$$\kappa^0 v^0 = k^0 \tag{5.54.12}$$

$$\kappa^h v^h = k^h \tag{5.54.13}$$

$$\dot{M}^h = p^i i^h - \frac{1}{\tau} M^h \tag{5.54.14}$$

This equation indicates the net capital imports on account of project h. It is assumed that during the construction period, capital is imported equal to the value of the imports of capital goods (I^h) needed for project h. During the operation period the loan (M^h) is repaid at an annual rate of $(1/\tau)M^h$ if τ is the period of repayment.

$$p(\dot{k}^0 + \Sigma \dot{k}^h) = S + F + \Sigma \dot{M}^h - \delta(k^0 + \Sigma k^h)p \tag{5.54.15}$$

This is a balance equation for capital formation. The left-hand side of the equation indicates the value of net capital investments, the right-hand side the available sources to finance these investments. The balance-of-payments deficit F is defined in the following equation:

$$F = p^i(i^0 + \Sigma i^h) - p(e^0 + \Sigma e^h) + m^i \Sigma M^h - \Sigma \dot{M}^h \tag{5.54.16}$$

The deficit F is not the deficit on current account, but on current and capital account to the extent that the capital items have a planned character.

5.55 The following variables are explained by the model: Y^0, v^0, c^0, e^0, i^0, S^0, k^0, Y^h, v^h, c^h, e^h, i^h, S^h, $\dot{M}^h$, S, and F. These $9 + 7H$-variables are equal in number to the number of equations. The investments k^h made for each project and the prices p^i, p, and m^i are assumed to be given.

The economic structure of the model may be explained as follows. The existing capital stock and the additions to it determine the level of gross production and the direct contribution of each project to national income. Each project generates savings which become available for financing the investments in the rest of the economy in addition to its own savings and make possible a further increase in production and income. This shows that, according to the model, an investment program influences total national income not only directly but also indirectly via the influence on total savings.

The model enables us to determine the development of total national income $(Y^0 + \Sigma Y^h)$ over time for any given investment program.

Different programs may lead to different development patterns. In order to be able to choose between alternative programs, a criterion is needed which summarizes the development patterns. As such a criterion, we could choose the discounted value of the future total national product $\bar{Y}_1$.

$$\bar{Y}_1 = \sum_{1}^{\infty}{}^{t} \frac{Y_t^0 + \Sigma Y_t^h}{\prod\limits_{2}^{t} (1 + \bar{m}_{t'})}$$

The rate of discount $\bar{m}$ is made variable but may also be a constant.[1] If the total volume of the investment program is given, the selection of the projects to be included in the program has to be made in such a way as to maximize $\bar{Y}_1$. This combination of projects can be found only by trial and error.

5.6. Nonlinear Partial Models

5.61 Indivisibilities play their role at an innumerable number of spots in any economy. In a single factory, each machine is an example, and examples can be found all the way up to the really giant projects that sometimes must be part of a development process. For the economist engaged in macroplanning, it is not usually feasible to go into all the details of these indivisibilities. Looked at in a more comprehensive way, they show themselves in the nonlinear shape of some cost functions. Plotting total costs against the volume of production of a factory, we shall often find that they must be represented by a curve rather than a straight line. Depending on the form of the cost curve, there may or may not be some optimum size of the factory or of any other investment project, a phenomenon of considerable importance for development programming. Thus, a new industry should not be started as a rule if the market for its product does not permit a size of the factory in the neighborhood of the optimum size.

Though theoretically attractive, it would be practically impossible to introduce curvilinear cost functions as a regular feature for most sectors in a development model. Mathematical, and even numerical, treatment of such functions soon becomes impossible if their numbers increase. This is another reason for applying the device of planning in stages and for singling out the problems of deciding about the size of single plants or projects from the general programming problem by first considering a

[1] Here $\prod\limits_{2}^{t} (1 + \bar{m}_{t'})$ stands for $(1 + \bar{m}_2)(1 + \bar{m}_3) \cdots (1 + \bar{m}_t)$.

partial problem, when necessary with the aid of a partial model. The justification to do so is that in many cases the dimensions of single projects are small in comparison with the economy as a whole and that therefore variations in the size of such projects do not affect the market and other variables of the economy as a whole. Therefore the latter can be treated as given and the partial problem solved without considering all the interconnections of the economy. We shall give some examples of this type of partial research in the subsequent subsections.

5.62 As already explained, the many almost hidden indivisibilities playing roles in any single factory may give rise to a cost function of a curvilinear nature, as shown in the textbooks on business economics. Let total costs be a quadratic function of production v, for example,

$$\psi_0 + \psi_1 v + \psi_2 v^2$$

Then unit costs will be a minimum for

$$\frac{d}{dv}\left(\frac{\psi_0}{v} + \psi_1 + \psi_2 v\right) = 0$$

or

$$-\frac{\psi_0}{v^2} + \psi_2 = 0$$

or $v = \sqrt{\psi_0/\psi_2}$, representing the optimum size of the enterprise.

5.63 In some cases, transportation costs may be a decisive element in finding the optimum size of an enterprise. The larger the volume of production, the longer the average distance over which the product must be transported in order to reach the customer. Let demand be proportional to the surface over which the commodity can be supplied; if it is supplied over a circle with radius ρ, the average distance as well as the marginal one is proportional to ρ. Since the total quantity demanded varies with the surface of the circle, which is $\pi\rho^2$, the radius of the circle is proportional to $\sqrt{v}$. So are transportation costs per unit. If production costs proper are of the usual linear type $\psi_0 + \psi_1 v$, total costs, including transportation, are of the shape $\psi_0 + \psi_1 v + \psi_2 v\sqrt{v}$. Unit costs will then be $\psi_0/v + \psi_1 + \psi_2\sqrt{v}$, and these again will be a minimum if

$$-\frac{\psi_0}{v^2} + \frac{1}{2}\frac{\psi_2}{\sqrt{v}} = 0$$

or

$$v = \left(\frac{2\psi_0}{\psi_2}\right)^{2/3}$$

5.64 Frequently labor productivity in the initial stages of a new enterprise, especially in underdeveloped countries, will be too low to permit the enterprise to compete in the world market. At short notice it will then be more advantageous for the country to import the commodity considered—apart from questions of employment or balance of payments. Since it is a common experience, however, that after some years of exercise, productivity increases, a partial study may be made about the prospects at somewhat longer notice; it may well be that the enterprise turns out to be competitive in the long run. On such evidence the decision may be taken to establish the unit.

Among the appropriate subjects for partial studies market analyses for export products should be mentioned also. Such analyses may include an exploration into the factors determining the size of the market, such as incomes in customer countries, size of competing crops, price level at which the commodity is sold, changes in commercial or fiscal policy of the customer countries or the producing country, and so on. As far as possible the quantitative influence of such factors should also be determined. Since econometric studies of this kind are the subject of many publications and textbooks, the reader may be referred to such sources.