CHAPTER 4

THE ELEMENT OF SPACE IN NATIONAL DEVELOPMENT PLANNING; SECOND ORDER SUBDIVISION OF AN OPEN ECONOMY; TRANSPORTATION COSTS IMPLICIT

4.1. Choice of Regions and Their Frontiers

We are now going to discuss how the element of space can be dealt with systematically in national development planning. In practice this is the level at which planning is now done over large parts of the globe, simply because most real power is in the hands of national governments. As a consequence the nature of the problem changes: nations are autonomous in the choice of the international sectors they want to expand. Since most nations are supplying only a small fraction of total supply in most markets for international products, they can afford to disregard total demand for such products in the world at large, contrary to what a "world planner" should do. We will come back to this aspect in Section 4.11.

In national planning the element of space will show itself first of all if it is desired to split up a national plan into plans for parts of the country, which we will call regions. Next steps may consist of further subdivisions, finally ending up with such small spatial units as villages or quarters of a city. Using the terminology proposed in Chapter 1 we are concerned with a second-order spatial subdivision of an open economy if we consider a country, subdivided into regions, in relation to the rest of the world. This will be the subject of this chapter.

The first question to be answered in any concrete planning activity of this type is how to choose the regions. While as a matter of course some thought must be given to this question, its importance should not be overrated. One of the subsequent questions, namely what method of planning to use, is much

more important. There is even some danger in being too perfectionist while defining the regions, since such perfectionism will often lead to a large number of regions; and there is some virtue in not making their number very large. Working with a large number of regions at once will make the model very complicated and difficult to handle and to discuss. Here we also prefer to work stepwise; which means postponing a further subdivision until later.

The subdivision of a country into regions remains an art rather than a piece of pure science, since a compromise must be found between various principles, none of which can be given a complete application. Such compromises have often to be based on intuition rather than on explicit calculation.

The main principles to be satisfied as well as possible are the following.

(1) Regions should be as homogeneous as possible with regard to their main socio-economic characteristics. These characteristics may already be numerous. For political reasons, in some countries linguistic and other cultural differences stand out. Also, the endowment with natural resources may be an important characteristic. These must play an important role especially but not only for developing countries. Further characteristics to be considered may be the industrial structure and the level of wellbeing.

(2) As much as possible the frontiers of the regions should coincide with the most important obstacles to transportation. Such obstacles may not always be important, but for some countries they are; for instance if there are chains of mountains.

(3) Both for reasons of information and for reasons of implementation of the plan the frontiers of each region should coincide with administrative frontiers between provinces, departments, districts or whatever their name.

(4) The regions defined should be of comparable importance; that is, one should avoid too big differences in size, measured in whatever way (e.g. along the lines suggested in Chapter 1). At the same time their size should differ significantly from the size of the country itself in order to make the subdivision meaningful.

In general the number of regions should be chosen as small as possible, as long as it is compatible with these four principles in order to keep the

models to a reasonable size and not to loose sight of the reality behind this subdivision. As for the maximum number, it is the intuitive opinion of the authors that the number of regions in a first subdivision of a country should be not more than about twelve. In the case study to be discussed in Section 4.13, referring to Mexico, ten regions were defined, covering the 32 provinces of the country.

4.2. Choice of Sectors

The next question to be settled is what sectors to distinguish. In principle we encounter the same difficulties here as with the definition of regions: we must compromise between various principles. These principles are roughly the same as the ones proposed for the choice of regions. Sectors should also be as homogeneous as possible and not too different in importance either now or in the future structure. This implies that we may introduce sectors expected to be important in the future. Statistical material should be available as well as the possibility of implementing plans which means that some correspondence must exist with the organizational structure of the economy. With regard to sectors we can be some what more specific, because many sectors are the same in many countries. This implies that some of the characteristics we have discussed in Chapter 1 may serve as a basis for a more precise discussion in this section.

The homogeneity of the sectors should not only be of the type normally discussed in input-output models, that is a certain homogeneity with regard to inputs and outputs (cf. CHENERY *et al.*, 1959, pp. 34-39). They should also be homogeneous with regard to the mobility of their products, that is all products of one sector should be either regional or national or international (cf. Table 1.2). In addition they should be homogeneous with regard to their shiftability. Fortunately these requirements of homogeneity with regard to mobility and shiftability are mostly satisfied if the sectors are chosen according to the homogeneity with regard to inputs and outputs.

Regional sectors are relatively easy to deal with in our simplest model and as long as we stick to that model we can introduce, without creating many complications, as many regional sectors as we find useful. This is due to the assumption that the income increases in these sectors can be considered functions of the total income increase in each region and hence data as long as we consider the income increases in each region to be given aims.

Taking the number of regional sectors small, however, will make it more likely that this assumption is a realistic one. In a model with input-output coefficients not obeying any special conditions the situation is more complicated (cf. Section 4.8 and Chapter 7).

Among the national sectors we will as a rule consider separately electricity, central government, higher education; also some of the heavy products (fertilizer, building materials). Of these, some may be shiftable in our sense. Central government will be shiftable only in exceptional cases; electricity and some of the others may be so more often.

Among the international sectors we will have mining, agriculture and manufacturing. Often some major single mining and agricultural products will be considered separately and then they may be non-shiftable or only partially shiftable, if climate or soil conditions are very different in different regions (cf. Section 4.13). As we said above, sectors should also in general be homogeneous with respect to their shiftability. Some manufacturing industries will also be taken as separate sectors. Among these, the first stages of processing heavy materials may be non-shiftable. Light industries and the later stages of manufacturing will mostly be shiftable.

A practical choice of the number of sectors seems to be around 25 (cf. also Section 4.13).

It is essential for an orderly process of planning that once a choice has been made in which all relevant aspects have been considered, one should stick to the regional and sectoral subdivision. This implies that all information available and necessary for the model chosen should then be estimated for these sectors. For experts or politicians accustomed to think in concrete partial problems this is sometimes difficult and they may judge that such a model is unrealistic. The answer is that what we aim at at this stage is a first approximation of the plan, but a consistent one, that is, one without internal contradictions. Since consistency is the main feature of any co-ordination it should have a heavy weight in any method of planning.

4.3. Nature of the National Planning Problem with Regional Subdivision

In solving the national planning problem with a regional subdivision we will use the same set up as in Section 3.2. So we consider as the unknowns of our problem the income increases in each sector for each region and as

given aims the total income increases for each region and hence for the country at large. The criterion to be used is to minimize the costs of the increases in production, defined in the same way as in Section 3.1.

In addition we make a number of simplifying assumptions similar to those in Section 3.2. Thus, corresponding to Assumptions 3.1 and 3.2 we assume that the products (and sectors) are either regional or national or international and that all production processes can be described by linear relationships between outputs and inputs of all kinds.

In the simple version now under discussion we assume that the ratio of income to output for each national sector is the same in each region. In addition we assume that the demand for regional products in each region is a function of the regional income only and hence given. Hence we may leave the regional sectors largely out of consideration. Finally we assume that for each national sector the increase in total demand for its products and hence its total income increase for all regions taken together is a function of the income increase for the nation. Its row total is also given therefore. More specifically, we will even assume that the increase in demand for regional or national products is proportional to the increase in regional or national income respectively; but again, as in Section 3.1, this is not essential for even this simplest version of the problem. It follows that the total income increase for all international sectors taken together for the country at large is also known, but not each individual row total. This constitutes the difference between this and the simplest version of our world planning problem in Chapter 3.

The *assumptions* made can be listed as follows:

Assumption 4.1. (corresponding to Assumption 3.1). Each product can be classified as either regional, or national or international. For national or international goods transportation costs between regions or regions and countries respectively, can be neglected.

Assumption 4.2. (corresponding to Assumption 3.2). All production processes can be described by linear relationships between outputs and inputs of all kinds.

Assumption 4.3. (corresponding to Assumption 3.3). For each national product the ratio of income to output is the same in all regions.

Assumptions 4.4. (corresponding to Assumption 3.4). For each national or regional product the increase in demand is proportional to the increase in national or regional income respectively.

Assumption 4.5. (corresponding to Assumption 3.5). For all products differences in prices between regions can be neglected.

Accordingly the *data* needed for the numerical solution of this simplest version of the national planning problem are:

(1) Target values of increases in income for each region;

(2) The classification of products and their production processes into the categories regional, national and international;

(3) Cost figures, expressed per unit of income increase, of each product and in each region considered (marginal costs applying to new production units of optimal size). These figures can be derived from the appropriate cost-output figures together with the data under 4;

(4) The ratios of income to output for each sector; for the regional sectors these ratios also for each region separately;

(5) Ratios between the increases in demand (in terms of value added) and the increase in national income for each national sector. For each regional sector ratios between the increase in demand (in terms of value added) and the increase in regional income, for each region separately.

The difference between this and the simplest version of the world planning problem can best be illustrated by a similar table as in Chapter 3 which had $H-H_1$ rows (sectors) and R columns (continents). Now we assume R regions, H_1 regional sectors, numbered $1, \ldots, H_1$, H_2 national sectors, numbered $H_1+1, \ldots, H_1+H_2$, and H_3 international sectors, numbered $H_1+H_2+1, \ldots, H$, where H is the total number of sectors. Correspondingly, Table 4.1 has R columns (regions) and H_2+H_3 rows (sectors). The international sectors are placed at the bottom of the table.

Here again the ${}^r y'$, $r = 1 \ldots R$ represent the regional income targets after deduction of the incomes from the regional sectors.

TABLE 4.1

Program of income increases

Sectors		Regions 1	2	... r	...	R	Sectoral totals
National	H_1+1	${}^1y^{H_1+1}$	${}^2y^{H_1+1}$	... ${}^ry^{H_1+1}$	...	${}^Ry^{H_1+1}$	y^{H_1+1}
	H_1+2	${}^1y^{H_1+2}$	${}^2y^{H_1+2}$	... ${}^ry^{H_1+2}$	...	${}^Ry^{H_1+2}$	y^{H_1+2}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
	h	${}^1y^h$	${}^2y^h$	... ${}^ry^h$	...	${}^Ry^h$	y^h
	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
	H_1+H_2	${}^1y^{H_1+H_2}$	${}^2y^{H_1+H_2}$	... ${}^ry^{H_1+H_2}$	...	${}^Ry^{H_1+H_2}$	$y^{H_1+H_2}$
International	H_1+H_2+1	${}^1y^{H_1+H_2+1}$	${}^2y^{H_1+H_2+1}$	... ${}^ry^{H_1+H_2+1}$	...	${}^Ry^{H_1+H_2+1}$	$\sum_{h=H_1+H_2+1}^{H} y^h$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	
	H	${}^1y^H$	${}^2y^H$	... ${}^ry^H$	...	${}^Ry^H$	
Regional income targets*		${}^1y'$	${}^2y'$	... ${}^ry'$	...	${}^Ry'$	Total* y'

*) After deduction of the incomes of the regional sectors.

Again for the sake of completeness we give here the algebraic formulation of the model of which Table 4.1 represents a reduced version:

Minimize

$$z = \sum_{r=1}^{R} \sum_{h=1}^{H} {}^{r}\tilde{c}^{h}\, {}^{r}y^{h} \tag{4.3.1}$$

subject to

$${}^{r}y^{h} = {}^{r}\tilde{\eta}^{h}\, {}^{r}y \qquad r = 1, \ldots, R;\ h = 1, \ldots, H_1 \tag{4.3.2}$$

$$\sum_{r=1}^{R} {}^{r}y^{h} = \tilde{\eta}^{h} y = y^{h} \qquad h = H_1+1, \ldots, H_1+H_2 \tag{4.3.3}$$

$$\sum_{h=1}^{H} {}^{r}y^{h} = {}^{r}y \qquad r = 1, \ldots, R \tag{4.3.4}$$

$${}^{r}y^{h} \geqslant 0 \qquad r = 1, \ldots, R;\ h = 1, \ldots, H \tag{4.3.5}$$

Here

- ${}^{r}y^{h}$: the increase in income or value added of sector h in region r
- ${}^{r}y$: the increase in income of region r
- y^{h} : the increase in income or value added of sector h in the country
- y : the increase in income of the country
- ${}^{r}\tilde{c}^{h}$: the total cost per unit increase in income of sector h in region r
- ${}^{r}\tilde{\eta}^{h}$: the increase in total demand for product h in region r, measured in value added, per unit increase of region r's income
- $\tilde{\eta}^{h}$: the increase in total demand for product h in the country, measured in value added, per unit increase of the country's income
- $\sim$: this symbol is used to remind the reader that all variables in the model are expressed in income or value added terms.

In each region and for each regional product equations (4.3.2) make supply and demand (measured in terms of value added) equal to each other. Equations (4.3.3) do the same, but now for each national sector and for the country as a whole. Equations (4.3.4) say that for each region the sum of the sectoral income increases must be equal to the income target of that region.

The inequalities (4.3.5) express the assumption that in a growing economy it will not be efficient for any sector to decrease its output in any region. For a discussion of this assumption see Section 7.2.

Note that for the international sectors no balancing equations appear in the model. They may be included in the model, but then it turns out that they are superfluous in the sense that they are not used in determining the optimal amounts to be produced. They are used solely in order to calculate the amount of exports and imports once the optimal solution has been found (cf. Section 4.8).

* *4.4. Solution of the Simplest Version of the National Planning Problem*

Only a slight modification of the method used in Chapter 3 is needed, in order to solve this problem. We can again apply the northwest-corner rule to find a first basic feasible solution. The difference with the world planning problem of Chapter 3 is that for e.g. the first international sector row a total is not given but a maximum equal to the total income increase of all international sectors together is. Once the elements of that first international sector have been chosen the maximum value in the next international sector row is less by the amount already chosen in the first international row. Algebraically the difference between our present problem and the one of Chapter 3 is that, with an equal number of sectors (H), the number of primary restrictions is smaller. In other words there are more degrees of freedom. Therefore, when we introduce again the numbers u^h and ${}^rv$ as in Section 3.5, the number of equations from which they have to be determined seems also to be smaller. We must, however, notice that the u^h corresponding to the international sector rows in fact correspond to only one restriction and should therefore be all equal. Consequently, the number of equations from which the numbers u^h and ${}^rv$ must be determined is in fact the same as in Section 3.5 and a unique solution can again be obtained by arbitrarily choosing one of them. Clearly it is convenient to take one u of the international sector rows – and consequently all u's corresponding with the international sectors – equal to zero and to derive then all other u^h and ${}^rv$.

With the u^h and ${}^rv$ obtained we calculate again the indirect cost table (cf. the numerical example in this section) ${}^r\bar{c}^h = u^h + {}^rv$ and we compare

${}^{r}\tilde{c}^{h}$ with ${}^{r}\bar{c}^{h}$. If all ${}^{r}\tilde{c}^{h} \geq {}^{r}\bar{c}^{h}$, the solution started with is a minimum basic feasible solution which means that the problem has been solved then.

If there is one ${}^{r}\tilde{c}^{h} < {}^{r}\bar{c}^{h}$ the corresponding ${}^{r}y^{h}$ must enter into the basis; if there are more ${}^{r}\tilde{c}^{h} < {}^{r}\bar{c}^{h}$ the ${}^{r}y^{h}$ corresponding with the maximum value of ${}^{r}\bar{c}^{h} - {}^{r}\tilde{c}^{h}$ is selected. The level at which the new ${}^{r}y^{h}$ has to be introduced and the variable which is to be dropped are determined in exactly the same way as in Chapter 3 (Section 3.5).

With the new basic solution a new indirect cost table is calculated and the whole procedure is repeated until we have a solution where each ${}^{r}\tilde{c}^{h} \geq {}^{r}\bar{c}^{h}$.

In general, however, a simplification is possible here which may save some work. This simplification is based on a property of the optimal solution, which can be easily proved, namely that in the optimal solution in each region at the most one variable related to an international sector will be positive. This variable will be the one with the lowest cost coefficient of all the international sector variables in that region. In order to take advantage of this fact one has to replace all the international sector rows by one row composed of the smallest cost coefficients of the international sectors in each column. The problem has then been reduced to the normal Hitchcock problem dealt with in Chapter 3 and the normal uv-method can be applied. Once the optimal solution is known it is easy to construct the complete

	Sectors	Regions 1	2	3	Sectoral totals
	1	1	3	4	3
	2	2	3	5	3
${}^{r}\tilde{c}^{h}$:	3	4	2	4	1
	4	4	4	2	
	5	4	1	4	8
	6	3	4	2	
	Regional totals	5	5	5	Total 15

final tableau, i.e. with all international sectors. In Sections 4.10 and 6.2 we will deal with cases where this is indeed desirable.

The following numerical example illustrates the solutions now discussed. We assume three regions, three international and three national sectors. In the table on page 58 the elements within the 6×3 matrix are the cost coefficients while the elements of the right-hand and bottom margins are the sectoral totals and regional targets respectively.

Using the northwest-corner rule we obtain the first basic feasible solution.

	Sectors	Regions 1	2	3	Sectoral totals
	1	3			3
	2	2	1		3
$^{r}y^{h}$:	3		1		1
	4		3	5	8
	5				
	6				
	Regional totals	5	5	5	Total 15

Next we determine for those variables in the first basic feasible solution 6 numbers u^h and 3 numbers ^{r}v such that

$$
\begin{aligned}
u^1 + {}^1v &= 1 \\
u^2 + {}^1v &= 2 \\
u^2 + {}^2v &= 3 \\
u^3 + {}^2v &= 2 \\
u^4 + {}^2v &= 4 \\
u^4 + {}^3v &= 2 \\
u^4 &= u^5 = u^6
\end{aligned}
$$

We have eight equations in nine variables which leaves us with one degree of freedom. Therefore we set the three numbers u^h corresponding with the international sectors $= 0$ and solve the remaining u's and v's. This yields:

$$\begin{array}{ll} u^4 = 0 & u^3 = -2 \\ u^5 = 0 & {}^1v = 3 \\ u^6 = 0 & {}^2v = 4 \\ u^1 = -2 & {}^3v = 2 \\ u^2 = -1 & \end{array}$$

The reader can easily verify that only in one case is the optimality criterion not satisfied: ${}^2\tilde{c}^5 < {}^2\bar{c}^5$.

First introducing ${}^2y^5$ at an unknown level θ and next taking $\theta = 3$ leads to the following new solution:

Sectors	Regions 1	2	3	Sectoral totals
1	3			3
2	2	1		3
3		1		1
4		$3-\theta$	5	8
5		θ		
6				
Regional totals	5	5	5	Total 15

Sectors	Regions 1	2	3	Sectoral totals
1	3			3
2	2	1		3
3		1		1
4			5	8
5		3		
6				
Regional totals	5	5	5	Total 15

We repeat the whole procedure and calculate new numbers u^h and ${}^rv$ using the new basic feasible solution. Next the indirect costs are compared with the real cost coefficients.

One can easily show now that all ${}^r\bar{c}^h \leq {}^r\tilde{c}^h$ which means that we have obtained an optimal basic feasible solution. From the fact that ${}^3\tilde{c}^6 = {}^3\bar{c}^6$ we can see that there is more than one optimal solution. Introduction of ${}^3y^6$ into the basis leads to another solution with ${}^3y^6 = 5$ and ${}^3y^4 = 0$, all other variables remaining the same. Also, weighted sums of both optimal solutions with nonnegative weights adding up to 1, form again optimal solutions. Looking in this way for all optimal solutions can be useful as one

may make a choice on the basis of preferences which are not part of the objective function.

The simplification of replacing the international sector rows by one single row consisting of the smallest cost coefficients of the international sectors in each column yields the following table:

Sectors	Regions			Sectoral totals
	1	2	3	
1	1	3	4	3
2	2	3	5	3
3	4	2	4	1
4	3	1	2	8
Regional totals	5	5	5	Total 15

If we now use the northwest-corner rule it turns out that we obtain the optimal solution at once. So we have to compute the numbers u^h and ${}^r v$ only once and the number of iterations is reduced by one. The methods already mentioned in Chapter 3 to reduce the number of iterations, that is, the row minimum, the column minimum and the matrix minimum method can also be applied when dealing with the national planning problem. *

* *4.5. Normalized Form of Problem; Problem with Predetermined Zeros*

Also in the present case we may try to reduce the problem to one of a simpler type by the normalizing procedure discussed in Section 3.6, that is, by an appropriate choice of the regions and the sector groups, resulting in a problem where as many of the column and row totals as possible are equal to one. All the column totals can become one; for the row totals this is only possible for the national sectors. The sector groups corresponding to the

international sectors should be so chosen that their number equals the total amount of units of income increase for all the international sectors together, and that each international sector is represented in at least one sector group.

Again we could have reduced the original simple version of the problem to a Hitchcock problem by replacing all the rows of the international sectors by one row composed of the minimum values of all international sectors in each column. Then the normalizing procedure becomes exactly the same as the one discussed in Section 3.6. However, we cannot interpret this one new row as corresponding to one sector, nor can we interpret its disaggregation as a disaggregation into more sectors. Therefore we consider here only the case where this reduction to a Hitchcock problem is not carried out.

As this problem is not a pure optimal assignment problem, we cannot apply the computational method dealt with in Section 3.6. We can, however, apply the uv-method and a first basic feasible solution consisting of the figure 1 in each cell of the main diagonal and of zeros for all other cells is possible here too. As an illustration we will deal with an example with seven columns and seven rows of which the last four rows correspond to the international sectors.

This is a degenerate case, the number of primary restrictions being ten while the number of basic variables different from zero amounts to seven :

Sectors	Regions							Sectoral totals
	1	2	3	4	5	6	7	
1	1							1
2		1						1
3			1					1
4				1				
5					1			
6						1		4
7							1	
Regional totals	1	1	1	1	1	1	1	Total 7

If we want to check for optimality we can write the following equations for the seven basic variables in the main diagonal:

$$\begin{array}{ll} u^1 + {}^1v = {}^1\tilde{c}^1 & u^5 + {}^5v = {}^5\tilde{c}^5 \\ u^2 + {}^2v = {}^2\tilde{c}^2 & u^6 + {}^6v = {}^6\tilde{c}^6 \\ u^3 + {}^3v = {}^3\tilde{c}^3 & u^7 + {}^7v = {}^7\tilde{c}^7 \\ u^4 + {}^4v = {}^4\tilde{c}^4 & \end{array}$$

Next we choose three of the variables with zero values and we consider them to be also in the basic feasible solution at hand. In this case, this can be done somewhat arbitrarily but it is clear that one can best select the three variables with smallest costs. Suppose these are ${}^2y^1$, ${}^3y^2$ and ${}^4y^3$. We can then add the equations:

$$\begin{aligned} u^1 + {}^2v &= {}^2\tilde{c}^1 \\ u^2 + {}^3v &= {}^3\tilde{c}^2 \\ u^3 + {}^4v &= {}^4\tilde{c}^3 \end{aligned}$$

In addition we have again: $u^4 = u^5 = u^6 = u^7$.

Now we have 13 equations for 14 numbers u^h and ${}^rv$. Just as in Section 4.4 we choose one number u corresponding to an international sector $= 0$ and solve the system for the remaining u^h and ${}^rv$. For the rest the optimization procedure has already been described in the preceding section. In the optimal solution there may now be more than one 1 in some international row and hence all zeros in another international row.

In the national problem as well as in the world problem we may again have a number of cells with zero ${}^ry^h$ values as pre-determined values. This may again reduce the quantity of work, since it excludes some iterations beforehand. Whether the reduction in volume of work is important again depends to a high degree on the precise place of the zeros and their number. The remark may be repeated that we can again raise the number of zeros by excluding variables with high cost figures. We refer to the discussion of the merits of this devise in Section 3.7. *

4.6. Costs of Regional Policy

So far we have assumed that the income increases for each of the regions are given and reflect a deliberate policy of the national government. This

assumption is realistic for an increasing number of countries. The policy implied stands at variance with a policy where the only target is to increase national income and where the distribution of the income increase over the regions is determined by a national minimum of investment costs or of costs generally. In simple language the latter policy is one of developing the regions which are "most promising" from the viewpoint of the nation as a whole. As long as there is perfect mobility of the population among the regions, there is no reason to deviate from this nationally optimal policy. As soon as this mobility is not perfect or its imperfection or its costs are better understood, a regional policy makes sense. In actual fact there are many imperfections in mobility and there is an increasing awareness of them. Concomitantly there will be growing pressures from some regional authorities or populations exerted on the national government. For a proper decision making in this field it is important to know the (material) costs of any regional policy deviating from the national optimum.

The nature of the regional targets and the implications of determining them should be understood properly. While the choice of a target with regard to the increase in the national income is in practice a decision of the policy makers (cf. Section 2.1), theoretical studies are undertaken to find a basis for an optimal choice of such a target by assuming utility functions for the country as a whole (e.g. INAGAKI, 1969). The regional targets, however, are of a different nature as they relate to a policy of regional income distribution in which a compromise has to be found for the possibly opposing interests of the different regions. We are not going to deal here in detail with the institutional procedure for reaching such a compromise, but we will make a few general remarks. Once the regional targets have been fixed, the allocation of the sectors to the regions takes place on the basis of efficiency criteria. If the regional targets have not been set in advance, there is a danger that opposing interests of the regions lead to bargaining about the spatial allocation of the sectorial increases. While such bargaining might result in the same regional incomes as when regional targets had been set in advance, it will very probably do so at much higher costs, i.e. by using more scarce factors. Obviously, it would be exceptional if the bargaining process leads to the same optimal solution as could be calculated from the regional income targets. So in order to avoid waste of scarce factors it is wise to bring opposing regional interests into agreement with each other at that level of decision making where regional income targets can be set.

Another feature of setting regional targets is that in the optimal solution each national or international sector will no longer be expanded in its cheapest region, in contrast with what might seem to be efficient. This stems from the fact that lump sum transfers of income from one region to another in order to redistribute national income among the regions, are in general not practical instruments of economic policy. Rather, the regional distribution of production is used as an instrument for attaining a certain regional distribution of income. This may also have a more lasting effect. Precisely the fact that each sector is not expanded in its cheapest region creates costs attached to such a policy of regional income distribution.

The methods developed in the previous and the present chapter enable us to estimate these costs. Supposing an increase of national income $=\bar{y}$ we can calculate the costs involved in any set of regional targets by first computing the costs associated with this income increase $\bar{y}$ taking account of the regional targets. Next, from these costs the costs of a plan with the same increase in national income $\bar{y}$ but without regional targets, thus showing the national minimum of costs, must be deducted. In the latter case the total income increase for each national sector will be located in the cheapest region – according to the type of costs chosen – and the total income increase for all the international sectors together will be chosen in the sector and region showing the lowest costs per unit of income to be created. Thus, in the example treated in Section 4.4 the lowest costs needed in order to reach the national income increase target will be 19. Since the actual costs with regional targets amount to 25, the costs of the set of these targets appear to be 6.

The cost problem can be refined by asking what the cost is of increasing a region's income by one extra unit while at the same time reducing another region's income by the same amount, thus keeping the income increase for the country as a whole the same. The problem is an example of sensitivity analysis with regard to the regional targets, well known from the general method of linear programming. The changes in total costs can be expressed in terms of the numbers ${}^{r}v$ associated with the columns (regional targets) of the table of income increases[1]). In the same example the values of the three numbers ${}^{r}v$ in the optimal solution are:

[1]) For such an interpretation of the ${}^{r}v$, which were introduced in Sections 3.5 and 4.4, cf. e.g. DANTZIG (1963), Chapters 12 and 14.

$$^{1}v = 0$$
$$^{2}v = 1$$
$$^{3}v = 2$$

The changes in costs by changing the regional income targets can be calculated by multiplying these changes – taken positive if the change is an increase and negative if the change is a decrease – by the ^{r}v's associated with the corresponding columns and adding up these products.

This has been done in Table 4.2 where an increase or decrease of income by one unit is indicated by $+1$ and -1 respectively.

TABLE 4.2

Changes of costs caused by changing the income targets of each region dealt with in Section 4.4 in the ways indicated under *A-F*

Changes in income targets of:	A	B	C	D	E	F
Region 1	+1	+1		−1		−1
Region 2	−1		+1	+1	−1	
Region 3		−1	−1		+1	+1
Changes of costs: increase (+) or decrease (−)	−1	−2	−1	+1	+1	+2

It should be kept in mind, however, that these figures refer only to small changes. For larger changes in regional targets the ^{r}v themselves change. One can expect in general that for larger changes in regional targets towards (or away from) the values in the solution without regional targets the changes in costs per unit change in targets will be smaller (or larger) in absolute amounts while the signs will be the same.

As a final remark we may notice that there might be an other than purely socio-political reason to set regional income targets. In some cases it may be expected that, while in the planning period the cost figures in a certain region are high, in the very long run, after some "breakthrough" period, they would become lower. This favourable very long run perspective might lead

to an emphasis on the development of the region in the planning period which can be expressed somewhat crudely in these models by a relatively high income increase target. A favourable very long run perspective may be based on the expectation of e.g. favourable effects of external economies or economies of scale or on a high marginal savings rate. These factors are not explicitly expressed in the models of this chapter. If some regional income targets are chosen on this basis, it remains interesting to use the methods described in this section for calculating the extra costs during the planning period due to the choice of these targets, in order to weigh these extra costs against the benefit to be expected in the long run.

4.7. Introduction of Employment Targets

In many countries a high level of employment is an even more important target of economic policy than a maximum level of income although these two targets are evidently interconnected. In this section we will briefly discuss how to take explicit account of these employment targets in the context of the models presented in this chapter, i.e., national development planning with a spatial subdivision. We will do so by proposing three types of models which are all minimization problems, i.e., the use of scarce factors is minimized, subject to a number of restrictions. Of course this problem can be changed into a maximization problem, but then account has to be taken of what was said in Section 3.1 about this reversing procedure.

The first and simplest model is exactly the same as the model dealt with in Section 4.3, but now completely in terms of employment. So all income variables have to be replaced by corresponding employment variables. The same holds for the cost coefficients in the objective function which are now cost-employment ratios. Evidently the assumptions and data needed have to be changed correspondingly.

The second method we propose is first to solve the so-called "income models" dealt with so far in this chapter and next to calculate for each region the consequences with regard to employment by means of employment-output ratios. If the result of an "income model" turns out to be not satisfactory with regard to the employment situation in one or more regions, one may change the regional income targets and solve the model again. Obviously, one may just as well first solve the "employment model" and

then calculate the consequences as for income. This approach however involves the drawback that one has to change the original targets (income or employment respectively) while this is clearly not the only way of also attaining a given additional target.

So, in order to avoid this drawback, we propose a third and more sophisticated approach i.e., a model in which the costs of scarce factors are again minimized subject to:

(1) A number of restrictions in the form of equalities. These equations correspond to the income targets and the regional and national sectors; e.g. the restrictions of the model in Section 4.3.

(2) A number of restrictions in the form of inequalities. These restrictions correspond to the regional employment targets and tell that the level of employment must be at least equal to a certain minimum.

In this way one takes account of the two targets simultaneously. One must, however, be careful not to set the employment targets too high which might make the problem unsolvable. This is easily identified when solving the model. However, in this case the simple uv-method can be applied no longer, as the second type of restrictions contain coefficients unequal to one. For the solution of this problem the reader is referred to the linear programming literature. We will only add that one can apply sensitivity analysis (DANTZIG, 1963, Chapter 12; HADLEY, 1962, Chapter 11) here also. The resulting shadow prices of employment in each region then can be used to indicate the costs of the employment targets as well as the costs of a regional policy dealt with in the preceding section.

Still other approaches are possible and more targets can be introduced. These problems however will not be dealt with in this study.

4.8. Introduction of Input-Output Relations

The assumptions made in Section 3.2 were necessary in order to give the world planning problem the relatively simple form to which the methods dealt with in Chapter 3 could be applied. It is easy to see that e.g. Assumption 3.4 has been made in order not to take explicit account of interindustry deliveries, and Assumption 3.3 in order to reduce the statistical work to be done (cf. Chapter 7). We are then dealing only with functions which relate increases in value added of sectors to increases in continental or world

income. The coefficients appearing in these functions can easily be estimated from time series on these variables. On the other hand, however, if we consider structural changes in the economy of a spatial unit or in general future alternatives which deviate considerably from the existing pattern, Assumption 3.4 might easily not be satisfied. This would lead to the necessity of introducing input-output relations explicitly into the model. When we do introduce these input-output relations, Assumption 3.3, which says that for each mobile product the ratio of income to output is the same in all continents, can be dropped or replaced by the assumption that there are no intercontinental differences in the input-output coefficients. Moreover, we can replace Assumption 3.4, which relates total demand of a sector to continental or world income, by Assumption 3.4′:

Assumption 3.4′. The increase in final demand for each immobile or mobile product is proportional to the increase in continental or world income respectively.

This implies that we have to add a new assumption:

Assumption 3.6. The inputs into intercontinental transportation can be neglected.

The new *data* which are needed are:

(1) Functions relating the increase in final demand for each product to the increase in income (again in a proportional way),

(2) Input-output coefficients for each continent.

We will however not deal with the input-output model for the world planning problem but do so only for the problem dealing with an open economy: national development planning with regional subdivision.

The assumptions for the national planning problem have been mentioned in Section 4.3. Using the same arguments as in the case of world planning we drop Assumption 4.3 and replace Assumption 4.4 by a new Assumption 4.4′:

Assumption 4.4′. The increase in final demand for regional or national or international products is proportional to the increase in regional or national income respectively.

Accordingly the new *data* needed are:

(1) Ratios between the increases in final demand for regional or national and international products and increases in regional or national income respectively.

(2) Input-output coefficients for each region.

This implies that we have to add:

Assumption 4.6. The inputs into interregional and international transportation can be neglected.

On the basis of these assumptions we can write down the complete model with input-output relations. If for the different regions no separate input-output tables are available, one has no other choice than to make use of the same national input-output table for each region. In many cases this will give a reasonably good approximation. It is interesting to note that this implies the former Assumption 4.3.

Our national planning problem with a regional subdivision is again an optimization problem with restrictions. We want to minimize total production costs, given regional income targets and, for each product, given functions for intermediate and final demand as well.

We will deal with an example with three regions and with one regional, one national and three international sectors. The programming problem is then:

$$\text{Minimize } z = \sum_{r=1}^{3} \sum_{h=1}^{5} {}^{r}c^{h}\, {}^{r}x^{h} \tag{4.8.1}$$

subject to:

$$ {}^{1}\varphi_{11}\, {}^{1}x^{1} + {}^{1}\varphi_{12}\, {}^{1}x^{2} + {}^{1}\varphi_{13}\, {}^{1}x^{3} + {}^{1}\varphi_{14}\, {}^{1}x^{4} + {}^{1}\varphi_{15}\, {}^{1}x^{5} + {}^{1}\gamma^{1}\, {}^{1}y = {}^{1}x^{1} \tag{4.8.2}$$

$$ {}^{2}\varphi_{11}\, {}^{2}x^{1} + {}^{2}\varphi_{12}\, {}^{2}x^{2} + {}^{2}\varphi_{13}\, {}^{2}x^{3} + {}^{2}\varphi_{14}\, {}^{2}x^{4} + {}^{2}\varphi_{15}\, {}^{2}x^{5} + {}^{2}\gamma^{1}\, {}^{2}y = {}^{2}x^{1} \tag{4.8.3}$$

$$ {}^{3}\varphi_{11}\, {}^{3}x^{1} + {}^{3}\varphi_{12}\, {}^{3}x^{2} + {}^{3}\varphi_{13}\, {}^{3}x^{3} + {}^{3}\varphi_{14}\, {}^{3}x^{4} + {}^{3}\varphi_{15}\, {}^{3}x^{5} + {}^{3}\gamma^{1}\, {}^{3}y = {}^{3}x^{1} \tag{4.8.4}$$

$$\begin{aligned} &{}^{1}\varphi_{21}\, {}^{1}x^{1} + {}^{2}\varphi_{21}\, {}^{2}x^{1} + {}^{3}\varphi_{21}\, {}^{3}x^{1} + {}^{1}\varphi_{22}\, {}^{1}x^{2} + {}^{2}\varphi_{22}\, {}^{2}x^{2} + {}^{3}\varphi_{22}\, {}^{3}x^{2} + \\ &+ {}^{1}\varphi_{23}\, {}^{1}x^{3} + {}^{2}\varphi_{23}\, {}^{2}x^{3} + {}^{3}\varphi_{23}\, {}^{3}x^{3} + {}^{1}\varphi_{24}\, {}^{1}x^{4} + {}^{2}\varphi_{24}\, {}^{2}x^{4} + \\ &+ {}^{3}\varphi_{24}\, {}^{3}x^{4} + {}^{1}\varphi_{25}\, {}^{1}x^{5} + {}^{2}\varphi_{25}\, {}^{2}x^{5} + {}^{3}\varphi_{25}\, {}^{3}x^{5} + \\ &+ {}^{1}\gamma^{2}\, {}^{1}y + {}^{2}\gamma^{2}\, {}^{2}y + {}^{3}\gamma^{2}\, {}^{3}y = {}^{1}x^{2} + {}^{2}x^{2} + {}^{3}x^{2} \end{aligned} \tag{4.8.5}$$

$$
\begin{aligned}
&{}^1\varphi_{31}\,{}^1x^1 + {}^2\varphi_{31}\,{}^2x^1 + {}^3\varphi_{31}\,{}^3x^1 + {}^1\varphi_{32}\,{}^1x^2 + {}^2\varphi_{32}\,{}^2x^2 + {}^3\varphi_{32}\,{}^3x^2 + \\
&\quad + {}^1\varphi_{33}\,{}^1x^3 + {}^2\varphi_{33}\,{}^2x^3 + {}^3\varphi_{33}\,{}^3x^3 + {}^1\varphi_{34}\,{}^1x^4 + {}^2\varphi_{34}\,{}^2x^4 + \\
&\quad + {}^3\varphi_{34}\,{}^3x^4 + {}^1\varphi_{35}\,{}^1x^5 + {}^2\varphi_{35}\,{}^2x^5 + {}^3\varphi_{35}\,{}^3x^5 + \\
&\quad + {}^1\gamma^3\,{}^1y + {}^2\gamma^3\,{}^2y + {}^3\gamma^3\,{}^3y + e^3 - m^3 = {}^1x^3 + {}^2x^3 + {}^3x^3 \qquad (4.8.6)
\end{aligned}
$$

$$
\begin{aligned}
&{}^1\varphi_{41}\,{}^1x^1 + {}^2\varphi_{41}\,{}^2x^1 + {}^3\varphi_{41}\,{}^3x^1 + {}^1\varphi_{42}\,{}^1x^2 + {}^2\varphi_{42}\,{}^2x^2 + {}^3\varphi_{42}\,{}^3x^2 + \\
&\quad + {}^1\varphi_{43}\,{}^1x^3 + {}^2\varphi_{43}\,{}^2x^3 + {}^3\varphi_{43}\,{}^3x^3 + {}^1\varphi_{44}\,{}^1x^4 + {}^2\varphi_{44}\,{}^2x^4 + \\
&\quad + {}^3\varphi_{44}\,{}^3x^4 + {}^1\varphi_{45}\,{}^1x^5 + {}^2\varphi_{45}\,{}^2x^5 + {}^3\varphi_{45}\,{}^3x^5 + \\
&\quad + {}^1\gamma^4\,{}^1y + {}^2\gamma^4\,{}^2y + {}^3\gamma^4\,{}^3y + e^4 - m^4 = {}^1x^4 + {}^2x^4 + {}^3x^4 \qquad (4.8.7)
\end{aligned}
$$

$$
\begin{aligned}
&{}^1\varphi_{51}\,{}^1x^1 + {}^2\varphi_{51}\,{}^2x^1 + {}^3\varphi_{51}\,{}^3x^1 + {}^1\varphi_{52}\,{}^1x^2 + {}^2\varphi_{52}\,{}^2x^2 + {}^3\varphi_{52}\,{}^3x^2 + \\
&\quad + {}^1\varphi_{53}\,{}^1x^3 + {}^2\varphi_{53}\,{}^2x^3 + {}^3\varphi_{53}\,{}^3x^3 + {}^1\varphi_{54}\,{}^1x^4 + {}^2\varphi_{54}\,{}^2x^4 + \\
&\quad + {}^3\varphi_{54}\,{}^3x^4 + {}^1\varphi_{55}\,{}^1x^5 + {}^2\varphi_{55}\,{}^2x^5 + {}^3\varphi_{55}\,{}^3x^5 + \\
&\quad + {}^1\gamma^5\,{}^1y + {}^2\gamma^5\,{}^2y + {}^3\gamma^5\,{}^3y + e^5 - m^5 = {}^1x^5 + {}^2x^5 + {}^3x^5 \qquad (4.8.8)
\end{aligned}
$$

$$
{}^1y = {}^1\varphi_{01}\,{}^1x^1 + {}^1\varphi_{02}\,{}^1x^2 + {}^1\varphi_{03}\,{}^1x^3 + {}^1\varphi_{04}\,{}^1x^4 + {}^1\varphi_{05}\,{}^1x^5 \qquad (4.8.9)
$$

$$
{}^2y = {}^2\varphi_{01}\,{}^2x^1 + {}^2\varphi_{02}\,{}^2x^2 + {}^2\varphi_{03}\,{}^2x^3 + {}^2\varphi_{04}\,{}^2x^4 + {}^2\varphi_{05}\,{}^2x^5 \qquad (4.8.10)
$$

$$
{}^3y = {}^3\varphi_{01}\,{}^3x^1 + {}^3\varphi_{02}\,{}^3x^2 + {}^3\varphi_{03}\,{}^3x^3 + {}^3\varphi_{04}\,{}^3x^4 + {}^3\varphi_{05}\,{}^3x^5 \qquad (4.8.11)
$$

and: all variables $x \geq 0$ (4.8.12)

where x : production increases
y : income increase targets
e : export increases
m : import increases
γ : (incremental) final demand coefficients
c : cost coefficients
${}^r\varphi_{hh'}$: input-output coefficients

$${}^r\varphi_{0h'} = 1 - \sum_{h=1}^{5} {}^r\varphi_{hh'}: \text{ value added coefficients.}$$

sector 1: regional sector
sector 2: national sector
sectors 3, 4 and 5: international sectors

The reader will note that for the regional sector three equations must hold; this is because of the non-transportability of regional goods. Further analysis

of the model shows that for its solution equations (4.8.6), (4.8.7) and (4.8.8) which correspond to the international sectors are superfluous (cf. Section 4.3). They do not influence the optimal amounts to be produced and are used solely to calculate exports and imports afterwards. Therefore there are only seven real primary restrictions. This simplification stems from the separation of short- and long-term policies (see Sections 2.1, 2.4 and 2.6).

Strictly speaking, there is no a priori reason why exports or imports would not decrease, as it was argued for production in Section 3.4. It could certainly be efficient to decrease the imports in a sector in a process of import substitution. Fortunately, the fact that the corresponding variables are not restricted in sign does not play a role in the optimization as there are no positive cost coefficients attached to these variables. Only the $e^h - m^h (h = 3, \ldots, 5)$, the changes in net exports of sector h, are determined by the equations (4.8.6.)–(4.8.8.), once the other variables have been determined. A more general treatment is necessary if the variables for exports and imports carry explicit costs in the model. This case is presented in the Chapters 5, 6 and 7.

* Such a problem can be solved by means of the simplex method. For this, the reader is referred to the literature already quoted in Chapter 3. We will only add here a few remarks on the amount of computational work involved. In linear programming problems this amount of work is closely related to the number of iterations to be carried out in order to find the optimal solution. Generally the primary restrictions of a linear programming problem are formulated as inequalities. Here, however, we are dealing with primary restrictions in the form of equalities which means that we have to apply the so-called "artificial basis technique" (GASS, 1958, pp. 61-63) if an easy starting solution is not available. The number of iterations required then amounts in general to approximately twice the number of primary restrictions.

Of course the number of iterations can in general be reduced by choosing in the first basic feasible solution those variables which can reasonably be expected to appear in the optimal solution. Clearly, in the models presented here the production increases of the immobile or regional sectors of each continent or region respectively are bound to appear in the optimal solution. Consequently we can save a number of iterations by choosing these variables in the first basic feasible solution. *

4.9. Decomposition of the National Planning Problem

Here we will discuss an approximate decomposition of the national planning problem with a regional subdivision and with input-output relations into two problems which can be solved successively. Such a decomposition can have a number of computational and other advantages which will be discussed in Section 5.5.

In order to make such a decomposition possible, we must introduce three additional assumptions.

Assumption 4.7. There are no interregional differences in input-output coefficients.

Assumption 4.8. In each region the differences between different solutions as regards the increase in intermediate demand for each regional product due to the expansion of different national or international sectors can be neglected.

Assumption 4.9. There are no interregional differences in final demand coefficients.

In the first problem we neglect the regional subdivision and consider only the country as a whole. Therefore, only the national income increase target is used. Using the input-output table we can calculate the sectoral production increases corresponding to this national income increase target which yield the national minimum of costs. As the national cost coefficients one can take e.g. for each sector the smallest regional cost coefficient.

In the second problem the regions and the regional income targets are introduced. We now have to locate the sectoral production increases corresponding to the national minimum of costs in such a way that the regional income targets are fulfilled and production costs are minimized. Assumptions 4.8 and 4.9 make it possible to solve this problem in an easy way. It is obvious that if the sector totals are expressed in terms of income this problem has been reduced to the traditional Hitchcock problem with given row and column totals, which can be solved easily by means of the uv-method after the regional sector totals have been distributed over the regions in proportion to the regional targets.

An example with the same three regions and five sectors as before may

illustrate this decomposition procedure. The smallest cost coefficient in each sector h is represented by c^h.

The first problem can now be formulated as follows

$$\text{Min } z = c^1x^1 + c^2x^2 + c^3x^3 + c^4x^4 + c^5x^5 \tag{4.9.1}$$

subject to:

$$\varphi_{11}x^1 + \varphi_{12}x^2 + \varphi_{13}x^3 + \varphi_{14}x^4 + \varphi_{15}x^5 + \gamma^1 y = x^1 \tag{4.9.2}$$

$$\varphi_{21}x^1 + \varphi_{22}x^2 + \varphi_{23}x^3 + \varphi_{24}x^4 + \varphi_{25}x^5 + \gamma^2 y = x^2 \tag{4.9.3}$$

$$y = \varphi_{01}x^1 + \varphi_{02}x^2 + \varphi_{03}x^3 + \varphi_{04}x^4 + \varphi_{05}x^5 \tag{4.9.4}$$

and:

$$x^h \geqslant 0 \qquad \text{for} \qquad h = 1, \ldots, 5. \tag{4.9.5}$$

Here:

x^h : the increase in production of sector h in the whole country measured in output terms ($h = 1, \ldots, 5$)

c^h : the cost per unit increase in production of sector h measured in output terms ($h = 1, \ldots, 5$)

$\varphi_{hh'}$: the increase in inputs from sector h into sector h' per unit increase in output of sector h' ($h = 1, \ldots, 5$; $h' = 1, \ldots, 5$)

γ^h : the increase in final demand for product h per unit increase in national income ($h = 1, \ldots, 5$)

y : the increase in national income.

The equations for the international sectors have been left out because again they define only exports and imports and do not play a role in the optimization process. Applying the simplex method we can obtain the sector totals x^1 through x^5 which minimize total costs given the national income target y. We must, however, notice that this problem can also be solved by means of the semi-input-output method dealt with in Chapter 2. In fact, if we choose immediately production increases in the regional and national sectors as basic variables in our first basic feasible solution the two methods come to the same thing.

For the second problem we first express the sector totals of the solution of the first problem in terms of income: y^1 through y^5. Next we distribute according to our last three assumptions the income of the regional sector among the three regions proportionally to the regional income targets.

Given these regional income targets we can now construct the well known table of income increases:

Sectors	Regions 1	2	3	Sectoral totals
1	${}^1y^1$	${}^2y^1$	${}^3y^1$	y^1
2				y^2
3				y^3
4				y^4
5				y^5
Regional totals	${}^1y$	${}^2y$	${}^3y$	Total y

The solution of such a problem by means of the uv-method has already been dealt with in Chapter 3. The advantage of this decomposition procedure in our example is that we are now dealing with two small and relatively simple problems instead of with a rather big model. Apart from the disadvantage of three additional simplifying assumptions there is another drawback: the solution of the decomposed problem is of course in general not the optimal solution of the complete model. This will be dealt with in Appendix III together with an iterative procedure which enables us to reach the optimal solution in many cases.

4.10. The Estimation of Interregional Transport Flows[2)]

From the models dealt with in the preceding chapters we can compute the spatial distribution of the increases in production; adding these increases to the existing production yields the spatial distribution of total production at the end of the planning period. The corresponding distribution of demand can also be calculated as we will discuss later on. Now the question is what are the transport flows corresponding to this spatial distribution of production and demand?

[2)] In this section transport always means interregional and international transport.

So far transport costs have not been dealt with explicitly in our planning problems. Implicitly this has been done by dividing the sectors of the economies of the spatial units into categories according to the mobility of the goods they produce. This will be discussed in more detail in the next chapter.

If there are no transport costs, many different patterns of transport flows without any difference in costs are possible. However, we have not assumed that there are no transport costs, but only that they can be neglected in solving the problem of regional distribution of production. In reality there are always some transport costs. Introducing these costs now, after the allocation problem has been solved makes it possible to determine the optimal pattern of transport flows. As a consequence we will assume in this section that transport costs between continents, or regions for the mobile national and international sectors respectively are proportional to the amounts of goods to be transported[3]). We just repeat that these transport costs will not influence the spatial distribution of production.

This is a new but (at this stage) independent extension of our planning problems. We are faced with a new optimization problem, that is, to determine the transport flows between the spatial units in such a way that total transport costs are minimized.

The data needed in order to solve this problem are the spatial distribution of production and demand for each of the relevant sectors together with the unit transport costs between the spatial units for each relevant product h, indicated by ${}^{rr'}\tau^{h}$. If production and demand are expressed in value added terms instead of in output terms we use the symbol ${}^{rr'}\tilde{\tau}^{h}$ for the unit transport costs. As stated above, the spatial distribution of production can be derived from the solution of the planning models dealt with in the preceding sections; the transport cost coefficients are assumed to be given. In order to determine the spatial distribution of demand we must in addition know for each spatial unit the (incremental) demand coefficients – final demand coefficients in the case of input-output models and total demand coefficients for the models without input-output relations – for each mobile national and international sector respectively, and add the increases in demand to existing demand.

* If these necessary data are thus available our problem will turn out to be a Hitchcock problem which can be solved again by means of the uv-method.

[3]) For a discussion on this representation of transport costs cf. Section 6.3.

We will illustrate this with an example for a national sector and five regions. The data for production and demand can be found below:

Region	1	2	3	4	5
Production	12	20	20	10	9
Demand	16	12	12	17	14

Our problem now is to fill up a two-way table where each cell represents a transport flow in such a way that transport costs are minimized:

Regions	Regions					Demand
	1	2	3	4	5	
1						16
2						12
3						12
4						17
5						14
Production	12	20	20	10	9	71

The corresponding matrix of transport cost coefficients is characterized by the fact that its main diagonal consists of zero elements. This is because transport costs within the spatial units do not play a role in determining the interregional pattern of transport flows.

Clearly, in many cases the unit transport costs for the two possible transport flows between two regions r and r' – one from r to r' and the other vice-versa – will be the same. Our matrix of cost coefficients is then a symmetric matrix. For the solution of the problem, however, it is not necessary to assume this symmetry. When we consider the table of transport cost

coefficients, it is easy to see that the elements in the main diagonal of the matrix of transport flows in general appear in the optimal solution[4]).

Regions	Regions 1	2	3	4	5
1	0	${}^{12}\tau$	${}^{13}\tau$	${}^{14}\tau$	${}^{15}\tau$
2	${}^{21}\tau$	0	${}^{23}\tau$	${}^{24}\tau$	${}^{25}\tau$
3	${}^{31}\tau$	${}^{32}\tau$	0	${}^{34}\tau$	${}^{35}\tau$
4	${}^{41}\tau$	${}^{42}\tau$	${}^{43}\tau$	0	${}^{45}\tau$
5	${}^{51}\tau$	${}^{52}\tau$	${}^{53}\tau$	${}^{54}\tau$	0

The best thing to do then, is to construct the first basic feasible solution in such a way that these diagonal elements appear in it with their maximum possible values. This means that we should not use the northwest-corner rule but the matrix minimum method. Our first basic feasible solution then could be:

Regions	Regions 1	2	3	4	5	Demand
1	12	4				16
2		12				12
3			12			12
4		4	3	10		17
5			5		9	14
Production	12	20	20	10	9	71

4) This implies the condition: ${}^{rr''}\tau^h \leqslant {}^{rr'}\tau^h + {}^{r'r''}\tau^h$. If this is not ascertained we should construct the more general transshipment problem (cf. Section 5.4).

The uv-method can now be applied again in order to arrive at the optimal solution.

For an international sector where imports or exports play a role an additional row or column appears in the tables. In the matrix of transport flows a column is added for imports and "production" should be replaced by "supply". In the case of exports an extra row related to for. ign demand is added. Of course a corresponding column or row is added to the matrix of transport cost coefficients. For the rest, the procedure remains the same. *

4.11. Introduction of Bounds for Some Variables

Applying the models presented so far one can expect a strong specialization of the country as a whole in a few international sectors and an even stronger specialization of each region separately in a few national or international sectors. This can be shown as follows. Assuming again R regions, H_1 regional, H_2 national and H_3 international sectors, the optimal solution of any model discussed so far, will in general contain H_2+R variables corresponding to the national and international sectors at a positive level[5]). Evidently at least H_2 of these H_2+R variables will correspond to the national sectors. This leaves at the most R variables appearing in the optimal solution which correspond to the international sectors. In addition these positive international variables do not necessarily correspond to different international sectors. So the optimal solution will show that, of the international sectors, at the most R, but in all probability less than R, will be expanded at all in the country.

Moreover, it is obvious that in each region at least one positive variable corresponding to a national or international sector must occur. This means that in one region at the most H_2+1 national or international sectors can be expanded. In general each region will expand only $(R+H_2)/R = 1+H_2/R$ national or international sectors. As in most cases the number of national sectors (H_2) will not be larger than the number of regions (R), the optimal

[5]) This is so because the optimal solution of any linear programming problem contains in general as many positive variables as there are constraint equations, in this case H_1R+H_2+R. In addition we know that the H_1R variables corresponding to the regional sectors are bound to appear in this optimal solution. So only H_2+R variables related to the other sectors will have a positive value.

solution will indicate a strong specialization of each region. Corresponding to this property of the optimal solution large interregional and international trade-flows will occur. This is quite understandable as it has been assumed that transport costs were irrelevant for the solution of the problem. Removal of this assumption makes the model more realistic in the sense that the optimum solution will not yield so strong a specialization (cf. Table 5.7).

Apart from neglecting transport costs, there are several other reasons why the calculated optimal solution may not be realistic:

(1) A large expansion of a sector in one region may be impossible for technical or social reasons. Moreover, the scarcity of some factors of production (e.g. certain types of skilled labour, or land) may increase considerably when output in certain sectors increases very fast, either leading to diminishing returns while retaining essentially the same technology, or leading to a radical change in technology which renders a recomputation necessary of the cost coefficients.

(2) It may also be that the solution indicates that a sector which traditionally has not exported its products but nevertheless has been correctly classified as international, should start to export. If in practice this sector would not be able to compete in the world market, e.g. because of the inferior quality of its products, then an expansion of its productive capacity beyond the level necessary for the domestic market would be a bad policy. Obviously, this point depends on how narrowly the sectors have been defined. In practice, however, it is often impossible to take account of quality differences in defining the sectors.

(3) Finally, a sector which already exports might not be able to increase its exports more than in proportion to the increase in world demand. E.g. because of an existing quota system or a possible depressing influence on the world market price which might in its turn affect exports. Similarly, a national government might restrict the increase in imports of products which can be or are produced domestically. Such reasons might necessitate the imposition of bounds on the variables of the models or on combinations of these variables. The first reason mentioned could lead to imposing upper bounds on the variables of the type:

$$ {}^{r}y^{h} \leq {}^{r}a^{h} \qquad r = 1, \ldots, R;\ h = 1, \ldots, H \tag{4.11.1} $$

The second reason could lead to excluding the possibility of exporting for a

number of international sectors

$$\sum_{r=1}^{R} {}^{r}y^{h} \leq \eta^{h} y \qquad h = H_1 + H_2 + 1, \ldots, H \tag{4.11.2}$$

The third reason could lead to the imposition of upper bounds either on the exports of a number of international sectors

$$\sum_{r=1}^{R} {}^{r}y^{h} \leq \eta^{h} y + A^{h} \qquad h = H_1 + H_2 + 1, \ldots, H \tag{4.11.3}$$

or on the imports of a number of international sectors

$$\sum_{r=1}^{R} {}^{r}y^{h} \geq \eta^{h} y - B^{h} \qquad h = H_1 + H_2 + 1, \ldots, H \tag{4.11.4}$$

where A^h, and B^h are the maximum increase in exports and imports respectively of sector h.

The possibility of adding bounds on certain variables or on sums of variables also allows introducing in the models considerations other than those mentioned above. One may for example want to ascertain the expansion of a sector in a region because such an expansion is already decided on in advance of the plan or because it is even under way. This can be done by setting a lower bound on the corresponding variable:

$$ {}^{r}y^{h} \geq {}^{r}b^{h} \tag{4.11.5}$$

Theoretically, the introduction of extra restrictions like (4.11.1)-(4.11.5) does not render a linear programming problem more difficult, but only increases its size. In principle one can always apply a general solution method like the simplex method to the new problem. However, as we will see in practice things may turn out not to be that easy. Before discussing this we will first indicate which effect such bounds have in general on the nature of the solution. When solving a model with such bounds, first transform the inequalities into equalities by subtracting from or adding to the left hand side of each inequality an unknown nonnegative variable, a so-called slack variable, e.g.

$$ {}^{r}y^{h} + {}^{r}s^{h} = {}^{r}a^{h} \qquad r = 1, \ldots, R; \; h = 1, \ldots, H \tag{4.11.6}$$

In general for each new restriction a new variable is bound to appear in the solution with a positive value. However, this variable may be a slack variable which actually means that the newly added restriction is not really binding.

As the rationale for adding bounds is that the new restrictions should change the original solution – that is that they are binding – it is meaningful to observe that in general, if N such binding constraints are added to the model, the new optimal solution will have N more positive variables than the original solution. The result is that, now, on the average $1+(H_2+N)/R$ national or international sectors instead of $1+(H_2/R)$ will be expanded in each region and hence less specialization in the regions (and in the country) will occur.

For practical computations however, the introduction of new constraints may cause some difficulties, either because the model becomes too large to be solved with the available computational facilities, or because some very efficient computational procedures (like the uv-method) are not directly applicable any more. Some more efficient methods than simply adding constraints and slack variables and dealing with them in the same way as with the original constraints have been developed, both for general linear programming models (upper bounds for variables) and for the Hitchcock problem. For a full description of these methods the reader is referred to the existing literature (DANTZIG, 1963, Chapter 18). Here we confine ourselves to two remarks on the introduction of bounds in a Hitchcock problem.

* First, each Hitchcock problem with upper bounds on some variables or on sums of variables corresponding to the same international (or national) sector can be transformed into a Hitchcock problem with some zero variables. This transformation procedure is best illustrated by an example. We take the case of a closed economy model with two sectors and three spatial units, where in one sector the sum of the increases in two regions has an upper bound.

Minimize

$$z = \sum_{r=1}^{3} \sum_{h=1}^{2} {}^r\tilde{c}^h \, {}^r y^h \tag{4.11.7}$$

subject to

$$\sum_{r=1}^{3} {}^r y^h = y^h \qquad h = 1, 2 \tag{4.11.8}$$

$$\sum_{h=1}^{2} {}^r y^h = {}^r y \qquad r = 1, 2, 3 \tag{4.11.9}$$

$$^{1}y^{1} + {}^{2}y^{1} \leq d \tag{4.11.10}$$

$$^{r}y^{h} \geq 0 \tag{4.11.11}$$

The transformation into a Hitchcock problem is made as follows. Two new variables both with a zero cost coefficient are introduced, a slack variable $s^1 (= d - {}^1y^1 + {}^2y^1)$ and $y_p^1 (= {}^1y^1 + {}^2y^1)$ the lower index p for "partial". Next (4.11.10) and the first equation of (4.11.8) are replaced by

$$^{1}y^{1} + {}^{2}y^{1} + s^{1} = d \tag{4.11.12}$$

$$y_p^1 + s^1 = d \tag{4.11.13}$$

$$y_p^1 + {}^{3}y^{1} = y^{1} \tag{4.11.14}$$

Now one can construct the following tableau for the program of income increases:

Sectors		Regions 1	2	3	
1	s^1	$^1y^1$	$^2y^1$		d
	y_p^1			$^3y^1$	y^1
2		$^1y^2$	$^2y^2$	$^3y^2$	y^2
	d	^1y	^2y	^3y	y

where the empty cells represent the zero variables. Evidently this is the same problem as dealt with in Section 3.7. The same devise can be used in transforming models of an open economy type.

The second remark concerns the case of a Hitchcock problem with bounds only on single variables. If many such bounds are present, the transformation into a pure Hitchcock problem may be cumbersome. The following procedure, described more fully in the literature mentioned earlier, is applicable to this case. In each cell of the Hitchcock problem, the bounds are also indicated. Then the matrix minimum method is applied with the additional rule that no variable may exceed its bound. If a variable has to be set at its bound it is considered to be non-basic although it has a value larger than zero, unless it is simultaneously limited by a row or column constraint in

which case it is considered to be basic. When a basic feasible solution has been found, the u's and v's are re-calculated, using the cost coefficients of the basic variables, and the indirect cost-coefficients are calculated. The criterion for optimality is only now somewhat more complicated. The solution is optimal if and only if the non-basic variables which are zero or at their lower bound have ${}^r\tilde{c}^h \geq {}^r\bar{c}^h$, and those non-basic variables which are at their upper bound have ${}^r\tilde{c}^h \leq {}^r\bar{c}^h$. If one or more of these conditions is not satisfied, the non-basic variables with the largest difference between ${}^r\tilde{c}^h$ and ${}^r\bar{c}^h$ should be introduced into the basis, and the θ-method should be used to find a new basic feasible solution. Notice that introduction into the basis of a non-basic variable which is at its upper respectively lower bound means decreasing respectively increasing its value by θ. In this way a number of iterations will yield the optimal solution. Solving a model with explicit input-output relations and some bounds on single variables requires a comparably simple extension of the methods (e.g. the simplex method) for solving such a model without bounds, but it will not be described here. *

The introduction of upper and lower bounds for variables is an important and very flexible tool to increase the realism of these models. However, it must be done carefully as these bounds strongly influence the optimal solution of the problem. Their imposition should be based on sectoral studies. Later on, one can apply sensitivity analysis in order to see which bounds have the greatest influence on the optimal value of the objective function. Then it can again be considered whether these bounds are realistic or whether they should be changed.

4.12. Integration of Countries into an Area

The models presented so far have been formulated in terms of a spatial unit (the world or a country) subdivided into a number of subunits (continents or regions). The same concepts and models as in Chapter 4 can be applied to a different type of problem on a somewhat intermediate level, namely the integration of a number of spatial units (e.g. countries) into a higher unit (which we will call an area). This is more meaningful according as there is more trade possible between the units, because then the degree of openness of the area as a whole is significantly lower than of any of the countries separately.

The least one can say is that it becomes useful to investigate whether the separate national plans together constitute a consistent plan for the area as a whole; in particular whether the assumptions made with respect to the possibilities of export to and import from the other countries of the area are consistent with the plans of the other countries. A next step would be to apply a model as presented here for a country with a subdivision into regions, to an area subdivided into countries. In such a model the income increase targets of the national plans can be taken as the income increase targets of the countries constituting the area. It could easily turn out that a more efficient use of scarce factors might be attainable than by just adding up the national plans.

The classification of the sectors according to the mobility of their products would be a little bit different (e.g. national government). Another difference compared with a national plan with regional subdivisions might be that in practice some instruments which in the national planning process could be used at the level of the higher spatial unit, can now only be used at the level of the lower spatial unit. Capital in particular might be less mobile within the area concerned than within each of the composing countries, unless a high degree of co-operation between the regions existed. Even if capital were largely mobile within the area, the income created in some country by the use of capital from another country (direct investment) will probably not stay completely within the former country. These institutional aspects should be taken into consideration when constructing such a model for an area.

A completely different type of advantage from economic integration could be taken account of if economies of scale play a role. Then countries should co-operate in distributing different sectors with economies of scale among themselves, thereby avoiding several countries expanding the same sector that has economies of scale and none of them being able to do so efficiently. This feature, however, has not been dealt with in the models presented here.

4.13. A Case Study: Mexico

In this section we will discuss briefly a case study of Mexico which will be published separately (R. CARRILLO ARRONTE, 1969). In this study the 32

provinces of Mexico have been grouped together into ten regions. In doing so the main principles mentioned in Section 4.1 have been followed. Account has been taken of the existing administrative units, of geographical barriers like mountain ranges and of a number of common socio-economic characteristics like per capita income, electricity consumption etc.

As for the choice of sectors, attention has been paid in particular to the homogeneity of the products with regard to their mobility (cf. Section 4.2). With the use of 1960 census data, the Mexican economy has been divided into 31 sectors. The list of sectors together with their classification according to the mobility of their products and the regions can be found in Appendix II, Fig. II.1 and Table II.1.

In the same Appendix II, Table II.2 gives capital-income ratios for each sector and each region with the exclusion of a number of cells which represent the predetermined zeros of the model (cf. Section 4.5). Because capital is the only scarce factor of production taken into consideration, these capital-income ratios can be taken to approximately represent the cost coefficients. In this study the predetermined zeros correspond only to non-shiftable industries. These non-shiftabilities have been taken up in the model on the basis of a careful analysis of the existing structure of production together with the possibilities of expansion. This existing structure of production is given in Table II.3 of Appendix II. These are the main data necessary in order to apply the methods discussed so far.

The purpose of the study was to apply the type of models dealt with in this chapter for a five year period, 1960-1965. Only models where input-output relations are not explicitly represented are used so that only cases where the models can be formulated as the semi-Hitchcock problem of Section 4.3 are considered. Two versions of the models have been constructed differing only with regard to the regional income increase targets.

(1) The first version, called the "historical model" assumes an annual rate of growth of national income of 6.5% as an overall target. This corresponds to the rate of growth during the period 1950-1960. Then, regional income targets have been set assuming constant elasticities (calculated from the preceding period) of the regional incomes with respect to total national income. Next, all variables which correspond to the regional sectors and which are functions of the corresponding regional incomes have been calculated. The sectoral totals of the national sectors have been computed by the same method of constant growth elasticities with respect to national

income as has been used when calculating the regional incomes. Finally, for all international sectors together, one overall rate of growth has been obtained as a residual.

(2) In the second version, called the "social model" different regional income targets have been introduced while the same annual rate of growth of national income has been assumed. According to this model, regions with a lower actual level of development must grow at a much faster rate than the country as a whole: 11.7% annually. The more developed regions will maintain about the same annual rate of growth as in the preceding period, i.e., 5.6%. But Mexico City in particular must slow down its rate of growth to a mere 2% per annum.

When these two versions were solved by means of the modified uv-method dealt with in Section 4.4, both their optimal solutions showed a heavy concentration in a few international sectors, i.e., mainly agricultural products. In order to get rid of this unrealistic feature (cf. Section 4.11) of the models, upper bounds were imposed on the expansion of a number of international sectors. These upper limits were calculated assuming that, for these co-called traditional sectors, their growth elasticities with respect to national income cannot increase. Then the two models were solved again. Now the results turned out to be much more satisfactory. Appendix II, Table II.4 gives a survey of these results. Note, however, that the international sector totals on which no upper bounds have been imposed show exactly the same values in both the solutions of the "historical model" and of the "social model". This is really exceptional and is a pure coincidence. In general these totals should be different in the solutions of the different models. The crossed cells in this table correspond to the empty cells of the table of capital-income ratios (Table II.2). They represent the predetermined zeros of the model. As the reader can verify himself, the introduction of regional income targets in the "social model" causes some striking changes in the regional pattern of location of some industries. Another interesting point is how much additional investment the introduction of these regional income targets of the "social model" would have implied. It turns out that investment in the "social model" is 8.1% higher than in the "historical model". One may conclude from this figure that in this case the cost of a regional policy would have been rather limited. The next steps in this study are to fix also the total increases of the other international sectors and to introduce upper bounds for all variables related to national or international

sectors, which prevented any increases of such sectors in any region during the planning period of more than 100 per cent of the existing production at the beginning of the planning period. For the results of these extended models and for a more detailed discussion, the reader is referred to the original publication.