CHAPTER 3

THE ELEMENT OF SPACE IN WORLD PLANNING; SECOND ORDER SUBDIVISION OF A CLOSED ECONOMY; TRANSPORTATION COSTS IMPLICIT

3.1. Introductory; Objective of Chapter; Nature of Problem

By the introduction of space the main problem of development planning acquires an additional dimension. If before we were only interested in a subdivision of the economy into sectors, a subdivision into geographical areas adds a second dimension. We want to know how much to invest in each sector and in each spatial unit. In more sophisticated planning there may be further complications; thus, we may distinguish several scarce factors and other means to attain the aims. Moreover, the spatial subdivision itself may be one of higher order. To begin with, however, we will concentrate our attention on the simplest version of the planning problem, where a two-dimensional subdivision is needed: one according to sectors and space units, representing a second order subdivision of a closed economy.

Our central example in this chapter will be the problem of planning for the world at large, with a subdivision into sectors and macro-spaces. For simplicity, we will call these macro-spaces "continents". But in actual practice such "continents" may not be the traditional ones. Some of the larger continents may be subdivided, for instance into centrally planned and other areas. We may think of Professor KIRSCHEN'S (1962) twelve parts of the world.

The reasons why we choose the world at large as our central example are (i) that this problem is beginning to attract political attention and (ii) that all products must then be used within the area considered. In other words, their demands are endogenous variables of the problem. This gives a simpler form to the problem than the corresponding problem at the national level, where part of the demand is exogenous.

To be sure, at present planning at the national level is what actually happens. For practical purposes therefore that type of planning problem is much more important. We will deal with it in Chapter 4 and subsequent chapters. World planning in its extreme form, starting from targets for the continental income increases, would imply the imposition on lower spaces of the rate of expansion of the sectors whose products can move outside the continents; the rate of expansion of the other sectors then follows from the model to be presented and does not need to be imposed from above. Even in the case of less extreme forms of world planning, for instance in the case where there would be a "co-ordination" of continental plans, in principle the same problem will arise, albeit with possibly different target values for the continental income increases. This makes it didactically attractive to consider the problem. It is also didactically attractive because it constitutes a simpler type of problem.

The nature of the problem now is to fill in a two-way table, arranged according to continents and sectors. We will arrange the continents horizontally and the sectors vertically. Two types of sectors are assumed to exist: immobile and mobile sectors, while for the latter transportation costs between continents can be neglected. As our main variables to be tabulated we will take the income increases, at constant prices, to be attained during the planning period, in each continent and each sector. If we assume production processes described by linear relations between outputs and inputs of any type, there will be constant ratios between any income increase and the corresponding investment. A two-way investment table can then be derived immediately from our main table. Similarly, a table showing production or output increases can be derived from our main table which shows income increases. In our simplest version of the problem we will not only assume that linear relations are a sufficiently accurate description of the production processes (to be called linear production processes); we will assume in addition that for mobile products the ratios between income or value added and output do not differ between continents. For the immobile products the assumption that the income-output ratio is the same in each continent is not necessary. For large areas the first assumption is likely to apply; over such areas indivisibilities, which give rise to non-linear relationships, will not play an important role. The second assumption will be only approximately correct and may have to be removed later on (cf. Chapter 7).

Our table (cf. Table 3.1) shows the main unknowns of the problem.

Still we have a number of different ways of formulating the planning problem. Generally speaking it will always be an optimization problem with restrictions. The objective function may be chosen in different ways, however. Again we will first try to choose a formulation which is as simple as possible without deviating too much from what is relevant in practice. The first version will be posed as follows: we consider as given the total income increases aimed at for each continent; this also implies that the total income increase is given. We then add the assumption that the demand for each mobile product is a function of the world income and that the demand for each immobile product in each continent is a function of that continental income. In the models to be presented we will, even more specifically, assume proportionality between the corresponding income and demand variables. However, the only essential point is that the future demand can be estimated on the basis of the income increases. As the ratio between income and output is given for all sectors, the income increase per continent in each immobile sector is also given. As a consequence of our previous assumption that for each mobile product this income-output ratio is the same, the row totals of our matrix are also given for the mobile products. They indicate the total income increase in each mobile sector. The restrictions to the unknowns are now very simple: all the column totals and all the row totals are known. We may then define the optimal solution as the one showing a minimum of costs of scarce factors. As the fixed policy targets are stated as increases in income over the planning period, the costs should be taken as annual investment and production costs of these income increases per sector and per region[1]). Here again we will introduce a simplifying assumption, namely that these costs, per unit of income to be created are given for each cell of the matrix. For the case of one scarce factor this does not add any further assumption: the incremental capital-output ratios have

[1]) There is some difference between this definition of costs and that of the definition of costs in Section 2.6. This last definition is more refined, but cannot be used in this comparative static model. The framework of the model does not allow distinguishing between sectors according to whether investments occur mainly in the beginning of the plan period or at its end. Neither is it possible to account completely for differences in gestation periods between sectors, but in principle the planning period should be so long that for all projects in the plan the gestation period falls within the plan period; otherwise approximative corrections for costs and outputs at the end of the planning period could be made. A brief discussion on the definition of costs and their role in the models is given in Appendix VI.

already been assumed to be constant. If there are more scarce factors, however, our assumption of given costs implies that the shadow prices of factors are known independently of the solution of the problem. Again this can only be an approximation. Clearly a more refined formulation of the problem is to introduce a relationship between these shadow prices and the quantities of each of the scarce factors used, which would make it a non linear programming problem however. Also the shadow prices for immobile scarce factors may be different in different continents.

Finally we make the assumption that for products, price differences between continents are negligable, in other words, that the contribution of intercontinental transportation to the income targets can be neglected.

There are other alternative formulations of the problem. Among the many conceivable alternatives we mention the following. We consider as given (*a*) the quantities of scarce factors available, either for each continent for immobile factors or for the world at large for mobile factors and (*b*) the ratios between the income increases aimed at; and we aim at maximizing the total income increase.

In the case of only one mobile scarce factor this formulation would lead in general to the same type of solution as results from the formulation given above. In the case of more mobile scarce factors or of one or more immobile scarce factors the solutions become in general different with different formulations.

3.2. Summarizing the Assumptions and the Data of the Simplest Version

The simplest version of the main world planning problem, as formulated in the preceding section, consists of finding a matrix of income increases by sector and by continent.

The *assumptions* made are:

Assumption 3.1. Some products are mobile between continents and others are not; for mobile products transportation costs between continents can be neglected.

Assumption 3.2. All production processes can be described by linear relationships between outputs and inputs of intermediate goods and scarce factors.

Assumption 3.3. For each mobile product the ratio of income to output is the same in all continents.

Assumption 3.4. The increase in demand for each mobile or immobile product is proportional to the income increase of the world or the various continents respectively.

Assumption 3.5. For products, price differences between continents can be neglected.

The *aims* set are given increases in total income for each continent.
The *criterion* to be used is to minimize total costs of scarce factors.

Accordingly, the *data* needed for the numerical solution of this simplest version of the main problem are:

(1) Target values of increases in income for each continent;

(2) Cost figures, expressed per unit of income increase for each product and each continent considered; essentially these costs must be marginal costs in the macro-sense, that is, costs applying to new production units of optimum size. These figures can be derived from incremental cost-output ratios combined with the data under 4;

(3) The classification of products and their production processes into the categories mobile and immobile between continents;

(4) The ratios of income to output for each sector;

(5) Ratios between the increase in demand (in terms of value added) and the increase in continental or world income for each immobile or mobile sector respectively.

3.3. Factors Determining Comparative Advantages

The cost figures entering our problem are an indication of the so-called comparative advantages which each space unit possesses in each of the activities considered. For any type of optimum distribution of activities over areas these comparative advantages are strategic, as is illustrated by theoretical as well as empirical research in this field.

In this section we will list the main factors which determine the costs at which any activity can be carried out.

These costs depend, first of all, on the availability of the natural resources needed. Natural resources have been often lumped together as one factor of production, "nature" or "land" even. For this factor more than for the others this suggestion of homogeneity is misleading. The diversity of natural resources is extremely wide. Rightly the oldest examples of trade and the corresponding division of labour in production have been taken from this diversity. Brazil's coffee, Egypt's cotton and Britain's coal exports are or were based on their natural resources. For developing countries natural resources are the most important factor of production, although we must hope that they will not remain so preponderant. Natural resources readily available will be cheap; many others will just not be available.

The next factor influencing costs is the factor of human labour. This factor again is much more diversified than the general phrase suggests. It can be present in hundreds of types and grades. With unskilled labour abundant it will be very cheap. In the same countries where this is so, the developing countries, many types of skilled or qualified labour will be scarce and hence expensive; similarly, research will be expensive since it requires labour with very high qualifications.

The third factor influencing costs is the availability of capital. Even this may take several specific forms as soon as fluid capital is invested; if it is still fluid, that is, available in the form of not yet invested money, it is almost homogeneous. For longer-term decisions this may be assumed to be the situation. The quantity available will determine its price.

As far as these scarce factors, natural resources, skilled labour and capital are immobile, their shadow prices may differ from continent to continent. These differences in shadow prices cause differences in costs, which in general can only partly be counterbalanced by using different combinations of these factors in producing the same good. If we consider fluid capital as largely mobile in principle, its shadow prices would differ less from continent to continent and these differences would contribute less to the differences in costs.

Most production units – farms, factories, offices – are a combination of the factors so far enumerated. The cost resulting for any type of product may then still depend on an element we want to mention separately, namely the experience of the unit and its leadership. Of course this could have been classified under the quality of labour. For practical purposes it is important, however, to state that some regularities have been found empirically in what is called the "learning process" and that cost estimates may take these into account.

In many analyses of the problem of the optimal division of labour the three factors listed above and their relative abundance have been indicated as the determinants of an area's specialization. Continents or countries should specialize in products which contain relatively more of the factors available in abundance. Apart from natural resources this implies that developed countries should specialize in capital-intensive and research-intensive products, developing countries in labour-intensive products[2]).

This rule of thumb must be qualified to the extent that complementarity exists between the natural resources needed and, say, capital or some types of labour. The combination may still be cheap if the influence of natural resources is considerable and rather capital-intensive activities may be attractive for a developing country[3]).

Another factor which complicates the picture is transportation costs, the element typical of our main subject. If it is difficult to transport a product essential to a given area's consumption or to its production structure, that product may have to be produced in that area. In our simpler models immobile products are examples; irrespective of their costs they will be produced within the area of use. Thus we will find that some products must be produced in developing countries even if they are capital-intensive, because their transportation costs are high. In our more complicated models we will deal with transportation costs explicitly, in particular with a number of so-called heavy sectors (cf. Section 1.2 and Chapters 5 and 6).

The picture we obtain from this sketch of the factors determining costs is more complicated than some simple rules of thumb suggest.

If all or an important part of the decisions about the production structure of a country are taken by private entrepreneurs, they will only decide to produce a given product if its production costs are lower than its world market price (including transportation costs) on the spot. For starting an exporting industry the even stronger condition must hold that production

[2]) This is the well-known Heckscher-Ohlin version of the doctrine of comparative costs.

[3]) The opposite case has been discovered by W. Leontief who found United States exports to be slightly less capital-intensive than its competitive imports (LEONTIEF, 1953-2 and 1956).

and transportation costs together are at most equal to the world market price. This comparison is influenced by the rate of exchange between local currency and some international currency. For several countries this exchange rate is maintained at a level which makes too few commodities competitive. This is true in particular for developing countries, but occasionally for developed countries also. It may then be necessary to adjust the exchange rate so as to make a sufficient number of products competitive.

For purposes of actual planning at a world level it is desirable to have at least a crude indication of the cost level for a number of products. The question arises whether there are possibilities of arriving at such estimates. Cost figures are difficult to obtain from direct statistical observation. Several authors (e.g., VERDOORN, 1954) who have faced the problem have chosen in favour of an indirect method based on import duties or export subsidies. The assumption made is that import duties are fixed at a level to bridge the gap between the world market price (c.i.f. at the frontier of the country considered) and the internal cost level. The latter can then be estimated by adding up the import duty to the world market price. This only applies to goods with an import surplus for the country considered. In the case of a product showing an export surplus the cost level may be estimated by adding up world market price (f.o.b.) and the export subsidy – if any – paid to producers. Clearly these estimates may be wrong; some producers may have been more successful in seeking protection than others. For illustrative purposes, however, one may use figures of the kind described.

3.4. Formulation of the Simplest Version as a Transportation or Hitchcock Problem

Returning to the main problem of this chapter and first to its simplest version we may reformulate it in the following words. We have to fill up a matrix of income increase figures referring to continents (columns) and sectors (rows), for which all row totals and all column totals are given. The former indicate, in a particular way geared to our problem, the demand totals for each product, and the latter the income increase aims set for the various continents. For the immobile products even the individual cells of their rows can be determined: they are a function of the given income increases for each continent. Hence we can omit these rows and we can deduct from the column totals the income increases for the immobile sectors in each continent.

The matrix elements must now be chosen so as to satisfy the primary restrictions (vertical and horizontal sums are given) and the secondary restrictions (all variables should be nonnegative) and to minimize total costs. Indicating by H_1 the number of immobile sectors, numbered $1, \ldots, H_1$, by H_2 the number of mobile sectors, numbered $H_1+1, \ldots, H$, (H being the total number of sectors), by ${}^ry^h$ the element of the continent (column) r and the activity (row) h, by ${}^ry'$ the column sum for column r (after deduction of the income from immobile sectors) and by y^h the row sum for row h, with $r = 1, \ldots, R$ and $h = H_1+1, \ldots, H$, the unknowns matrix may be written out as follows:

TABLE 3.1
Program of income increases

Sectors	Continents 1	2	... r ...	R	Sectoral totals
H_1+1	${}^1y^{H_1+1}$	${}^2y^{H_1+1}$	$\ldots$	${}^Ry^{H_1+1}$	y^{H_1+1}
H_1+2	${}^1y^{H_1+2}$	${}^2y^{H_1+2}$	$\ldots$	${}^Ry^{H_1+2}$	y^{H_1+2}
$\vdots$					$\vdots$
h	$\vdots$	$\vdots$	${}^ry^h$	$\vdots$	y^h
$\vdots$					$\vdots$
H	${}^1y^H$	${}^2y^H$	$\ldots$	${}^Ry^H$	y^H
Continental income increase targets*)	${}^1y'$	${}^2y'$	$\ldots$ ${}^ry'$ $\ldots$	${}^Ry'$	y'

*) After deduction of the increase in income from immobile sectors.

Here y' indicates the total increase in income of the world, minus the income increase in immobile sectors. Notice that the upper index h refers to the immobile sectors for $h = 1, \ldots, H_1$ so that one can immediately write:

$$ {}^ry' = {}^ry - \sum_{h=1}^{H_1} {}^ry^h \qquad r = 1, \ldots, R $$

where ${}^ry =$ the total income increase target of continent r.

The costs of each ${}^{r}y^{h}$ will be indicated by ${}^{r}\tilde{c}^{h}$ and can be also presented in a two-way table; all the elements of this matrix are given.

The problem has now been reduced to what is known as the transportation problem (see e.g., DANTZIG, 1951, 1963; GASS, 1958). In order to avoid confusion with transport problems in another sense, which will be dealt with more fully later on, we will call this transportation problem the Hitchcock problem. This corresponds to the name of the author who first formulated the problem in this way (HITCHCOCK, 1941).

It represents a simpler sub-class of linear programming problems than the general linear programming problem. This enables us to use a simpler version of the simplex method than is usually applied when solving general linear programming problems. This simpler method like the simplex technique was discovered by G.B. Dantzig and is generally called the transportation method. However, as we have just said, we will deal with transport problems later on. So again in order to avoid confusion we will replace the expression "transportation method" by "uv-method" corresponding to the symbols used in this method.

In order to be complete we give here in algebraic form the model dealt with in this section, of which Table 3.1 represents a reduced version.

Minimize
$$z = \sum_{r=1}^{R} \sum_{h=1}^{H} {}^{r}\tilde{c}^{h}\, {}^{r}y^{h} \tag{3.4.1}$$

subject to

$${}^{r}y^{h} = {}^{r}\tilde{\eta}^{h}\, {}^{r}y \qquad r = 1, \ldots, R;\ h = 1, \ldots, H_1 \tag{3.4.2}$$

$$\sum_{r=1}^{R} {}^{r}y^{h} = \tilde{\eta}^{h} y = y^{h} \qquad h = H_1+1, \ldots, H \tag{3.4.3}$$

$$\sum_{h=1}^{H} {}^{r}y^{h} = {}^{r}y \qquad r = 1, \ldots, R \tag{3.4.4}$$

$${}^{r}y^{h} \geq 0 \qquad r = 1, \ldots, R;\ h = 1, \ldots, H_1 \tag{3.4.5}$$

Here

${}^{r}y^{h}$: the increase in income or value added of sector h in continent r

${}^{r}y$: the increase in income of continent r

y^h : the increase in income or value added of sector h in the world
y : the increase in income of the world
${}^r\tilde{c}^h$: the total cost per unit increase in income of sector h in continent r
${}^r\tilde{\eta}^h$: the increase in total demand for product h in continent r (measured in value added) per unit increase of continent r's income
$\tilde{\eta}^h$: the increase in total demand for product h (measured in value added) in the world per unit increase of the world's income
$\sim$: this symbol is used to remind the reader that all variables in the model are expressed in income or value added terms.

Equations (3.4.2) and (3.4.3) correspond to the immobile and mobile sectors respectively, equating supply in terms of value added to demand also in terms of value added. Equations (3.4.4) make the sum of the sectoral income increases equal to the regional income targets. The inequalities (3.4.5) express the assumption that in a growing economy it will not be efficient for any sector to decrease its output. For a discussion of this assumption see Section 7.2.

Before turning to a discussion of solution methods of this model, we notice that the model could be made more realistic in at least three ways. First one could also take account of the transportation costs of a number of mobile products, the so-called heavy products. Secondly one could introduce upper bounds for a number of variables in the case when only a limited expansion of a sector in a continent is possible. Thirdly input-output relations could be introduced. However, we will not introduce these refinements here. The reader is referred to Sections 4.8, 4.9, 4.11 and Chapters 5, 6 and 7.

* 3.5. *Solution of the Simplest Version with the Aid of the uv-Method*

The solution procedure consists of three steps:

I : Finding a basic feasible solution.

II : Checking this solution for optimality.

III: Finding by iteration an improved basic feasible solution, if necessary.

The last two steps may be repeated several times until the optimal solution has been found. We describe the steps for the reduced model of Table 3.1.

(*Step I*) Similarly to what is done in most methods available for the solution of the general programming problem, first a basic feasible solution is found. A basic feasible solution is a solution whose number of positive variables equals at most the number of independent primary restrictions and which satisfies all the restrictions. Its costs are not necessarily minimal however. Next, the optimal solution must be found by iteration.

There are many ways of finding a first basic feasible solution, but an easy and well-known method is the northwest-corner rule (CHARNES and COOPER, 1954).A value is given to ${}^1y^1 = \min\{y^1, {}^1y'\}$. If ${}^1y' < y^1$ then, ${}^1y^1 = {}^1y'$. All other ${}^1y^h$ are then taken equal to zero. Next ${}^2y^1$ is chosen = min $(y^1 - {}^1y', {}^2y')$. Say this is $y^1 - {}^1y'$, then all other ${}^ry^1$ must be taken equal to zero. The next element to be determined is then ${}^2y^2 = \min({}^2y' - {}^2y^1, y^2)$ and if this is ${}^2y' - {}^2y^1$, all other ${}^2y^h = 0$. In a similar way we can proceed until we have a basic feasible solution. If ${}^1y' > y^1$, then ${}^1y^1 = y^1$ and all other ${}^ry^1$ are taken equal to zero. Next we proceed in a similar way as described above.

If ${}^1y' = y^1$, then ${}^1y^1 = {}^1y' = y^1$ and either all other ${}^1y^h$ or all other ${}^ry^1$ are taken equal to zero and considered non-basic. The ${}^ry^1$ or ${}^1y^h$, which are not considered basic in this way remain eligible as basic variables although their value will be equal to zero anyway.

(*Step II*) In order to find the optimal solution we must always check whether the basic feasible solution at hand is already the optimal solution. It has been shown (DANTZIG, 1951, 1963) that this can be done in a relatively simple way by calculating "indirect" cost figures for each element not appearing in the basic solution at hand and comparing them with the actual cost figures ${}^r\tilde{c}^h$ already mentioned. The indirect cost figures ${}^r\bar{\tilde{c}}^h$ can be found by introducing new variables u^h and ${}^rv$ and defining

$${}^r\bar{\tilde{c}}^h = u^h + {}^rv$$

The ${}^hu$ and ${}^rv$ themselves are defined by

$${}^r\tilde{c}^h = u^h + {}^rv$$

where the direct cost figures ${}^r\tilde{c}^h$ correspond to the variables appearing in the basic feasible solution at hand. This set of equations leaves us with one more unknown than we have equations; the number of unknowns being $R+H$ and the number of equations $R+H-1$. We choose arbitrarily one of the unknowns, give it an arbitrary value, preferably zero, and then we are able to derive all other u^h and ${}^r v$. These we use to calculate the ${}^r\bar{c}^h$ for the variables not appearing in the basic feasible solution at hand. This basic feasible solution at hand will be optimal if and only if for all its elements

$$ {}^r\tilde{c}^h \geq {}^r\bar{c}^h $$

If for some elements

$$ {}^r\tilde{c}^h < {}^r\bar{c}^h $$

iterations are necessary in order to find the optimal solution (*Step III*).

(*Step III*) A positive value θ is given to the ${}^r y^h$ with the largest positive $({}^r\bar{c}^h - {}^r\tilde{c}^h)$, that is, this variable is introduced into the basis. The values of the other variables in the basic feasible solution at hand must be changed so as to still satisfy the restrictions (given row totals and column totals). The value θ should be as high as possible within the limits set by the restrictions. Thus another basic feasible solution will be found. By repeating step II this new solution can again be checked for optimality. If it is not yet optimal another iteration is made. Thus after a number of iterations the optimum will be found.

We shall illustrate the procedure with a simple 3×2 problem. In the problem we have three mobile sectors with row totals of $y^1 = 4$, $y^2 = 6$ and $y^3 = 8$ and two continents with column totals of ${}^1y' = {}^2y' = 9$. The costs per unit of income increases are given in the following cost matrix

	Sectors	Continents	
		1	2
	1	2	3
${}^r\tilde{c}^h$:	2	4	3
	3	1	1

(*Step I*) Using the northwest-corner rule we obtain the first basic feasible solution:

	Sectors	Continents 1	Continents 2	
	1	4		4
${}^ry^h$:	2	5	1	6
	3		8	8
		9	9	

The value of the objective function is 39.

(*Step II*) Next we determine for the elements appearing in this first basic feasible solution 3 numbers u^h and 2 numbers ${}^rv$ such that

$$\begin{aligned} u^1 + {}^1v &= {}^1\tilde{c}^1 = 2 \\ u^2 + {}^1v &= {}^1\tilde{c}^2 = 4 \\ u^2 + {}^2v &= {}^2\tilde{c}^2 = 3 \\ u^3 + {}^2v &= {}^2\tilde{c}^3 = 1 \end{aligned}$$

Here we have five variables in four equations. In order to determine a solution we choose arbitrarily one of the variables u^h and ${}^rv$ equal to zero, e.g. ${}^1v = 0$. Then $u^1 = 2$, $u^2 = 4$, ${}^2v = -1$ and $u^3 = 2$. Now we compute the indirect costs for the elements not appearing in the basic feasible solution: ${}^1\bar{\tilde{c}}^3 = u^3 + {}^1v = 2$ and ${}^2\bar{\tilde{c}}^1 = u^1 + {}^2v = 1$. Next we compute differences ${}^r\bar{\tilde{c}}^h - {}^r\tilde{c}^h$: ${}^1\bar{\tilde{c}}^3 - {}^1\tilde{c}^3 = 1$ and ${}^2\bar{\tilde{c}}^1 - {}^2\tilde{c}^1 = -2$. The direct costs of ${}^1y^3$ being smaller than its indirect costs it pays to introduce ${}^1y^3$ into the basic feasible solution.

(*Step III*) We do this first at an unknown nonnegative level θ (top page 43). Since we must keep the row and column sums correct we have to add and subtract θ from a number of ${}^ry^h$. As the largest possible size of θ is most advantageous, the size of θ is determined by the smallest ${}^ry^h$ from which it is subtracted, so $\theta = 5$. The new basic feasible solution is shown next.

	Sectors	Continents 1	Continents 2	
	1	4		4
${}^r y^h$:	2	$5-\theta$	$1+\theta$	6
	3	θ	$8-\theta$	8
		9	9	

	Sectors	Continents 1	Continents 2	
	1	4		4
${}^r y^h$:	2		6	6
	3	5	3	8
		9	9	

The objective function for this solution is equal to 34.

(*Step II*) In order to see whether this new basic feasible solution is optimal we have to repeat the whole procedure. This has to be done until after a finite number of iterations a solution is obtained for which all ${}^r\bar{z}^h - {}^r\bar{c}^h \leq 0$.

In our example the optimal solution has already been obtained after the first iteration as the reader can easily verify himself. *

* 3.6. *Reducing the Volume of Work*

For practical purposes it is desirable to reduce to a minimum the number of iterations necessary by trying to determine a first basic feasible solution as close as possible to the optimal solution. A number of procedures which determine such an initial basic feasible solution have already been described in the existing literature on the Hitchcock problem (e.g. HADLEY, 1962, p. 304-309). Some of these methods, known as the row minimum, the column minimum and the matrix minimum method will be described briefly here. Note, however, that no guarantee exists that these methods lead to less work in all cases. But for hand computations they generally appear to be efficient.

Row minimum. Beginning with row 1, we choose the element with the smallest cost coefficient in this row. Let this one occur in column k. We set ${}^ky^1 = y^1$ if $y^1 \leq {}^ky'$ or ${}^ky^1 = {}^ky'$ if $y^1 > {}^ky'$. In the first case we have allocated all the y^1 units and go on to the second row after changing ${}^ky'$ to ${}^ky' - y^1$. Next we find the element with the smallest cost coefficient in the second row and repeat the process. In the second case only ${}^ky'$ units of the y^1 have been allocated. Hence we change y^1 to $y^1 - {}^ky'$ and ${}^ky'$ to zero and find the element with next smallest cost coefficient in the first row. Suppose it occurs in column s. We set ${}^sy^1 = y^1 - {}^ky'$ if $y^1 - {}^ky' \leq {}^sy'$ or ${}^sy^1 = {}^sy'$ if $y^1 - {}^ky' > {}^sy'$. We repeat this procedure till the first row constraint is satisfied. Then we move to the second row. We continue in this way until all the row constraints are satisfied.

Column minimum. Exactly the same reasoning is followed but now we start with the first column and proceed to the last column. So we choose the element with the smallest cost coefficient in column 1. Suppose it occurs in row i. Then we set ${}^1y^i = {}^1y'$ if ${}^1y' \leq y^i$ and move to column 2. If ${}^1y' > y^i$, we set ${}^1y^i = y^i$ and change ${}^1y'$ and y^i to ${}^1y^i - y'$ and zero respectively. Next we choose the element with the next highest cost in the first column. Suppose it occurs in row j. We set ${}^1y^j = y^j$ if $y^j \leq {}^1y' - y^i$ or ${}^1y^j = {}^1y' - y^i$ if $y^j > {}^1y' - y^i$. We continue in this way till the first column constraint is satisfied. Then we move to the second column. The procedure is repeated until all the column constraints are satisfied.

Matrix minimum. Here we determine the element with the smallest cost coefficient in the entire matrix. Suppose this occurs for element ${}^ky^i$. Now we set ${}^ky^i = \min({}^ky', y^i)$. If ${}^ky^i = {}^ky'$ we decrease y^i by ${}^ky'$ and ${}^ky'$ to zero; if ${}^ky^i = y^i$ we decrease ${}^ky'$ by y^i and y^i to zero. Then the whole process is repeated for the resulting matrix. If a row and a column constraint are satisfied simultaneously we make in general an arbitrary choice: ${}^{k}y^{i} = {}^{k}y'$ or $=y^{i}$.

A reduction to a problem of simpler type can be achieved by redefining the rows and columns. We may combine two or more of them or we may subdivide them into parts, in a way comparable to the choice of units in many problems of applied mathematics. This normalizing procedure may be undertaken in such a way as to give a very special case of the Hitchcock problem, namely the one where all row totals and all column totals are equal, implying that the number of rows and columns is equal or $R = H$. In the normalized form where all column and row totals are equal to 1, the problem is known as the optimal assignment problem (cf. DANTZIG, 1963, Chapter 15, Section 1).

Applicaton of the northwest-corner rule leads to the first basic feasible solution. This consists of the figures 1 in each cell of the main diagonal and zero's everywhere else. This represents a degenerate solution, since the number of non-zero values ${}^{r}y^{h}$ is now $R = H$ only instead of $R+H-1$ in the general Hitchcock problem. The iterations which may be necessary will lead to a ${}^{r}y^{h}$ matrix with one 1 in each row and one 1 in each column, but not necessarily on the main diagonal. Clearly any iteration and hence the optimal solution can now be obtained by a permutation of either the rows or the columns.

The simplification obtained is that the value of the objective function is now $\Sigma\,\Sigma\ {}^{r}\tilde{c}^{h}$ extended only over the cells where ${}^{r}y^{h} = 1$.

For the solution of an optimal assignment problem in addition to the uv-method a computational procedure can be used which, although itera-

	Sectors	continents			
		1	2	3	4
	1	4	8	8	9
${}^{r}\tilde{c}^{h}$:	2	6	6	2	9
	3	7	5	5	2
	4	6	8	4	1

tions may be necessary, in many cases gives at once the optimal solution. This is best explained with an example. Suppose we have to find an optimal assignment for the cost matrix given at the bottom of page 45.

For the solution we make use of the following theorem which holds for any Hitchcock problem and which the reader should verify for himself.

If in an assignment problem we add a positive or negative constant to every element of a row or column in the cost matrix, then an assignment minimizing total costs in one matrix also minimizes total costs in the other matrix.

We now proceed to subtract the minimum element in each row from all the elements in its row, yielding

Sectors	Continents			
	1	2	3	4
1	0	4	4	5
2	4	4	0	2
3	5	3	3	0
4	5	7	3	0

Next we subtract the minimum element in each column from all the elements in its column, obtaining

Sectors	Continents			
	1	2	3	4
1	0	1	4	5
2	4	1	0	2
3	5	0	3	0
4	5	4	3	0

It is clear that if we can choose an assignment which has a zero total, there cannot be an assignment with a lower total. Thus if all assignments can be made to positions where zeros appear, the solution is an optimal solution.

In our example with row and column totals = 1, the optimal solution is:

	Sectors	Continents 1	2	3	4
r_{yh} :	1	1	0	0	0
	2	0	0	1	0
	3	0	1	0	0
	4	0	0	0	1

When no complete assignment can be found at this stage iterations must be made. For a complete description of this method the reader is referred to the existing literature (cf. SASIENI *et al.*, 1959). *

3.7. The Model with Pre-Determined Zero Activities

As stated already, it is sometimes realistic to impose zero income increases in some sectors in some continents. Absence of mineral resources of a given type in such continents may be an obvious reason to impose zero activity increases in some sectors.

These "zero activity" restrictions as we will call them for brevity's sake, represent additional restrictions. These have disadvantages as well as advantages. One disadvantage is that they may make a solution impossible. If we stick to the assignment problem as the special form of our "simplest version", imposed zeros for two different continents in all sectors but one, present such an example. It will not then be possible to solve the problem. The essential feature of such a case is that we can imagine two areas which can both undertake production only in the same few sectors, e.g. because they both lack the natural resources or other inputs for other sectors. If then their income targets surpass the total increase in demand – in terms of value added – in these few sectors, the problem can not be solved. Fortunately such a situation is unlikely to occur for continents. It may show up at lower levels, however.

A further disadvantage is that although the problem may be solvable, pre-determined zeros render it more difficult to determine an initial basic feasible solution. For instance in the assignment problem one of the imposed zeros may be on the main diagonal. Hence a first basic feasible solution along the diagonal may be impossible. In most cases we can escape this difficulty by a rearrangement of either the columns or the rows. Anyway, the extra difficulty of getting a first basic feasible solution when predetermined zeros are present can always be avoided if we assume "very high" cost figures for the cells concerned.

The advantage of the presence of pre-determined zeros is, of course, that they restrict the amount of work per iteration while they may restrict the number of necessary iterations. An extreme example is the presence of pre-determined zeros everywhere outside the main diagonal before or after rearrangement of the columns of rows.

Because of this it may be useful to raise the number of zeros by excluding some sectors in some continents because of the higher costs of production these continents show. This can save some work as has been indicated above: one does not need to compute the indirect cost figures for such zero places for each iteration and the number of iterations might become smaller. However, the saving on the number of iterations depends on the method one uses to get a first basic feasible solution. If one uses the northwest-corner rule we might particularly expect some saving, although we have seen that precisely this rule might need some change if zeros are present. The point is that if one guesses correctly that the variable with the higher cost coefficient will not be in the optimal basis, by excluding it in advance one might save some work if the solution method would tend to introduce it into one of the bases before the optimal solution has been reached.

It is another question whether one has excluded the variable correctly or not. If not, the final solution is not the optimal one. This, however, can always be checked easily in the final table by computing the indirect costs for the excluded variables and comparing them with the actual costs. If one or more of these actual costs turn out to be lower, one should introduce the corresponding variable into the basis etc. In this way one can always get the really optimal solution. Whether the temporary exclusion of variables leads to less work after all depends then apparently also on the criterion for "higher" cost used in excluding the variables. If one does not check whether the first final solution is the optimal one, one saves work, but at the cost of possibly having only an approximately optimal solution. In view of the easy way to avoid this it seems worth the work to use the check.