Appendix A

Population, income per head and the percentage growth of income per head in twelve world areas.

Areas	Population in 1957 (millions)	Income per head in 1957 (dollars)	Percentage growth of income per head, 1950–60
A. Developed 'Capitalist' countries			
A1 North America and Australia	200	1993	1·5
A2 European Economic Community	166	911	4·7
A3 The rest of Europe	86	1123	2·6
A4 Other countries	107	425	7·5
B. European 'Communist' countries			
B 1 Soviet Union	200	605	8·7
B 2 The rest of Eastern Europe	114	508	7·5
C. Developing countries			
C1 'Communist' Asia	666	102	8·6
C2 Africa	172	143	1·6
C3 The rest of Asia	733	147	1·7
C4 Latin America	185	312	1·4
C5 Petroleum countries	39	320	3·9
C6 Mediterranean countries	138	355	1·3

Source: E. S. Kirschen, 'Vers un modèle prévisionnel mégisto-économique', *Cahiers Economiques de Bruxelles* (1962), No. 16, p. 471.

Appendix B

Relationship between production (net value added) per head in major sectors and 1 income per head and 2 size of the population, expressed in elasticities of the production in respect of 1 and 2.

Sector	Elasticity in respect of 1	Elasticity in respect of 2
I Primary production	0·49	−0·09
a. Agriculture	0·47	−0·08
b. Mining	0·94	0·13
II Industry	1·36	0·05
a. Manufacturing industry	1·44	0·20
b. Building industry	1·15	−0·06
III Transport and communications	1·29	−0·05
IV Other services	1·07	0·01

Source: H. B. Chenery, 'Patterns of Industrial Growth', *The American Economic Review* L (1960), p. 624, and especially p. 634.

Appendix C

Relationship between production (net value added) per head in certain industries and 1 income per head and 2 size of the population, expressed in elasticities of the production in respect of 1 and 2.

Sectors	Elasticity in respect of 1	Elasticity in respect of 2
Food and drink	1·13	0·00
Tobacco	0·93	0·23
Textiles	1·44	0·40
Clothing	1·69	0·07
Wood working	1·77	0·08
Paper	2·69	0·52
Printing	1·70	0·18
Leather working	1·64	−0·03
Rubber	2·00	0·44
Chemical products	1·66	0·26
Petroleum products	2·22	1·04
Non-metallic minerals	1·62	0·16
Metals, etc.	2·14	0·42
Engineering, etc.	2·80	0·32
Means of transport	2·33	0·26
Industry, total	1·62	0·09
Industry, total*	1·44	0·20

*excluding India, Pakistan and Ceylon

Source: H. B. Chenery, 'Patterns of Industrial Growth', *The American Economic Review* (1960), p. 624, and especially p. 633.

Appendix D

Further data about marginal capital-output ratios for entire countries during recent periods of five to ten years.

Norway	9·8	Burma	3·0
Denmark	7·3	Columbia	3·7
United Kingdom	6·5	Ecuador	2·3
Belgium	5·0	Thailand	2·8
United States	5·0	Brazil	2·3
France	4·0	India	2·2
Italy	3·6	Chile	4·2
West Germany	2·8	Argentine	15·0
Portugal	4·0	Algeria	2·5
Austria	3·8	Jamaica	2·9
Venezuela	2·1	Korea	2·6
Turkey	2·6	Nicaragua	3·5
Greece	2·2	Nigeria	2·8
The Philippines	1·0		

Source: Werner Baer and Isaac Kerstenetzky, 'Import Substitution and Industrialisation in Brazil', *The American Economic Review* LIV (1964), (Papers and Proceedings), p. 411; the figure for Turkey has been adjusted by the author of this book.

Appendix E

Further data about capital-output ratios for various sectors in India.

Building industry, urban	0·14	Forestry products	1·50
Leather and leather working	0·35	Motorised transport	1·50
Processed foods	0·38	Stock farming and fishing	1·50
Petroleum products	0·50	The rest of agriculture	1·50
Rubber products	0·50	Other minerals	1·65
Jute weaving	0·63	Synthetic rubber	1·84
Electrical capital goods	0·65	Coal	2·00
Glass, wooden and non-metallic mineral products	0·65	Hydro-electricity	2·00
Transport equipment	0·70	Cement	2·30
Cotton and other textiles	0·73	Railways	2·30
Plantations	0·90	Iron and steel	2·35
Non-electrical capital goods	1·00	Other metals	3·00
Chemical products	1·04	Iron ore	3·30
Artificial fertilisers	1·50	Thermo-electricity	5·75
Cereals	1·50	Crude oil	6·87

Source: Allen S. Manne, *A Consistency Model of India's Fourth Plan, Studies in the Structure of the Indian Economy*, Report No. 1, M.I.T. Center for International Studies (1964). Table X-3a.

Appendix F

An example of the use of linear programming in the middle phase.

To give the reader an idea of the kind of problem that has to be solved in the middle phase, I have provided in this appendix an extremely simple example which is in fact more like a crossword puzzle than a real planning problem. Let us assume that there are only three regions and five sectors. These regions and sectors are shown by upper indices, placed respectively before and after the symbols used to indicate the economic variables under consideration. Thus, ${}^3y^1$ means the contribution to the national income made by sector 1 in region 3. The planning problem consists in ascertaining all the numbers in the following table:

		Regions		
		1	2	3
	1	${}^1y^1$	${}^2y^1$	${}^3y^1$
	2	${}^1y^2$	${}^2y^2$	${}^3y^2$
Sectors	3	${}^1y^3$	${}^2y^3$	${}^3y^3$
	4	${}^1y^4$	${}^2y^4$	${}^3y^4$
	5	${}^1y^5$	${}^2y^5$	${}^3y^5$

All these y's represent increases in respect of the initial situation, since planning is concerned with such increases.

In order to do justice to as many of the aspects of the problem discussed in chapter 7 as possible in this simple example, let us assume that the sectors have the following characteristics. Sector 1 represents all the regional activities. Sectors 2 and 3 are the 'other national' sectors, with the proviso that sector 2 can be present in each of the three regions, whereas sector 3 can be present only in region 1 (for example, port activities or central government services). Sectors 4 and 5 are the international sectors. Let us assume further that sector 4 can be present in each of the three regions, whereas sector 5 can be present only in region 2.

Let us also choose simpler laws of production than those which apply in the input-output method, and assume that the increase in volume of the regional sectors is dependent only on the increase in income in the region in which they are located. We may also assume that the increase in the total volume of the national sectors is dependent on the increase in the national income. In a first example, which also forms the basis of figure 29, let us assume the following values:

$$ {}^1y^1 = 0{\cdot}2\,{}^1y\,;\ {}^2y^1 = 0{\cdot}2\,{}^2y\,;\ {}^3y^1 = 0{\cdot}3\,{}^3y, $$

in which ${}^1y$, ${}^2y$ and ${}^3y$ represent the increases in income in regions 1, 2 and 3.

Let us further assume that the following applies in sector 2:

$$^1y^2 + {}^2y^2 + {}^3y^2 = 0{\cdot}5\,y,$$

in which y is the increase in the whole national income: thus

$$y = {}^1y + {}^2y + {}^3y.$$

Finally, let us assume that for sector 3:

$$^1y^3 = 0{\cdot}1\,y.$$

In this equation, $^2y^3$ and $^3y^3$ do not appear, since sector 3 cannot be present in regions 2 and 3. The determination of the numerical values of the coefficients and of the regions in which certain sectors cannot occur is a technical operation that may or may not take place in consultation with the industries concerned.

We shall therefore state the planning problem that the income aims for each of the regions are first ascertained, probably in consultation between the government and the regional authorities. The table of increases in income mentioned above is shown under 1 in figure 29. The income aims chosen are shown under 2, the second stage in planning – each region has to achieve an increase of 1, and thus the country as a whole an increase of 3. All the technical data previously referred to are entered under 3. The fact that the total increases in income in sector 2 only must be related to the total increase in the national income is also expressed. To this is added another economic datum that is assumed to have been found in the analysis of the sectors, namely that sector 4 is less attractive to the country than sector 5 and is therefore not expanded (case A).

It is therefore possible to distribute sector 2 over the regions in such a way that the aims are achieved. Furthermore, the sum of the incomes in the sectors shown in the vertical column must be equal to the regional figure given in the first line. The calculations resulting in the first place from the given mathematical structure of the problem are shown under 4, and the remaining steps are shown under 5, 6 and 7. Case A, which has been selected here, is so simple that it is already solved as soon as the conditions or restrictions already mentioned are taken into account. There is no degree of freedom left. This is because of the great number of nils assumed on technical and economic grounds and introduced in stage 3.

This will not occur very often. The particular case chosen here and set out in figure 29 is intended only as an introduction to the problem of linear programming to be discussed below. It should also be noted that figure 29 at the same time aims to show in a fairly graphic form that the tasks and the procedure of planning are determined by the mathematical structure of the problem.

The example of linear programming provided in case B is different from case A in that the assumption is now made that sector 4 and not sector 5 should be chosen on a basis of the analysis made of the sectors. Since sector 4 can be carried out in all the regions, we cannot know in advance in which region or regions this sector should be expanded. There will consequently be two more unknowns, and the number of restrictions will remain the same, that is, the four already mentioned. It is now possible to use the degrees of freedom in such a way that the total investment costs are as small as possible.

The unknowns that now remain are the three variables ${}^1y^2$, ${}^2y^2$ and ${}^3y^2$ of the national sector 2 and the three variables ${}^1y^4$, ${}^2y^4$ and ${}^3y^4$ of the international sector 4. These must satisfy the restrictions already mentioned. Let us take a different numerical value for the coefficient 0·5 – let us choose 0·2. The restrictions are then:

$$\begin{aligned} &{}^1y^2 + {}^1y^4 = 0{\cdot}5 && (1)\\ &{}^2y^2 + {}^2y^4 = 0{\cdot}8 && (2)\\ &{}^3y^2 + {}^3y^4 = 0{\cdot}8 && (3)\\ &{}^1y^2 + {}^2y^2 + {}^3y^2 = 0{\cdot}6 && (4)\end{aligned}$$

With the help of these equations, we can express four variables in the two that remain. For the remaining two, let us choose ${}^1y^4$ and ${}^2y^4$. Hence

$$\begin{aligned} &{}^1y^2 = 0{\cdot}5 - {}^1y^4 && (1')\\ &{}^2y^2 = 0{\cdot}8 - {}^2y^4 && (2')\\ &{}^3y^2 = 0{\cdot}6 - {}^1y^2 - {}^2y^2 = -0{\cdot}7 + {}^1y^4 + {}^2y^4 && (4')\end{aligned}$$

from which it follows that:

$$ {}^3y^4 = 0{\cdot}8 - {}^3y^2 = 1{\cdot}5 - {}^1y^4 - {}^2y^4 \qquad (3')$$

It is now a question of making the total investment costs as small as possible. If we indicate the capital-output ratios of each industry in each region by the letter x, and use the same indices as those used in the case of y, these investment costs will be:

$$ j = {}^1x^2\,{}^1y^2 + {}^2x^2\,{}^2y^2 + {}^3x^2\,{}^3y^2 + {}^1x^4\,{}^1y^4 + {}^2x^4\,{}^2y^4 + {}^3x^4\,{}^3y^4 \qquad (5)$$

There is no need to take the investment costs for the already established industries 1 and 3 into account, since no change can be made in their volume. By means of the above equations (1′) –(4′), it is now possible to express j in the remaining unknowns ${}^1y^4$ and ${}^2y^4$ thus:

$$ j = ({}^1x^4 + {}^3x^2 - {}^1x^2 - {}^3x^4)\,{}^1y^4 + ({}^2x^4 + {}^3x^2 - {}^2x^2 - {}^3x^4)\,{}^2y^4 = P\,{}^1y^4 + Q\,{}^2y^4$$

The values that must be chosen for ${}^1y^4$ and ${}^2y^4$ will depend on the numerical values of P and Q.

The problem can be clarified by making a graph in which all the possible values of ${}^1y^3$ and ${}^2y^3$ can be shown (see figure 13). A great number of all the possible values are not feasible. As I shall show, only the trapezium within the shaded surfaces is feasible, since the numerical values of our unknowns must satisfy more conditions than those already mentioned, i.e. equations (1)–(4), namely that no increase in production must be negative. If this is so, it means that wrong investments were made initially. (It is possible to make this part of the method of dealing with the problem rather more subtle and to admit limited negative values as 'adjustments' to earlier investments.)

The following conditions have then to be added to the restrictions (1)–(4):

${}^1y^2 \geqslant 0$	or, with (1′):	${}^1y^4 \geqslant 0{\cdot}5$	(7)
${}^2y^2 \geqslant 0$	or, with (2′):	${}^2y^4 \geqslant 0{\cdot}8$	(8)
${}^3y^2 \geqslant 0$	or, with (4′):	${}^1y^4 + {}^2y^4 \geqslant 0{\cdot}7$	(9)
${}^1y^4 \geqslant 0$			(10)
${}^2y^4 \geqslant 0$			(11)
${}^3y^4 \geqslant 0$	or, with (3′):	${}^1y^4 + {}^2y^4 \geqslant 1{\cdot}5$	(12)

Certain parts of the plane ${}^1y^4 - {}^2y^4$ are 'forbidden' by each of these. Some of the forbidden parts are, however, already excluded because of other defences. The four restrictions (7), (8), (9) and (10) are shown as shaded in figure 4 – these shaded areas are forbidden. The restrictions (11) and (12) are, however, superfluous. The admissible area that remains is therefore the trapezium indicated by the letter *T*.

For each of the combined values of *P* and *Q* in (6), there is one point in *T* that produces the lowest value for *j*. If, for example, *P* is positive and greater than *Q*, which is also positive, then the point indicated by II is the one that produces the minimum for *j*. This can be shown most simply by working out a few numerical examples. Let $P = 2$ and $Q = 1$. Thus

$$j = 2\,{}^1y^4 + {}^2y^4 \qquad (13)$$

At point II, we find then that $j = 0 + 0{\cdot}7 = 0{\cdot}7$.

If ${}^1y^3$ and ${}^2y^3$ are both taken to be greater than in (11), then *j* will of course be greater. If ${}^1y^4$ is greater, but ${}^2y^4$ is as low as possible, then there is a chance of *j* falling, though this would not appear to be the case if we remain within the area *T*. The lowest level to which ${}^2y^4$ can possibly fall is therefore the lower limit of *T*. There is, however, the slope – 1 and that means that ${}^2y^3$ falls as much as ${}^1y^4$ increases. As, however, the ${}^1y^4$ in (13) has a coefficient 2 as against a coefficient of 1 for the ${}^2y^4$, the result is always that *j* is higher. Point II is therefore the point with the lowest *j*. There are, however, other possible situations, and these are shown in the table which follows by the roman figures I–VI. The corresponding minimum points for j are shown in figure 13. As soon as ${}^{1}y^{3}$ and ${}^{2}y^{3}$ are known, it is possible to determine the values of the other unknowns with the help of (1)–(4). Their values are included in the following table

Possible signs and relative size of P and Q and the corresponding solutions

Situation	P		Q	${}^{1}y^{4}$	${}^{2}y^{4}$	${}^{3}y^{4}$	${}^{1}y^{2}$	${}^{2}y^{2}$	${}^{3}y^{2}$
I	+	>	+	0·5	0·2	0·8	0	0·6	0
II	+	<	+	0	0·7	0·8	0·5	0·1	0
III	+	<	−	0	0·8	0·7	0·5	0	0·1
IV	−	>	+	0·5	0·2	0·8	0	0·6	0
V	−	>	−	0·5	0·8	0·2	0	0	0·6
VI	−	<	−	0·5	0·8	0·2	0	0	0·6

Appendix G

The causes of industrialisation according to Chenery (in percentage of the growth above a proportionate growth).

Sector	Import replacement	Final demand	Inter-industrial supplies
Group A. Investment goods and allied products			
Machinery	94	5	1
Transport	86	11	3
Metals	55	6	39
Other minerals	31	54	15
Together	72	11	17
Group B. Other producers' goods			
Paper	85	13	2
Petroleum	98	0	2
Rubber	73	19	8
Chemical products	50	−5	55
Textiles	67	12	21
Together	66	7	27
Group C. Consumers' goods			
Wood products	19	51	30
Clothing	15	13	72
Printing	2	11	87
Leather products	29	16	56
Food and drink	9	77	14
Together	13	45	42
GRAND TOTAL	**50**	**22**	**28**

Source: H. B. Chenery, 'Patterns of Industrial Growth', *The American Economic Review* L (1960), p. 624, and especially p. 643.

Appendix H

An example of an input-output table.

Input-output table for Japan ($a_{ij} \times 10^4$). Totals of the rows and columns in 100 million yen.

Coefficients

	1	2	3	4	5	6	7	8	9	10
1 Apparel	866			1		22	52			
2 Shipbuilding					81		97			
3 Leather and products	1		3726			2	1	3	78	
4 Processed foods				774	315	18	3			
5 Fishing				96						
6 Grain mill products				963	57	9		11		
7 Transport	36	103	226	150	57	101	206	114	120	18[illegible]
8 Industry n.e.c.	157	13	58	5			3	552	2	
9 Transport equipment							111		1036	
10 Rubber products	65	60	148	7	147	1	152	14	578	2[illegible]
11 Textiles	5868	144	158	13	811	20	119	443	28	22[illegible]
12 Machinery	29	1998	2	1			55	35	547	
13 Iron and steel		2691		135	59		276	250	2366	
14 Nonmetallic mineral products		62	10	95	5		23	67	130	
15 Lumber and wool products	5	358	10	55	61		104	229	160	[illegible]
16 Chemicals	137	140	36	268	15		53	1051	55	14[illegible]
17 Printing and publishing	1	1		12			22	23	1	
18 Agriculture and forestry	14		3261	1207	29	8768	28	324		
19 Nonmetallic minerals				4			5	22		[illegible]
20 Petroleum products	4	69	5	8	437		465	35	59	[illegible]
21 Nonferrous metals	1	75		10	7		80	536	492	
22 Metal mining										
23 Coal products	1	5	4	6	5		11	15	14	
24 Trade	328	302	338	190	165	347	136	267	460	3[illegible]
25 Paper and products	74	14		138	14	20	36	383	26	[illegible]
26 Electric power	33	70	22	49		31	102	57	55	[illegible]
27 Coal mining	9	68	45	101	16	2	586	105	27	1[illegible]
28 Services	261	283	189	138	245	98	729	348	282	2[illegible]
29 Petroleum and natural gas							10			
30 Interindustry total (U_j)	916	646	327	2914	442	6641	1794	399	899	4[illegible]
31 Total production (X_j)	1161	999	397	6583	1749	7035	5180	830	1380	8[illegible]
32 Imports (M_j)	12	91	10	198	15	445	0	19	60	
33 Total supply (Z_j)	1173	1090	407	6781	1764	7480	5180	849	1440	8[illegible]

Source : H. B. Chenery and P. G. Clark, Interindustry Economics (New York 1959), pp. 216, 217

	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
					25	1		23	3	3		8				14
	18	15				1		1								
				4		167		4								
						226										
	28			1		21	1	123							7	
84	40	101	79	346	409	172	88	54	114	37	120	89	498	98	297	302
		38	1		6									22		
	6	30	9	48	1	45	28	49	194	30	7	257	2	59	42	28
01	3740	36	7	77	2	40	44	43	33	11	150	23	2		140	4
99	40	1369	41	3	3	5	53	23	6		3	16		2	3	84
1																
	6	2767	5511	204	7	63	4	18	697	100	32	828	180	85	68	189
	1	91	81	775	33	73	1	13	24	12	44	17	36		6	27
	10	132	21	198	1239	19	1	9	102	16	11	20		42	47	197
15	863	107	8	372	120	2361	307	683	124	237	93	357	83		266	59
·76																
	1	2				18			1	9	3	2		2	87	5
	2436		16	71	4443	581	13	285	176		3	207		33	635	41
		14	11	596		53				8	1	1			63	
13	4	29	37	48	2	14	28	14	198	395	48	57	13	4	9	33
54																
	1	834	133	45	19	199	22	4	134		2845	235	3		35	312
1		3	255	11		106					1030					
		17	410	80	2	234	20	1	95	2	68		1969	4	3	
	166	294	93	235	117	268	329	183	161	47	166	171	151	59	193	198
359																
	15	76	4	507	18	452	3561	10		27	17		11	131	3665	9
32	40	54	93	176	120	198	50	17	88	30	117	259	40	10	146	119
47	62	50	99	1235	22	293	12		199	41	81	34	4552	5	258	2533
21	329	283	131	179	231	511	982	193	220	265	173	511	79	1054	255	610
255				6						4995			7			
—	8877	2287	8044	776	1327	3584	568	2159	72	328	1046	98	871	1639	1457	589
134	11372	3575	11425	1485	1946	5853	1026	12347	281	524	2082	316	1142	10180	2319	1237
357	33	61	34	5	8	254	9	3840	34	127	76	267	0	0	121	0
1	11405	3636	11459	1490	1954	6107	1035	16187	316	651	2158	582	1142	10180	2440	1237
358																

Table continued overleaf

Appendix H Input-output *table continued from pages 246–7*

				Total intermediate demand	Final demand			Total demand
					Exports	Domestic	Total	
	27	28	29	30	31	32	33	34
1 Apparel		11		194	141	837	979	1173
2 Shipbuilding				64	61	965	1026	1090
3 Leather and products		2		192	7	208	215	407
4 Processed foods		338		1145	115	5521	5636	6781
5 Fishing		34		242	60	1462	1522	1764
6 Grain mill products		89		971	7	6503	6510	7481
7 Transport	183	71	61	1256	601	3324	3924	5180
8 Industry n.e.c.		45		172	151	525	677	849
9 Transport equipment		87		319	48	1073	1121	1440
10 Rubber products	63	20		482	30	346	376	858
11 Textiles	17	35		5649	1850	3906	5755	11404
12 Machinery	156	165	6	1197	223	2216	2439	3636
13 Iron and steel	495	22	970	8569	717	2172	2889	11458
14 Nonmetallic mineral products	21	16		457	184	849	1033	1490
15 Lumber and wool products	102	15		678	78	1198	1276	1954
16 Chemicals	171	215		4276	250	1581	1831	6107
17 Printing and publishing	3	176	16	299	4	732	736	1035
18 Agriculture and forestry	344	105	106	11902	72	4213	4284	16186
19 Nonmetallic minerals		2		164	2	150	152	316
20 Petroleum products	34	16	84	516	1	134	134	650
21 Nonferrous metals	16	27		1444	184	530	714	2158
22 Metal mining				571	2	10	12	583
23 Coal products	2	17		963	—	179	180	1143
24 Trade	94	211	100	2039	429	7713	8141	10179
25 Paper and products		17		1957	62	421	483	2440
26 Electric power	372	45	266	728	0	509	509	1237
27 Coal mining	186	24		1959	1	54	55	2014
28 Services	296	708	625	4375	45	9250	9296	13671
29 Petroleum and natural gas			1038	274	0	15	15	289
30 Interindustry total (U_j)	469	3433	16	53054	5325	56596	61919	114973
31 Total production (X_j)	1838	13670	49					
32 Imports (M_j)	176	0	240					
33 Total supply (Z_j)	2014	13670	289					

Appendix I

A hypothetical model of the geographical spread of economic activities.

The question of fundamental importance in the spatial distribution of industry is what is the best combination in villages and towns of the industries that are necessary to a country. So far, little research has been done in connection with this problem. It has been established empirically that the distribution of hamlets, villages, towns, cities and world cities of different sizes is fairly regular. No explanation has as yet been provided for this distribution. It is probable that economic forces play an important part in it, but not certain.

In order to give an idea of what these problems imply, a model has been prepared that, to all appearances, would involve a minimum of transport costs, at least under certain conditions. This model may serve the scientific purpose of providing 1 a basis for comparison with reality, 2 a basis for the formulation of the conditions under which the transport costs are in fact at a minimum and 3 a starting-point for finding other models which will bring about an optimum under more general conditions.

The model is based on a number of assumptions. There are $H + 1$ industries shown by the index h which runs from 0 to H. Agriculture is shown by the index 0, and it is assumed that production in agriculture is evenly distributed over the territory of the country in question. It has then been assumed that each of the remaining industries is characterised by a certain size, expressed in the value of the product. They are also characterised by the number of plants of optimal size required to satisfy the demand. For the sake of convenience, foreign trade has been disregarded. The industries have been arranged in such a way that the value $h = 1$ is given to the industry with the greatest number of plants, the value $h = 2$ to that with the next greatest number, and so on, the industry $h = H$ having only one enterprise. The income of the country is represented by Y and the demand for the product h by ahY. In the assumed state of equilibrium, then,

$$a_0 + a_1 + a_2 + \ldots\ldots\ldots\ldots + aH = 1.$$

The word used for all villages and towns is centre, and there are as many centre types as there are industries. These can also be represented by an index h', which may be called the rank, since the centres form a hierarchy. All the industries with $h = 1, 2 \ldots$ up to and including h' occur in a centre with the index h'. Agriculture is regarded as a centre with $h' = 0$. There is generally more than one centre of the same kind, although there is only one with $h' = H$. The centres are composed in such a way that only the industry with the highest value for h supplies other centres, all the other industries in a centre supplying the local market only.

On the basis of these assumptions, the income of each category of centre with a given value of h' can be calculated as well as the number of these centres, assuming that the balance of payments is in a state of equilibrium in each centre.

A numerical example may help to make the model clearer. Let us assume that $Y = 1000$ and $H = 5$. Let $a_0 = a_3 = a_4 = a_5 = 0{\cdot}2$; $a_1 = a_2 = 0{\cdot}1$ and the number of businesses in each industry: $n_1 = 900$; $n_2 = 200$; $n_3 = 39$; $n_4 = 10$; $n_5 = 1$.

In this case, it can be calculated that $N_1 = 200$; $N_2 = 50$; $N_3 = 13$; $N_4 = 5$; $N_5 = 1$ and $Y_0 = 200$; $Y_1 = 22$; $Y_2 = 28$; $Y_3 = 83$; $Y_4 = 167$; $Y_5 = 500$ show the incomes of each category of centre.

The composition of each category of centre according to the number of undertakings is given below.

		Industry					Number of
		1	2	3	4	5	centres
Rank	1	200					200
of	2	25	50				50
the	3	75	17	13			13
centres	4	150	33	6·5	5		5
h'	5	450	100	19·5	5	1	1
Total		900	200	39	10	1	

It will be clear from the example that the condition is not satisfied that in each centre the number of enterprises is an integer. This is only one of the many complications that still remain to be investigated. I do not propose to go further into this question here.

Appendix J

Share of the labour incomes in the net value added for a number of industries in the Netherlands. (This share may be regarded as the elasticity of production in respect of the volume of labour, if the Cobb-Douglas function is assumed.)

Classes of industry	Description	1950	1960
		per cent of the value added at factor cost	
01/02	Agriculture and forestry	21	21
04	Fishing	66	70
11	Coal mining	99	90
13/19	Extraction of oil and salt, peat-cutting, quarrying	36	32
20 a	Processed foodstuffs (animal produce)	77	54
20	Processed foodstuffs (other produce)	56	54
21/22	Production of drinks and tobacco	66	59
23	Textile industry	52	65
24	Footware and clothing industry	60	64
25/26	Wood and furniture industry	68	64
27	Paper industry	58	56
28	Printing and publishing	60	65
29/30	Leather and rubber industry	58	67
31	Chemical industry	44	56
32	Petroleum refineries		
33	Manufacture of pottery, glass, lime and bricks	72	64
34	Metallurgical industry	59	55
35/36	Manufacture of metal products, mechanical eng.	72	67
37	Electrical industry	62	54
38	Transport equipment	70	65
39	Other industries	61	58
40	Building industry	72	65
51/52	Electricity supply, gasworks, waterworks	99	63
61	Wholesale trade	45	46
	Retail trade	21	20
62	Banks and clearing houses	43	49
63	Insurance industry	64	58
71/72	Transport	70	60
73	Communications	79	88
82	Medical and health services	39	42
	Other liberal professions and industries	58	55
83	Entertainments	61	62
84	Hotels, cafés, restaurants, etc.	64	66
	Other personal services	76	74
	Collective contributions to superannuation		

Source: Netherlands Central Statistical Office.