APPENDIX I

THE CHARACTERIZATION OF SPATIAL UNITS

In Section 1.1. it was stated that we are in need of a criterion to measure the size of spatial units which is based on economic factors and not on a political concept such as a nation or on physical size. In this appendix we will give some attention to this problem both by discussing some of its theoretical aspects and by reporting an empirical investigation of it. However, our contribution will be confined to offering some tentative ideas which may be useful in further discussion and which have served as a rationale for the empirical investigation, but which certainly must not be considered as a final answer to the problem mentioned above.

In principle there are two types of factors which lead to an economically meaningful classification of spatial units: first, differences in natural endowments and in availability of other factors of production; secondly, differences in mobility of goods in the broadest sense. We will be mainly interested in this second type of factors.

For this purpose we will first introduce the concept of a "corresponding region" of a product[1]), defined as the region around a point of production which can be supplied with the products produced at that point. The reason that a corresponding region is limited is the limited mobility of the product concerned due to technical, cultural or political factors, in other words, to its unit costs of transportation in the widest sense.

The significance of this concept of corresponding regions is clear: if the corresponding region of a good is smaller than a spatial unit, then this good has to be produced within the spatial unit – at least to a certain degree – in order to satisfy the spatial unit's demand for that good. It seems, therefore,

[1]) This concept, though not the phrase, was first introduced by W. Leontief and elaborated by W. Isard (cf. LEONTIEF *et al.*, 1953 and ISARD, 1960).

It is also closely related to Professor Klaassen's concept of a "relevant region" (cf. KLAASSEN, 1967, p. 78 ff.).

useful to define the size of a spatial unit in such a way that it coincides more or less with the corresponding region of at least one good, and preferably of a number of goods.

In reality the size of some product's corresponding region will depend on a number of factors inside as well as outside of the region and which will be different for different parts of the world. We will, as a first approximation, abstract from such differences and assume one unique corresponding region for each product. In addition we will assume that all corresponding regions have a regular shape, which will be approximately a circle.

If, therefore, we may assume that a corresponding region can be characterized by one number i.e. its "corresponding distance", which is normally the radius of a circular corresponding region, then we can, in principle, rank all products – or sectors – in order of decreasing relevant distances. This rank will be indicated by means of the symbol k_h; the good with the largest corresponding distance gets the highest rank, k_H. Now if we consider a spatial unit of about the same size as the corresponding region of commodity k_h, we know that all commodities with rank $\leq k_h$ must be produced within the spatial unit. This implies that total imports of this spatial unit cannot exceed its total demand for goods with a rank $> k_h$. More precisely, at least one good with rank $> k_h$ should be produced within the spatial unit concerned. This good then will be exported in order to finance the imports of the other goods with rank $> k_h$. Taking account of this we can calculate a spatial unit's maximum possible amount of imports and exports – divided by two – in relation to its total production or income, which seems to be a good criterion for measuring – in an economic sense – its size. We will call it a spatial unit's *Maximum Potential Openness* (M.P.O.).

In practice there are a number of reasons why a spatial unit should not realize its M.P.O., such as the existence of transportation costs and the differences in endowment with natural resources within the spatial unit. If account could be taken of these factors it would turn out that the spatial unit would normally reach a lower level of openness than its M.P.O. We will call this level a spatial unit's *Potential Openness* (P.O.). However, this concept cannot be clearly defined yet since we would then need a theory of an optimal distribution of labour in the world. For this and other reasons, such as the lack of data, we are not yet able to give an empirical basis to the theoretical concepts mentioned above. The statistical data which are available are exports and imports of nations. This brings us to our statistical

exercise[2]) which is therefore only indirectly related to the theoretical concepts mentioned above, namely, as far as these concepts more or less justify our interest in the degree of openness of a spatial unit.

The purpose of this statistical exercise was to find a number of factors which could be considered as "explanatory" variables for the *Actual Openness* of a spatial unit, defined as the sum of exports and imports divided by twice the national income: $(E+M)/2Y$, where

E = exports;
M = imports;
Y = national income.

A cross sectional analysis was applied; the sample referred to the year 1962 and consisted of 56 countries covering about 73 percent of world trade. First, several variables all representing geographical size were tried but they all yielded insignificant regression coefficients. Then, population was taken as the main "explanatory" variable; it also represents the size of the spatial unit (i.e. the nation) concerned. Next a number of other variables were introduced into the regression equation such as: degree of industrialization, degree of concentration of exports, isolated location and British Preference. It turned out that the regression with population, isolated location (defined as the average distance of a country from it's nearest trading partners) and British Preference as "explanatory" variables yielded the best result, although the coefficient of the British Preference variable was not significant. The relation in logarithmic form is:

$$\log \frac{E+M}{2Y} = 1.89 - \underset{(0.02)}{0.21} \log P - \underset{(0.03)}{0.20} \log I + \underset{(0.054)}{0.35} \log B \qquad \underset{(0.149)}{} \qquad R = 0.77,$$

where P = population;
I = isolation;
B = British Preference.

The combined influences of the three explaining variables constitute some "Estimated Actual Openess" but its relation to the "Potential Openness" is not yet clear. Evidently, further empirical analysis with more data and more

[2]) This exercise was carried out by Mr. B. Herman.

TABLE I.1

Actual Openness and Estimated Actual Openness[a)]

Country	Actual Openness	Estimated Actual Openness	Country	Actual Openness	Estimated Actual Openness
1 Argentina	12.2	11.3	29 Ireland	34.6	35.3
2 Australia	15.2	15.8	30 Israel	23.2	25.2
3 Austria	22.6	23.8	31 Italy	15.5	13.1
4 Barbados	36.5	47.3	32 Jamaica	30.0	33.2
5 Belgium + Luxemburg	38.0	25.8	33 Japan	11.0	8.4
6 Bolivia	19.3	18.3	34 Jordan	24.1	33.3
7 Brazil	5.8	9.1	35 Mexico	7.9	12.4
8 Burma	20.4	15.8	36 Morocco	21.1	17.5
9 Canada	17.5	16.4	37 Netherlands	42.0	25.7
10 Ceylon	27.1	19.3	38 New Zealand	20.7	21.9
11 Chile	15.8	15.2	39 Nigeria	17.7	14.7
12 Colombia	9.6	15.6	40 Norway	27.4	26.7
13 Cyprus	36.6	42.3	41 Pakistan	8.1	11.1
14 Denmark	29.3	24.0	42 Paraguay	16.9	20.3
15 Dominican Republic	22.3	22.4	43 Peru	27.2	14.9
16 Ecuador	14.3	18.8	44 Portugal	19.1	19.7
17 El Salvador	25.7	22.6	45 Rhodesia	32.8	25.0
18 Finland	24.7	21.4	46 South Africa	17.2	15.6
19 France	12.2	17.0	47 Sudan	21.4	15.9
20 Germany (West)	17.3	16.3	48 Sweden	23.4	20.4
21 Ghana	22.6	21.0	49 Switzerland	26.6	26.7
22 Greece	14.3	19.5	50 Syria	26.5	21.3
23 Guatemala	18.5	23.1	51 Thailand	19.9	10.9
24 Haïti	11.1	21.6	52 Trinidad + Tobago	59.5	35.3
25 Honduras	19.9	24.7	53 United Kingdom	16.5	19.9
26 Iceland	37.9	47.4	54 U.S.A.	3.7	8.1
27 India	5.7	8.1	55 Uruguay	12.3	17.4
28 Iraq	30.5	15.3	56 Venezuela	25.1	17.9

a) Actual Openness $= [(E+M)/2Y] \cdot 100$

Estimated Actual Openness $= 1.89 - 0.21 \log P - 0.20 \log I + 0.35 \log B$.

refined methods is necessary in order to find an economic criterion for classifying spatial units based on the concepts dealt with above.

It should be again emphasized that the concepts and the exercise presented are only meant as a rough indication for the direction of further research. In addition the reader should be reminded of what was said in Section 1.1, namely that the usefulness of a characterization of spatial units depends on the purpose for which it is used. In this case especially we have a useful hierarchical ordering of spaces in mind where a subdivision of a spatial unit is useful only if the openness of the smaller unit is larger than that of the larger unit. For other purposes, e.g. planning the transport system in a country, one may be equally or even more interested in the geographical size of the country.

Finally, Table I.1 of this appendix gives for each of the countries of the sample its Actual Openness together with its Estimated Actual Openness.

APPENDIX II

DATA AND SOLUTIONS OF THE MEXICAN MODEL

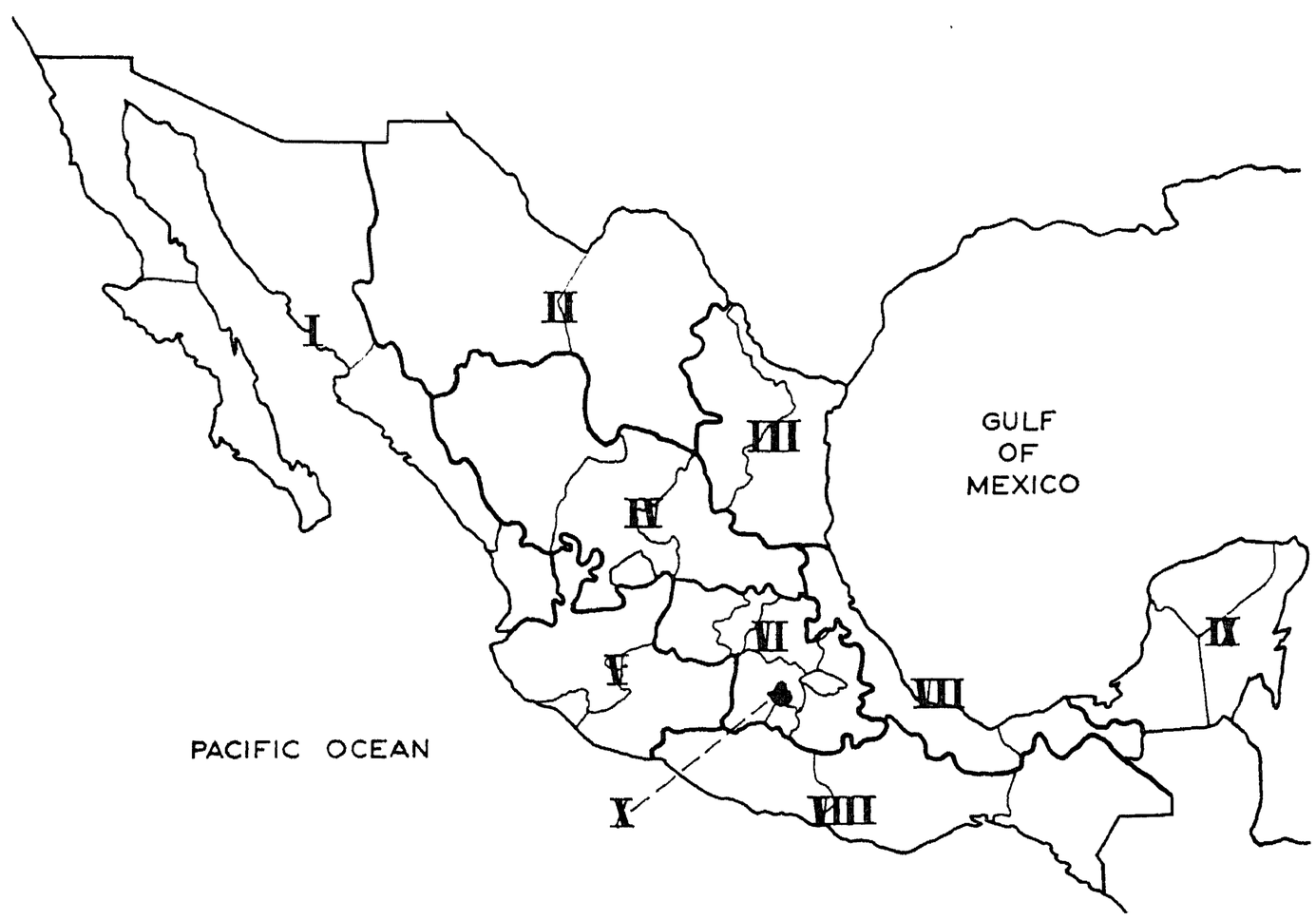

Fig. II.1. Regional subdivision of the Mexican Republic.

I = North-pacific
II = Northern
III = North-gulf
IV = Centre-north
V = Centre-pacific
VI = Central
VII = Centre-gulf
VIII = South-pacific
IX = Peninsular
X = Mexico City

TABLE II.1

Sectors and their classification for the Mexican economy

Number	Sector	Classification
1	Slaughtery, dairy, grain milling and bakery	Regional
2	Construction	Regional
3	Domestic and other personal services	Regional
4	Trade and commerce	Regional
5	Rent of houses and buildings	Regional
6	Educational and medical care	Regional
7	Non-metallic mineral products	National
8	Basic metal foundries	National
9	Electricity (generation and distribution)	National
10	Transportation and communication	National
11	Banking and insurance	National
12	Government (incl. all levels; excl. government enterprises)	National
13	Cotton	International
14	Coffee	International
15	Sugar	International
16	Sisal (Henequen)	International
17	Minor export-crops	International
18	Domestic-consumption crops	International
19	Livestock	International
20	Fishing	International
21	Exploitation of metallic ores	International
22	Exploitation of non-metallic ores	International
23	Petroleum and coal products (excl. petrochem.)	International
24	Other foodstuffs, beverages and tobacco	International
25	Textiles (hard and soft fibres)	International
26	Footwear, clothing, leather and leather products	International
27	Wood, paper and printing	International
28	Heavy chemicals (rubber, plastics and basics)	International
29	Light chemicals (medicines, cosmetics and sim.)	International
30	Metal products (machinery, apparatus, etc.)	International
31	Transport equipment	International

TABLE II.2

Capital-income ratios for the Mexican economy by sector and by region

Sector \ Region	I	II	III	IV	V	VI	VII	VIII	IX	X
1	3.00	3.20	2.86	—	1.55	2.63	1.11	—	—	2.31
2	7.27	2.05	1.31	—	2.17	1.70	—	—	—	1.57
3	1.23	2.13	1.77	—	1.59	1.00	1.50	—	—	1.45
4	1.50	1.33	1.31	1.30	1.37	1.21	1.10	1.51	1.13	1.94
5	6.91	18.19	5.26	6.32	5.79	6.33	6.81	6.16	2.43	5.77
6	1.74	1.24	1.51	1.69	1.28	1.50	—	1.61	1.48	1.47
7	—	—	3.31	—	3.34	3.26	—	2.80	—	2.26
8	—	4.37	14.33	—	—	5.69	7.05	—	—	1.44
9	4.83	—	—	—	6.95	—	—	—	—	12.42
10	3.70	5.91	3.37	3.02	1.33	3.18	5.80	3.76	—	4.09
11	9.88	17.22	6.66	—	—	—	—	—	—	13.66
12	8.65	11.33	6.78	9.66	7.85	7.04	8.00	16.06	7.92	2.62
13	2.29	3.09	2.68	2.38	—	—	—	—	—	—
14	—	—	—	1.35	—	—	.67	.42	—	—
15	.50	—	—	—	2.31	1.68	.58	.40	—	—
16	—	—	—	—	—	—	—	—	.18	—
17	.18	—	.23	—	—	.75	.16	—	—	—
18	.11	.10	.09	.26	.35	.58	.12	.06	.12	2.65
19	3.18	2.15	2.83	4.14	3.89	3.98	1.75	3.09	1.94	1.53
20	1.25	—	—	—	—	—	—	—	1.40	—
21	—	.55	.71	1.23	—	1.23	—	—	—	—
22	—	1.36	2.81	3.48	—	—	5.21	4.47	—	—
23	—	3.55	2.76	—	—	1.60	5.57	—	—	5.88
24	2.93	3.72	2.50	3.02	2.82	3.35	2.38	2.66	2.41	2.50
25	3.19	2.45	4.07	2.13	3.27	3.21	1.97	—	2.82	2.43
26	.85	1.13	2.51	2.21	1.77	1.80	1.12	1.80	—	1.56
27	2.30	3.01	4.14	1.19	4.35	4.59	—	1.66	2.10	1.92
28	4.33	—	2.97	—	3.38	2.77	—	—	—	2.28
29	—	—	—	—	3.04	2.43	—	—	—	1.76
30	2.63	2.30	2.42	1.92	2.20	3.03	—	—	—	2.10
31	—	1.38	2.18	—	.79	2.53	—	—	—	1.21

TABLE II.3

Existing structure of income for the Mexican economy, 1960
(Mexican pesos $\times 10^8$)

Sector \ Region	I	II	III	IV	V	VI	VII	VIII	IX	X	Σ
1	2	2	3	2	4	6	2	1	1	17	40
2	2	2	3	1	2	5	3	1	—	39	58
3	4	2	5	1	2	3	2	1	1	36	57
4	42	32	41	20	44	61	31	20	9	162	462
5	12	9	9	8	13	20	9	10	3	34	127
6	2	1	2	1	2	3	2	2	—	11	26
7	—	—	2	—	1	4	—	1	—	2	10
8	—	5	2	—	—	2	1	—	—	6	16
9	1	—	—	—	1	—	—	—	—	8	10
10	3	1	2	1	3	3	2	1	—	25	41
11	1	1	2	—	—	—	—	—	—	8	12
12	5	3	2	2	3	5	3	1	1	36	61
13	7	4	3	2	—	—	—	—	—	—	16
14	—	—	—	1	—	—	4	5	—	—	10
15	1	—	—	—	1	1	2	1	—	—	6
16	—	—	—	—	—	—	—	—	4	—	4
17	2	—	1	—	—	1	1	—	—	—	5
18	13	7	6	15	13	24	11	16	3	1	109
19	9	10	5	8	8	10	19	7	2	1	79
20	2	—	—	—	—	—	—	—	1	—	3
21	—	8	2	3	—	1	—	—	—	—	14
22	—	3	1	1	—	—	1	1	—	—	7
23	—	1	16	—	—	5	13	—	—	8	43
24	7	3	7	2	6	8	8	2	3	15	61
25	2	2	1	1	1	8	2	—	2	7	26
26	1	1	2	1	2	5	1	1	—	12	26
27	1	2	2	1	2	3	—	1	1	9	22
28	1	—	3	—	2	5	—	—	—	5	16
29	—	—	—	—	1	1	—	—	—	13	15
30	1	1	3	1	1	7	—	—	—	15	29
31	—	1	1	—	1	2	—	—	—	6	11
Σ	121	101	126	72	113	193	117	72	31	476	1,422

TABLE

A. Optimal solution of the historical model (Mexican pesos × 10^8)

Sector \ Region	I	II	III	IV	V	VI	VII	VIII	IX	X	Σ
1	1	1	1	x	1	2	1	x	x	8	15
2	1	1	2	x	1	2	x	x	x	18	25
3	1	1	2	x	1	2	1	x	x	17	25
4	12	8	12	4	12	21	7	5	3	73	157
5	4	3	3	2	4	8	2	3	1	17	47
6	1	1	1	1	1	2	x	1	x	9	17
7	x	x		x			x		x	5	5
8	x			x	x			x	x	15	15
9	6	x	x	x		x	x	x	x		6
10					15				x		15
11			7	x	x	x	x	x	x		7
12										15	15
13				4	x	x	x	x	x	x	4*
14	x	x	x		x	x		3	x	x	3*
15		x	x	x				2	x	x	2*
16	x	x	x	x	x	x	x	—		x	0*
17		x		x	x		1	—	x	x	1*
18						11	1	5	6		23*
19										19	19*
20		x	x	x	x	x	x	x	1	x	1*
21	x	14	14		x	29	x	x	x	x	57
22	x				x	x			x	x	
23	x			x	x			x	x		
24							3			23	26*
25							5	x			5*
26	12						6		x		18*
27				4			x				4*
28		x		x			x	x	x		
29	x	x	x	x			x	x	x		
30							x	x	x		
31	x			x	1		x	x	x	13	14
Σ	38	29	42	15	36	77	27	19	11	232	526

*) Considered as a "traditional" sector.

II.4

B. Optimal solution of the social model (Mexican pesos × 10^8)

Sector \ Region	I	II	III	IV	V	VI	VII	VIII	IX	X	Σ
1	1	1	1	1	2	4	1	2	x	2	15
2	1	1	2	1	1	3	x	2	x	5	16
3	1	1	2	1	1	2	1	2	x	4	15
4	14	8	14	12	27	37	7	25	3	18	165
5	4	3	3	5	8	14	2	14	1	4	58
6	1	1	1	1	2	5		4	x	2	17
7	x	x		x			x	5	x		5
8	x	14		x	x	1		x	x	0	15
9	6	x	x	x		x	x	x	x		6
10					15				x		15
11			7	x	x	x	x	x	x		7
12										15	15
13				4	x	x	x	x	x	x	4*
14	x	x	x		x	x		3	x	x	3*
15		x	x	x				2	x	x	2*
16	x	x	x	x	x	x	x	x		x	0*
17		x	1	x	x			x	x	x	1*
18				11				12			23*
19							15		4		19*
20		x	x	x	x	x	x	x	1	x	1*
21	x		11		x	46	x	x	x	x	57
22	x				x	x			x	x	
23	x			x	x			x	x		
24								24	2		26*
25				5				x			5*
26	10					7	1		x		18*
27				4			x				4*
28		x		x			x	x	x		
29	x	x	x	x			x	x	x		
30							x	x	x		
31	x			x	14		x	x	x		14
Σ	38	29	42	45	70	119	27	95	11	50	526

APPENDIX III

MATHEMATICAL REMARKS ON THE DECOMPOSITION PROCEDURES IN CHAPTERS 4 AND 6

In this appendix we do not intend to give formal mathematical proofs of the decomposition procedures presented in Chapters 4 and 6. Rather, we aim at providing in an informal way some insight into the mathematical background of these procedures. While some of the concepts introduced are more mathematical than in the main text of the book, in particular with respect to Chapter 6, it is hoped that they also help to make the main text more understandable to the non-mathematician.

III.1. Decomposition of the National Planning Problem in Section 4.9.

In Section 4.9 it was stated that the successive solution method proposed there had some drawbacks: Assumptions 4.7, 4.8 and 4.9 had to be satisfied and the solution finally found might not be optimal. Here we will make some suggestions for partially overcoming these drawbacks, at the cost of a few extra calculations.

Assumptions 4.7 and 4.9, that there are no regional differences in the input-output and final demand coefficients, will not in most cases be a very strong one. In fact we assume that such differences can be neglected and since in most cases only a national input-output table will be available, this assumption will be not only computationally convenient, but also a practical necessity.

If it is felt that Assumption 4.8, that the increase in the intermediate demand for regional products in each region is independent of which sectors are expanded in each region, and that the increases in final demand for

regional products in the regions are also proportional to the increases in the regional incomes, is far from realistic, then it is possible, nevertheless, to use it in the first round of the calculations, but to adjust the solution found so that it is more in accordance with reality. After the first solution has been found, one recalculates the corresponding increases in the regional sectors by means of equations (4.8.2)-(4.8.4), in which only the increases of the regional sectors are now considered as unknowns. Next, one inserts these values into the table with the program of income increases of Section 4.9., and a new solution for the second problem can then be calculated. The solution so found is less inconsistent with respect to equations (4.8.2)-(4.8.4), but if one still considers the inconsistency too large, one can repeat this adjustment procedure, which amounts to largely giving up Assumption 4.8. It remains an approximation method, which quickly gives a good approximate solution, but not a really consistent one.

Finally, there is the possible non optimality of the solution. This possibility is due to the fact that in the first problem of Section 4.9 allowance was not made for regional differences in cost coefficients within one national or international sector, while the existence of regional targets may make it necessary to extend a sector not only in its cheapest region. Always in the combined models of Section 4.9, for example, only one international sector will be extended, since in the first problem only one sector will be selected for expansion, while it may be optimal to extend different international sectors in different regions. It is, however, not easy to reach an optimal and consistent or feasible solution when starting from the successive solution of Section 4.9 even apart from the inconsistencies with regard to the regional sector, discussed above. The difficulty is that if one allows the totals of the international sectors in the second problem to be changed, one must go back to the first problem in order to find a new consistent total for the national sector. In addition, the exact formula for checking whether the solution can be improved at all is complicated and will not be given here.

Fortunately, a reasonable approximation method is available. It consists of first checking approximately whether the solution is optimal by comparing in the solution of the second problem the values of the variables u^h corresponding to the international sectors: if the u^h value of the international sector selected in the first problem is larger than any of the u^h values of the other international sectors, then the solution is not optimal. In that case one solves again the second problem, but now without the given total income

increases of each international sector, in order to improve the optimality of the solution. The results are used to recalculate from equation (4.9.3) the total income increase of the national sector (and if desired the regional income increases of the regional sector from equations (4.8.2)-(4.8.4)), now taking the values of the variables of the international sectors as given. This is done in order to improve the consistency of the solution. Finally, one recalculates the solution of the second problem, now taking again the new total income increase of the national sector (and possibly the new income increases of the regional sector) as given. This solution is in general nearer to the optimal one than the first solution. It could itself be improved further by once again applying the same adjustment procedure, or calculating the corresponding solution as indicated below.

This method will be illustrated briefly by a *numerical example* corresponding to the model of Sections 4.8 and 4.9. Sector 1 is a regional and sector 2 a national sector; sectors 3, 4 and 5 are international sectors. The main data can be found in Tables III.1 and III.2.

TABLE III.1
Technical coefficients

Sector	Sector 1	2	3	4	5
1	0.040	0.050	0.040	0.100	0.100
2	0.020	0.040	0.100	0.100	0.050
3	0.030	0.100	0.050	0.100	0.020
4	0.060	0.010	0.020	0.020	0.030
5	0.050	0.100	0.040	0.030	0.150
φ_{0h}	0.800	0.700	0.750	0.650	0.750

TABLE III.2
Cost coefficients

Sector	Region 1	2	3
1	4.000	4.800	4.800
2	3.150	2.800	3.500
3	2.250	1.500	2.400
4	2.600	2.600	1.950
5	1.800	2.250	3.000

The regional income targets and the demand coefficients are as follows:

$$^1y = {}^2y = {}^3y = 1.000; \quad {}^1\gamma^1 = {}^2\gamma^1 = {}^3\gamma^1 = 0.200; \quad \gamma^2 = 0.200.$$

The first problem corresponding to (4.9.1)-(4.9.4), and where for each sector the lowest regional cost coefficient is used, reads as follows:

$$\text{Min. } z = 4.000\, x^1 + 2.800\, x^2 + 1.500\, x^3 + 1.950\, x^4 + 1.800\, x^5.$$

Subject to:

$$\begin{aligned} 0.960\, x^1 - 0.050\, x^2 - 0.040\, x^3 - 0.100\, x^4 - 0.100\, x^5 &= 0.6 \\ -0.020\, x^1 + 0.960\, x^2 - 0.100\, x^3 - 0.100\, x^4 - 0.050\, x^5 &= 0.600 \\ 0.800\, x^1 + 0.700\, x^2 + 0.750\, x^3 + 0.650\, x^4 + 0.750\, x^5 &= 3.000 \\ x^h \geq 0, \quad h = 1, \ldots, 5. \end{aligned}$$

The solution computed by e.g. the simplex method, is:

$$x^1 = 0.769; \quad x^2 = 0.885; \quad x^3 = 2.353.$$

The resulting second problem is represented in Table III.3 by means of a table of cost coefficients together with the sectoral and regional totals expressed in value added terms.

TABLE III.3

Data of the second problem

Sectors	Regions			Totals
	1	2	3	
1	5.600	6.400	6.400	0.615
2	4.500	4.000	5.000	0.620
3	3.000	2.000	3.200	1.765
4	4.000	4.000	3.000	0
5	2.400	3.000	4.000	0
Totals	1.000	1.000	1.000	3.000

The uv-method gives the following solution:

TABLE III.4

Solution of the second problem[a])

Sectors	u^h	Regions 1	2	3	Totals	Costs
		^rv				
		3.000	2.000	3.200		
1	x[b])	0.205	0.205	0.205	0.615	3.487
		(−0.006)	(0.003)	(0.003)		
2	1.500	0.620	—	—	0.620	2.789
3	0	0.175	0.795	0.795	1.765	4.659
4	−0.200	—	—	0[c])	0	0
5	−0.600	0[c])	—	—	0	0
Totals		1.000	1.000	1.000	3.000	10.935

[a]) The numbers between brackets represent the differences between this and a completely consistent solution i.e., where equations (4.8.2)-(4.8.4) are satisfied exactly.

[b]) There is no u^h value for the regional sector 1 since its income increases are calculated independently of the optimizing problem.

[c]) In each non-selected international sector a variable must be considered to be basic in order to give a complete basis for calculating all the u^h and ^rv values.

As the value of u^3 is larger than u^4 and u^5, this is an indication that the solution is not optimal. Therefore, we drop the given totals for the international sectors and obtain the following changed solution: (see Table III.5). Recalculating $^1x^1$, $^2x^1$, $^3x^1$ and x^2 by (4.8.2)-(4.8.4) and (4.9.2) gives: $^1x^1 = 0.319$; $^2x^1 = 0.262$; $^3x^1 = 0.336$; $x^2 = 0.848$. These values have to be expressed in value added terms. The new second problem, with the same cost coefficients as in Table III.3, but now with different income data, for the regional sector also, then becomes : (see Table III.6).

TABLE III.5

Changed solution of the second problem [a)]

Sectors	u^h	Regions 1	2	3	Totals	Costs
		r_v				
		2.400	2.000	3.000		
1	x	0.205	0.205	0.205	0.615	3.487
		(−0.048)	(−0.006)	(−0.061)		
2	2		0.620		0.620	2.479
					(0.026)	
3	0		0.175			
4	0			0.795	1.765	4.643
5	0	0.795				
Totals		1.000	1.000	1.000	3.000	10.609

[a)] See Table III.4, footnotes a and b.

TABLE III.6

Income data for the second problem after adjustment

Sectors	Regions 1	2	3	Totals
1	0.255	0.210	0.269	0.734
2				0.594
3				
4				1.672
5				
Totals	1.000	1.000	1.000	3.000

The final solution of the second problem after the adjustment calculations is given in Table III.7.

TABLE III.7

Final solution after adjustment[a)]

Sectors	u^h	Regions 1	2	3	Totals	Costs
		r_v				
		2.400	2.000	3.000		
1	x	0.255 (0.005)	0.210 (−0.001)	0.269 (0.008)	0.734 (0.012)	4.149
2	2.000	—	0.594	—	0.594 (0.006)	2.376
3	0	—	0.196	—	1.672	0.392
4	0	—	—	0.731		2.193
5	0	0.745	—	—		1.788
Totals		1.000	1.000	1.000	3.000	10.898

[a)] See Table III.4, footnotes a and b.

Finally, it is interesting to compare the final solution after adjustment with the consistent solution with the same positive variables. Once it is known which variables are basic, or positive, in the solution, one can solve the original model of Section 4.8 after setting all non-basic variables = 0. It turns out that the remaining system of 7 equations in 7 variables is relatively easy to solve. The result is given in Table III.8.

Notice that this solution might not be the really optimal one, as the optimality check with the uv-method neglects the effects of interindustry relations. In most cases, however, it will also be the optimal solution. An example of an exceptional case would be that an international sector would appear in a region and require considerable intermediate deliveries from a regional sector, which in that region would be considerably more expensive

TABLE III.8

Consistent solution with the same positive variables as in the final solution after adjustment

Sectors	u^h	Regions 1	2	3	Totals	Costs
		r_v				
		2.400	2.000	3.000		
1	x [a)]	0.250	0.211	0.261	0.722	4.082
2	2.000	—	0.589	—	0.589	2.356
3	0	—	0.200	—		0.400
4	0	—	—	0.739	1.689	2.217
5	0	0.750	—			1.800
Totals		1.000	1.000	1.000	3.000	10.855

[a)] See Table III.4, footnotes a and b.

than in other regions. Then, in order to find out in such cases the really optimal solution, one would have to use more complicated methods for solution, e.g. solving the model of Section 4.8 simultaneously by the simplex method.

In drawing conclusions from this numerical example one must be aware that the results depend on the given numerical values and cannot be simply generalized. Comparing the solutions of Tables III.4, III.5, III.7 and III.8, we see that the solution of Table III.5 is indeed considerably cheaper than that of Table III.4. However, it is also less consistent (measured by the resulting surpluses). The adjustment procedure improves the consistency of the solution considerably, but the costs also rise again, as shown in Table III.7. Making the solution completely consistent as in Table III.8 results in costs which are roughly 1% lower than the original solution in Table III.4. If we may assume that Table III.8 is the optimal solution, then it shows

that the original solution was not very bad, from the point of view either of consistency, or of optimality. More computational experience would be necessary for generalizing this conclusion. This would be especially important for cases where upper bounds, as discussed in Section 4.11, would have increased the realism of the model.

III.2. The Decomposition Methods in Chapter 6

This section discusses some of the basic ideas behind the decomposition methods of Sections 6.1, 6.2 and the solution method of Section 6.3. We shall deal mainly with the type of problems of Sections 6.1 and 6.2. Only at the end of this section is the solution method in Section 6.3 drawn into the discussion.

If one first solves the continental (or regional) planning and next the transport (or transshipment) problem, like in Sections 5.2 or 6.2, obtaining in this way the so-called successive solution, then the transport variables can be considered to be functions of the (heavy) production variables, since the production variables of the heavy sector(s) determine the surpluses and deficits of production over demand, which make transportation necessary. Certain small changes in these (heavy) production variables, however, may not change the optimal basis of the transport problem – that is the set of basic variables in the optimal solution – but they do change the numerical values of these basic variables. Now geometrically speaking, we will call the collection of all feasible points in the space of production variables which give rise to the same optimal basis in the transport problem a feasible subspace.

As long as the optimal basis of the transport problem remains the same, the cost of transportation is a (linear) function solely of the basic variables, which in their turn are a (linear) function of the (heavy) production variables. Therefore, for all values of the (heavy) production variables within one feasible subspace, the corresponding minimum transport costs are a linear function of these (heavy) production variables.

This linear function, expressing the transport costs in the (heavy) production variables, can be found in two ways. If we have solved the transport problem by the original simplex method, then it is indeed easiest to express the basic transport variables in terms of the (heavy) production variables.

However, if we have solved the transport problem by the simplex method using multipliers, then it is easier to use a theorem of linear programming, which is easy to prove and which says that the minimum costs of the transport problem are equal to the scalar product of the vector of constants of the transport problem and the vector of simplex multipliers corresponding to the optimal basis of the transport problem[1]) The constants themselves are linear functions of the heavy product variables. Notice that the simplex multiplier values remain the same as long as the basis remains the same, and therefore, as long as the (heavy) production variable values remain within the same feasible subspace.

The feasible subspaces are closed convex[2]) sets and together they fill up the whole feasible space. Neighbouring feasible subspaces have boundary points in common, so some feasible points may belong to two or more feasible subspaces at the same time. Since transport costs are defined as a linear function, over each feasible subspace, they are defined as a piecewise linear function over the whole feasible space. In addition, this piecewise linear function is continuous, since at the boundary points of the feasible subspace there are multiple degenerate solutions of the transport problem, that is, there is more than one basis, each basis corresponding to a feasible subspace, but these different bases of the transport problem, and the different multiple solutions, yield the same value of the costs of transportation.

The most important property of this piecewise linear function, expressing the transport costs, is that it is convex over the whole feasible space[3]). This can be demonstrated as follows. If the vectors y_a and y_b are two feasible production expansion programmes, and $y = \lambda y_a + (1-\lambda) y_b$ also constitutes,

[1]) Call the column vector of basic variables in the optimal solution of the transport problem $\tilde{t}_B$, the square matrix of their coefficients in the restrictions B, the row vector of their cost coefficients $\tilde{\tau}_B$, the column vector of constants b, and the row vector of the corresponding simplex multipliers u_B. Then, by definition: $u_B \cdot B = \tilde{\tau}_B$, and, because the non-basic variables are $= 0$, $B \cdot \tilde{t}_B = b$. Then the transport costs are

$$z_T = \tilde{\tau}_B \cdot \tilde{t}_B = \tilde{\tau}_B \cdot {}_B^{-1} \cdot b = u_B \cdot b.$$

[2]) A convex set is defined by the following property: if two points y_a and y_b belong to the set, then any point $\lambda y_a + (1-\lambda) y_b$, $0 \leqslant \lambda \leqslant 1$ also belongs to it.

[3]) A function $f(y)$ is convex if it is defined over a convex space and if for any point y_1 in that region, which is a convex combination of two points y_a and y_b in that region (that is: $y_1 = \lambda y_a + (1-\lambda) y_b$ for a certain $0 \leqslant \lambda \leqslant 1$ and some y_a and y_b),

$$f(y_1) \leqslant \lambda f(y_a) + (1-\lambda) f(y_b).$$

by virtue of the convexity of the feasible space a feasible solution of the (continental) regional planning problem, then $\tilde{t} = \lambda\tilde{t}_a + (1-\lambda)\tilde{t}_b, 0 \leq \lambda \leq 1$, where the vectors $\tilde{t}_a$ and $\tilde{t}_b$ are the optimal transport solutions corresponding to y_a and y_b, constitutes a feasible solution of the transport problem. In other words, the transportation necessary because of solution y, can be carried out by λ times the optimal transport programme corresponding to solution y_a and $(1-\lambda)$ times the optimal transport programme corresponding to solution y_b. Its costs are:

$$z_T(\tilde{t}) = z_T(\lambda\tilde{t}_a + (1-\lambda)\tilde{t}_b) = z_T(\lambda\tilde{t}_a) + z_T(1-\lambda)\tilde{t}_b = \lambda z_T(\tilde{t}_a) + (1-\lambda) z_T(\tilde{t}_b),$$

where $z_T(\tilde{t})$ is the value of the transport cost function for the vector $(\tilde{t})$. Because $\tilde{t}_a$ and $\tilde{t}_b$ are linear functions of y_a and y_b, we can also write $z_T(\tilde{t}) = \lambda\bar{z}_T(y_a) + (1-\lambda)\,\bar{z}_T(y_b)$, where $\bar{z}_T(y)$ represents the minimum transport costs corresponding to the solution y, the above-mentioned piecewise linear function.

Since $\tilde{t}$ constitutes a feasible transport programme corresponding to solution y, the minimal transport costs corresponding to y are certainly not higher than $z_T(\tilde{t})$: $\bar{z}_T(y) \leq z_T(\tilde{t}) = \lambda\bar{z}_T(y_a) + (1-\lambda)\,\bar{z}_T(y_b)$.

This proves that $\bar{z}_T(y)$ is convex.

In fact $\tilde{t}$ is not necessarily the optimal transport programme corresponding to the given solution y. In particular if $\tilde{t}_a$ and $\tilde{t}_b$ contain transport flows of the same product in different directions, $\tilde{t}$ contains return flows of the same product, which can be cancelled advantageously.

The reason why the convexity of the piecewise linear function is so important, is that we can then make use of the fact that a local minimum is also a global minimum, that is, if no very small change of the solution variables can make total costs smaller, then a larger change cannot do so either. We can illustrate this fact, and the solution method based on it, graphically and schematically as follows. (See Figure III.1).

The interpretation of Figure III.1 is as follows. One wants to minimize total costs $z(y)$. In principle one can calculate the minimum within each feasible subspace – corresponding to what we sometimes called in Chapter 6 a case – and take the minimum of these minima as the overall minimum. The successive solution method computes first the minimum of z_p in the feasible space ($E'E$ when y at E). Next the corresponding minimum of the transport costs is computed ($E''E$, the transport variables are not represented here). The decomposition method constructs a new (continental) regional

planning problem within the feasible subspace DE and with $D'''E'''$ as objective function. D''' turns out to be the minimum of this problem, when y is at D. Then one only needs to look at the neighbouring problem(s), in this

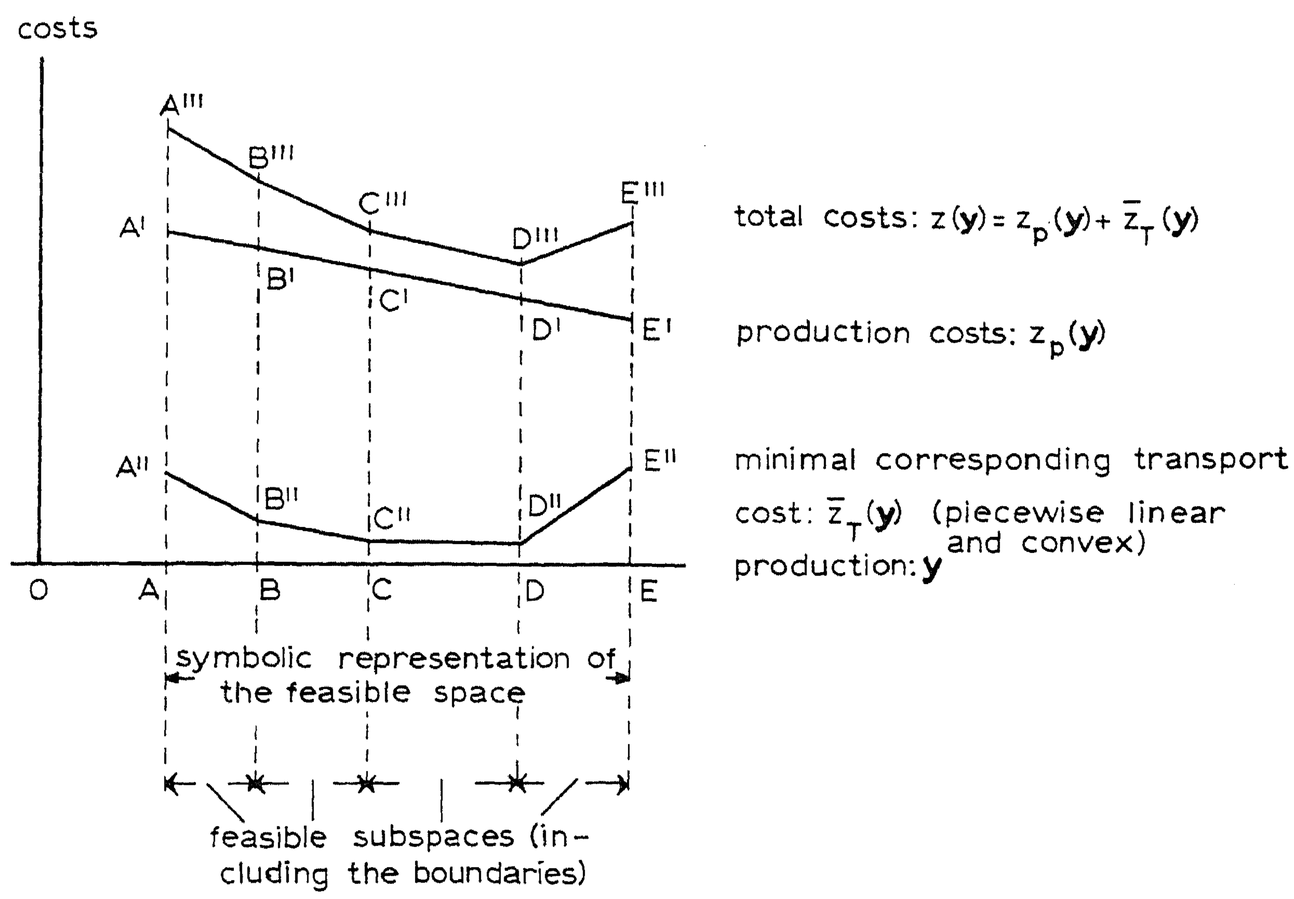

Fig. III.1. Schematic graphical representation of the feasible subspaces, production costs and transport costs defined over the y-space.

case within feasible subspace CD and with objective function $C'''D'''$ in order to see whether D is also the minimum for that problem.

If this is the case then D represents the overall optimum. If D was not such an overall minimum, one would have to solve the new problem again, reach C as solution and would apply the same check there, etc., until the overall optimum is found.

Finally, in Fig. III.2 we give as an illustration a graphical representation of how the solution by means of the decomposition method proceeds in order to find the overall optimal solution. It should be compared with Tables 6.3 and 6.4. We can only represent the problem in three dimensions and show, therefore, only the intersection of the feasible y space with the ${}^1y^4$, ${}^2y^4$, ${}^3y^4$ space. Notice that $\bar{z}_T(y)$ in this case is only a function of the variables ${}^1y^4$, ${}^2y^4$, ${}^3y^4$, as all other variables have the coefficient $= 0$. The intersection with the feasible space is the figure $ABCDEF$. The total

number of feasible subspaces would be a maximum of 2^3, as there are 3 regions, which can be either deficit or surplus areas. However, the condition

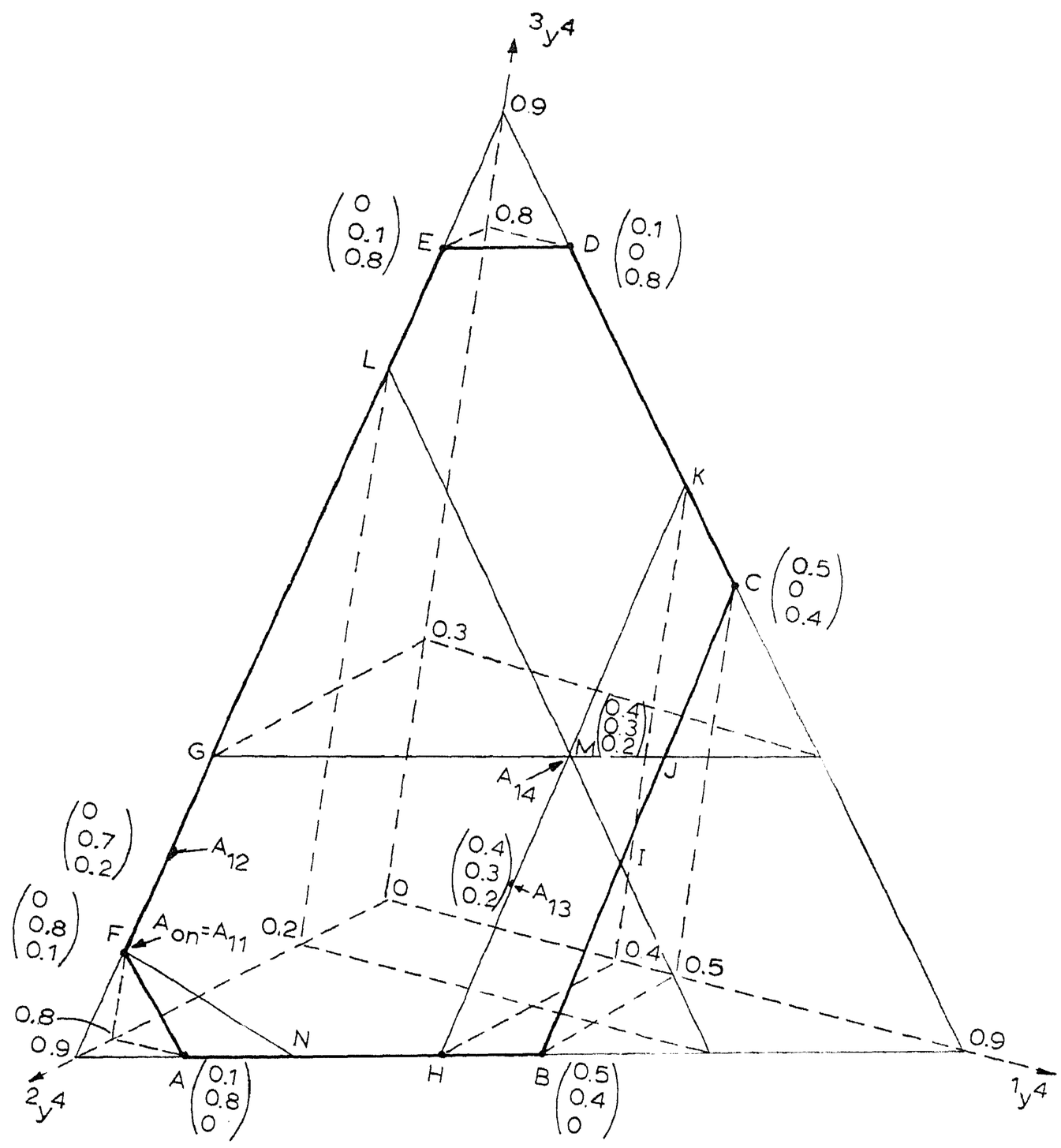

Fig. III.2. Graphical representation of solving the problem of Section 5.6., by decomposition. For explanation: see text.

$^1y^4 + {}^2y^4 + {}^3y^4 = 0.9$ (cf. Section 5.2) excludes the possibility of all regions being deficit regions or of them all being surplus regions. So 6 feasible subspaces remain with which the intersections are: *GFAHM*, *HBIM*, *IJM*,

JCKM, *KDELM* and *LGM*. The solution of the continental distribution problem is in point F; that is, point F gives the coordinates of ${}^1y^4$, ${}^2y^4$ and ${}^3y^4$ in this solution. Next, solving the transport problem gives the relevant part of the $\bar{z}_T(y)$ function: $\bar{z}_T(y) = 0.\ {}^1y^4 + 0.5\ {}^2y^4 - 0.2\ {}^3y^4$, which is determined apart from a constant. *FN* is an example of a line in which the feasible intersection of each point gives the same value to $\bar{z}_T(y)$. A parallel shift of this line in the direction of M does not change this property, only the $\bar{z}_T(y)$ value decreases. In round A_1 we deal with the subproblem, restricted to the feasible subspace of which *GFAHM* is the intersection and with a cost function to which $\bar{z}_T(y)$ has now been added. Via the solutions of iteration A_{12} and A_{13}, M is found as the solution for this subproblem. But M also belongs to the 5 other feasible subspaces, and we must check whether M is also optimal for the problems corresponding to them (cf. Table 6.4 check 1-5). The δ_1 and δ_2 here represent small shifts from M into the other feasible subspaces. M turns out to be the overall optimum, that is to say, it is the projection of the overall optimum in the ${}^1y^4$, ${}^2y^4$, ${}^3y^4$ space.

The method of solving the transport problem with dominant flow costs proceeds along the same lines. Here, the feasible space of transport variables is however, directly subdivided into feasible subspaces, each corresponding to a specific combination of dominant flows along the routes. The objective function is again piecewise linear and convex over the feasible space. The convexity can be proved in the same way as for the case dealt with above. The application of the iterative improvement method after the optimal solution of the transport problem has been reached also follows the same basic lines as indicated above.

A model with heavy sectors and explicit representation of input-output relations, as dealt with in Chapter 7 (the Basic Model), can also be solved by the decomposition method. In fact a decomposition procedure like either the one in Section 4.9 or the one in Section 6.1 or 6.2 could be applied. The same basic principles as are discussed in this appendix and in Chapter 6 can probably be applied. However, an explicit description of the method for this case, the calculation of a numerical example and an investigation of the efficiency of this method is left for later research.

We will not compare the decomposition method of Chapter 6 with existing decomposition methods (see e.g. DANTZIG, 1963; KORNAI, 1967; KÜNZI *et al.*, 1966), but merely present it in a schematical form to facilitate

such a comparison. The problems of Section 6.1, and similarly those of 6.2 and 7.3, can be schematically represented as follows:

$$\begin{array}{lcccccc}
\text{Variables} & \boxed{y} & & \boxed{t} & \geq 0 & & \\
\text{Minimize:} & \boxed{c^1 y} & + & \boxed{c^2 t} & & & \\
\text{Constraints:} & \boxed{A^1 y} & & & = & \boxed{b^1} \\
 & \boxed{A^2 y} & + & \boxed{A^3 t} & = & \boxed{b^2}
\end{array}$$

Here a distinction is made between the group of variables which appear in al the constraints, represented by the vector y, and the group of variables which appear only in those parts of the constraints which refer to heavy sectors, represented by the vector t. The vectors c^1 and c^2 represent the coefficients in the objective function for these two groups of variables, A^1 and b^1 the matrix of coefficients and the vector of constants in the constraints in which only the y-variables appear, and A^2, A^3 and b^2 the matrices of the coefficients of the y-variables and the t-variables and the vector of constants in the other constraints.

The decomposition method consists of solving the first problem:

$$\begin{array}{lcc}
\text{Minimize:} & \boxed{c^1 y} & \\
\text{Constraints:} & \boxed{A^1 y} = & \boxed{b^1}
\end{array}$$

and then solving the second problem:

$$\begin{array}{lcccccc}
\text{Minimize:} & \boxed{c^2 t} & & & & & \\
\text{Constraints:} & \boxed{A^3 t} & = & \boxed{b^1} & - & \boxed{A^2} & \boxed{y}
\end{array}$$

in which $\boxed{y}$ represents the solution of the first problem.

Finally, the value of the optimal solution of the second problem is expressed in the y-variables and added to the objective function of the first problem. This one is then solved again with this new objective function under extra constraints which ascertain that the expression for the objective function remains valid. If some of these constraints become active then some iterations consisting of switching between the first and the second problem as described in Section 6.1 are necessary before the optimal solution of the whole problem is reached.

In principle such a procedure could be applied to any problem of this structure. However, some special features in this case make the procedure here rather efficient. The main feature is that the cost coefficients of the t-variables are considerably smaller than those of the y-variables. Therefore, the optimal solution of the first problem is a good approximation of the values of the y-variables in the optimal solution of the whole problem. Two other simplifying features are that the A^2 matrix contains only a few non-zero elements, which are in addition $= 1$, and that the A^3 matrix is such that the second problem always has a feasible solution.

APPENDIX IV

HEAVY PRODUCTS[1])

This appendix will give some information on the relative share of five commodity groups – agricultural products, fertilizers, building materials, minerals, fuels – in the total inland goods traffic. The first part deals with figures for the Netherlands, France, Norway and Western Germany. The second part gives some figures with regard to developing countries.

IV.1. Commodity classification

The five commodity groups are, classified according to the CSTE (Commodity Classification for Transport Statistics in Europe), aggregates of the following groups: (page 297)

IV.2. Principal means of inland transport

Four means of transport are distinguished: roads, railways, waterways and oil pipelines. Only in the case of road transport was it possible to get statistics on goods traffic by commodity groups both in tons and ton-kilometers. However, a good comparison between countries is hindered by differences in definitions concerning short-distance road transport. The Netherlands and France define road transport as all intercity transport, while Norway and Western Germany exclude short-distance transport. But in Norway a short distance is defined as at the most 30 km, while in Western Germany it is at the most 50 km.

[1]) This investigation has been carried out by Mr. A. Kuyvenhoven, M.A.

Group		CSTE classification
Agricultural products	1	Cereals
	2, 3	Food products and feeding stuffs
	4	Seed, oil nuts, oils, fats
	5	Wood and cork
Fertilizers	6	Fertilizers
Building materials	7	Crude minerals other than ore
	15	Lime, cement and other minerals
Ores	8, 9	Ore and scrap
Fuels	11	Solid fuels
	12, 13	Petroleum and petroleum products
Other products	10	Other crude materials
	14	Chemicals
	16	Metals
	17	Manufactured metal articles
	18	Machines, transport equipment
	19	Miscellaneous manufactured articles
	20	Special transactions

Fortunately, there were Norwegian statistics available on freight transport on railways both in tons and ton-kilometers. The statistics of the other countries have been adjusted in accordance with these figures.

In the case of inland water transport it was not possible to derive ton-kilometer figures in a reasonable way. Some corrections have been applied in the light of the railways correction.

IV.3. The Netherlands

TABLE IV.1

Relative importance of the principal means of inland transport, 1963, the Netherlands

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Road	231	78	34.5	7,967	50
Railways	17	6	152 [a)]	2,608	17
Waterways	48	16	109 [a)]	5,260	33
Total	296	100	53.4	15,835	100

[a)] Figure of 1962.

TABLE IV.2

Share of commodity groups in the total inland goods traffic, 1963, the Netherlands

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
By road					
Agricultural products	56.6	24.5	50	2,825	35.5
Fertilizers	3.9	1.7	31	120	1.5
Building materials	102.1	44.2	16	1,652	20.7
Ores	1.0	0.4	30	29	0.4
Fuels	22.1	9.6	33	727	9.1
Heavy products	185.7	*80.4*	29	5,353	*67.2*
Other products	45.3	19.6	58	2,614	32.8
Total	231.0	100.0	34.5	7,967	100.0
By railways					
Agricultural products	1.3	7.3			
Fertilizers	1.0	5.8			
Building materials	2.2	12.6			
Ores	0.3	1.5			
Fuels	9.4	54.8			
Heavy products	14.1	*82.0*	148	2,086	*80*
Other products	3.1	18.0	169	522	20
Total	17.1	100.0	152	2,608	100
By waterways					
Agricultural products	6.2	12.9			
Fertilizers	1.4	3.0			
Building materials	35.5	73.5			
Ores	0.4	0.7			
Fuels	3.3	6.9			
Heavy products	46.8	*97.0*	109	5,102	*97*
Other products	1.4	3.0	109	158	3
Total	48.3	100.0	109	5,260	100

TABLE IV.3

All means of transport and the share of heavy products, 1963, the Netherlands

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Heavy products	246.6	*83*	51	12,541	*80*
Other products	49.8	17	66.1	3,294	20
Total	296.4	100	53.4	15,835	100

Source: C.B.S., Maandstatistiek voor verkeer en vervoer, 1963 en 1964.

IV.4. France

TABLE IV.4

Relative importance of the principal means of inland transport, 1962, France

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Road	944	75	35	32,952	31
Railways	231	19	263	60,727	57
Waterways	72	6	157	11,320	10
Pipelines	n.a.	n.a.	n.a.	2,201	2
Total	1,247	100	86	107,200	100

TABLE IV.5

Share of commodity groups in the total inland goods traffic, 1962, France

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
By road					
Agricultural products	228	24.1	49.5	11,263	34.2
Fertilizers	32	3.4	14	451	1.4
Building materials	469	49.7	15	7,062	21.4
Ores	12	1.3	35	431	1.3
Fuels	70	7.4	40	2,778	8.4
Heavy products	811	*85.9*	27	21,985	*66.7*
Other products	133	14.1	82	10,967	33.3
Total	944	100.0	35	32,952	100.0
By railways					
Agricultural products	27	11.6			
Fertilizers	14	6.1			
Building materials	18	7.9			
Ores	51	22.3			
Fuels	48	20.7			
Heavy products	158	*68.6*	257	40,687	*67*
Other products	73	31.4	276	20,040	33
Total	231	100.0	263	60,727	100
By waterways					
Agricultural products	8.1	11.2			
Fertilizers	2.4	3.3			
Building materials	28.2	39.1			
Ores	2.4	3.3			
Fuels	12.1	16.8			
Heavy products	53.2	*73.7*	153	8,150	*72*
Other products	18.9	26.3	168	3,170	28
Total	72.1	100.0	157	11,320	100

TABLE IV.6

All means of transport and the share of heavy products, 1962, France

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Heavy products	1,022	*82*	71.5	73,023 [a)]	*68*
Other products	225	18	152	34,177	32
Total	1,247	100	86	107,200	100

[a)] Including 2,201 ton-kilometers pipeline traffic.
Source: Annuaire Statistique de la France, 1964.
Annual Bulletin of Transport Statistics in Europe, U.N., 1964.

IV.5. Norway

TABLE IV.7

Relative importance of the principal means of inland transport, 1963, Norway.

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Road	3.056	15	38	116	7
Excluding short-distance transport	0.246	1.2	97	24	1.5
Railways	17.9	85	85	1,525 977	93
Excluding Ofoten railway [a)]	4.2	20	231	977	60
Total	20.96	100	78	1,641	100

[a)] Transport of iron ore on the Norwegian part of the Ofoten railway to Sweden. Distance up to the Swedish frontier: 40 km.

TABLE IV.8

Share of commodity groups in the total inland goods traffic, 1963, Norway

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
By road					
Agricultural products	0.105	42.8	91.5	9.6	40.5
Fertilizers	0.008	3.4	68	0.6	2.4
Building materials	0.023	9.5	100	2.3	9.8
Ores	0.016	6.4	59.5	0.9	3.9
Fuels	0.017	6.8	90.5	1.5	6.3
Heavy products	0.169	*68.9*	88	14.9	*62.9*
Other products	0.077	31.1	115.5	8.9	37.1
Total	0.246	100.0	97	23.8	100.0
By railways					
Agricultural products	1.1	6.3	252	282	18.5
Fertilizers	0.3	1.8	185	60	3.9
Building materials	0.4	2.3	210	87	5.7
Ores	13.9	77.4	42	586	38.5
Fuels	0.2	1.2	132	28	1.8
Heavy products	15.9	*89.0*	66	1,043	*68.4*
Other products	2.0	11.0	245	482	31.6
Total	17.9	100.0	85	1,525	100.0
By railways excluding Ofoten railway					
Agricultural products	1.1	26.6	252	282	28.9
Fertilizers	0.3	7.7	185	60	6.1
Building materials	0.4	9.8	210	87	8.9
Ores	0.2	4.2	215	38	3.9
Fuels	0.2	5.0	132	28	2.9
Heavy products	2.2	*53.3*	225	495	*50.7*
Other products	2.0	46.7	245	482	49.3
Total	4.2	100.0	231	977	100.0

TABLE IV.9

All means of transport and the share of heavy products, 1963, Norway[a)]

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Agricultural products	2.4	12	137	328	20
Fertilizers	0.4	2	158	63	4
Building materials	0.7	3	140	98	6
Ores	14.1	67	42	593	36
Fuels	0.3	2	105	34	2
Heavy products	18.0	*86*	62	1,116	*68*
Other products	3.0	14	180	525	32
Total	21.0	100	78	1,641	100

a) Weighted average. Weighting according to the relative shares of total transport of road and railways in inland transport (see Table IV.7), so, in an approximate way, including short-distance road transport.

Source: Statistisk Sentralbyrå, Samferdselsstatistikk, Oslo 1966.
Lastetiltransport, Oslo 1964.
United Nations, Annual Bulletin of Transport Statistics of Europe, 1964.

IV.6. Western Germany

TABLE IV.10

Relative importance of the principal means of inland transport, 1964, Western Germany

	1,000,000 ton	%	Average distance (km)[a])	1,000,000 ton-km	%
Road[b])	110	22	244	26,864	26
Railways	245	50	210	51,492	49
Waterways	96	20	221	18,816	18
Pipelines	38	8	189	7,106	7
Total	489	100	213	104,278	100

[a]) Average distance for total transport, including international transport. The share of inland transport in the total transport is, for road transport 89%, for railways 73%, for waterways 52% and for pipelines 100%. It was not possible to correct the distance figures in a reasonable way.

[b]) Excluding short-distance road transport.

TABLE IV.11

Share of commodity groups in the total inland goods traffic, 1964, Western Germany

	Road		Railways		Waterways	
	1,000,000 ton	%	1,000,000 ton	%	1,000,000 ton	%
Agricultural products	29	26	19	8	4	4
Fertilizers			13	5	1	1
Building materials	26	24	34	14	42.5	44
Ores	1	1	32	13	8	8
Fuels	9	8	98	40	35.5	37
Heavy products	65	*59*	195	*80*	91	*94*
Other products	45	41	50	20	5	6
Total	110	100	245	100	96	100

TABLE IV.12

All means of transport and the share of heavy products, 1964, Western Germany[a)]

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Heavy products	389	*80*	210	81,679	*78*
Other products	100	20	226	22,599	22
Total	489	100	213	104,278	100

[a)] Including pipeline figures of Table IV.10. The means of transport are weighted according to the distance figures of Table IV.10 in order to obtain the ton-kilometer totals. No weighting is applied to the separate commodity groups. First, because there are no average-distance data available for commodity groups, secondly, because the differences between ton and ton-kilometers disappear when the average distance per means of transport becomes greater.

Source: Statistisches Jahrbuch für die Bundesrepublik Deutschland, 1966.

IV.7. Conclusions

We can summarize our findings for the developed countries in the following way:

(1) The share of heavy products in total inland transport measured in ton-kilometers, varies between 68% and 80%.

(2) This share becomes higher, between 80% and 86%, when measured in tons.

(3) This difference in share, as measured in tons or in ton-kilometers, becomes larger as the average distance declines.

So, we can conclude that there exists a natural tendency to transport heavy goods over shorter distances than "normal" goods.

Next, we will consider three less-developed countries: Korea, Argentina and Tchad. In these countries the statistics on inland goods traffic are less reliable than in the countries we have been dealing with so far. For that reason the conclusions should be interpreted with somewhat more care.

IV.8. Korea

TABLE IV.13

Relative importance of the principal means of inland transport, 1964, Korea

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Road	12.2	40	40	484	10
Railways	16.8	54	232	3,907	82
Waterways [a)]	2.0	6	200	389	8
Total	31.0	100	154	4,780	100

[a)] Including coastal shipping.

TABLE IV.14

Share of commodity groups in the total inland goods traffic, 1964, Korea

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
By road[a])					
Agricultural products	2.91	23.9	50	145.5	30.1
Fertilizers[b])	0.65	5.3	30	19.5	4.0
Building materials			(15)		
Ores	0.62	5.1	30	18.6	3.8
Fuels	2.92	23.9	40	116.8	24.2
Heavy products	7.10	*58.2*	42	300.4	*62.1*
Other products	5.10	41.8	36	183.6	37.9
Total	12.20	100.0	40	484.0	100.0
By railways					
Agricultural products	2.04	12.1	229	466	12.0
Fertilizers[b])	1.24	7.4	202	250	6.4
Building materials	1.13	6.7	287	324	8.3
Ores	0.90	5.4	257	231	5.9
Fuels	8.81	52.4	217	1,911	48.9
Heavy products	14.12	*84.0*	225	3,182	*81.5*
Other products	2.68	16.0	260	725	18.5
Total	16.80	100.0	232	3,907	100.0
By waterways			c)		
Agricultural products	0.25	12.8	90	22	5.7
Fertilizers[b])	0.06	3.1	100	6	1.5
Building materials	0.06	3.1	80	5	1.3
Ores	0.01	0.5	100	1	0.3
Fuels	1.29	66.1	257	332	85.3
Heavy products	1.67	*85.6*	220	366	*94.1*
Other products	0.28	14.4	80	23	5.9
Total	1.95	100.0	200	389	100.0

[a]) In order to obtain these figures, some crude approximations are made:

TABLE IV.15

All means of transport and the share of heavy products, 1964, Korea

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Agricultural products	5.20	17	122	633	13
Fertilizers	1.95	6	142	276	6
Building materials	1.19	4	276	329	7
Ores	1.53	5	164	251	5
Fuels	13.02	42	181	2,359	49
Heavy products	22.89	*74*	168	3,848	*80*
Other products	8.06	26	116	932	20
Total	30.95	100	154	4,780	100

Source: BCEOM-NEDECO-NEI-SOFRERAIL, Korea Transportation Survey Draft Report, 1966.

(1) the relative share of the commodity groups, measured in tons, is supposed to be the same for short and long run trips (intercity traffic being 65% of 18.72 ton = 12.20 ton, and 95% of 510 ton-km = 484 ton-km);
(2) agricultural products are aggregated in accordance with the aggregate for railways;
(3) average distance figures are estimates.

b) Including limes, used in agriculture and for the manufacture of fertilizer, cement and iron. The ratio of fertilizers to limes is 3 : 7.

c) Estimates of BCEOM-NEDECO-NEI-SOFRERAIL.

IV.9. Argentina

TABLE IV.16

Relative importance of the principal means of inland transport, 1960, Argentina

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Road	48.3	51	350 [a]	13,800	35
Railways	25.9	28	585	15,158	39
Waterways	19.8 [b]	21	518	10,250	26
Total	94.0	100	417	39,208	100

[a]) Estimate based on a sample.

[b]) Extrapolation of the 1958 figure.

TABLE IV.17

Share of commodity groups in the total inland goods traffic, 1960, Argentina

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
By railways					
Agricultural products	15.5	60.0	518	8,051	53.1
Building materials	3.2	12.2	567	1,787	11.8
Fuels	3.2	12.2	728	2,309	15.2
Heavy products	21.9	*84.4*	556	12,147	*80.1*
Other products	4.0	15.6	745	3,011	19.9
Total	25.9	100.0	585	15,158	100.0
By waterways[a]					
Agricultural products	0.456	36.5	774	353	42.6
Building materials	0.244	19.5	471	115	13.9
Fuels	0.339	27.2	723	245	29.6
Heavy products	1.039	*83.2*	686	713	*86.1*
Other products	0.210	16.8	549	115	13.9
Total	1.249	100.0	663	828	100.0

[a]) Sample

Although there was a sample of road transport during one week in 1961, it was not possible to derive suitable figures according to our classification. It seems reasonable to estimate that at least 60%, measured in ton-kilometers, of all road traffic refers to heavy products. Taking the same figures for tons, we can derive Table IV. 18.

TABLE IV.18

All means of transport and the share of heavy products, 1960, Argentina

	1,000,000 ton	%	Average distance (km)	1,000,000 ton-km	%
Heavy products	67.3	*72*	435	29,250	*75*
Other products	26.7	28	373	9,960	25
Total	94.0	100	417	39,210	100

Source: A Long Range Transportation Plan for Argentina, 1962.

IV.10. Tchad

TABLE IV.19

Share of commodity groups in the total inland goods traffic
Projections for 1970, 1975 and 1985. By railways

	1970		1975		1985	
	1,000 ton	%	1,000 ton	%	1,000 ton	%
Agricultural products	83	61	103	58	127	48
Building materials	35	25	48	28	96	37
Fuels	6	4	8	4	15	6
Heavy products	124	*90*	159	*90*	238	*91*
Other products	13	10	17	10	24	9
Total	137	100	176	100	262	100

Source: Rapport de la mission conjointe pour l'étude du projet de prolongement du chemin de fer transcamerounais vers le Tchad méridional, 1964.

IV.11. Conclusions

It is interesting to compare the figures of four developed European countries – The Netherlands, France, Norway and Western Germany – with two developing countries: Korea and Argentina. This comparison reveals some similarities, but also some striking differences.

(1) The share of heavy products in total inland transport, measured in ton-kilometers, in Korea is 80% and in Argentina 75%. These two figures lie within the interval of 68-80%, found in the case of the four developed countries.

(2) When measured in tons, this share is 74% for Korea and 72% for Argentina. So it becomes not higher, as in the developed countries, but lower.

(3) The cause of this phenomenon is the high average distance over which fuels are transported. In a sense this reflects the under-developed character of the economy of the two countries, namely too high a distance between the mining fields and the areas where fuel consuming industries are (traditionally) located.

(4) In developed as well as in developing countries, there is a rising share of heavy products in total inland transport as one goes from road to railways and from railways to waterways transport. This tendency is most pronounced when transport is measured in ton-kilometers.

(5) The main conclusion of IV.7 that there exists a natural tendency to transport heavy products over shorter distances than "normal" products, clearly refers only to developed economies.

APPENDIX V

CLASSIFICATION OF SECTORS

TABLE V.1

Classification of Sectors according to the mobility of their products

ISIC[a] numbers	International sectors
011	Agriculture and livestock production
021	Forestry
022	Logging
030	Hunting, trapping and game propagation
041	Ocean and coastal water fishing excluding factory-vessel fishing
042	Factory-vessel fishing
043	Inland water fishing
110	Coal mining
121	Iron ore mining
122	Metal mining except iron ore mining
130	Crude petroleum and natural gas
192	Chemical and fertilizer mineral mining
199	Non-metallic mining and quarrying not elsewhere classified
201	Slaughtering, preparation and preserving of meat
202	Manufacture of dairy products
203	Canning and preserving of fruits and vegetables
204	Canning and preserving of fish and other sea foods
205	Manufacture of grain mill products
207	Sugar factories and refineries
208	Manufacture of cocoa, chocolate and sugar confectionery
209	Manufacture of miscellaneous food preparation (excluding perishable products)
211	Distilling, rectifying and blending of spirits
212	Wine industries
213	Breweries and manufacturing of malt
214	Soft drinks and carbonated water industries

TABLE V.1 (continued)

ISIC[a)] numbers	International Sectors
220	Tobacco manufactures
231	Spinning, weaving and finishing textiles
232	Knitting mills
233	Cordage, rope and twine industries
239	Manufacture of textiles not elsewhere classified
241	Manufacture of footwear
243	Manufacture of wearing apparel, except footwear
244	Manufacture of made-up textile goods, except wearing apparel
251	Sawmills, planing and other wood mills
252	Wooden and cane containers and cane small ware
259	Manufacture of cork and wood products not elsewhere classified
260	Manufacture of furnitures and fixtures
271	Manufacture of pulp, paper and paperboard
272	Manufacture of articles of pulp, paper and paperboard
280	Printing, publishing and allied industries
291	Tanneries and leather finishing plants
292	Manufacture of fur products, except wearing apparel
293	Manufacture of leather products, except footwear and other wearing apparel
300	Manufacture of rubber products
311	Basic industrial chemicals including fertilizers
312	Vegetable and animal oils and fats
313	Manufacture of paints, varnishes and lacquers
319	Manufacture of miscellaneous chemical products
321	Petroleum refineries
332	Manufacture of glass and glass products
333	Manufacture of pottery, china and earthenware
334	Manufacture of cement
341	Iron and steel basic industries
342	Non-ferrous metal basic industries
350	Manufacture of metal products except machinery and transport equipment
360	Manufacture of machinery except electrical machinery
370	Manufacture of electrical machinery, apparatus, appliances and supplies
381	Ship building and repairing
382	Manufacture of railroad equipment
383	Manufacture of motor vehicles
385	Manufacture of motorcycles and bicycles
386	Manufacture of aircraft
389	Manufacture of transport equipment not elsewhere classified

TABLE V.1 (continued)

ISIC[a] numbers	National sectors
391	Manufacture of professional, scientific, measuring and controlling instruments
392	Manufacture of photographic and optical goods
393	Manufacture of watches and clocks
394	Manufacture of jewellery and related articles
395	Manufacture of musical instruments
399	Manufacturing industries not elsewhere classified
715	Ocean transport
717	Air transport
719	Transport not elsewhere classified
841	Motion picture production and distribution (Cf. regional sectors)
	National sectors
012	Agricultural services
140	Stone quarrying, clay and sand pits
191	Salt mining and quarrying
329	Manufacture of miscellaneous products of petroleum and coal
331	Manufacture of structural clay products
339	Manufacture of non-metallic mineral products not elsewhere classified
511	Electric light and power
512	Gas manufacture and distribution
513	Steam, heat and power
611	Wholesale trade
620	Banks and other financial institutions
630	Insurance
640	Real estate
711	Railway transport
713	Road passenger transport, except omnibus operators
716	Water transport except ocean transport
810	Government services
823	Research and scientific institutions
826	Trade associations and professional and labour organizations
827	Libraries, museums and botanical and zoological gardens
829	Community services not elsewhere classified
856	Portrait and commercial photographic studios
	Regional sectors
206	Manufacture of bakery products
209	Manufacture of miscellaneous food preparations (only perishable products)

TABLE V.1 (continued)

ISIC[a] numbers	Regional sectors
242	Repair of footwear
384	Repair of motor vehicles
400	Construction
521	Water supply
522	Sanitary services
612	Retail trade
712	Tramway and omnibus operators
714	Road transport not elsewhere classified
718	Services incidental to transport
720	Storage and warehousing
730	Communication
821	Education services
822	Medical and other health services
824	Religious organizations
825	Welfare institutions
831	Legal services
832	Accounting, auditing and bookkeeping services
833	Engineering and technical services
839	Business services not elsewhere classified
841	Motion picture projection (Cf. international sectors)
843	Recreation services except theatres and motion pictures
851	Domestic service
852	Restaurants, cafes, taverns, and other drinking and eating places
853	Hotels, rooming houses, camps, and other lodging places
854	Laundries and laundry services; cleaning and dyeing
855	Barber and beauty shops
859	Personal services not elsewhere classified

[a] "International Standard Industrial Classification" Statistical Papers, Series M, Nr. 4, Rev. 1, United Nations, New York 1958.

APPENDIX VI

COST COEFFICIENTS

The calculation of the cost coefficients to be used in the models of Chapters 3-7 is probably one of the most difficult tasks in the field of data collection. We will, therefore, make a few remarks on this point without even attempting to discuss it extensively.

The cost coefficients are used to determine which sectors should be expanded in which regions in order to attain the regional income targets in such a way that the costs of the use of scarce resources is minimized.

As the world market prices of the international products and the domestic prices of the national and regional sectors are determined by total costs which include investment and current production costs, it is most logical to use in our models cost coefficients based on both types of costs.

If we distinguish capital and skilled labour as scarce factors of production three difficulties have to be overcome. First, investment costs, which are in terms of stocks, have to be converted into flows, i.e. interest flows. The average lifetime of the projects in a sector is taken account of when calculating this sector's income-output ratio by deducting the depreciation flows together with the material inputs from total output to be produced. Another possibility for estimating the capital component of the cost coefficient is to use the interest component of the value added row of input-output tables. Unfortunately, input-output tables do not generally contain such detailed information and often depreciation together with profit and interest appear in one figure.

Our second difficulty is one of the same nature but now with regard to skilled labour. In input-output tables the labour component of value added is generally given: wages and salaries. The share of skilled labour, however, is very difficult to determine so that our estimate of labour input will generally have a considerable unskilled labour component.

A third difficulty is that, even if the costs of the use of scarce factors of production both for investment and production would be known, these scarce factors should be valued at shadow prices, reflecting their scarcity or their opportunity costs. As these shadow prixes may deviate from the market prices, they should, strictly speaking, be estimated independently. A rough and ready method is to use "practical judgment" for correcting the market prices, e.g. by guessing 10% as the shadow rate of interes, tor by assuming it to be equal to the target rate of growth and by making some estimate of the wage rate for skilled labour. Another method which is theoretically more sound, would be to use the reversed form of the models in this book, as it was indicated in Section 3.1, but then with only a few sectors in order to keep the model relatively easy. The dual prices for the extra amounts of scarce factors available at the end of the planning period as compared with the beginning of the planning period, would give a reasonable approximation of the shadow prices to be used.

In order to get rid of these difficulties and because the methods dealt with in this book are most appropriate for distributing over the spatial units an investment fund which becomes available during the planning period it might be best to take into account solely investment costs. The limited availability of skilled labour can then be taken account of by means of additional restrictions, if so desired.

If we confine ourselves to investment costs in our models, it is also less necessary to convert them into flows and we could use incremental capital-output ratios (see Section 9.5).

In Section 3.1 we have indicated already that the benefit/cost ratios of project evaluation, as discussed in Section 2.6, use a more refined definition of benefits and costs, in so far as also the time profile of benefits and costs during and after the plan period are taken into account. This appears too refined a concept, however, to use it already at the middle stage.