Seasonal Smooth Transition Autoregression

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Abstract

In this paper we put forward a new time series model, which describes nonlinearity and seasonality simultaneously. We discuss its representation, estimation of the parameters and inference. This seasonal STAR (SEASTAR) model is examined for its practical usefulness by applying it to 18 quarterly industrial production series. The data are tested for smooth-transition nonlinearity and for time-varying seasonality. We find that the model fits the data well for 14 of the 18 series. We also consider out-of-sample forecasting where we compare forecasts from the SEASTAR models with forecasts from nested models. It turns out that the SEASTAR model sometimes outperforms the other models, particularly for large horizons. Finally, we compare the SEASTAR models with STAR models for the 14 corresponding seasonally adjusted series, and we find that the estimated business cycle chronologies can be markedly different.

Key words: nonlinearity, smooth transition autoregression, seasonality, forecasting.

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1 Introduction

Additional to a trending pattern, several (quarterly observed) macroeconomic time series variables display pronounced seasonal fluctuations (at least, if they are not seasonally adjusted) and sometimes also signs of nonlinearity. The latter feature can be visualized by the apparent presence of business cycle regimes, where typically recessions cover only a few quarters while expansions last much longer. Examples of such variables are industrial production and employment. These variables are usually examined in order to investigate business cycle fluctuations in the individual series and to study possible dynamic correlations across the variables.

It is well known that the quality of the analysis of business cycle fluctuations in individual series can depend on the way one takes care of the trend and the seasonal fluctuations. For example, Canova (1994) shows that various detrending methods lead to different business cycle turning points. In a similar vein, Franses (1996) shows that various ways to treat seasonality lead to different business cycle chronologies. Furthermore, using empirical and simulated data Franses and Paap (1999) show that nonlinear time series models suggest different regime-switches, depending on whether the data are seasonally adjusted or not.

In this paper we propose a univariate time series model which incorporates explicit descriptions of seasonality and nonlinearity. As the concept of a trend in a nonlinear environment is not yet well defined, we assume that one can take care of a trend by first differencing the log-transformed data and converting the analysis to growth rates. To keep matters tractable, for the nonlinear part we apply a Smooth Transition Autoregression [STAR] model, see Teräsvirta (1994, 1998) and Granger and Teräsvirta (1993). To describe the seasonal fluctuations, we consider a combination of deterministic seasonal dummies and lags at the seasonal frequencies. As we wish to allow for the possibility that seasonal fluctuations change over time, and perhaps change according to the business cycle as is suggested by the empirical results in Canova and Ghysels (1994), we introduce a second function that indicates regime-switching behavior, where this function addresses seasonality. The final model is called a Seasonal STAR [SEASTAR] model. In a sense, this model is a specific version of the Multiple-Regime STAR model, introduced in Van Dijk and Franses (1999). In this paper, we examine the potential applicability of the SEASTAR model for various quarterly industrial production series.

The outline of the paper is as follows. In Section 2, we discuss various details concerning the SEASTAR model for quarterly data. First, we discuss representation, and next we outline a potentially useful specification strategy. We illustrate this specification strategy, which builds on the method proposed in Teräsvirta (1994) for the basic STAR model, for 18 country-specific quarterly industrial production series. Based on the relevant p-values of the LM-type test statistics, we find ample evidence of STAR-type nonlinearity and of non-constant seasonal fluctuations. In Section 3, we outline how one can estimate the parameters in the SEASTAR model and which diagnostic measures can be used to evaluate the empirical adequacy of the SEASTAR model. We present the estimation and diagnostic results for 14 of the 18 variables, as the model does not seem to fit well for 4 series. To save space, we give graphs for only three countries, that is, for the US, Canada and Italy. Detailed results for the other countries can be obtained from the authors. In Section 4, we outline how one can generate one-step and multi-step ahead forecasts from a SEASTAR model. A comparison of these forecasts for the 14 series with forecasts from linear models and from STAR models with constant seasonality shows that the SEASTAR model certainly deserves attention in practice. In Section 5, we investigate if STAR models can be fitted to the seasonally adjusted equivalents of the 14 variables considered, where we specifically compare the estimated business cycles. Finally, in Section 6, we conclude the paper with some remarks.

2 Representation and specification

In this section we first briefly discuss a STAR model, then we put forward our representation of a SEASTAR model, and finally we discuss an empirical specification strategy. We describe all material while having an application to quarterly growth rates of industrial production in mind. In the final subsection we focus on our illustrative variables.

2.1 A STAR model

Our SEASTAR model is a rather straightforward extension of the well-known STAR model (Teräsvirta, 1994). A STAR model of order p for a non-trending time series variable y_t is defined as

$$\begin{aligned} y_t \;=\;& F(z_{t-d};\gamma,\mu)(\phi_0^E + \phi_1^E y_{t-1} + \phi_2^E y_{t-2} + \ldots + \phi_p^E y_{t-p}) \\ &+ [1 - F(z_{t-d};\gamma,\mu)](\phi_0^R + \phi_1^R y_{t-1} + \phi_2^R y_{t-2} + \ldots + \phi_p^R y_{t-p}) + \epsilon_t, \end{aligned} \tag{1}$$

where $\epsilon_t \sim IID(0,\sigma_\epsilon^2)$. The transition function $F(\cdot)$ can be defined as the logistic function

$$F(z_{t-d};\gamma,\mu) = \frac{1}{1+\exp[-\gamma(z_{t-d}-\mu)]},\ \gamma > 0, \tag{2}$$

and this is also what we will do in the sequel. The transition variable z_{t-d} can be, for example, a lagged variable y_{t-d}. The function value approaches 0 if $z_{t-d} << \mu$. For increasing values of z_{t-d}, $F(z_{t-d};\gamma,\mu)$ attains increasing values between 0 and 1, where $F(z_{t-d};\gamma,\mu) = 0.5$ for $z_{t-d} = \mu$. The function $F(z_{t-d};\gamma,\mu)$ approaches 1 if $z_{t-d} >> \mu$. The parameter γ determines the smoothness of the function $F(\cdot)$. A large value of γ, relative to the values of z_{t-d}, implies less smoothness.

The STAR model is capable of describing time series data which experience two different regimes. Within these regimes, the data are described by two different AR processes, and across the regimes transitions can occur more or less smoothly. When analyzing economic time series variables, we are tempted to call the two regimes recessions and expansions. If z_{t-d} is a monotonical increasing function of past growth in industrial production, implying that when z_{t-d} is below some μ there is not enough growth, one might consider the corresponding regime a recession. Therefore, we label the parameters in (1) with the superscripts R and E.

2.2 Representation

The STAR model can be useful for nonseasonal (or seasonally adjusted) data, which display STAR type nonlinearity. The SEASTAR model nests the STAR model, but it is also applicable to time series data with a seasonal pattern. In this paper we will focus on quarterly data, but a generalization of the model for data with a different frequency is straightforward.

Denote $D_{s,t}$ $(s = 1, 2, 3, 4)$ as quarterly seasonal dummy variables with $D_{s,t} = 1$ when time t corresponds with season s, and $D_{s,t} = 0$ otherwise. The seasonal STAR [SEASTAR] model, for a time series variable y_t, is now given by

$$\begin{aligned} y_t \quad = \quad & F_s(w_{t-d_s}; \gamma_s, \mu_s)(\delta_1^E D_{1,t} + \delta_2^E D_{2,t} + \delta_3^E D_{3,t} + \delta_4^E D_{4,t}) \\ & + [1 - F_s(w_{t-d_s}; \gamma_s, \mu_s)](\delta_1^R D_{1,t} + \delta_2^R D_{2,t} + \delta_3^R D_{3,t} + \delta_4^R D_{4,t}) \\ & + F_c(z_{t-d_c}; \gamma_c, \mu_c)(\phi_1^E y_{t-1} + \phi_2^E y_{t-2} + \ldots + \phi_p^E y_{t-p}) \\ & + [1 - F_c(z_{t-d_c}; \gamma_c, \mu_c)](\phi_1^R y_{t-1} + \phi_2^R y_{t-2} + \ldots + \phi_p^R y_{t-p}) + \epsilon_t, \end{aligned} \tag{3}$$

where ϵ_t has been defined before, and where w_t also denotes a transition variable. Notice that, compared to (1), the intercepts ϕ_0^R and ϕ_0^E are redundant because of the inclusion of two times four seasonal dummies in the model. We define the two transition functions $F_s(\cdot)$ for changing deterministic seasonal variation and $F_c(\cdot)$ for the business cycle by the logistic functions

$$F_s(w_{t-d_s}; \gamma_s, \mu_s) = \frac{1}{1 + \exp[-\gamma_s(w_{t-d_s} - \mu_s)]}, \; \gamma_s > 0 \tag{4}$$

and

$$F_c(z_{t-d_c}; \gamma_c, \mu_c) = \frac{1}{1 + \exp[-\gamma_c(z_{t-d_c} - \mu_c)]}, \; \gamma_c > 0. \tag{5}$$

The transition variables w_t and z_t in (3) can be a function of $(y_t, y_{t-1}, \ldots)$, like for example $\Delta_4 y_t = y_t - y_{t-4}$, or a linear deterministic trend t. When $w_t = t$, the function value of the seasonal transition function $F_s(\cdot)$ changes smoothly from 0 to 1 as time progresses, and thus represents a smooth structural break in the deterministic seasonality in the data. The delay parameters $d_s \geq 1$ and $d_c \geq 1$ are included to represent the possibility that the dependent variable y_t correlates with some delay with changes in the respective transition variables. In principle, $\gamma_c \neq \gamma_s$, $d_c \neq d_s$ and $\mu_c \neq \mu_s$, although parameter restrictions can be imposed in practice. Notice that the SEASTAR model focuses on changing deterministic seasonality. Obviously, if a SEASTAR model is truly generating the data, one may find the presence of seasonal unit roots in an otherwise linear model.

The SEASTAR model is a straightforward generalization of the STAR model (1)-(2). When $\gamma_s = 0$ in (4) the function value of $F_s(\cdot)$ is constant and equal to 0.5 for all values of w_t. In this case the seasonal pattern in the data is described by the four seasonal dummies with respective coefficients $(\delta_s^E + \delta_s^R)/2$, for $s = 1, 2, 3, 4$. The resulting model is a STAR model without regime switches in the seasonal component, that is,

$$\begin{aligned} y_t &= \delta_1 D_{1,t} + \delta_2 D_{2,t} + \delta_3 D_{3,t} + \delta_4 D_{4,t} \\ &\quad + F_c(z_{t-d_c}; \gamma_c, \mu_c)(\phi_1^E y_{t-1} + \phi_2^E y_{t-2} + \ldots + \phi_p^E y_{t-p}) \\ &\quad + [1 - F_c(z_{t-d_c}; \gamma_c, \mu_c)](\phi_1^R y_{t-1} + \phi_2^R y_{t-2} + \ldots + \phi_p^R y_{t-p}) + \epsilon_t. \end{aligned} \tag{6}$$

Another noteworthy restricted variant of the general SEASTAR model in (3) is the model with $\gamma_s = \gamma_c = 0$. In this case, (3) results in the linear seasonal model, that is,

$$\begin{aligned} y_t &= \delta_1 D_{1,t} + \delta_2 D_{2,t} + \delta_3 D_{3,t} + \delta_4 D_{4,t} \\ &\quad + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t, \end{aligned} \tag{7}$$

which is a model with constant seasonal dummy parameters and AR(p) dynamics. Furthermore, one can consider a restricted SEASTAR model with $F_c(\cdot) = F_s(\cdot)$, that is,

$$\begin{aligned} y_t &= F(w_{t-d}; \gamma, \mu)(\delta_1^E D_{1,t} + \delta_2^E D_{2,t} + \delta_3^E D_{3,t} + \delta_4^E D_{4,t} + \phi_1^E y_{t-1} + \ldots + \phi_p^E y_{t-p}) \\ &\quad + [1 - F(w_{t-d}; \gamma, \mu)](\delta_1^R D_{1,t} + \delta_2^R D_{2,t} + \delta_3^R D_{3,t} + \delta_4^R D_{4,t} + \phi_1^R y_{t-1} + \ldots + \phi_p^R y_{t-p}) \\ &\quad + \epsilon_t. \end{aligned} \tag{8}$$

In this last model it is assumed that deterministic seasonality changes with the business cycle. Additional to the SEASTAR model, we will also use the three restricted variants (6), (7) and (8) in our empirical study below.

2.3 Specification strategy

Before fitting a specific nonlinear model to time series data, it is common practice first to test whether this model can be suitable for the data instead of a linear model. Now, the null hypothesis of linearity can be expressed as equality of the autoregressive parameters in the two business cycle regimes and at the same time equality of the seasonal parameters in the two seasonal regimes in (3), that is $H_0 : \phi_i^E = \phi_i^R, (i = 1, \ldots, p)$ and $\delta_j^E = \delta_j^R, (j = 1, 2, 3, 4)$. This is to be tested against the alternative hypothesis $H_1 : \phi_i^E \neq \phi_i^R$ and/or $\delta_j^E \neq \delta_j^R$ for at least one value of i, j. Notice that under H_0 the parameters γ_s, μ_s, γ_c and μ_c are unidentified. To circumvent this problem we propose a test that expands on the one proposed in Teräsvirta (1994), which tests for STAR-type nonlinearity in nonseasonal time series data. Our test discriminates between the linear seasonal model (7) and the SEASTAR model (3). The procedure is as follows. First, one should decide which order p is appropriate for a linear seasonal model for the data. A possible method is to choose the AR-order p which corresponds with a minimum value for AIC. Another possibility is to a priori set the order at a chosen value, for example $p = 4$ or $p = 5$ for quarterly data. Second, one should decide which variables may be suitable as transition variables z_{t-d_c} and w_{t-d_s} in (3). Previous experience with the STAR model has indicated that $z_{t-d_c} = \Delta_4 y_{t-d_c}$ can be a good choice for the business cycle part, see Teräsvirta and Anderson (1992) for example. As the seasonal transition variable w_{t-d_s}, one can opt for $\Delta_4 y_{t-d_s}$ or t. The last option results into what is called a Time-Varying STAR model. Notice that the proposed transition variables should be, at least approximately, free of seasonality. This is important for the assumption that there are smooth transitions from one regime to the other.

The linearity tests make use of the following auxiliary regression, which is obtained from the SEASTAR model (3)-(5) by replacing the transition functions by a Taylor approximation, that is,

$$
\begin{aligned}
y_t &= \sum_{s=1}^{4}(\delta_s D_{s,t} + \delta_{s,1} D_{s,t} w_{t-d_s} + \delta_{s,2} D_{s,t} w_{t-d_s}^2 + \delta_{s,3} D_{s,t} w_{t-d_s}^3) \\
&\quad + \sum_{i=1}^{p}(\phi_i y_{t-i} + \phi_{i,1} y_{t-i} z_{t-d_c} + \phi_{i,2} y_{t-i} z_{t-d_c}^2 + \phi_{i,3} y_{t-i} z_{t-d_c}^3) + u_t. \qquad (9)
\end{aligned}
$$

The test amounts to the familiar F-test for the significance of the cross-product variables $D_{s,t}w_{t-d_s}$, $D_{s,t}w_{t-d_s}^2$, $D_{s,t}w_{t-d_s}^3$, $(s = 1, \ldots, 4)$ and/or the cross-product variables $y_{t-i}z_{t-d_c}$, $y_{t-i}z_{t-d_c}^2$, $y_{t-i}z_{t-d_c}^3$, $(i = 1, \ldots, p)$, where the first test concerns the seasonal cycle (F_s) and the second the business cycle (F_c). If these variables are significant, a SEASTAR model can be considered for the data.

2.4 Quarterly industrial production

To illustrate the test procedure, we consider quarterly seasonally unadjusted data on industrial production for 18 OECD countries, that is, Austria, Belgium, Canada, Finland, France, Greece, Ireland, Italy, Japan, Luxemburg, The Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom (UK) and the United States (US). The data source is Datastream. The sample period covers 1960:1-1997:1. The series for Canada and Spain start in 1961:1, and for Greece in 1962:1. The series for Austria ends in 1995:4, for Canada, Greece, Norway and Sweden in 1995:1, for Ireland in 1994:4, and for Luxemburg and Switzerland the last observation is 1996:4. The data are transformed by taking logs and first differences. The seasonal difference of a logarithmic transformed series represents the seasonal growth rate, and, as mentioned before this variable may be useful as a transition variable.

The first-differenced and seasonally differenced log-transformed series of 3 G7 series, that is Canada, Italy and the US, are depicted in Figures 1 to 3. For Canada (Figure 1) there are no clear signs of changing seasonality for the first-differenced log-transformed series, although the mean growth rate appears to decline over the sample period. The seasonally differenced log-transformed series indicates a clear recession in 1983 and 1991.

For Italy (Figure 2) seasonality is clearly not constant (upper panel), and we see also pronounced business cycle fluctuations (lower panel). Finally, for the US we see in Figure 3 an outlier in 1975 and recessions in or around the years 1970, 1975, 1982 and 1991. Also, changing seasonality may be noticed in the upper panel.

We apply the test for SEASTAR nonlinearity to the 18 series. For convenience, the order p is fixed at 5. We denote x_t as the original industrial production series. We confine ourselves to the cases where $z_{t-d_c} = w_{t-d_s} = \Delta_4 \ln x_{t-d} (d_c = d_s = d \in \{1, \ldots, 4\})$, and $z_{t-d_c} = \Delta_4 \ln x_{t-d_c} (d_c \in \{1, \ldots, 4\}), w_{t-d_s} = t$. For these eight cases we perform the auxiliary regression (9) and we test for the significance of the cross-product variables. The resulting p-values are given in Table 1. When for a series the p-values under F_s and F_c are both small, a SEASTAR model with the combination of the relevant transition variables is considered suitable for the data. We see that, for the choice $w_t = t$ in the right-hand panel of Table 1, nonlinearity is detected for the seasonal part F_s in all cases. Notice also that there are series (Belgium, Spain, France, Italy and the US) for which no combination results in a p-value smaller than 0.10 for both F_s and F_c. As our main interest lies in examining the possible applicability of the SEASTAR model, we will nevertheless choose a combination for each of the 18 countries, usually by considering the lowest p-values.

3 Parameter estimation and diagnostics

In this section we elaborate on the estimation of the parameters in the SEASTAR model, and we discuss some diagnostics on the residuals of a fitted SEASTAR model.

3.1 Parameter estimation

Consider again the SEASTAR model (3)-(5). Call θ the vector of model parameters, that is,

$$\theta = \{\phi_1^R, \ldots, \phi_p^R, \phi_1^E, \ldots, \phi_p^E, \delta_1^R, \ldots, \delta_4^R, \delta_1^E, \ldots, \delta_4^E, \gamma_s, \mu_s, \gamma_c, \mu_c\}. \tag{10}$$

When the last four elements of θ are fixed, the model reduces to a model, which is linear in the other parameters. In this case, the remaining parameters can be estimated by ordinary least squares. When we choose a reasonable grid of values for $\gamma_s, \mu_s, \gamma_c, \mu_c$ and calculate the residual variance of every corresponding fitted linear model, we find a suboptimal model, in the sense of least residual variance. The resulting model parameters are then taken to be the starting values for the next step, that is, nonlinear least squares (NLS) optimization. This NLS boils down to minimizing the residual sum of squares of a fitted SEASTAR model, that is,

$$\sum_{t=1}^{T}(y_t - G(I_t;\theta))^2, \tag{11}$$

with respect to θ. Here, I_t is the information set $y_t, y_{t-1}, \ldots$, and $G(I_t;\theta)$ is the so-called skeleton of the SEASTAR model, that is, the right hand side of (3) without the noise term ϵ_t. When the error process ϵ_t in (3) is normally distributed, NLS is equivalent to maximum likelihood estimation.

For the empirical data, we fit SEASTAR models, where the transition variables w_{t-d_s} and z_{t-d_c} are set according to the results in Table 1. In some cases, simply taking the combination of transition variables, which resulted in the smallest p-values for the linearity test, does not automatically result in satisfactory estimation results. Therefore, we take the combination of transition variables which correspond with small p-values ($< .15$) in Table 1, with transition functions in which the two regimes actually appear, and with residual diagnostics (to be discussed below) that do not suggest misspecification. The order p in the AR part of the model is determined by AIC. In a few cases it turns out that this p is larger than the length of all occurring regimes where the business cycle function F_c attains the value 0. Considering this to be undesirable, we reduce the AR-order in this regime, and this leads to satisfactory results. Indeed, for Belgium, Canada, Greece, Japan, Luxemburg, Switzerland and the UK this approach is successful.

For many series it appears impossible to fit a satisfactory SEASTAR model, in the sense that the residuals show no significant serial correlation and that there are enough data points in both regimes. For these cases it proves helpful to eliminate the influence of one or a few aberrant data points by including dummy regressors dum_{t,t^*}, defined by

$$dum_{t,t^*} = \begin{cases} 1 & \text{if } t = t^* \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where we implicitly assume that the outliers are innovation outliers. For four series this procedure is still not sufficient. The French industrial production series has two very dominant additive outliers. They concern the quarters 1963.1 and 1968.2. We remove them by replacing the log-value of an outlying observation at time t^* by the average of the log-values at $t^* - 4$ and $t^* + 4$. As this can be rather influential to the outcome of the SEASTAR test, we redo the test for France with the modified series. After these corrections, we find an acceptable SEASTAR model for the French series.

Norway exhibits a prominent additive outlier in the second quarter of 1975. Removing this in the same manner as for France, however, does not result in an acceptable SEASTAR model. The estimated F_c-function attains the value 0 almost everywhere, which would suggest a permanent recession. For Finland and Sweden the fitted SEASTAR models feature estimated parameters in both regimes which do not differ much from each other in both the business cycle part and the seasonal part. This indicates that no distinction between the regimes can be made, and thus that a SEASTAR model is not adequate for these data sets. The estimated transition function values $F_c(z_{t-d_c})$ are almost always equal to 0. Finally, for Japan we find AR parameters which suggest explosive behavior. Hence, we decide to discard the four countries in our further analysis. The location of the supposed outliers in the remaining 14 series is given in Table 2.

The results of our estimation procedure can be found in Table 3. As we want to analyze the out-of-sample forecasting performance of the models, we only estimate the parameters for samples ending in 1988.4. Notice that no standard errors are given for the estimated smoothness parameters γ_s and γ_c. The reason for this is that it can be extremely difficult to estimate these parameters accurately. Particularly for large values of the smoothness parameter, the associated transition function approaches the indicator function. Comparing the parameters $\hat{\phi}_i^R$ with $\hat{\phi}_i^E$ and $\hat{\delta}_j^R$ with $\hat{\delta}_j^E$, we see that in almost all cases there are different regimes in seasonality. Only the seasonal regimes for Austria show some similarity.

In Figures 4 to 6 we present the estimated transition functions for Canada, Italy and the US. For Canada (Figure 4) we see that F_c and F_s are almost the same. This comes as no surprise, as the transition variables z_{t-d_c} and w_{t-d_s} are equal and also the estimated γ_c and γ_s as well as μ_c and μ_s are almost equal, see Table 3. The F_c for Italy (Figure 5) shows regular fluctuations until 1981, after which a long recession follows. Finally, the transition functions for the US in Figure 6 indicate four very clear recessions: 1971, 1975, 1981 and 1983.

3.2 Diagnostics

Evaluating estimated SEASTAR models involves the properties of the resulting residuals, for example by testing whether the residuals are serially correlated. The standard Ljung-Box test for residual serial correlation does not apply when the data are generated by a STAR, see Eitrheim and Teräsvirta (1996). In the same paper, these authors therefore propose a proper LM-test for residual correlation, and we will follow their suggestion. Testing whether the residuals are approximately Gaussian by calculating the Jarque-Bera statistic may also indicate the correctness of the model specification. Aberrant observations in the data may have an impact on the estimation results. Outliers have already been taken into account in the estimation phase, but only in an informal manner. Therefore, we also look at the skewness of the residuals. If the residuals are skewed, this may indicate the presence of remaining outliers. Finally, we test for ARCH.

The residuals of the 14 fitted models are tested for normality, autocorrelation and ARCH. The resulting p-values can be found in Table 4. We see that most test results do not give reason for concern, when utilizing a confidence level of 95%. An exception is the ARCH test result for Canada. In sum, it seems that we can successfully fit SEASTAR models to 14 of the 18 industrial production series initially considered.

4 Forecasting

In this section we examine if the fitted SEASTAR models perform well in terms of forecasting. First, we outline how one can generate out-of-sample forecasts.

4.1 Generating forecasts

For ease of notation, in this section we write $F_s(\cdot)$ for $F_s(\cdot;\gamma_s,\mu_s)$, and $F_c(\cdot)$ for $F_c(\cdot;\gamma_c,\mu_c)$. We assume that all the parameters in (3)-(5) are known. Of course, this is in practice generally not the case, giving cause to extra uncertainty. Call T the forecast origin, and I_T the information set available at time T, that is $I_T = y_T, y_{T-1}, \ldots$. The minimum mean squared error (MMSE) conditional h-step-ahead point forecast $\hat{y}_{T+h|T}$ is given by

$$\hat{y}_{T+h|T} = E[y_{T+h}|I_T]. \tag{13}$$

For $h = 1$, this results in

$$\begin{aligned}\hat{y}_{T+1|T} \quad = \quad & F_s(w_{T+1-d_s})(\sum_{i=1}^{4}\delta_i^E D_{i,T+1}) + [1-F_s(w_{T+1-d_s})](\sum_{i=1}^{4}\delta_i^R D_{i,T+1}) \\ & + F_c(z_{T+1-d_c})(\sum_{i=1}^{p}\phi_i^E y_{T-i+1}) + [1-F_c(z_{T+1-d_c})](\sum_{i=1}^{p}\phi_i^R y_{T-i+1}),\end{aligned} \tag{14}$$

where we have used that $E[f(y_T)|I_T] = f(y_T)\,\forall f, T$ and $E[\epsilon_{T+j}|I_T] = 0$, $(j > 0)$. This means that the 1-step-ahead MMSE forecast can be calculated straightforwardly. As all terms on the right hand side of (14) are known at T, the variance of this prediction is equal to the variance of ϵ_{T+1}, which is σ_ϵ^2.

For h-step-ahead forecasting, matters become more complicated. In case $d_s, d_c \geq h$, $F_s(w_{T+h-d_s})$ and $F_c(z_{T+h-d_c})$ belong to I_T, and are thus given. In this case, one has

$$\begin{aligned}\hat{y}_{T+h|T} \quad = \quad & F_s(w_{T+h-d_s})(\sum_{i=1}^{4}\delta_i^E D_{i,T+h}) + [1-F_s(w_{T+h-d_s})](\sum_{i=1}^{4}\delta_i^R D_{i,T+h}) \\ & + F_c(z_{T+h-d_c})(\sum_{i=1}^{p}\phi_i^E \hat{y}_{T+h-i|T}) + [1-F_c(z_{T+h-d_c})](\sum_{i=1}^{p}\phi_i^R \hat{y}_{T+h-i|T}).\end{aligned} \tag{15}$$

Note that if $p \geq h$, $\hat{y}_{T|T}, \ldots, \hat{y}_{T+h-p|T}$ can be replaced by their actual values $y_T, \ldots, y_{T+h-p}$.

When $d_s < h$ or $d_c < h$, forecasting is not as straightforward, that is,

$$\begin{aligned} \hat{y}_{T+h|T} \;&=\; E[F_s(w_{T+h-d_s})|I_T](\sum_{i=1}^{4}\delta_i^E D_{i,T+h}) + \; E[1 - F_s(w_{T+h-d_s})|I_T](\sum_{i=1}^{4}\delta_i^R D_{i,T+h}) \\ &+ E[F_c(z_{T+h-d_c})(\sum_{i=1}^{p}\phi_i^E y_{T+h-i})|I_T] + \; E[(1 - F_c(z_{T+h-d_c}))(\sum_{i=1}^{p}\phi_i^R y_{T+h-i})|I_T]. \end{aligned} \tag{16}$$

As $F_s(\cdot)$ and $F_c(\cdot)$ are nonlinear functions, we can not replace $E[F_s(\cdot)]$ by $F_s(E[\cdot])$, or $E[F_c(\cdot)]$ by $F_c(E[\cdot])$. A few approximating methods have been proposed, see for example De Gooijer and de Bruin (1998). A first is the naive method, where for every forecast horizon h the noise term ϵ_{T+h} is put to zero. A second method is the Monte Carlo method. The expected value of y_{T+h} conditional on I_T can be approximated with the help of computer simulations. Let M be a large number, and $y_{T+1}^{(i)}, (i = 1, \ldots, M)$ be simulated realizations of y_{T+1}, making use of (3), where the noise terms, say $\epsilon_{T+1}^{(i)}$, are produced by a random number generator and drawn from the assumed distribution of the noise terms. In a similar vein, $y_{T+2}^{(i)}, (i = 1, \ldots, M)$ can be produced, where in the underlying process y_{T+1} is filled in by $y_{T+1}^{(i)}$. Continuing this way to h steps, the Monte Carlo h-step-ahead forecast is the geometric mean, given by

$$\hat{y}_{T+h}^{mc} = \frac{1}{M}\sum_{i=1}^{M} y_{T+h}^{(i)} \tag{17}$$

Finally, one can consider the bootstrap method. This is similar to the Monte Carlo method, but this time the noise terms $\epsilon_{T+h}^{(i)}$ are not produced by a random number generator, but drawn randomly from the residuals of the fitted model. This has the advantage that no assumptions have to be made about the noise distribution. As our diagnostic test results do not reject the approximate normality of the estimated residuals, we consider the Monte Carlo method in the sequel.

4.2 Empirical results

The Monte Carlo forecasting method is applied to the 14 industrial production series at hand. As we are dealing with quarterly data, we are interested in 1-step-ahead, in 4-, 8- and 12-step-ahead forecasts. As mentioned before, for the specification and estimation procedure of SEASTAR models, the data up to and including 1988.4 (in the following indicated by T) are used. When an empirical series contains, say, N data points, then there are for every series still a sufficient number, $N-T$, of data available for out-of-sample forecasting purposes. We do not re-estimate the parameters in the models when the forecast origin shifts from $T+1$ to $N-1$.

In order to quantify the accuracy of forecasts produced by the SEASTAR model for a series, a comparison is made between the forecasts of a SEASTAR model and the restricted model (6), that is, the STAR model with constant seasonality and (7), that is, the linear model. These restricted models are also fitted to the industrial production data up to 1988.4, where we take the same value of p as for the SEASTAR model. For some countries we also fit (8) when the transition variables and delay parameters are the same. In Table 5, we give the AIC and BIC values for the various models. Upon using AIC, one would prefer the SEASTAR model in 13 of the 14 cases. In case of Canada, one would select the model with seasonality changing with the business cycle. Interestingly, and in contrast to many diagnostic test results for nonlinearity in Table 1, when using BIC one would prefer a linear model in 8 of the 14 cases. Still, for Spain, Italy, Luxemburg, The Netherlands, Portugal and the UK one would select a SEASTAR model based on BIC.

We generate forecasts for the SEASTAR model and for (6) and (7). The mean squared prediction error is calculated. For h-steps-ahead, this MSPE is defined as

$$MSPE = \frac{1}{N-T-h+1} \sum_{j=h}^{N-T} (\hat{y}_{T+j|T+j-h} - y_{T+j})^2 \tag{18}$$

The forecast evaluation results are given in Table 6. For one-step-ahead forecasts, the SEASTAR model outperforms its competitors for Belgium, France, The UK and the US. The linear model is seen to generate rather good forecasts, as compared with the SEASTAR and STAR model. The SEASTAR model yields considerably poor results for the Netherlands.

The forecast performance of the SEASTAR improves considerably if the forecast horizon is extended to four, eight and twelve steps. The model seems particularly useful for Belgium, Italy, The UK and the US. In general we can conclude that the SEASTAR model can sometimes outperform the STAR model and the linear model. For some countries, however, the linear model is hard to beat.

5 Effects of seasonal adjustment

In this penultimate section we briefly investigate the influence of seasonal adjustment on estimated regime shifts in the data. For this purpose we obtain the corresponding seasonally adjusted industrial production data for the 14 countries. We test for STAR type nonlinearity in the growth rates, following the method described in Teräsvirta (1994). This test is essentially the same as the SEASTAR test in subsection 2.3, where now all seasonal dummies are left out. The resulting p-values can be found in Table 7. STAR models are fitted, where the delay of the transition variable, which is the annual growth rate of the seasonally adjusted series, follows from Table 7.

The resulting transition function values for the STAR models for adjusted data are compared with the function values of F_c for the SEASTAR models for the corresponding seasonally unadjusted series. A measure of agreement across the functions is calculated. The results can be found in Table 8. We find that, apart from Austria and France, there is substantial agreement, but also that this agreement is not perfect. The values of kappa differ significantly from 1 for all series. This can be interpreted as that there are quite a number of observations which get different classifications. Apart from Austria, this usually amounts to about 5% to 12% of the data points. Hence, it seems that the SEASTAR model for unadjusted data and the STAR model for adjusted data can lead to different business cycle chronologies.

6 Concluding remarks

In this paper we introduced a new STAR-type nonlinear model with two components, that is, a component for the business cycle and a component for changing deterministic seasonality. When illustrated for 14 quarterly industrial production series, we found that the resultant SEASTAR model had good in-sample fit and did not get rejected when using residual diagnostics. Its out-of-sample forecasts turned out to be a good competitor in some cases to more restricted models. Finally, the estimated business cycle from a SEASTAR model could be quite different from that estimated from a STAR model for adjusted data. In general, we found almost no evidence that seasonal fluctuations changed with the business cycle.

In our further work we aim to extend the univariate SEASTAR model to a multivariate setting. This can be useful for the analysis of common business cycles and/or common seasonal fluctuations across variables.

Table 1: The p-values of SEASTAR type nonlinearity tests for 18 quarterly industrial production series (based on all observations)

	$z_{t-d_c} = w_{t-d_s} = \Delta_4 \ln x_{t-d}$								$z_{t-d} = \Delta_4 \ln x_{t-d}, w_t = t$							
	$d=1$		$d=2$		$d=3$		$d=4$		$d=1$		$d=2$		$d=3$		$d=4$	
Country	F_s	F_c	F_s	F_c	F_s	F_c	F_s	F_c	F_s	F_c	F_s	F_c	F_s	F_c	F_s	F_c
Austria	.082	.088	.819	.117	.462	.169	.772	.429	.001	.006	.005	.235	.002	.288	.009	.764
Belgium	.718	.116	.338	.685	.166	.529	.602	.446	.000	.156	.000	.463	.000	.164	.000	.115
Canada	.000	.001	.101	.012	.565	.295	.687	.382	.000	.175	.000	.110	.000	.176	.001	.337
Spain	.111	.252	.156	.220	.377	.610	.551	.433	.020	.805	.009	.668	.003	.208	.009	.118
Finland	.058	.111	.532	.280	.603	.922	.537	.397	.001	.170	.001	.052	.002	.093	.001	.033
France	.103	.001	.268	.123	.225	.485	.403	.199	.000	.247	.000	.632	.000	.345	.000	.338
Greece	.368	.438	.019	.034	.084	.088	.941	.562	.000	.692	.000	.820	.002	.086	.000	.057
Ireland	.691	.454	.120	.120	.033	.026	.422	.754	.000	.198	.000	.194	.000	.210	.000	.311
Italy	.165	.122	.564	.838	.839	.751	.798	.311	.001	.611	.000	.907	.001	.149	.005	.373
Japan	.745	.796	.156	.150	.322	.011	.881	.916	.000	.000	.000	.004	.000	.027	.000	.530
Luxemburg	.941	.475	.962	.727	.204	.269	.056	.008	.010	.003	.015	.106	.002	.331	.013	.045
The Netherlands	.569	.166	.124	.043	.068	.009	.007	.012	.076	.331	.004	.070	.047	.233	.011	.444
Norway	.094	.226	.649	.701	.530	.198	.097	.037	.001	.006	.001	.024	.001	.066	.017	.383
Portugal	.251	.236	.729	.430	.074	.024	.123	.810	.002	.384	.002	.045	.000	.054	.009	.874
Sweden	.539	.774	.654	.845	.109	.104	.507	.178	.023	.858	.007	.999	.011	.168	.000	.001
Switzerland	.002	.007	.037	.530	.509	.813	.601	.420	.058	.178	.082	.223	.058	.113	.058	.060
UK	.200	.038	.260	.465	.214	.338	.025	.084	.000	.011	.001	.883	.000	.505	.000	.132
US	.235	.061	.845	.735	.830	.671	.517	.291	.005	.109	.003	.126	.046	.223	.032	.193

Note: The test procedure is discussed in subsection 2.3. The AR order p in (9) is fixed at 5. F_s means that in this column one can find the p-values of the test for redundant cross-product variables with seasonal components. F_c means the same, but now with respect to the cyclical cross-product variables.

Table 2: Observations for which a dummy regressor is added to the SEASTAR model

Country	
Austria	1972.4,1973.1
Belgium	-
Canada	-
Spain	-
France	1974.4
Greece	1974.2
Ireland	1969.1, 1969.2
Italy	1969.4, 1970.1, 1974.4
Luxemburg	-
The Netherlands	-
Portugal	1972.3, 1972.4
Switzerland	1974.4, 1975.1, 1975.4
UK	1972.1, 1972.2, 1974.1
US	1974.4, 1975.1

Table 3: Estimated parameters in SEASTAR models for 14 industrial production series (all samples end in 1988.4)

	Country						
	Austria	Belgium	Canada	Spain	France	Greece	Ireland
z_{t-d_c}	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-3}$	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-4}$	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-2}$	$\Delta_4 \ln x_{t-3}$
w_{t-d_s}	$\Delta_4 \ln x_{t-1}$	t	$\Delta_4 \ln x_{t-1}$	t	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-2}$	$\Delta_4 \ln x_{t-3}$
$\hat{\gamma}_c$	488	10827	137.8	335	88.63	8718	311
$\hat{\mu}_c$	.01 (.00)	.03 (.00)	.03 (.01)	.07 (.00)	-.01 (.00)	.05 (.00)	.06 (.00)
$\hat{\gamma}_s$	1566	8.89	147.1	1724	115.2	8718	90.83
$\hat{\mu}_s$	.01 (.00)	.26 (.10)	.03 (.01)	.46 (3.87)	.00 (.01)	.09 (.07)	.06 (.01)
$\hat{\phi}_1^E$	-.04 (.10)	.06 (.11)	.11 (.15)	-.03 (.14)	-.33 (13)	-.22 (.11)	.11 (.14)
$\hat{\phi}_2^E$		.37 (.12)	-.26 (.15)	-.19 (.10)	.13 (.14)	-.24 (.11)	-.38 (.15)
$\hat{\phi}_3^E$		.10 (.10)	-.21 (.14)	.01 (.10)	-.02 (.13)	.28 (.10)	.24 (.16)
$\hat{\phi}_4^E$		-.08 (.10)	.61 (.14)	.07 (.10)	.36 (.13)	.51 (.11)	.28 (.14)
$\hat{\phi}_5^E$		-.07 (.11)	-.31 (.13)	-.25 (.14)	.07 (.11)		
$\hat{\phi}_6^E$		-.39 (.10)					
$\hat{\phi}_1^R$	.80 (.20)	-.06 (.11)	-.05 (.18)	-.31 (.11)	.65 (.25)	.03 (.13)	-.22 (.11)
$\hat{\phi}_2^R$		-.16 (.10)	.58 (.22)	.00 (.10)	-.53 (.25)	-.70 (.10)	-.21 (.10)
$\hat{\phi}_3^R$		-.07 (.11)		.26 (.10)	-.45 (.21)	-.31 (.12)	-.20 (.12)
$\hat{\phi}_4^R$		-.16 (.10)		.32 (.11)	.85 (.36)		.47 (.11)
$\hat{\phi}_5^R$		-.13 (.11)		.30 (.10)	-1.53 (.35)		
$\hat{\phi}_6^R$							
$\hat{\delta}_1^E$	-.07 (.01)	.01 (.03)	.01 (.01)	-.01 (.02)	.07 (.02)	-.04 (.02)	-.09 (.02)
$\hat{\delta}_2^E$	.08 (.01)	.04 (.02)	.02 (.01)	.05 (.02)	-.03 (.02)	.00 (.01)	.10 (.02)
$\hat{\delta}_3^E$	-.07 (.01)	-.15 (.02)	.02 (.01)	-.14 (.02)	-.11 (.02)	.05 (.02)	-.11 (.02)
$\hat{\delta}_4^E$	.11 (.01)	.13 (.02)	.00 (.01)	.12 (.02)	.09 (.03)	.03 (.01)	.10 (.02)
$\hat{\delta}_1^R$	-.17 (.02)	-.06 (.03)	-.05 (.01)	.04 (.02)	.09 (.04)	-.02 (.01)	.01 (.02)
$\hat{\delta}_2^R$	.15 (.02)	.07 (.02)	.08 (.01)	.06 (.02)	.02 (.05)	.05 (.01)	.04 (.02)
$\hat{\delta}_3^R$	-.14 (.01)	-.08 (.03)	.05 (.01)	-.07 (.02)	.02 (.06)	-.01 (.01)	.02 (.02)
$\hat{\delta}_4^R$	.20 (.02)	.13 (.02)	-.05 (.01)	.09 (.02)	-.07 (.09)	.02 (.01)	.02 (.02)

(continued on next page)

Table 3: *(continued)*.

	Country						
	Italy	Luxemburg	The Netherlands	Portugal	Switzerland	UK	US
z_{t-d_c}	$\Delta_4 \ln x_{t-3}$	$\Delta_4 \ln x_{t-4}$	$\Delta_4 \ln x_{t-2}$	$\Delta_4 \ln x_{t-3}$	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-1}$	$\Delta_4 \ln x_{t-2}$
w_{t-d_s}	t	t	t	t	$\Delta_4 \ln x_{t-1}$	t	t
$\hat{\gamma}_c$	126	6222	166.6	5299	539.8	4897	2600
$\hat{\mu}_c$	.07 (.01)	.05 (.00)	.02 (.01)	.01 (.01)	-.03 (.00)	.00 (.09)	-.01 (1.81)
$\hat{\gamma}_s$	20.18	1724	6.98	16.90	112.2	1724	108.8
$\hat{\mu}_s$	.57 (.02)	.50 (1.54)	.60 (.03)	.40 (.02)	.00 (.01)	.66 (1.99)	.64 (.01)
$\hat{\phi}_1^E$	.18 (.16)	.74 (.19)	-.14 (.13)	-.11 (.09)	-.12 (.11)	-.07 (.09)	.47 (.10)
$\hat{\phi}_2^E$	.18 (.10)	-.29 (.16)	.13 (.11)	.06 (.09)	-.07 (.11)	-.04 (06)	-.05 (.12)
$\hat{\phi}_3^E$	.20 (.11)	.30 (.12)	.30 (.11)	-.17 (.08)	-.01 (.12)	.22 (.07)	.23 (.12)
$\hat{\phi}_4^E$	-.06 (.09)	-.02 (.10)	.14 (.11)	.18 (.09)	.39 (.11)		.09 (.11)
$\hat{\phi}_5^E$	-.51 (.15)	-.51 (.18)	-.09 (.13)				-.26 (.10)
$\hat{\phi}_6^E$							
$\hat{\phi}_1^R$	-.09 (.09)	.21 (.11)	-.30 (.11)	-.01 (.17)	-.18 (.16)	.02 (.09)	.37 (.12)
$\hat{\phi}_2^R$	.19 (.08)	-.21 (.10)	-.36 (.16)	.37 (.14)	.01 (.12)		-.46 (.13)
$\hat{\phi}_3^R$	-.02 (.08)	.09 (.10)	-.19 (.17)	.00 (.16)	.57 (.37)		.23 (.13)
$\hat{\phi}_4^R$	-.07 (.08)		-.26 (.17)	.20 (.15)			-.39 (.14)
$\hat{\phi}_5^R$	-.21 (.09)		-.65 (.16)				-.13 (.13)
$\hat{\phi}_6^R$							
$\hat{\delta}_1^E$	.13 (.03)	-.05 (.02)	.22 (.05)	.05 (.02)	-.04 (.01)	.02 (.01)	.01 (.00)
$\hat{\delta}_2^E$	-.02 (.02)	.10 (.02)	-.11 (.04)	-.03 (.02)	.03 (.01)	-.05 (.01)	-.01 (.00)
$\hat{\delta}_3^E$	-.23 (.02)	-.16 (.02)	-.29 (.05)	-.10 (.02)	-.02 (.01)	-.05 (.01)	.02 (.00)
$\hat{\delta}_4^E$	.14 (.03)	.14 (.02)	.15 (.04)	.12 (.02)	.04 (.01)	.10 (.01)	-.01 (.00)
$\hat{\delta}_1^R$	.05 (.02)	-.01 (.01)	-.02 (.02)	-.04 (.01)	-.11 (.03)	.00 (.01)	-.01 (.01)
$\hat{\delta}_2^R$	.03 (.01)	.07 (.01)	.03 (.02)	.06 (.01)	.07 (.03)	.00 (.01)	.03 (.01)
$\hat{\delta}_3^R$	-.13 (.01)	-.07 (.01)	-.08 (.02)	.04 (.01)	-.02 (.02)	-.10 (.01)	-.02 (.01)
$\hat{\delta}_4^R$	.11 (.02)	.04 (.01)	.12 (.02)	.00 (.01)	.12 (.04)	.12 (.01)	.02 (.01)

Note: The SEASTAR model is given in (3)-(5). The model is fitted to the data up to and including 1988.4. The standard errors are given in parentheses.

Table 4: Diagnostics on residuals of fitted SEASTAR models: p-values

	Serial correlation				JB	SK	ARCH			
Country	1	2	3	4			1	2	3	4
Austria	.32	.49	.67	.26	.13	.05	.18	.20	.35	.44
Belgium	.51	.59	.65	.37	.91	.49	.08	.17	.10	.00
Canada	.28	.54	.50	.36	.73	.42	.00	.00	.00	.00
Spain	.43	.17	.30	.12	.14	.17	.52	.69	.83	.84
France	.70	.71	.52	.66	.13	.04	.51	.69	.71	.27
Greece	.25	.35	.53	.45	.90	.34	.12	.04	.06	.09
Ireland	.09	.10	.10	.01	.15	.42	.32	.54	.74	.72
Italy	.30	.46	.56	.33	.54	.24	.18	.14	.19	.14
Luxemburg	.26	.27	.28	.45	.39	.08	.49	.62	.05	.04
The Netherlands	.37	.46	.64	.68	.10	.09	.45	.64	.73	.59
Portugal	.08	.16	.29	.39	.44	.18	.76	.87	.69	.34
Switzerland	.23	.47	.67	.35	.52	.44	.89	.85	.84	.66
UK	.11	.06	.06	.11	.21	.45	.34	.36	.52	.20
US	.06	.07	.16	.29	.42	.13	.51	.73	.64	.73

Note: The LM test for serial correlation considers lag 1, lags 1-2, ..., and lags 1-4. JB denotes the Jarque-Bera test for normality, where SK concerns the skewness. ARCH test is considered up to order 4.

Table 5: Evaluating the fit of the SEASTAR model as compared to various nested models using AIC and BIC

	AIC				BIC			
Country	SEASTAR	STAR-S	Lin-S	$F_s = F_c$	SEASTAR	STAR-S	Lin-S	$F_s = F_c$
Austria	-8.120^*	-7.975	-7.969	-8.080	-7.718	-7.723	-7.793^*	-7.729
Belgium	-7.548^*	-7.470	-7.415	-	-6.970	-7.017	-7.164^*	-
Canada	-8.499	-8.421	-8.456	-8.535^*	-8.010	-8.061	-8.224^*	-8.097
Spain	-7.484^*	-7.119	-7.074	-	-6.917^*	-6.759	-6.868	-
France	-7.858^*	-7.832	-7.737	-7.845	-7.280	-7.404	-7.485^*	-7.317
Greece	-6.954^*	-6.868	-6.877	-6.829	-6.426	-6.499	-6.640^*	-6.355
Ireland	-6.774^*	-6.676	-6.739	-6.745	-6.221	-6.274	-6.488^*	-6.243
Italy	-7.258^*	-6.799	-6.779	-	-6.629^*	-6.346	-6.477	-
Luxemburg	-6.754^*	-6.438	-6.363	-	-6.252^*	-6.061	-6.137	-
The Netherlands	-7.335^*	-6.913	-6.838	-	-6.783^*	-6.511	-6.611	-
Portugal	-7.138^*	-6.695	-6.782	-	-6.585^*	-6.318	-6.531	-
Switzerland	-7.482^*	-7.467	-7.454	-7.400	-6.929	-7.040	-7.177^*	-6.923
UK	-7.635^*	-7.365	-7.339	-	-7.157^*	-6.988	-7.088	-
US	-8.421^*	-8.200	-8.174	-	-7.818	-7.747	-7.898^*	-

Note: The SEASTAR model is given in (3)-(5). The restricted models are the STAR model with deterministic seasonality (6), represented by STAR-S, the linear seasonal model (7), represented by Lin-S, and the SEASTAR model with $F_s = F_c$. A * indicates the lowest AIC or BIC values across the three (or four) models.

Table 6: Ratios of mean squared prediction errors

	1-step		4-step		8-step		12-step	
Country	M_1/M_2	M_1/M_3	M_1/M_2	M_1/M_3	M_1/M_2	M_1/M_3	M_1/M_2	M_1/M_3
Austria	1.131	1.170	0.919	0.921	1.012	0.993	1.015	1.004
Belgium	0.805	0.815	0.962	0.909	0.857	0.845	0.801	0.827
Canada	0.971	1.469	0.966	1.065	0.970	1.004	0.820	0.827
Spain	1.167	1.472	1.075	1.464	1.285	1.656	1.489	1.779
France	0.703	0.820	0.925	1.066	1.201	1.059	1.451	1.209
Greece	1.023	1.126	0.834	0.920	0.987	0.972	0.938	0.920
Ireland	1.249	1.210	1.017	1.105	0.893	0.976	0.990	1.033
Italy	1.387	1.419	1.058	0.934	0.800	0.760	0.707	0.683
Luxemburg	1.300	1.544	1.278	1.354	0.772	0.863	0.788	0.885
The Netherlands	13.051	13.862	7.848	10.182	5.165	11.336	2.315	8.389
Portugal	1.529	1.610	1.879	1.795	2.415	2.359	2.396	2.296
Switzerland	0.950	1.157	0.890	1.121	1.079	1.165	1.104	1.076
UK	0.487	0.545	0.626	0.610	0.531	0.517	0.511	0.516
US	0.930	0.775	1.107	0.850	0.828	0.676	0.881	0.674

Note: The elements are ratios of the MSPEs of the different models. M_1 denotes the MSPE of the SEASTAR model (3), M_2 denotes the MSPE of the restricted model with $\gamma_s = 0$ (6), M_3 denotes the MSPE of the linear seasonal model (7).

Table 7: The p-values of STAR-type nonlinearity tests for the growth rates of seasonally adjusted industrial production series

	$z_{t-d} = \Delta_4 \ln x_{t-d}$			
Country	$d = 1$	$d = 2$	$d = 3$	$d = 4$
Austria	.003	.311	.354	.639
Belgium	.147	.125	.270	.110
Canada	.045	.053	.171	.374
Spain	.561	.325	.079	.002
France	.000	.000	.002	.021
Greece	.104	.485	.476	.074
Ireland	.368	.059	.785	.483
Italy	.229	.235	.029	.136
Luxemburg	.001	.104	.074	.000
The Netherlands	.070	.092	.054	.748
Portugal	.325	.088	.010	.183
Switzerland	.001	.001	.007	.007
UK	.013	.444	.038	.000
US	.000	.009	.018	.076

Note: The test procedure can be found in Teräsvirta (1994). It is essentially the same as the SEASTAR test in subsection 2.3, where all seasonal dummies are left out. The AR order p is fixed at 5. Notice that z_t and x_t concern seasonally adjusted variables.

Table 8: Agreement on the nonlinear cycle in industrial production. The cells contain frequencies

	States indicated for SA and NSA data, respectively						
Country	$(0,0)^a$	(1,0)	(0,1)	(1,1)	Agreement[b]	Kappa[c]	(ase)[d]
Austria	0.236	0.547	0.009	0.208	0.443	0.136	(0.043)
Belgium	0.519	0.113	0.019	0.349	0.868	0.730	(0.066)
Canada	0.235	0.049	0.059	0.657	0.892	0.738	(0.074)
Spain	0.637	0.108	0.000	0.255	0.892	0.751	(0.069)
France	0.028	0.000	0.104	0.868	0.896	0.321	(0.142)
Greece	0.418	0.000	0.051	0.531	0.949	0.897	(0.045)
Ireland	0.304	0.000	0.152	0.543	0.848	0.685	(0.104)
Italy	0.689	0.028	0.019	0.264	0.953	0.885	(0.050)
Luxemburg	0.679	0.047	0.009	0.264	0.943	0.863	(0.054)
The Netherlands	0.340	0.047	0.019	0.594	0.934	0.859	(0.051)
Portugal	0.144	0.000	0.058	0.798	0.942	0.800	(0.077)
Switzerland	0.057	0.000	0.028	0.915	0.972	0.785	(0.119)
UK	0.142	0.000	0.123	0.736	0.877	0.629	(0.089)
US	0.123	0.000	0.066	0.811	0.934	0.751	(0.088)

Notes:

[a] (0,0) denotes that the switching function in the model for NSA data takes a value of 0 and that this also holds true for a similar function in the model for SA data. The switching function value is set to 0 when $F_c < 0.5$. The number in the cells is the number of observations with (0,0), divided by the total number of observations. The cells under the header (1,0), (0,1) and (1,1) are defined similarly.

[b] Agreement is defined as the sum of the percentages in the columns (1,1) and (0,0).

[c] The Kappa is defined as $(oa - ea)/(1 - ea)$, where oa denotes observed agreement and ea denotes expected agreement, see Cohen (1960).

[d] The large sample standard error (denoted as ase) is calculated along the lines suggested in Fleiss, Cohen and Everitt (1969), see also Schouten (1982).

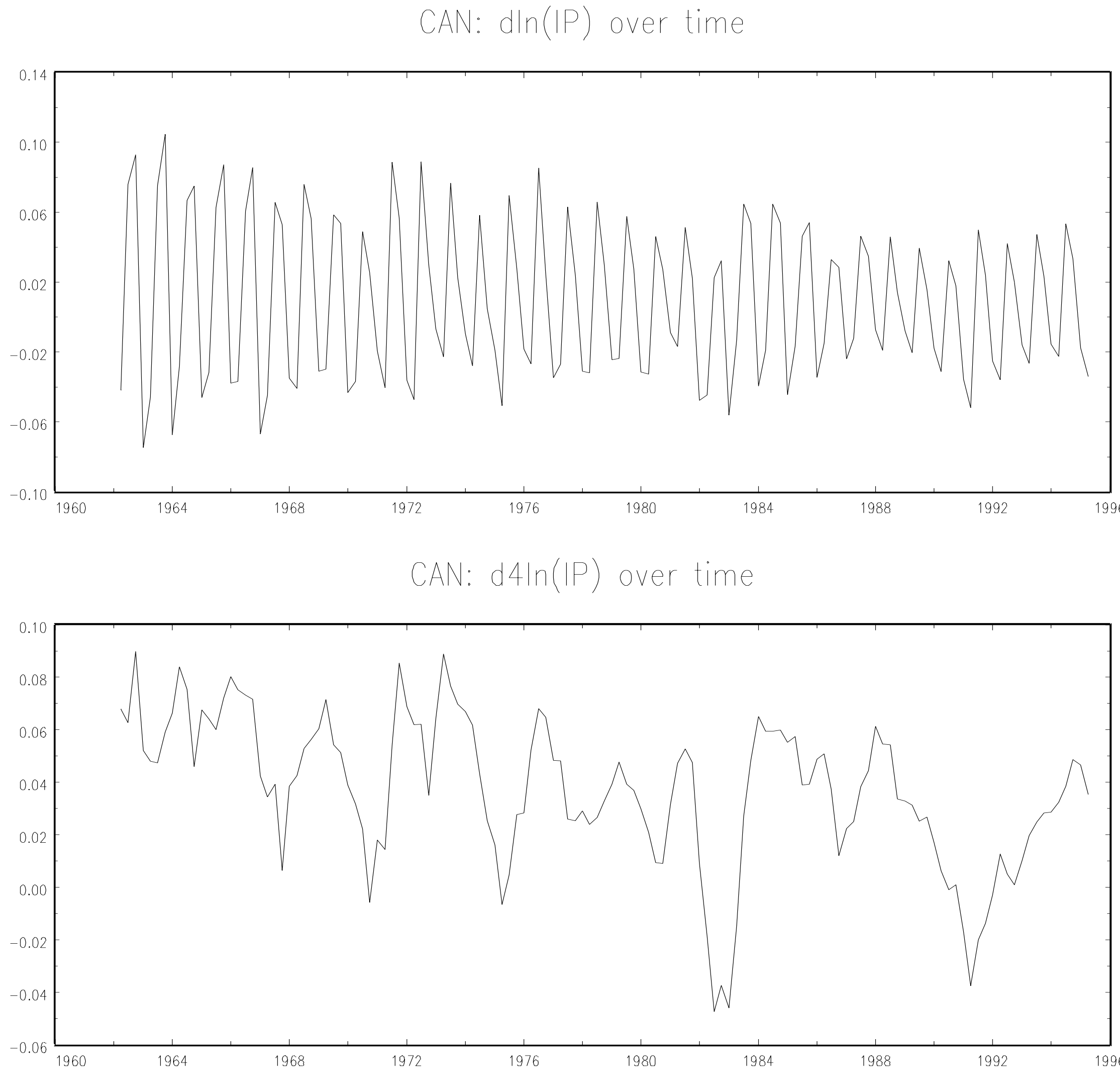

Figure 1: Industrial production: first-differenced logged (dln(IP)) and seasonally differenced log-transformed series (d4ln(IP)): Canada

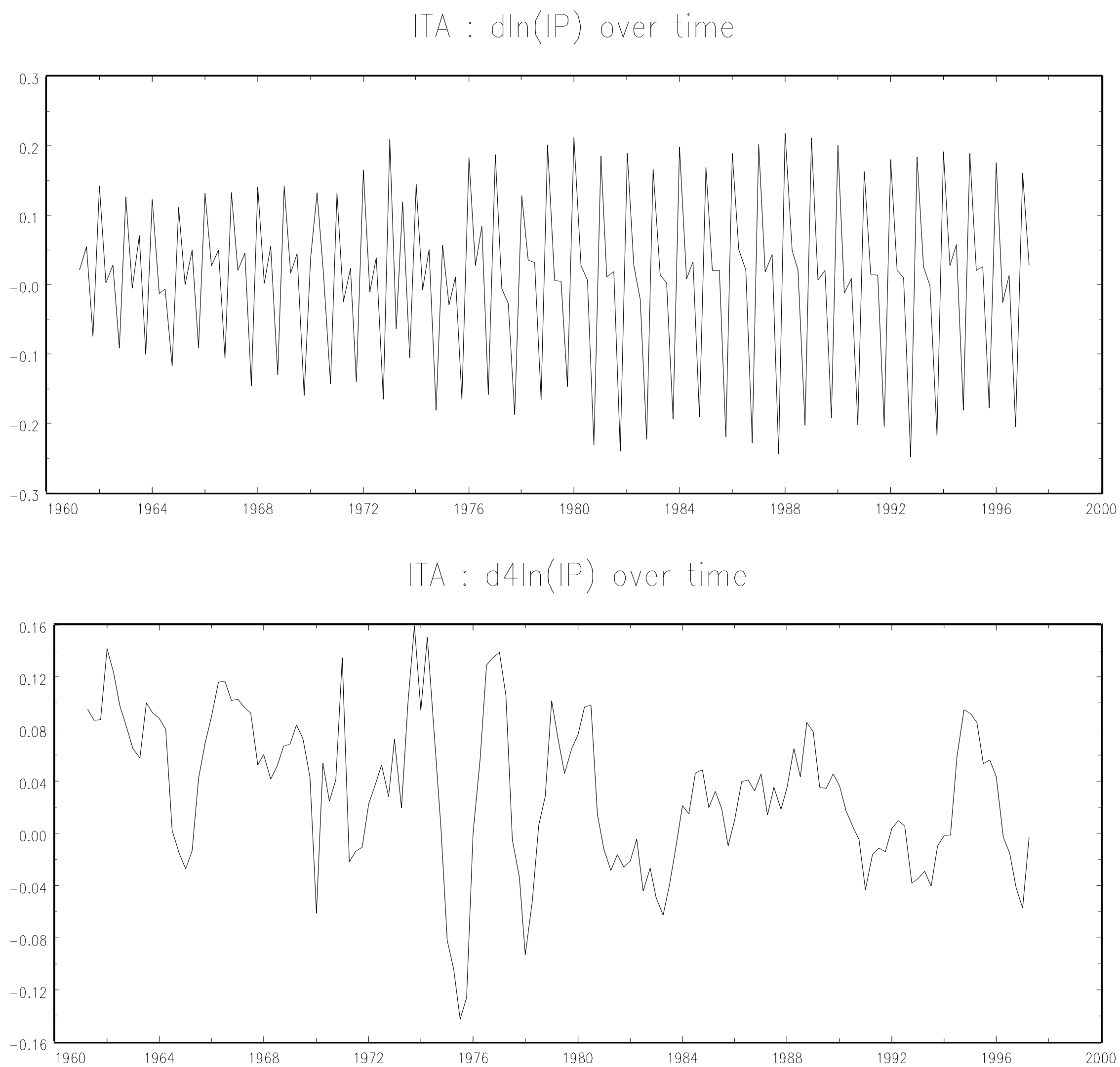

Figure 2: Industrial production: first-differenced logged (dln(IP)) and seasonally differenced log-transformed series (d4ln(IP)): Italy

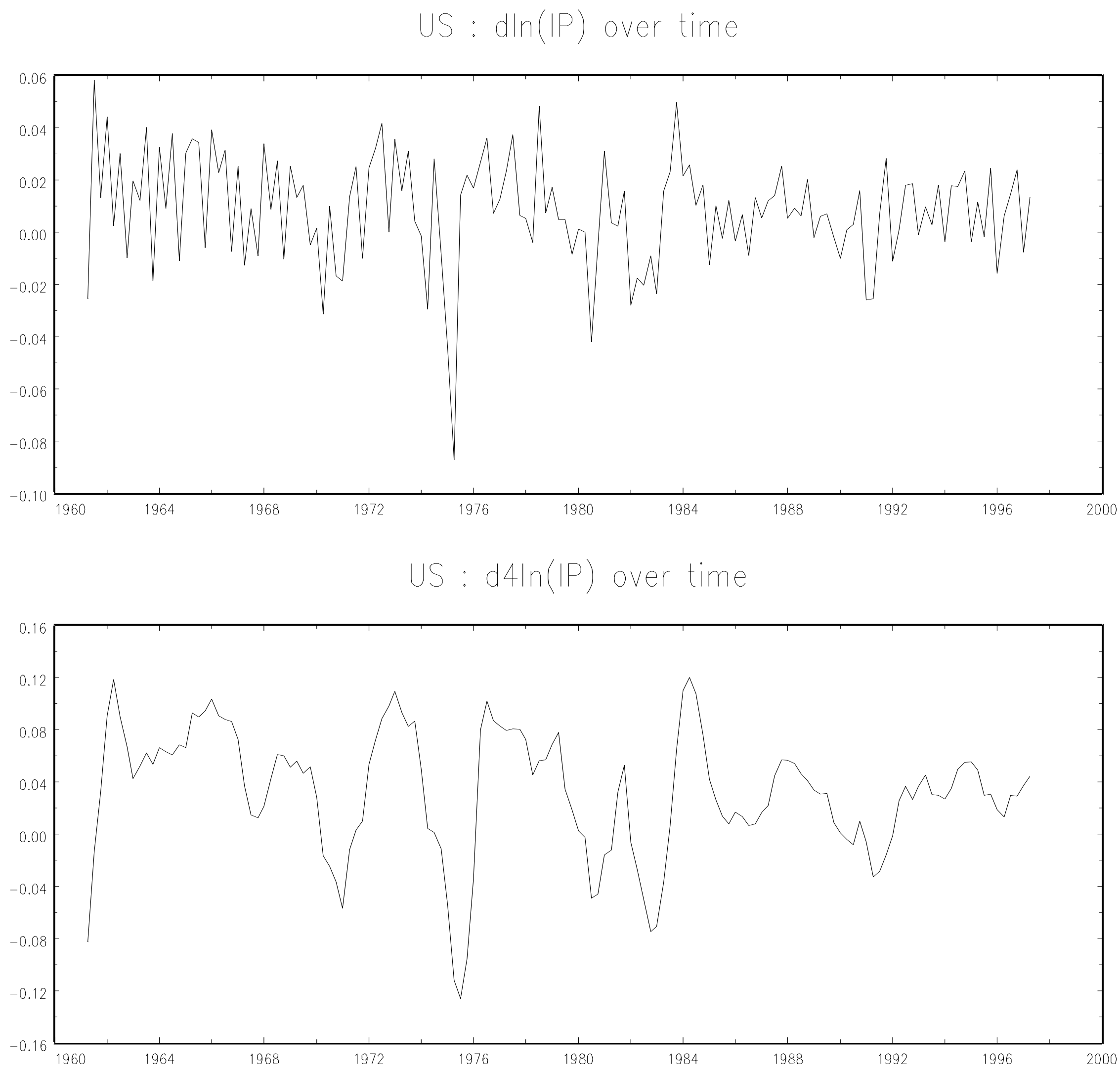

Figure 3: Industrial production: first-differenced logged (dln(IP)) and seasonally differenced log-transformed series (d4ln(IP)): The US

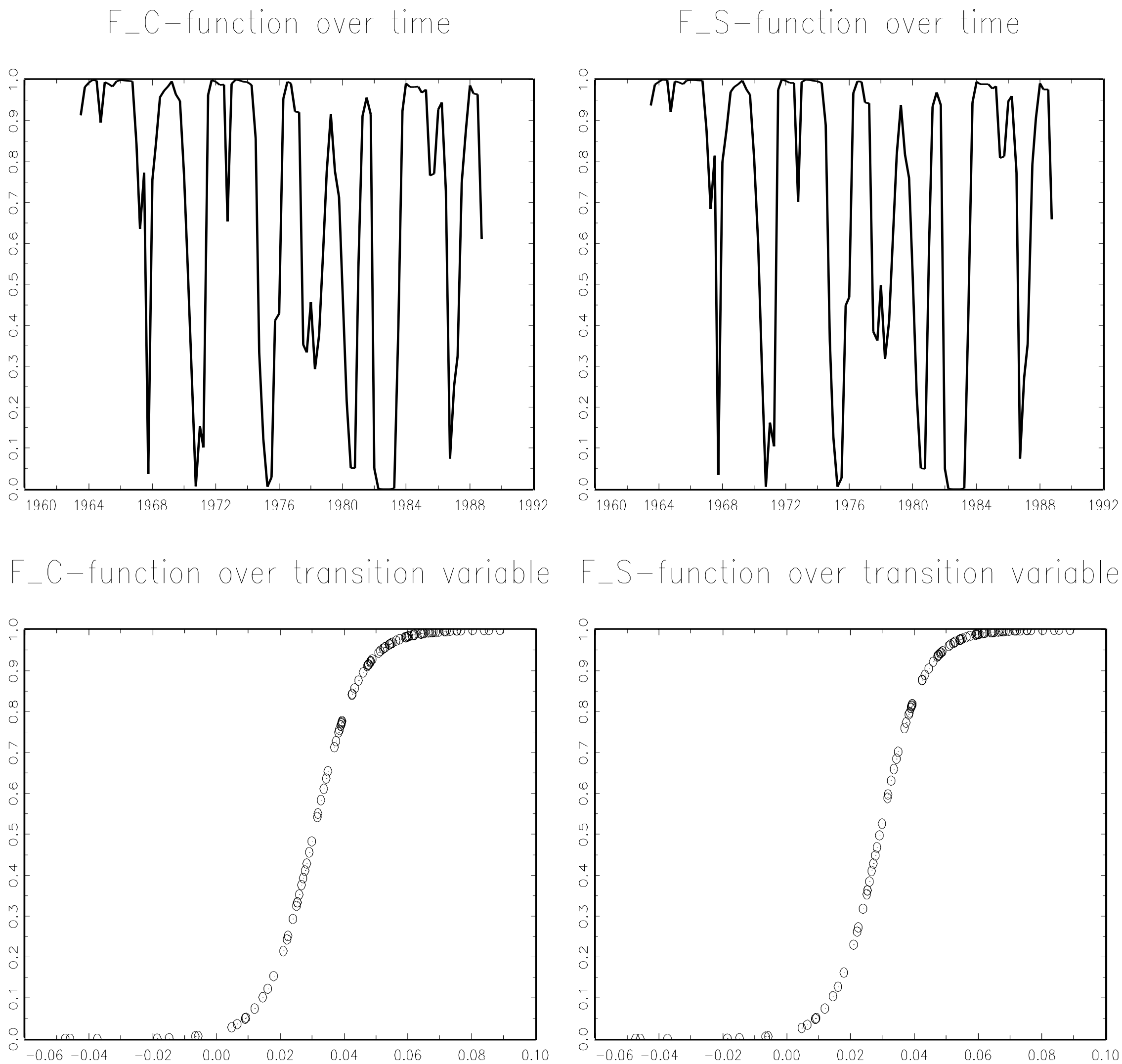

Figure 4: The estimated business cycle transition function F_c (left) and the seasonal transition function F_s (right) from the SEASTAR model : Canada

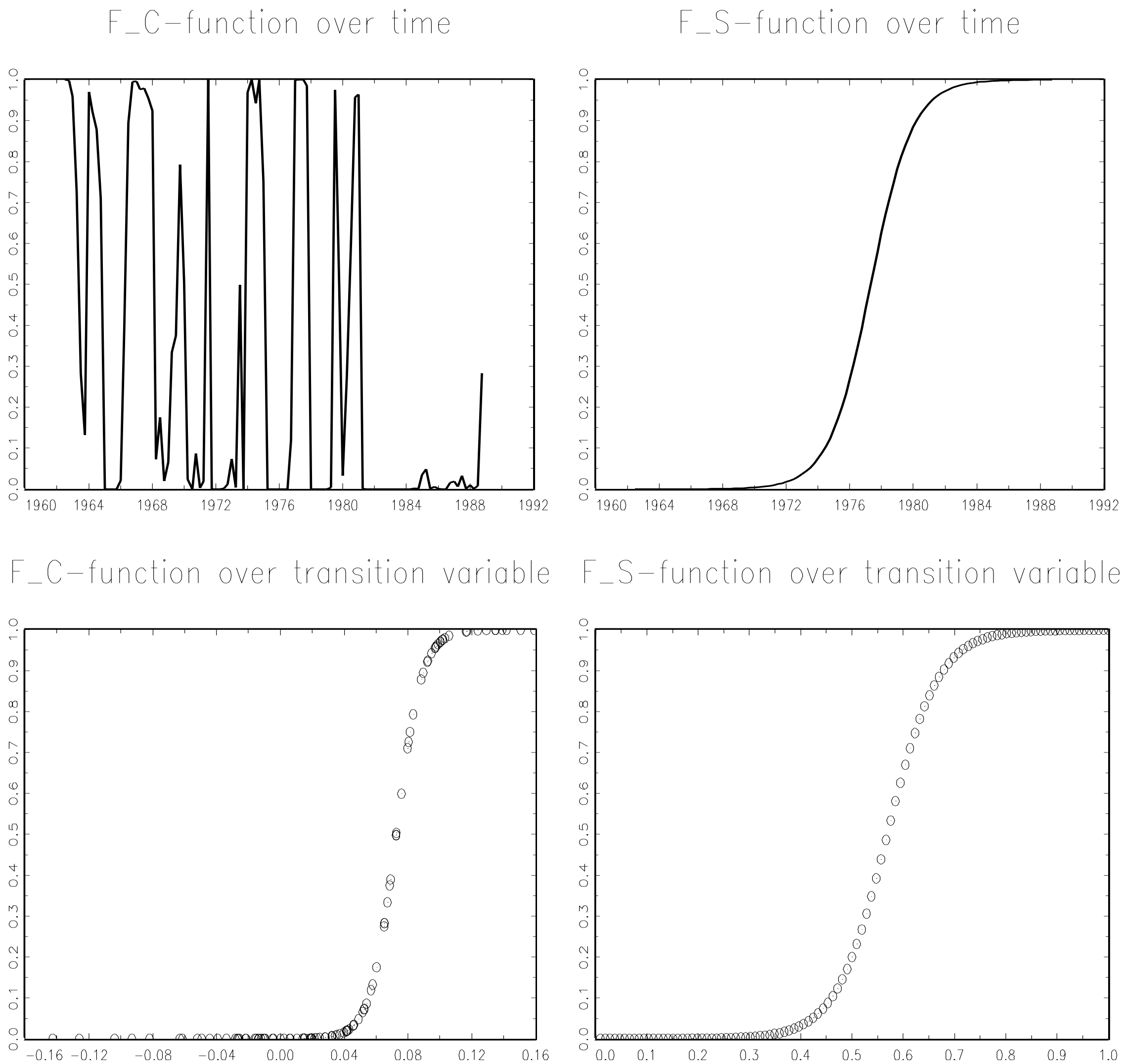

Figure 5: The estimated business cycle transition function F_c (left) and the seasonal transition function F_s (right) from the SEASTAR model : Italy

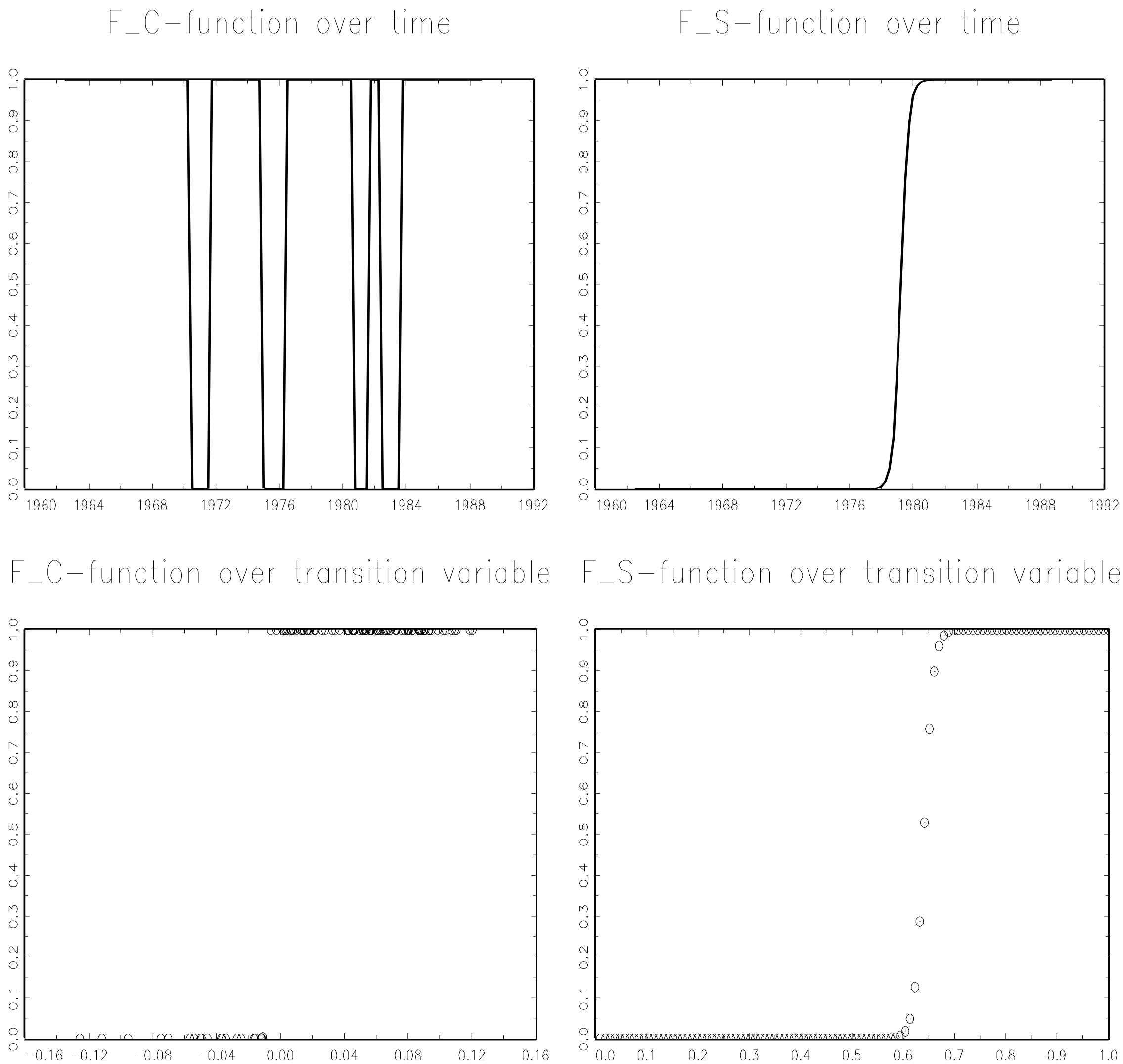

Figure 6: The estimated business cycle transition function F_c (left) and the seasonal transition function F_s (right) from the SEASTAR model : The US

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