Chapter 7

SEVERAL SECTORS AND SEVERAL REGIONS

7.1. Rigid Distribution of Inputs over Sectors of Origin

7.11 In the present chapter, models are discussed in which the economy is subdivided not only according to sectors but also according to geography. The geographical units will be called regions; they are parts of a country.[1] Regions will be indicated by prefixes (indexes placed in front of the main symbol) r, the total number of regions being R. Models of this type make it possible to consider problems of development policy and planning in which targets are set referring either to the relative position of the regions or to particular regions.

The subdivision of the economy into regions forces us to consider a number of phenomena which we need not discuss when we use models without a geographical subdivision, phenomena which are familiar to the student of international economic problems. B. Ohlin[2] has already pointed out many of the parallels and some differences between international and interregional economic intercourse.

A geographical subdivision of the economy introduces movements between regions, of both products and factors of production. In both categories we meet cases of high and of low mobility, and we have a choice between crude and more detailed approaches to these phenomena. The crudest, but often quite useful, approach is to distinguish between products or factors which cannot move on the one hand and which move freely on the other hand. Free movements will be present if no cost of transportation occurs. The crudest approach means that there are either

[1] The word "region" is also used for areas larger than countries; this meaning will, however, not be given to the word in this book.

[2] B. Ohlin, "Interregional and International Trade," p. 45, Cambridge, Mass., 1933.

prohibitive or no costs of transportation. We shall call those sectors the products of which cannot move outside the region *regional sectors;* sectors the products of which cannot move outside the economy, *national sectors.*

Of the factor movements, capital movements are the most important to us, since we assume that capital is a scarce factor. Population and labor movements, as long as this factor is abundant, are irrelevant to the economy. In reality they are not, of course; the simplest way of dealing with this phenomenon is to admit that population movements are creating boundary conditions to the production capacity of sectors, to the extent that labor, or certain types of labor, becomes scarce. In an economy where labor is scarce generally, labor movements will have to be treated with the same degree of precision as capital movements.

In a more refined approach, we shall deal with transportation costs for commodities as a quantitative variable, admitting all the shades that can play a role (see Sec. 7.3). This necessitates the introduction of prices, since transportation costs can work out only through the prices of the products transported.

7.12 In this first section, we shall present the simplest approach to regional problems that can be applied in the absence of knowledge about the economic forces which determine the distribution of inputs of any kind over the regions of origin. This approach has been based on the method of input-output analysis and is due to Professor Hollis Chenery.[1] It consists in observing the existing distribution and assuming that this distribution remains the same with other absolute levels of production. We shall use this first model as an appropriate introduction to our subject since it enables us to leave prices out of consideration just as we did in Chaps. 4 and 5. We can thus concentrate on the purely quantitative variables which have to be introduced in a regional model before dealing with the price aspect. It will be assumed that there is complete mobility in all sectors except some *regional sectors;* that newly saved capital is freely mobile between sectors; and that population movements are irrelevant.

7.13 The variables of the model are

- ${}^r v^h$ production of commodity h in region r
- ${}^r v^{hh'}$ input of commodity h into sector h' for current production in region r
- ${}^r w^{hh'}$ investment input of commodity h into sector h' in region r

[1] "The Structure and Growth of the Italian Economy," chap. 5, Rome, 1953. See also L. N. Moses, The Stability of Interregional Trading Patterns and Input-Output Analysis, *Amer. Econ. Rev.*, vol. 45, p. 803, 1955; and W. W. Leontief and others, "Studies in the Structure of the American Economy," New York, 1953.

${}^{r}c^{h}$ consumption of commodity h in region r
${}^{r}e^{h}$ exports of commodity h from region r to foreign countries
${}^{r'r}x^{h}$ flow of commodity h from region r' to region r
${}^{r}y$ income of region r
${}^{r}s$ savings of region r

7.14 The relations of the model are

$$ {}^{r}w_t^{hh'} = \frac{{}^{r}\kappa^{hh'}}{\theta} \left({}^{r}v_{t+\theta}^{h'} - {}^{r}v_t^{h'} \right) \tag{7.14.1} $$

The possibility has been kept open that partial capital coefficients are different between regions, although it may seldom be possible to go into all details. For the sake of simplicity, no differences between gestation periods have been assumed, not even between sectors.

$$ \sum_r {}^{r}s = \sum_r \sum_h \sum_{h'} {}^{r}w^{hh'} \tag{7.14.2} $$

This equation assumes free mobility of newly formed capital.

$$ {}^{r}s = \sigma\,{}^{r}y \tag{7.14.3} $$

No difference in savings rate σ between the regions is made, although this could easily be done.

$$ {}^{r}y = \sum_h {}^{r}v^{h} - \sum_h \sum_{h'} {}^{r}v^{hh'} \tag{7.14.4} $$

$$ {}^{r}v^{h} = \sum_{r'} {}^{rr'}x^{h} \tag{7.14.5} $$

The intermediary variable ${}^{rr'}x^{h}$ is an easy tool of analysis in that it summarizes all commodity movements between regions irrespective of the destination given to them (consumption, current inputs, investment inputs, or exports); production in each region has then to be equal to the sum total of all shipments to all regions.

$$ \sum_{r'} {}^{r'r}x^{h} = {}^{r}\gamma^{h}({}^{r}y - {}^{r}s) + {}^{r}\bar{c}^{h} + \sum_{h'} {}^{r}v^{hh'} + \sum_{h'} {}^{r}w^{hh'} + {}^{r}e^{h} \tag{7.14.6} $$

All the *needs* of the region for good h have to be covered from the flow of good h into region r (including the region's own production as far as retained, ${}^{rr}x^{h}$).

$$ {}^{r}v^{hh'} = {}^{r}\varphi^{hh'}\,{}^{r}v^{h} \tag{7.14.7} $$

$$ {}^{r'r}x^{h} = {}^{r'r}\xi^{h} \sum_{r'} {}^{r'r}x^{h} \tag{7.14.9} $$

This equation is numbered (.9) since in the other models it has the same number. Equation (.8), being already needed in Chap. 6, but not used here, refers to price formation. The particular version of Eq. (.9) used here is the rigid one, in which ${}^{r'r}\xi^h$ are constants. They will have to obey the condition

$$\sum_{r'} {}^{r'r}\xi^h = 1 \tag{7.14.10}$$

For regional sectors $\quad {}^{r'r}\xi^h = 0 \quad \text{for } r' \neq r \quad (7.14.11)$

and consequently ${}^{rr}\xi^h = 1$, that is, all needs are covered by production of the region itself.

The numbers of variables and of equations are, respectively, $2R + 3RH + 2RH^2 + R^2H$ and $1 + 2R + 2RH + 2RH^2 + R^2H$, indicating that there are $RH - 1$ degrees of freedom. Similarly to the model of Sec. 4.4, the variables ${}^re^h$ may be assumed to be given or to depend on some other variables, subject to the one condition that

$$\sum_r \sum_h {}^re^h = 0 \tag{7.14.12}$$

We should, however, add that for regional and national sectors the corresponding ${}^re^h$ all have to be zero, which restricts freedom. The remaining degrees of freedom may be used to maximize some policy aim, for example, national income.

7.15 As an illustration of the use of the model, this very example may again be taken up, as was done in Sec. 5.24. As before, savings s_0 can be invested in different sectors showing different capital coefficients and different contributions to national income per unit of product: φ^{0h}; in addition, there is a choice between regions now. In principle, the same problem arises, if all coefficients are constants: as a rule, there will be one sector in one region which is contributing most, per unit of investment, to future national income and in the absence of boundary conditions full specialization in that one sector would be the solution—with the proviso that a certain amount of investment will have to be used for regional and national sectors, the size of which is determined by consumption and other demands, depending on national income as a whole, directly and indirectly. Nevertheless, the approach is unrealistic, as was said of the approach in Sec. 5.24, and either boundary conditions or prices have to be introduced, as shown in the remainder of Chap. 5 and Chap. 6.

7.16 As a consequence of the subdivision into regions, our present model is already a little bit more realistic, as may be shown by the treatment of other policy problems in which the relation between regions plays a larger role.

If, for instance, the condition is added that incomes per head in the various regions must become as near equal as possible or rise by an equal percentage, the lopsided pattern first obtained will be corrected in a more realistic direction. Evidently the first problem, that of equalizing incomes per head, presupposes knowledge about the movement of population which may be either taken from other sources or assumed dependent on income differences. Both problems mentioned will tend to introduce into each region what is for that region the best industry, implying already a diversification of the production pattern.

A final example consists in the assumption that each region's savings are invested in that region. This changes the model since instead of Eq. (7.14.2) we now have R equations

$$ {}^{r}s = \sum_{h} \sum_{h'} {}^{r}w^{hh'} $$

7.17 In order to remove the unrealistic features mentioned in Sec. 7.15, we may introduce prices, as in Chap. 6, and assume a relationship between volumes of production and prices, representing *world demand.* Assuming perfect markets within the country considered, the price of any commodity will be uniform in all regions. Our model can be easily generalized in this way. Prices would come in in the same way as in Chap. 6, since for them no regional differentiation is needed.

7.2. Transportation Costs Neglected; Prices Varying between Regions

7.21 The next simplest way to introduce prices into regional models is also similar to the way followed in Chap. 6, meaning that we still neglect transportation costs, as is usual in the theory of international trade. We shall assume now that prices are not only variables, but even may differ, for the same commodity, between regions. In other words we assume imperfect markets. This raises a further interesting question, namely, how demand reacts to the possibility of buying the same type of commodity at different prices. Usually in imperfect markets the commodities offered by different producers are supposed not to be identical; otherwise only one price could prevail. It will be assumed that the quantities bought from the various regions depend on the ratio of each price to the average price. The price at which each region supplies its product will be assumed to depend on the demand exerted in the region as well as in general for that particular product.

7.22 The variables of the model are

${}^{r}v^{h}$	production of good h in region r
${}^{r}v^{hh'}$	input of product h into current production in sector h' in region r
${}^{r}w^{hh'}$	input of product h for investment in sector h' in region r
${}^{r}c^{h}$	consumption of commodity h in region r
${}^{r}e^{h}$	exports of commodity h by region r to foreign countries
${}^{rr'}x^{h}$	flow of commodity h from region r to region r'
${}^{r}Y$	income of region r

^rS savings of region r
$^rp^h$ price of product h originating from region r
$^r\bar{p}^h$ average price paid for product h in region r

7.23 The relations constituting the model are

$$^rw^{hh'} = \frac{^r\kappa^{hh'}}{\theta}\left(^rv^{h'}_{t+\theta} - {}^rv_t{}^{h'}\right) \tag{7.23.1}$$

$$\sum_r {}^rS = \sum_r \sum_h \sum_{h'} {}^rw^{hh'} \cdot {}^r\bar{p}^h \tag{7.23.2}$$

$$^rS = \sigma^rY \tag{7.23.3}$$

$$^rY = \sum_h {}^rp^h \left({}^rv^h - \sum_{h'} {}^r\varphi^{hh'}v^{h'}\right) \tag{7.23.4}$$

$$^rv^h = \sum_{r'} {}^{rr'}x^h \tag{7.23.5}$$

$$\sum_{r'} {}^{r'r}x^h = \frac{^r\gamma^h(^rY - {}^rS)}{^r\bar{p}^h} + {}^r\bar{c}^h \frac{\sum_h {}^r\bar{p}^h}{^r\bar{p}^h} + \sum_{h'} {}^rv^{hh'} + \sum_{h'} {}^rw^{hh'} + {}^re^h \tag{7.23.6}$$

The shape of this equation corresponds to that given to Eq. (6.23.6); for the sake of simplicity, we have omitted the terms with $\gamma^{hh'}$ in the latter equation; these should run

$$\sum_{h'} {}^r\gamma^{hh'} \frac{^r\bar{p}^{h'}}{^r\bar{p}^h}$$

The omission means that we disregard price elasticities in consumer demand.

$$^rv^{hh'} = {}^r\varphi^{hh'} \cdot {}^rv^{h'} \tag{7.23.7}$$

$$^rp^h = {}^r\pi^h(^rv^h)^{-\psi_1{}^h} \left(\sum_{r'} {}^{r'}v^h\right)^{-\psi_2{}^h} \tag{7.23.8}$$

As already mentioned, this is the equation for total demand; the price level of good h in region r is assumed to depend on demand exerted for region r's product as well as total demand for product h. The exponents are price flexibilities.

$$^{r'r}x^h = \left(\xi_0 - \xi_1 \frac{^{r'}p^h}{\sum_{r'} {}^{r'}p^h}\right) \sum_{r'} {}^{r'r}x^h \tag{7.23.9}$$

This is the relation also mentioned as determining the relative quantities bought in region r' as a function of the price ratio of region r' to the average of all prices for good h. Coefficients ξ_0 and ξ_1 must satisfy the condition[1]

$$R\xi_0 - \xi_1 = 1$$

$$^r\bar{p}^h = \frac{\Sigma^{r'r}x^h \cdot {}^{r'}p^h}{\Sigma^{r'r}x^h} \tag{7.23.10}$$

[1] Strictly speaking, there are also boundary conditions, namely, $^{r'r}x^h \geq 0$; we shall not pursue this issue further here, however.

7.24 With a model of this kind the problem of maximizing national income for year θ, with the side condition that savings S_0 in year 0 are given, can be treated in a way similar to the one set out in Chap. 6. Income for the economy as a whole can be expressed again as a function of all ${}^{r}v^{h}$, since prices ${}^{r}p^{h}$ can be so expressed with the aid of (7.23.8). It will depend to a high degree on the nature of the functions, that is, the coefficients and exponents in (7.23.8), whether solutions are obtained obeying the boundary conditions, but the chances are better than they would be without the introduction of prices.

Clearly there are other problems which can be treated with the present model, for example, those mentioned in Sec. 7.16.

7.3. Static Models with Transportation Costs

7.31 In this section we present two models with transportation costs, the element so far neglected. The models are of a very simple, static type and focus on the phenomena implied by the introduction of transportation costs.[1] In the next section the development planning model, with transportation costs, will be taken up again.

Transportation costs arise from the movement of goods between geographically separated regions or centers. Generally, the costs per unit of product depend on the distance between the regions, the means of transportation used (rail, road, water), and the nature of the goods transported. Costs may be assumed proportional to distance, but when we want to distinguish between the fixed costs, including those of loading and unloading, and the variable costs, they are not. For long distances, however, variable costs prevail. Furthermore, transportation costs can be an absolute markup on the producer's price or a proportional one. An ad valorem tariff is an example of the latter case. Here we shall make the assumption of proportionality, but, in principle, it is not difficult to introduce alternative assumptions.

The models of this section subdivide the economy into industries and regions. The impact which transportation costs can have on the economy is further given its full weight by assuming that both demand and supply are influenced by prices. Therefore, the models distinguish supply and demand functions for each product in each region as functions of the relevant prices and incomes.

Two alternative assumptions can be made with regard to the reactions of demand to price changes.

1. The demand will fully shift to the cheapest supplier if one of the

[1] J. Tinbergen, The Appraisal of Road Construction: Two Calculation Schemes, *Rev. Econ. Stat.*, vol. 39, p. 241, August, 1957.

supply prices changes (assumption of infinite substitution between competitors).

2. The demand will only partly shift to cheaper suppliers (assumption of finite elasticity of substitution between competitors).

Two different models correspond to these two cases. The problem to be solved with these models is to estimate the consequences of a change in transportation costs. Such a change may be due to the construction or improvement of roads. The present models, therefore, may be used in the appraisal of road construction. In such a case the change in the national product caused by the decrease in transportation costs may be considered as a measure of the national returns of the project.

We discuss first a model based on the assumption of an infinite substitution elasticity in demand.

7.32 The variables of the model are

$^{rr'}v^h$ volume of product h transported from region r to region r'
$^{rr'}V^h$ value of $^{rr'}v^h$ in region r'
$^{r}p^h$ price of product h in production region r

7.33 The relationships of the model are the following.

$$^{rr'}V^h = {}^{rr'}v^h \cdot {}^{r}p^h \cdot {}^{rr'}T^h \tag{7.33.1}$$

These equations define the value of $^{rr'}v^h$ as the price at the delivery region r', that is, inclusive of transportation costs. The *transportation coefficients* $^{rr'}T^h$ are given constants; they are equal to 1 for $r = r'$.

If r supplies product h to r'

$$^{rr'}V^h = {}^{r'}\xi^h \sum_{h'} \sum_{r''} {}^{r'r''}V^{h'} \tag{7.33.2}$$

If r'' does not supply product h to r'

$$^{r''r'}V^h = 0 \tag{7.33.3}$$

These two sets of equations (.2) and (.3) represent expenditure equations. In (.2) the expression $\Sigma\Sigma^{r'r''}V^{h'}$ represents the income of r', and the expenditure on product h is a constant fraction $^{r'}\xi^h$ of this income. The propensities to spend ξ have to fulfill the condition $\sum_h {}^{r'}\xi^h = 1$, since we are considering a static model. The equations (.2) are valid only for the demand for product h from the region which has the lowest supply price in r'. In (.3), r'' indicates the regions not delivering to r'.

$$^{r}p^h \sum_{r'} {}^{rr'}v^h = {}^{r}\bar{\sigma}^h \cdot {}^{r}p^h - {}^{r}\sigma^h \sum_{h' \neq h} {}^{r'}p^{h'} \cdot {}^{r'r}T^{h'} \tag{7.33.4}$$

In these supply equations the expression at the left-hand side represents the production value of product h supplied by region r to other regions, including itself. At the right-hand side ${}^r\bar{\sigma}^h$ stands for the capacity limit, and ${}^r\sigma^h$ is a coefficient related to the supply elasticity. The equation assumes the quantity supplied to be a hyperbolic function of the price relation ${}^rp^h/\left(\sum_{h' \neq h} {}^{r'}p^{h'} \cdot {}^{r'r}T^{h'}\right)$. The denominator of this relation can be considered to express in a rough way the costs of producing product h in r.

7.34 The use to be made of the model is to estimate what the effect of a change in one (or more) of the transportation coefficients will be. As will be understood from the assumption of an infinite substitution elasticity, such a change affects not only the prices and, consequently, the quantities supplied and demanded, but also the pattern of commodity flows between the regions. For example, region r' buying product h from r may shift its demand to region r'', after a reduction of the transportation costs for product h between r' and r''. This feature of the model makes no easy solution possible. First, a provisional choice has to be made as to which are the cheapest supplying regions for each product, and on the basis of this choice it has to be tested by solving the model whether each region is actually buying from the cheapest supplier. If this test is not satisfied, a new choice has to be made, etc. Only by trial and error can we find the correct solution.[1]

7.35 The second model in this section is based on the assumption of a finite substitution elasticity. If the supply price of product h from region r' in r falls below the price of h originating from r'', the demand will only partly shift from r'' to r'. Thus, the same product may be purchased by r from more than one region. Clearly, this will happen only in the case of an imperfect market or, in other words, in the case of product differentiation.

A second assumption of this model is that each region produces one product only. This assumption makes a separate index for the product superfluous and, accordingly, we simplify the notation.

- ${}^{rr'}v$ volume of the product produced in r and transported to r'
- ${}^{rr'}V$ value of ${}^{rr'}v$ at the price in r'
- p^r price of the product produced by r in the production region

7.36 The equations of the model are the following.

$$ {}^{rr'}V = {}^{rr'}vp^r \cdot {}^{rr'}T \tag{7.36.1} $$

$$ {}^{rr'}V = {}^{r'}\bar{\eta}^r \sum_{r''} {}^{r'r''}V - \eta^{r'}p^r \cdot {}^{rr'}T + \frac{\eta^{r'}}{n-1} \sum_{r'' \neq r} p^{r''} \cdot {}^{r''r'}T \tag{7.36.2} $$

In these expenditure equations $\sum_{r''} V^{r'r''}$ measures the income of region r',

[1] For a numerical example, see *ibid.*

and ${}^{r'}\bar{\eta}^{r}$ represents the propensity to spend on the product from r by region r'. Thus, the first term on the right-hand side of the equations represents the influence of the income of region r' on its expenditures on the product of region r. The two other expressions indicate the influence on the expenditures of the price of the product purchased and of the prices of the other products purchased by region r', respectively. These last two influences are formulated in such a way that they cancel out when the expenditures of one region on the products of all regions are added up. On the assumption that $\sum_{r} {}^{r'}\bar{\eta}^{r} = 1$, the condition of a static model is fulfilled: the income of each region is fully spent.

$$p^r \sum_{r'} {}^{rr'}v = \bar{\rho}^r p^r - \rho^r \sum_{r' \neq r} p^{r'} \cdot {}^{r'r}T \qquad (7.36.3)$$

These supply equations do not differ from those of the first model (Sec. 7.33).

7.37 The solution of this model is simpler than that of the first because the location of the supplying regions does not shift after a change in the transportation costs. The solution proceeds along the following lines. The expenditures of all regions together on the product of one region r can be derived from the demand functions and expressed as a function of the prices of all products and the total income of the regions, which can be considered as given. By taking account of the transportation costs, the producer's value can be derived from the expenditures on r. This value should be equal to the producer's value as given by the supply function for r. Thus, by equating for each product the producer's value as derived from the demand and from the supply side, we find a set of linear equations with the prices as the only unknowns, which enables us to solve, first, the prices and, next, the values and the volumes. Numerical examples are given elsewhere.[1]

7.4. Development Model with Transportation Costs

7.41 The concepts and relations described in Sec. 7.3, in order to deal with transportation costs and their impact on the economy, will now be used in a model for development planning in many sectors and regions. As before, the model will be an input-output model; it will be possible to introduce elements of linear programming for such sectors where this makes sense. The model will be a further generalization of the one shown in Sec. 7.2.

[1] See *ibid.* and H. C. Bos and L. M. Koyck, The Appraisal of Road Construction Projects: A Practical Example, *Rev. Econ. Stat.*, vol. 43, no. 1, pp. 13–20, February, 1961.

We will consider transportation as a separate sector, which we give the index $h = 1$. Like other sectors, this sector will have its own inputs for current production as well as for investment. Furthermore, transportation will be treated as a regional sector in that all the transportation of goods originating from region r is considered to be produced in that region, that is, ${}^{r'r}x^1 = 0$ for $r' \neq r$ and ${}^{rr}x^1 = {}^{r}v^1$, as well as ${}^{r}e^1 = 0$. It will be assumed, however, that the inputs of transportation services into other sectors are not following the usual input-output schedule of proportionality. It will rather be assumed that all ${}^{r}\varphi^{1h} = {}^{r}\kappa^{1h} = 0$ and that the value of transportation services rendered by region r is equal to

$$\begin{aligned} {}^{rr}x_t^1 \cdot {}^{r}p_t^1 = \sum_h \sum_{r'} {}^{rr'}x_t^h \cdot {}^{r}p_t^h({}^{rr'}T^h - 1) \\ + {}^{r}\gamma^1({}^{r}Y - {}^{r}S) + {}^{r}\bar{c}^1 \sum_h {}^{r}\bar{p}^h \quad (7.41.1) \end{aligned}$$

On the right-hand side we first have the value increase due to transportation of all flows ${}^{rr'}x_t^h$ of goods transported from sector r to other sectors, ${}^{r}p_t^h$ being the price of these goods in region r and ${}^{r}p_t^h \cdot {}^{rr'}T^h$ their price in region r'. The second and third term on the right-hand side represent consumers' demand for transportation. Equation (7.41.1) rests on the assumption already mentioned that all transports starting in sector r are counted as transportation services produced in sector r. One could of course have followed different principles of computation here.

With regard to the distribution of demand exerted by any sector r over the sectors of origin, we make the same assumption as was made in Sec. 7.2. This corresponds to what was called in Sec. 7.3 the method of *finite elasticities of substitution.*

7.42 The variables of the model are

${}^{r}v_t^h$	production of commodity h in region r
${}^{r}v_t^{hh'}$	input of product h into current production of sector h' in region r
${}^{r}w_t^{hh'}$	input of product h for investment in sector h' in region r
${}^{r}c_t^h$	consumption of commodity h in region r
${}^{r}e_t^h$	exports of commodity h from region r to foreign countries
${}^{rr'}x_t^h$	flow of commodity h from region r to region r'
${}^{r}Y_t$	income of region r
${}^{r}S_t$	savings of region r
${}^{r}p_t^h$	price in region r of product h originating in region r
${}^{r}p_t^h \cdot {}^{rr'}T_t^h$	price in region r' of product h originating in region r
${}^{r}\bar{p}_t^h$	average price paid for product h in region r

All variables refer to period t.

7.43 The relations of this model are

$$ {}^r w_t^{hh'} = \frac{{}^r\kappa^{hh'}}{\theta} ({}^r v_{t+\theta}^{h'} - {}^r v_t^{h'}) \tag{7.43.1} $$

$$ \sum_r {}^r S_t = \sum_r \sum_h \sum_{h'} {}^r w_t^{hh'} \cdot {}^r \bar{p}_t^h \tag{7.43.2} $$

$$ {}^r S_t = \sigma^r Y_t \tag{7.43.3} $$

$$ {}^r Y_t = \sum_h {}^r p_t^h ({}^r v_t^h - \sum_{h'} {}^r \varphi^{hh'} \cdot {}^r v_t^{h'}) \tag{7.43.4} $$

$$ {}^r v_t^h = \sum_{r'} {}^{rr'} x_t^h \tag{7.43.5} $$

$$ {}^{rr} x_t^1 \cdot {}^r p_t^1 = \sum_h \sum_{r'} {}^{rr'} x_t^h \cdot {}^r p_t^h ({}^{rr'} T_t^h - 1) + {}^r\gamma^1 ({}^r Y - {}^r S) + {}^r \bar{c}^1 \sum_h {}^r \bar{p}^h \tag{7.43.6'} $$

$$ \sum_{r'} {}^{r'r} x_t^h = \frac{{}^r\gamma^h ({}^r Y_t - {}^r S_t)}{{}^r \bar{p}_t^h} + {}^r \bar{c}^h \frac{\sum_h {}^r \bar{p}_t^h}{{}^r \bar{p}_t^h} + \sum_{h'} {}^r v_t^{hh'} + \sum_{h'} {}^r w_t^{hh'} + {}^r e^h \qquad h = 2, \ldots, H \tag{7.43.6''} $$

As before, we have neglected price elasticities of consumer demand. If we do not want to neglect them, a term should be added: $\sum_{h'} {}^r\gamma^{hh'} \frac{{}^r \bar{p}_t^{h'}}{{}^r \bar{p}_t^h}$.

$$ {}^r v_t^{hh'} = {}^r \varphi^{hh'} \cdot {}^r v_t^{h'} \tag{7.43.7} $$

$$ {}^r p_t^h = {}^r \pi^h ({}^r v_t^h)^{-\psi_1^h} \left(\sum_{r'} {}^{r'} v_t^h\right)^{-\psi_2^h} \tag{7.43.8} $$

$$ {}^{r'r} x_t^1 = 0 \qquad \text{for } r' \neq r \tag{7.43.9'} $$

$$ {}^{r'r} x_t^h = \left(\xi_0 - \xi_1 \frac{{}^{r'} p_t^h \cdot {}^{r'r} T_t^h}{\sum_{r'} {}^{r'} p_t^h \cdot {}^{r'r} T_t^h} \right) \sum_{r'} {}^{r'r} x_t^h \qquad h = 2, \ldots, H \tag{7.43.9''} $$

$$ {}^r \bar{p}_t^h = \frac{\sum_{r'} {}^{r'r} x_t^h \cdot {}^{r'} p_t^h \cdot {}^{r'r} T_t^h}{\sum_{r'} {}^{r'r} x_t^h} \tag{7.43.10} $$

For $h = 1$ we take ${}^r \bar{p}_t^1 = {}^r p_t^1$.

7.44 The model has become more complicated than those discussed in Chap. 6; it has become necessary to distinguish between producers' price ${}^r p^h$ and consumers' price ${}^r \bar{p}^h$ for the same commodity h in the same region r. The most important planning problem which must be solved, the one discussed in Sec. 6.2, becomes more complicated as a consequence. We shall discuss its solution in this section. As before, we consider as given, at a time t, the production capacities of all sectors in all regions, and we assume that they are fully used, meaning that all ${}^r v_t^h$ are known. Consequently also the ${}^r p_t^h$ are known from Eq. (7.43.8), the ${}^r v_t^{hh'}$ from (7.43.7), and ${}^r Y_t$ and ${}^r S_t$ from Eqs. (7.43.4) and (7.43.3). Our planning problem is to determine ${}^r v_t^{hh'}$, ${}^r w_t^{hh'}$, ${}^{rr'} x_t^h$, ${}^r e^h$, and we shall need the ${}^r \bar{p}_t^h$ for this purpose.

As before, we must know the ${}^r v_{t+\theta}^h$ in order to be able to determine the unknowns just enumerated, since the ${}^r w_t^{hh'}$ depend on them by virtue of (7.43.1). In Sec. 6.2, we determined the ${}^r v_{t+\theta}^h$ from a maximization of $Y_{t+\theta}$, given total savings σY_t at time t. It is this part of the problem which has now become more complicated, since the side condition (7.43.2), under which this maximization of $Y_{t+\theta}$ has to be carried out, contains the unknown ${}^r \bar{p}_t^h$.

The exact method of solution consists in expressing, with the aid of Eqs. (7.43.1), (7.43.5), (7.43.9), and (7.43.10), the ${}^r \bar{p}_t^h$ and the other unknowns ${}^r w_t^{hh'}$ and ${}^{rr'} x_t^h$ in terms of ${}^r v_{t+\theta}^h$ (and ${}^r v_t^h$) and in finding the side condition (7.43.2) imposed on the ${}^r v_{t+\theta}^h$. From the maximization of $Y_{t+\theta}$, under this latter side condition, the ${}^r v_{t+\theta}^h$ must be determined. Since the side condition is no longer linear, this will be a cumbersome process.

For practical purposes it may be better to assume tentative values for ${}^r \bar{p}_t^h$ in side condition (7.43.2), which reduces the problem to the simple shape it took in Sec. 6.2. This will lead to provisional values of ${}^r V_{t+\theta}^h$.

After the values of ${}^r v_{t+\theta}^h$ have been found, the remaining unknowns can be found, that is, the unknowns ${}^r \bar{p}_t^h$, ${}^r w_t^{hh'}$, ${}^{rr'} x_t^h$, and ${}^r e_t^h$.

In case the tentative values assumed for ${}^r \bar{p}_t^h$ do not coincide with the ones just found, a second round of calculations will have to be started on the basis of the second approximations found for ${}^r \bar{p}_t^h$; strictly speaking, it should be proved that such an iteration process will converge.

It should be added that again the values for ${}^r e_t^h$ will obey a condition which now runs $\sum\limits_r \sum\limits_h {}^r e_t^h \cdot {}^r \bar{p}_t^h = 0$.