CHAPTER 7

A SURVEY OF THE MODELS, THEIR ASSUMPTIONS AND SOLUTION METHODS

7.1. Introduction

In this chapter a survey of the models presented so far will be given, starting from the most general model with the smallest number of assumptions and proceeding from this General Model, via the Basic Model to the particular models by adding systematically more simplifying assumptions. The intention of such a presentation is on the one hand to provide insight into the interconnection of the different models and their generalizations. On the other hand, however, it is intended that planners who want to use these models as a framework for their planning models, may be guided by the systematic survey of simplifying assumptions and the resulting models to choose a model that constitutes a good compromise between the demands for realism and precise results, and the limitations of data and computational facilities, which may be different from place to place and case to case.

At each step, from the General to the Basic Model, and from the Basic Model to each particular model, we will distinguish as much as possible between the verbal discussion of the models and their simplifying assumptions on the one hand, and the mathematical formulation of the assumptions, the resulting models and some technical details on the other hand. The latter parts are starred, so that they can be omitted at a first reading.

We warn the reader once again that all these models, including the general model, are intended to be framework models. This means that in general they cannot be applied blindly to any actual situation, but that they may solely constitute the framework of a model to be built for practical use, which may contain a number of extensions or refinements of the framework models. For instance, in Section 4.7 it was observed that in addition to income targets one might also like to introduce employment targets, and in

Section 4.11 the desirability of introducing bounds on the variables or linear combinations of the variables in order to improve the realism of the model was discussed. Neither these, nor other refinements one can think of, will be dealt with in this chapter in order to keep the discussion as simple and straightforward as possible. Non-shiftabilities (cf. Section 3.7) are also not included in the formulation of the models in this chapter, but they can be included in practice, if appropriate. Transport cost functions dependent on the dominant flow along a route can also be introduced if appropriate (cf. Section 6.3). Another restriction on the discussion in this chapter is that we limit ourselves to (open and closed) models with a second order spatial subdivision, as higher order spatial subdivision models have not been dealt with so far. One could, however, extend these models directly to third or fourth order spatial subdivision models, e.g. distinguishing a world, continents, countries and regions (HERMAN *et al.*, 1968).

7.2. The General Model

Behind the models introduced in the Chapters 3, 4, 5 and 6 one can imagine a single *Basic Model*, from which these particular models can be derived by a number of simplifying assumptions. This basic model itself can in its turn be derived from a *General Model* by some simplifying assumptions. In this section we will discuss the assumptions and character of this General Model. Its mathematical formulation is given at the end of this section.

All the models included in this discussion can be divided into two broad categories, according to whether they deal with an open or with a closed economy. A particular model of the second category was dealt with in Chapter 3; particular models of the first category were discussed in Chapter 4, while in Chapters 5 and 6 models both for closed (Sections 5.2 and 5.3) and open (Sections 6.2 and 6.3) economies were introduced. Since the models for a closed economy can be considered as special cases of the models for an open economy in this chapter we will deal mainly with open economy models.

In order not to loose sight of the real problems which the models attempt to deal with, we will describe them not abstractly in terms of spaces of higher and lower order, but concretely in terms of countries and regions. Instead of the words country and regions we could also use the words continents and

countries, or, in case of a closed economy model, world and continents.

The General Model deals with the same logical type of planning problem as the particular models, i.e. one where, for each of the regions, fixed targets for the increases in the regional incomes are given, and where the aim is to attain these targets at a minimum cost, in terms of the use of scarce factors of production for investment and increase in production over the planning period, while these means of production are evaluated by means of appropriate shadow prices (see appendix VI).

Therefore these models are classified as models in comparative statics. At the end of Section 3.1 we noted already that an alternatively possible formulation would be to maximize the increase in national income, while the increases of the regional incomes should stand in fixed ratios to each other and the available quantities of scarce resources would be given. Such a formulation would still remain in the area of comparative statics. Here we may notice in addition that these models are not concerned with the intertemporal distribution of investments and of income increases over the planning period. Introduction of such considerations would complicate the models considerably, either by increasing the number of variables or by necessitating use of a different mathematical method for their solution. Therefore, we have omitted such considerations deliberately from this study, although they might become important at a certain stage.

The General Model deals simultaneously with three medium or long term planning problems, numbered I, II and III:

I. The national sector planning problem. Determine the optimal increases in the total domestic production of each sector, optimal in the sense that the national income increase target constituted by the sum of the regional income increases is attained at minimum cost.

II. The regional distribution problem. Determine the optimal regional distribution of the increases in total domestic production of each sector, optimal in the sense of attaining each regional income target separately, at minimum total costs.

III. The transport problem. Determine the optimal system of transportation of the products; optimal in the sense of balancing supply and demand at minimum total transport costs.

Of course we are equally interested in the investment corresponding to

these increases in production, but assuming that in the year before the planning period all capacities are fully utilised – as we do – makes the problem of determining the optimal investments in the sectors nearly identical to that of determining the increases in production of the sectors.

Together these three problems constitute the middle phase of the planning in stages procedure (cf. Section 2.3 and TINBERGEN, 1967). In the general model these three problems have merged into one simultaneous problem:

I+II+III. The national-regional and transport planning problem. Determine the optimal increases in production per sector and per region and the corresponding optimal transport system, optimal in the sense that the income increase targets for each region are attained and the total costs of production and transport are minimized.

The reason for formulating the three problems as one simultaneous problem is that in general, as it will turn out, one can get a better solution if one takes all three problems into account simultaneously, instead of solving them separately, that is in the order I, II and III, as we saw in Sections 4.9, 5.4, 6.2, and Appendix III. However, in the next sections some attempt will nevertheless be made to disentangle the simultaneous problem into the three separate problems, for reasons set out in Section 5.5. By then, however, the interconnection of the problems may be clearer.

The General Model and all the particular models derived from it are based on two basic assumptions and three more technical assumptions.

Assumption 7.1 (the same as the first part of Assumption 4.1 and as the first part of Assumption 3.1 for a closed economy and slightly different from Assumption 6.1 or from Assumption 5.2 for a closed economy). The sectors are distinguished qualitatively according to the mobility of their products, corresponding to whether these products can move only within the regions, outside of the regions but only within the country, or across the national border. The sectors and products are classified accordingly as regional, (other) national or domestic, and international (continental or world) sectors and products.

As the limitation of the mobility of the goods is partly determined by their transport costs, this qualitative distinction already largely represents the influence of transport costs in the model in an implicit way. Notice that e.g.

some regional products might cross a regional border in small amounts in regional border trade, without becoming national, however.

Assumption 7.2 (the same as Assumption 4.2 and as Assumption 3.2). All production processes can be described by linear relationships between outputs and inputs of intermediate goods and scarce factors. All kinds of inputs into interregional and international transport are proportional to its output.

As a result of the first part of this assumption all increases in all kinds of inputs into the sectors are proportional to the increases in production of these sectors. In addition, Assumption 5.1b is excluded.

The following three assumptions are of a more technical nature.

Assumption 7.3 (corresponding to Assumption 3.5 and to Assumption 4.5). Prices are fixed and price differences for national or international products between regions can be neglected.

The consequence of the second part of this assumption is that the income earned by interregional or international transport is neglected. While in certain cases one might like to abandon therefore this assumption – which is certainly possible without completely giving up the general model – we maintain it because of its simplifying nature. The amounts of the products can now be measured in units such that all prices are = 1 everywhere, and these units are the same everywhere for the same product.

Assumption 7.4 (similar to, but slightly stronger than Assumptions 3.4′ and 4.4′). The increases in final demand for regional or national and international products respectively in each region are proportional to the increase in income of the corresponding region.

This assumption is very convenient when writing down the models in mathematical terms, but in fact it is sufficient to assume that the increase in final demand for each product in each region is determined, once the increase in income is determined.

Assumption 7.5. No sector (apart from interregional and international transport) will reduce its output.

This assumption may not be too strong in a growing economy and it allows to maintain the nonnegativity of all variables which indicate increases in

the model. In fact one assumes this to be the case in the optimal or any otherwise useful solution, even if it is technically possible to reduce the output of a sector considerably. If one wishes to leave open the possibility of a reduction in the production of certain sectors, one may do so by redefining the variables representing the increases in production in those sectors as indicating increases measured from a lower level, which is considered in advance to be the lowest possible efficient level. In this case, however, one should examine carefully whether the costs attached to these variables can still be assumed to be proportional to the value of the variable concerned, which is essential for the linearity of the model.

The *construction of the General Model*, based on these assumptions, is as follows. In contradistinction to most multisectoral models the transport sector is dealt with in a special way [1]). It is split up into two parts: intraregional transport, and also interregional and international transport. The first part is treated just as a normal regional sector and whenever we use the word transport, we mean interregional and international transport. The variables corresponding to the transport sector are characterized both by the type of product transported, and by the regions of origin and destination, so that they have three indices: ${}^{rr'}t^h$, indicating the total amount of product h transported from region r to region r'. International transport of product h from, or to, region r is represented by ${}^{r}e^h$ or ${}^{r}m^h$ respectively, and unit cost coefficients are attached to these transport variables, like the other ones [2]). If direct transport between two regions – or a region and the outside world – is not possible or could not occur in any useful solution, then the corresponding variable is left out of the model [3]). If more than one route for direct transport exists, then only the cheapest one is included in the model [4]). The

[1]) Part of this paragraph has already been discussed in Section 5.1.

[2]) The reader is warned that the variables and coefficients used here are different from those in Chapter 5. In short: the variable t represents the total amount transported corresponding to the variable x (that is measured in terms of production or output), while $\tilde{t}$ also represents the total amount transported, but corresponding now to the variable y (that is measured in terms of value added). The same holds for the coefficients $\tilde{\tau}$ and τ.

[3]) If one were interested in e.g. the demand for loading or unloading capacity, one would define the transport variables exclusive of transit transport. Cf. Section 5.2.

[4]) Unless it is not determined in advance which route is the cheapest one, as in the case of dominant flow transport costs (cf. Section 6.3). This case is, however, excluded by Assumption 7.2.

reason for treating transportation differently from other sectors is that demand for its product does not depend only on the regional incomes or the output of other sectors, but also, and essentially, on the location of production and consumption. Another peculiar feature of transport in this model is that, as interregional price differences are neglected, transport produces no income, although it carries cost. Its function is therefore only to connect supply and demand spatially.

The variables for the production of the sectors – by which we do not mean interregional and international transport – indicate increases in production during the planning period. The reason for this is that these are the variables we are most interested in in development planning, and that for these variables the proportionality relationships hold best and the coefficients can be estimated better. Only the transport variables do not represent increases, but absolute amounts, since it is quite possible that it turns out to be useful to reduce the transportation of certain products on a certain route, for example when developing import substituting industries, whereas we prefer to have nonnegative variables in the model for mathematical convenience[5]).

The primary restrictions of the model are the equations balancing total supply and demand at the end of the planning period for each sector – not including the sector of interregional and international transportation – and each region. All existing supply and demand at the beginning of the planning period is assumed to be known and balancing each other. In addition, for each region the sum of the value added increases in all sectors must equal the target set for the increase in regional income. The secondary restrictions state that all variables are nonnegative. The objective function in the model is to minimize the sum of the increases in production costs and the total transport costs. The existing production costs are already fixed.

* Next we will define a number of variables, coefficients and indices used in the General Model. Moreover some technical details will be briefly mentioned.

Indices and special signs

H_1 : total number of regional sectors (indicated by $h_1 = 1, \ldots, H_1$), including intraregional transport.

[5]) The efficient solution methods for such models are based on this nonnegativity of the variables.

H_2 : total number of national sectors (indicated by $h_2 = H_1+1, \ldots, H_1+H_2$).

H_3 : total number of international sectors (indicated by $h_3 = H_1+H_2+1, \ldots, H$).

H : total number of sectors (indicated by $h = 1, \ldots, H$), excluding inter-regional and international transport.

R : total number of regions (indicated by r or $r' = 1, \ldots, R$).

$\sum_{r'=1}^{\prime}$: summation leaving out the term for which $r' = r$.

Variables

${}^{r}x^{h}$: increase in the production of sector h in region r, measured in output terms.

${}^{rr'}t^{h}$: amount of product h directly transported from region r to region r'.

${}^{r}e^{h}({}^{r}m^{h})$: amount of product h directly exported from (imported into) region r.

z_G : total costs of transport and of the increase in production.

Target variable

${}^{r}y$: increase in income of region r.

All variables are at constant prices of some base year.

Constants

${}^{r}\bar{x}^{h}$: total production of product h in region r existing at the beginning of the planning period.

${}^{r}\bar{d}^{h}_{G}$: total intermediate and final demand for product h in region r, excluding intermediate demand by transportation, existing at the beginning of the planning period. The lower index G is added to indicate that this is the constant in the General Model. The corresponding constant in other models has a slightly different definition.

Coefficients

${}^{r}c^{h}$: total costs of increasing the production of sector h in region r by one unit.

${}^{rr'}\tau^h$: total costs of directly transporting one unit of product h from region r to region r'.

${}^r\varepsilon^h({}^r\mu^h)$: total transport costs of directly exporting (importing) one unit of product from (into) region r.

${}^r\varphi_{h_i h}$: increase in current inputs from sector h_i into sector h per unit increase in output of sector h in region r.

${}^r\varphi_{0h} = \left(1 - \sum_{h_i=1}^{H} {}^r\varphi_{h_i h}\right)$: increase in value added (or income derived) per unit increase of production of sector h in region r.

${}^r\varphi_{h_i\, rr'\, t^h}({}^r\varphi_{h_i\, r'r\, t^h})$: current inputs from sector h_i, demanded in region r, for transportation of product h from region $r'(r)$ to region $r(r')$ per unit amount transported.

${}^r\varphi_{h_i\, re^h}({}^r\varphi_{h_i\, rm^h})$: current inputs from sector h_i, demanded in region r, for transportation of exports from (imports into) region r of product h per unit amount transported.

${}^r\gamma^h$: increase in final demand for product h in region r (measured in the same units as ${}^rx^h$) per unit increase in income in region r.

Some technical details

Our problem is clearly a linear programming problem. If there are R regions, H_1 regional, H_2 national and H_3 international sectors and if for all regions direct transport to and from other regions and the outside world is possible, then there are $R(H_1+H_2+H_3)+R$ equations in

$$R(H_1+H_2+H_3)+R(R-1)(H_2+H_3)+2RH_3$$

variables, not including the target variables ${}^ry$ and z_G. However, for each region one equation is redundant, so only

$$R(H_1+H_2+H_3)$$

equations are relevant.

Demand for investment products is assumed to be part of the final demand. One could deal with investment in a more refined way, by relating the increase in demand for investment goods to the increases in production of the sectors and to the increase in transport during the planning period, assuming some particular time-path for investments – e.g. exponential growth – during and beyond the planning period. Then a second matrix of

technical coefficients could relate all sectors to each other with respect to these capacity expansions.

The use of the coefficient ${}^{r}\varphi_{h_i\,rr'_t h}$ (or ${}^{r}\varphi_{h_i\,r'r_t h}$) implies that one can determine in advance which part of current inputs into transportation between the regions r and r' are demanded in region r and which part in r'.

The result of Assumption 7.3 is that prices need not appear explicitly in the model; the result of the Assumptions 7.2 and 7.5 is that the model is of the linear programming type, while the result of Assumption 7.4 is that, if all values of the ${}^{r}y$ are multiplied by the same factor λ, then in the solution the values of the production variables and those of the differences between the transport variables and the existing transport volumes are also multiplied by λ. Finally, Assumption 7.1 prevents the appearance of $R(R+1)H_1+2RH_2$ more transport variables in the model.

It has already been observed that for practical applications at least some bounds to certain variables or combinations of variables – for instance on the exports of certain products – should be introduced into the model (cf. Section 4.11). The result of Assumption 7.4, mentioned above, would then hold no longer.

The *mathematical formulation* of the General Model, based on the Assumptions 7.1–7.5 is as follows.

$$\text{Minimize } z_G = \underset{(1)}{\sum_{h=1}^{H}\sum_{r=1}^{R} {}^{r}c^h \cdot {}^{r}x^h} + \underset{(2)}{\sum_{h=H_1+1}^{H}\sum_{r=1}^{R}\sum_{r'=1}^{R}{}' {}^{rr'}\tau^h \cdot {}^{rr'}t^h} + \underset{(3)}{\sum_{h=H_1+H_2+1}^{H}\sum_{r=1}^{R} ({}^{r}\varepsilon^h\,{}^{r}e^h + {}^{r}\mu^h\,{}^{r}m^h)} \tag{7.2.1}$$

subject to

regional sectors:

$$\underset{(4)}{{}^{r}\bar{x}^{h_1}} + \underset{(5)}{{}^{r}x^{h_1}} = \underset{(6)}{\sum_{h=1}^{H} {}^{r}\varphi_{h_1 h}\,{}^{r}x^h} + \underset{(7)}{\sum_{h=H_1+1}^{H}\sum_{r'=1}^{R}{}' ({}^{r}\varphi_{h_1\,rr'_t h}\,{}^{rr'}t^h + {}^{r}\varphi_{h_1\,r'r_t h}\,{}^{r'r}t^h)} +$$

$$+ \underset{(8)}{\sum_{h=H_1+H_2+1}^{H} ({}^{r}\varphi_{h_1\,re^h}\,{}^{r}e^h + {}^{r}\varphi_{h_1\,rm^h}\,{}^{r}m^h)} + \underset{(9)}{{}^{r}\gamma^{h_1}\,{}^{r}y + {}^{r-h_1}d_G} \tag{7.2.2}$$

$r = 1, \ldots, R\,; h_1, = 1, \ldots, H.$ Number of equations: (RH_1)

national sectors:

$$
\begin{array}{l}
{}^{r}\bar{x}^{h_2} + {}^{r}x^{h_2} = \sum_{h=1}^{H} {}^{r}\varphi_{h_2 h}\, {}^{r}x^{h} + \sum_{h=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^{r}\varphi_{h_2\, rr't^h}\, {}^{rr'}t^{h} + {}^{r}\varphi_{h_2\, r'rt^h}\, {}^{r'r}t^{h}) + \\
\quad (4) \quad (5) \qquad (6) \qquad\qquad\qquad\qquad (7) \\
\qquad + \sum_{r'=1}^{R}{}' ({}^{rr'}t^{h_2} - {}^{r'r}t^{h_2}) + \sum_{h=H_1+H_2+1}^{H} ({}^{r}\varphi_{h_2\, re^h}\, {}^{r}e^{h} + {}^{r}\varphi_{h_2\, rm^h}\, {}^{r}m^{h}) + \\
\qquad\qquad\qquad (11) \qquad\qquad\qquad (8) \\
\qquad + {}^{r}\gamma^{h_2}\, {}^{r}y + \overset{r-h_2}{d_G} \qquad r = 1, \ldots, R;\ h_2 = H_1+1, \ldots, H_1+H_2 \qquad (7.2.3) \\
\qquad\qquad (9) \quad (10)
\end{array}
$$

Number of equations: (RH_2)

international sectors:

$$
\begin{array}{l}
{}^{r}\bar{x}^{h_3} + {}^{r}x^{h_3} = \sum_{h=1}^{H} {}^{r}\varphi_{h_3 h}\, {}^{r}x^{h} + \sum_{h=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^{r}\varphi_{h_3\, rr't^h}\, {}^{rr'}t^{h} + {}^{r}\varphi_{h_3\, r'rt^h}\, {}^{r'r}t^{h}) + \\
(4) \quad (5) \qquad (6) \qquad\qquad\qquad\qquad (7) \\
\qquad + \sum_{r'=1}^{R}{}' ({}^{rr'}t^{h_3} - {}^{r'r}t^{h_3}) + \sum_{h=H_1+H_2+1}^{H} ({}^{r}\varphi_{h_3\, re^h}\, {}^{r}e^{h} + {}^{r}\varphi_{h_3\, rm^h}\, {}^{r}m^{h}) + \\
\qquad\qquad\qquad (11) \qquad\qquad\qquad (8) \\
\qquad + {}^{r}e^{h_3} - {}^{r}m^{h_3} + {}^{r}\gamma^{h_3}\, {}^{r}y + \overset{r-h_3}{d_G} \qquad r = 1, \ldots, R;\ h_3 = H_1+H_2+1, \ldots, H \\
\qquad\qquad (12) \qquad (9) \quad (10) \qquad\qquad\qquad\qquad (7.2.4)
\end{array}
$$

Number of equations: (RH_3)

regional income increases:

$$
{}^{r}y = \sum_{h=1}^{H} {}^{r}\varphi_{0h}\, {}^{r}x^{h} \qquad r = 1, \ldots, R \qquad (7.2.5)
$$

Number of equations: (R)

All the variables x, t, e and $m \geqslant 0$. (7.2.6)

The numbers below the different terms of the equations refer to the following interpretation:

(1) increase in production costs.
(2) costs of interregional transport.

(3) costs of international transport.
(4) existing production.
(5) increase in production.
(6) increase in intermediate demand because of production increase.
(7) intermediate demand exerted by interregional transport.
(8) intermediate demand exerted by international transport.
(9) increase in final demand.
(10) existing total intermediate and final demand, excluding intermediate demand exerted by transport[6]).
(11) amount transported interregionally.
(12) amount transported internationally.
(13) increase in regional income.
(14) increase in income from production.

7.3. From the General Model to the Basic Model

In principle one could use the General Model for the problems mentioned in the earlier chapters; one could insert all the data and send the model to an appropriate computer for solution by one or another automatic method like the simplex method. In practice, however, we do not want to leave it at that for three reasons. Firstly, an enormous amount of data is assumed to be known and since in many practical cases this is an unrealistic assumption, one wonders whether the data requirements cannot be reduced considerably by adding a number of general assumptions which are not too unrealistic. Secondly, there is a great number of – in particular – transport variables, and it is doubtful whether all their values are relevant for the purpose of the model, namely medium (long) term planning. In fact many of their values might themselves be of little interest for giving a general picture, even of the necessary transport system (cf. Section 5.1). Moreover, they may also have only a small effect on the values of other, more interesting variables. The disadvantage of so many irrelevant variables is, in addition to their data and

[6]) In these models transport is always interregional and international transport and is not included in the "sectors" or "production". Intraregional transport, however, is just one of the regional sectors

computational requirements, that we may loose sight of the key elements of the model. Finally, in Section 5.5 we gave a few arguments in favour of breaking down such a model into a number of component problems, and those arguments apply again here, the component problems obviously being the problems I, II and III of the previous section. But for such a decomposition a number of further simplifying assumptions must be satisfied.

For all these reasons we successively introduce a number of further simplifying assumptions. The first series of them, to be discussed in this section, leads from the General Model to the Basic Model, which still precedes all models presented in the earlier chapters. The result of these assumptions will be mainly that quite a number of transport variables no longer appear in the model, at least not in what we will call the *main body* of the model. This main body also has fewer equations than the General Model. These results imply that all the cost and technical coefficients related to these variables no longer need to be known exactly. In addition, the remaining transport variables appear without technical coefficients.

Before introducing the simplifying assumptions in this and later sections of this chapter, their real meaning should be made clear. If, in order to attain a simplification, we say that we may neglect certain differences between the feasible solutions of the General Model, then this never means that such differences do not exist between all feasible solutions, but only that in the simplified model we will not obtain a solution which is far from the optimal solution of the general model. We should add that "far from the optimal solution" means displaying either considerably higher costs than corresponding to this optimal solution, or considerable inconsistencies, when the values of the optimal solution of the simplified model are inserted into the equations of the General Model. For instance, if we assume that for different feasible solutions the differences in intermediate demand by transport can be neglected, then this does not mean that, besides the optimal solution of the general model, say A, there would be no feasible solution, say B, in which the intermediate demand by transport would be considerably different from that in solution A, but it means that assuming away such differences will not make us end up with a solution B, which is far from optimal or considerably inconsistent. In fact, the five assumptions of the general model are of the same character: they say in fact that using them in the general model will not cause the optimal solution of the general model to be far from what is in reality the optimal solution.

We begin with two alternative assumptions about the irrelevance of differences in the costs of non-heavy transport. The alternatives concern the representation of these costs: whether they are estimated independently or by relating them to intraregional transport.

Assumption 7.6a (similar to the second part of the Assumptions 3.1 and 4.1 for non-heavy products). For a majority of the national and international products, called non-heavy products, differences between different solutions with regard to their transportation costs can be neglected in determining the values of the production variables. These costs themselves can be approximated by an independent estimate, possibly as a linear function of the increases in the regional incomes.

Assumption 7.6b (similar to the second part of the Assumptions 3.1 and 4.1 for non-heavy products). For a majority of the national and international products, called non-heavy products, differences between different solutions with regard to their transportation costs can be neglected in determining the values of the production variables. The variable part of these costs, that is the change in it compared with the existing costs, may be approximated by redefining intraregional transport so as to include non-heavy transport.

Notice that the first parts of these alternative assumptions, which are identical, although reflecting the intention of introducing the concept of non-heavy sectors, do not define this concept operationally.

If we were to define non-heavy products as those which satisfy this first part of the Assumptions 7.6a or 7.6b, we would be able to select all of them only after trying out a lot of alternative solutions. Neither would it be correct to define them as products with lower transport costs than the differences in production costs between regions, as argued in Section 5.4. Indeed, completely shiftable sectors may be non-heavy, because of the existence of regional targets. Therefore, they are defined as those products with a low transport cost/production cost ratio, which is convenient for statistical reasons. Transport costs are measured per unit distance.

The first part of both alternative assumptions appears to be rather realistic, as we argued in Section 5.1, when introducing the concept of heavy products. The choice between the two alternatives must be made together with a

corresponding choice with regard to the next two alternative assumptions, which relate to the irrelevance of differences in intermediate demand by non-heavy transport. As far as only these costs are concerned one does not need to keep them represented in the main body of the model, since it will soon turn out that all variables of non-heavy transport can be removed from the main body of the model and that their values, their costs and resulting intermediate demand can be calculated from the original equations in which they appear, after this main body of the model has been solved. Next we turn to the following assumptions.

Assumption 7.7a (part of Assumption 4.4 or of Assumption 3.4; similar to Assumption 4.6 for non-heavy products). Differences between different solutions of the model as regards intermediate demand, exerted by non-heavy transport for each product and in each region can be neglected. This intermediate demand can be approximated by an independent estimate, possibly as a linear function of the increase in income of each corresponding region.

Assumption 7.7b (part of Assumption 4.4 or of Assumption 3.4; similar to Assumption 4.6 for non-heavy transport). Differences between different solutions of the model as regards intermediate demand, exerted by non-heavy transport, for each product and in each region can be neglected. The variable part of this intermediate demand, that is, the change in it compared with the existing intermediate demand, can be taken account of sufficiently well by redefining intraregional transport, so as to include non-heavy transport.

These alternative assumptions also appear to be rather realistic, as the amounts involved are relatively small anyway. If Assumption 7.7b is made, Assumption 7.6b must also be made, and conversely. The choice between the two alternative sets of assumptions must be based on practical judgement and statistical convenience. The last reason might lead to the use of Assumptions 7.6b and 7.7b, if in the statistical sources distinguishing between intraregional and other transport is difficult. Then it might be most convenient to make separate studies of the transport of heavy products and deduct the results from those of the transport sector, which then must also include non-heavy transport. Notice that including non-heavy transport in the intraregional transport sector does away with the special treatment of transport, discussed in the former section, for this type of transport and does

not allow it to decrease. Although one could leave the intermediate demand exerted by non-heavy transport out of the main body of the model, just like its costs, this would necessitate a correction of the production figures afterwards. Therefore, there might be more reason this time to make a (preliminary) estimate of this intermediate demand according to the Assumptions 7.7a or 7.7b and include it in the main body of the model. Then the constants ${}^{r}\bar{d}_{G}^{h}$ could be appropriately redefined to include also existing intermediate demand in non-heavy transport.

As by each of these two assumptions the intermediate demand exerted by non-heavy transport is separated from the variables for non-heavy transport, these variables can be left out of the main body of the model, since their values are dependent on those of the other variables, but not vice versa. The way in which these variables are eliminated is a follows. For each non-heavy sector the set of corresponding equations is added together. For each national non-heavy sector one equation remains in which all transport variables have been eliminated. For each international non-heavy sector, however, only one equation is left; in this equation, however, the difference between the sum of the export variables and the sum of the import variables does appear. As to this difference, neither a sign restriction nor a cost is attached, the equation merely defines this difference, which appears in no other equation. Therefore, these equations for non-heavy international sectors can be left out of the main body of the model altogether. After the values of the production variables have been computed from the main body of the model, the original equations for the non-heavy sectors can be used in order to determine the values of the non-heavy transport variables as in Section 4.10 or 5.4.

Finally we also want to remove the technical coefficients of the heavy transport variables, not in order to remove these variables themselves from the main body of the model – at least not at this stage of simplification of the model – but in order to reduce the data requirements of the main body of the model still further and to make the main features of the model clearer by leaving to heavy transportation in the main body of the model only the function of transporting (at some costs) and no longer the function of requiring intermediate inputs. To this end we introduce two alternative assumptions, similar to the Assumptions 7.7a and 7.7b.

Assumption 7.8a (part of Assumption 4.4 or Assumption 3.4; similar to Assumption 4.6 for heavy transport). The differences between different solutions of the model as regards intermediate demand exerted by heavy transport can be neglected. This intermediate demand can be approximated by an independent estimate, possibly as a function of the income increases of the corresponding regions.

Assumption 7.8b (part of Assumption 4.4 or Assumption 3.4; similar to Assumption 4.6 for heavy transport). The differences between different solutions of the model with regard to intermediate demand exerted by heavy transport can be neglected. This intermediate demand can be sufficiently well taken account of by estimating it as a linear function of intraregional transportation.

The reason for distinguishing these alternative assumptions from the alternative Assumptions 7.7a and 7.7b is, that these Assumptions 7.8a and 7.8b are considerably stronger than the former ones. For that reason one might prefer, in certain cases, to use a model based on the Assumptions 7.1–7.7b instead of on the Assumptions 7.1-7.8b. In Assumption 7.8b it is impossible to take account of the intermediate demand by heavy transport by redefining the regional sector for intraregional transport to include also heavy transport, as in Assumption 7.7b, where it could be done for intermediate demand by non-heavy transport, as there are still explicit costs attached to the variables of heavy transportation. A redefinition of that intraregional transport sector would result in double counting the costs of heavy transport in two different ways.

In formulating below the Basic Model which results from the introduction of these assumptions we will use only the Assumptions 7.6b, 7.7b, and 7.8b, and leave it to the reader to formulate the Basic Model which would result from using one or more of the alternative formulations of these assumptions. In addition, we will present only those equations in the model which also influence the values of variables not contained in those equations themselves, and call it therefore the *main body* of the Basic Model (see further the explanation and technical details of the model at the end of this section).

* We will now give the mathematical formulation of the Assumptions 7.6a-7.8b and of the resulting Basic Model, together with some technical details.

Heavy and non-heavy sectors are distinguished by the indices h″ and h′ respectively. The reader is warned that by e.g. $\sum_{h'=H_1+1}^{H} \left(\sum_{h''=H_1+1}^{H} \right)$ we mean summation only over those national and international sectors which are non-heavy (heavy).

Assumption 7.6a

$$\sum_{h'=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' \, {}^{rr'}\tau^{h'} \cdot {}^{rr'}t^{h'} + \\ + \sum_{h'=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^{r}\varepsilon^{h'} \, {}^{r}e^{h'} + {}^{r}\mu^{h'} \, {}^{r}m^{h'}) \approx A_{T'} + \sum_{r=1}^{R} {}^{r}B_{T'} \, {}^{r}y$$

(total costs of non-heavy transport) (7.3.1)

$A_{T'}$ and ${}^{r}B_{T'}$ are the coefficients in the linear expression of the total costs of non-heavy transport in the variables ${}^{r}y$.

$\approx$: is approximately identical to.

Assumption 7.6b

$$\sum_{h'=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' \, {}^{rr'}\tau^{h'} \cdot {}^{rr'}t^{h'} + \\ + \sum_{h'=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^{r}\varepsilon^{h'} \cdot {}^{r}e^{h'} + {}^{r}\mu^{h'} \cdot {}^{r}m^{h'}) \approx \bar{z}_{T'} + \sum_{r=1}^{R} {}^{r}c_k \, {}^{r}\hat{x}^{k}$$

(total costs of non-heavy transport) (7.3.2)

k : index of the regional sector of intraregional transport.

${}^{r}\hat{x}^{k}$: the difference between the values of the new and the old variable representing intraregional transport, the new variable now including non-heavy transport.

$\bar{z}_{T'}$: existing costs of non-heavy transport.

Assumption 7.7a

$$\sum_{h'=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^{r}\varphi_{h\,rr't^{h'}} \, {}^{rr'}t^{h'} + {}^{r}\varphi_{h\,r'rt^{h'}} \, {}^{r'r}t^{h'}) + \sum_{h'=H_1+H_2+1}^{H} ({}^{r}\varphi_{h\,re^{h'}} \, {}^{r}e^{h'} + \\ + {}^{r}\varphi_{h\,rm^{h'}} \, {}^{r}m^{h'}) \approx {}^{r}A'_h + {}^{r}B'_h \, {}^{r}y \qquad h = 1, \ldots, H; \; r = 1, \ldots, R \quad (7.3.3)$$

${}^rA'_h$ and ${}^rB'_h$ are here the coefficients of the linear expression in the ${}^ry$ variables of intermediate demand for product h in region r exerted by non-heavy transportation.

It would be most convenient to estimate ${}^rA'_h$ as the existing intermediate demand for product h in region r exerted by non-heavy transport.

Assumption 7.7b

$$\sum_{h'=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^r\varphi_{h\,rr'\,th'}\,{}^{rr'}t^{h'} + {}^r\varphi_{h\,r'r\,th'}\,{}^{r'r}t^{h'}) + \sum_{h'=H_1+H_2+1}^{H} ({}^r\varphi_{h\,re^{h'}}\,{}^re^{h'} +$$

$$+ {}^r\varphi_{h\,rm^{h'}}\,{}^rm^{h'}) \approxeq {}^r\hat{d}'_h + {}^r\varphi_{hk}\,{}^r\hat{x}^k \qquad h = 1, \ldots, H;\ r = 1, \ldots, R \quad (7.3.4)$$

k : index of the regional sector of intraregional transport.

${}^r\hat{d}'_h$: the existing intermediate demand for product h in region r exerted by non-heavy transport.

${}^r\hat{x}^h$: the difference between the values of the new and the old variable representing intraregional transport, the new variable now including non-heavy transport.

Assumption 7.8a

$$\sum_{h''=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^r\varphi_{h\,rr'\,th''}\,{}^{rr'}t^{h''} + {}^r\varphi_{h\,r'r\,th''}\,{}^{r'r}t^{h''}) + \sum_{h''=H_1+H_2+1}^{H} ({}^r\varphi_{h\,re^{h''}}\,{}^re^{h''} +$$

$$+ {}^r\varphi_{h\,rm^{h''}}\,{}^rm^{h''}) \approxeq {}^rA''_h + {}^rB''_h\,{}^ry \qquad h = 1, \ldots, H;\ r = 1, \ldots, R \quad (7.3.5)$$

${}^rA''_h$ and ${}^rB''_h$ are here the coefficients of the linear expression in the ${}^ry$ variables of the intermediate demand for product h in region r exerted by heavy transport.

It would be most convenient to estimate ${}^rA''_h$ as the existing intermediate demand for product h in region r exerted by heavy transport.

Assumption 7.8b

$$\sum_{h''=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^r\varphi_{h\,rr'\,th''}\,{}^{rr'}t^{h''} + {}^r\varphi_{h\,r'r\,th''}\,{}^{r'r}t^{h''}) + \sum_{h''=H_1+H_2+1}^{H} ({}^r\varphi_{h\,re^{h''}}\,{}^r\varphi^{h''} +$$

$$+ {}^r\varphi_{hrm^{h''}}\,{}^rm^{h''}) \approxeq {}^r\hat{d}''_h + {}^r\hat{\varphi}''_{hk}({}^rx^k + {}^r\hat{x}^k) \quad h = 1, \ldots, H;\ r = 1, \ldots, R \quad (7.3.6)$$

k : indicates the intraregional transport sector.

${}^r\hat{\bar{d}}''_h$ and ${}^r\hat{\varphi}''_{hk}$: the coefficients of the linear expression in the variables $({}^rx^k + {}^r\hat{x}^k)$ of intermediate demand exerted by heavy transportation.

It would be most convenient to estimate ${}^r\hat{\bar{d}}''_h$ as the existing intermediate demand for product h in region r exerted by heavy transport, and ${}^r\hat{\varphi}''_{hk}$ as being related to ${}^r\varphi_{hk}$ in the same proportion as the existing intermediate demand for product h in region r exerted by heavy transport to the same existing intermediate demand exerted by intraregional transport. In the mathematical formulation of this assumption given above it has been assumed that Assumptions 7.6b and 7.7b have been made also and that, therefore, sector k includes non heavy transport.

In all cases of approximate expressions for the intermediate demand exerted by non-heavy and heavy transport one could, after the main body of the model has been solved and the variables outside this main body have been determined, recalculate this intermediate demand and correct the production figures taking account of this recalculated intermediate demand.

Basic Model

In formulating the Basic Model which results from these assumptions we will use the alternative forms 7.6b, 7.7b, and 7.8b of the assumptions. In doing so, we will assume that ${}^r\hat{\bar{d}}''_h$ represents indeed the existing intermediate demand for product h by heavy transport. In addition we will use a somewhat inelegant devise with regard to the intraregional transport sector in order to keep the notation simple and symmetric. Although in the Basic Model the intraregional transport sector variable is now interpreted as to include also non-heavy (interregional and international) transport, we will use for it the same symbol as in the General Model: ${}^rx^k$, $k \in \{H_1\}$. Also the technical coefficients ${}^r\varphi_{hk}$ of these variables should be interpreted to express the intermediate demand for product h in region r by intraregional and non-heavy and heavy (interregional and international) transport per unit increase of sector k in region r; nevertheless we will use for it the same symbol as in the General Model.

Explanation of the symbols of the basic model

N.B. For all symbols which are the same as those in the General Model and

which have kept the same meaning, the reader is referred to the explanation in the preceding section.

Indices and special notations

h' : non heavy sector.

h'' : heavy sector $\left(\sum_{h''=H_1+H_2+1}^{H} \text{ concerns only heavy sectors}\right)$.

$\in \{H_2'\}$ or $\in \{H_2''\}$ or $\in \{H_3''\}$: belongs to the collection of non-heavy national or heavy national or heavy international sectors.

k : regional sector of intraregional transport, including non-heavy (interregional and international) transport.

Variables

z_B : total costs of heavy transport, of the increase in production, and of the increase in non-heavy transport.

Coefficients and Constants

${}^r\bar{d}_B^{h_2''}$, $({}^r\bar{d}_B^{h_3''})$: at the beginning of the planning period the existing total demand for product h_2'' (or h_3''). The lower index B indicates that these are the constants in the Basic Model. Note that

$$ {}^r\bar{d}_B^{h''} = {}^r\bar{d}_G^{h''} + {}^r\hat{\bar{d}}_{h''}' + {}^r\hat{\bar{d}}_{h''}''. $$

${}^r\varphi_{hk}$: intermediate demand in region r for product h by intraregional and (interregional and international) non-heavy and heavy transport per unit increase of production in sector k.

The Basic Model (Main Body)

$$ \text{Minimize}: z_B = \underbrace{\sum_{h=1}^{H} \sum_{r=1}^{R} {}^rc^h\, {}^rx^h}_{(1)} + \underbrace{\sum_{h''=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' {}^{rr'}\tau^{h''} \cdot {}^{rr'}t^{h''}}_{(2)} + $$

$$ + \underbrace{\sum_{h''=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^r\varepsilon^{h_3''}\, {}^re^{h_3''} + {}^r\mu^{h_3''}\, {}^rm^{h_3''})}_{(3)} \qquad (7.3.7) $$

subject to:

regional sectors:

$$\underset{(4)}{{}^{r}x^{h_1}} = \underset{(5)}{\sum_{h=1}^{H} {}^{r}\varphi_{h_1h}\,{}^{r}x^{h}} + \underset{(6)}{{}^{r}\gamma^{h_1}\cdot{}^{r}y} \qquad \begin{array}{l} r=1,\ldots,R; \\ h_1=1,\ldots,H_1 \end{array} \tag{7.3.8}$$

Number of equations: (RH_1)

national non-heavy sectors:

$$\underset{(4)}{\sum_{r=1}^{R} {}^{r}x^{h_2'}} = \underset{(5)}{\sum_{r=1}^{R}\sum_{h=1}^{H} {}^{r}\varphi_{h_2'h}\,{}^{r}x^{h}} + \underset{(6)}{\sum_{r=1}^{R} {}^{r}\gamma^{h_2'}\cdot{}^{r}y} \tag{7.3.9}$$

$h_2' \in \{H_2'\}$ Number of equations: (H_2')

national heavy sectors:

$$\underset{(7)}{{}^{r}\bar{x}^{h_2''}} + \underset{(4)}{{}^{r}x^{h_2''}} = \underset{(5)}{\sum_{h=1}^{H} {}^{r}\varphi_{h_2''h}\,{}^{r}x^{h}} + \underset{(8)}{\sum_{r'=1}^{R}{}' ({}^{rr'}t^{h_2''} - {}^{r'r}t^{h_2''})} + \underset{(6)}{{}^{r}\gamma^{h_2''}\cdot{}^{r}y} + \underset{(9)}{{}^{r}\bar{d}_{\mathrm{B}}^{h_2''}} \tag{7.3.10}$$

$r=1,\ldots,R;\quad h_2'' \in \{H_2''\}$ Number of equations: (RH_2'')

international heavy sectors:

$$\underset{(7)}{{}^{r}\bar{x}^{h_3''}} + \underset{(4)}{{}^{r}x^{h_3''}} = \underset{(5)}{\sum_{h=1}^{H} {}^{r}\varphi_{h_3''h}\,{}^{r}x^{h}} + \underset{(8)}{\sum_{r'=1}^{R}{}' ({}^{rr'}t^{h_3''} - {}^{r'r}t^{h_3''})} + \underset{(10)}{{}^{r}e^{h_3''} - {}^{r}m^{h_3''}} + \underset{(6)}{{}^{r}\gamma^{h_3''}\cdot{}^{r}y} + \underset{(9)}{{}^{r}\bar{d}_{\mathrm{B}}^{h_3''}}$$

$r=1,\ldots,R;\ h_3'' \in \{H_3''\}$ (7.3.11)

Number of equations: (RH_3)

regional income increases:

$$\underset{(11)}{{}^{r}y} = \underset{(12)}{\sum_{h=1}^{H} {}^{r}\varphi_{0h}\,{}^{r}x^{h}} \qquad r=1,\ldots,R \tag{7.3.12}$$

Number of equations: (R)

All variables x, t, e and $m \geqslant 0$ (7.3.13)

The numbers below the terms refer to the following interpretations:

(1) increase in production costs.
(2) total costs of heavy interregional transport.
(3) total costs of heavy international transport.
(4) increase in production.
(5) increase in intermediate demand because of an increase in production.
(6) increase in final demand.
(7) existing production.
(8) amount transported interregionally.
(9) existing total demand.
(10) amount transported internationally.
(11) increase in regional income.
(12) increase in income from production.

Explanation and technical details

The way the constants disappear from the equations without transport is as follows. If we use the Assumptions 7.7b (or 7.7a) and 7.8b (or 7.8a) to replace the expressions in the general model for intermediate demand exerted by transport, we add to the constant ${}^r\bar{d}^h_G$ in the general model the constants ${}^r\hat{\bar{d}}'_h$ (or ${}^rA'_h$) and ${}^r\hat{\bar{d}}''_h$ (or ${}^rA''_h$), which – if conveniently estimated, as we suppose – together represent the existing intermediate demand exerted by transport. The three constants together then represent total final and intermediate demand. For regional products this total existing demand must equal the total existing production in each region, since we use the model for describing the difference between two equilibrium situations. For national goods, total demand must equal total production for all regions together. Therefore, the constants on both sides of these equations cancel out.

In addition to this main body the equations (7.2.3) and (7.2.4) for non-heavy sectors are still available in order to compute, if so desired, the values of the non-heavy transport variables, together with the exactly corresponding intermediate demands and costs. In these equations one could, as an approximation, use expressions like in (7.3.9) for estimating the intermediate demand exerted by transport, if the appropriate coefficients for it were not available.

The main body of the basic model contains

$$R(H_1+H_2+H_3)+R(R-1)\,(H''_2+H''_3)+2RH''_3$$

variables, not counting the $'y$ and z_B and

$$R(H_1+H_2''+H_3'')+H_2'+R$$

equations, which are all relevant now. This means that in the main body of the Basic Model there are

$$R(R-1)(H_2'+H_3')+2RH_3'$$

variables and

$$H_2'(R-1)+H_3'R$$

equations less than in the General Model, which is indeed a considerable simplification. *

7.4. Derivation of the Models of Chapters 3-6 from the Basic Model

In this section we will show how the models presented in the Sections 3.4, 4.3, 4.8, 4.9, 5.2 and 6.2 can all be derived from the Basic Model[7]) by additional simplifying assumptions. In fact the models in the Sections 3.4 and 5.2 are basically the same as those in Sections 4.3 and 6.2; only they relate to a closed instead of an open economy. Therefore, we will occupy ourselves only with the models in Sections 4.3, 4.8, 4.9 and 6.2. The first three models have in common that they neglect the influence of explicit transport costs on the regional distribution of production increases; they will be discussed first and we will do so by following a decreasing order of complexity, that is in the order Section 4.8, 4.9 and 4.3. Together they constitute, in the terminology of Chapter 1, first approximation models.

The model of Section 4.8

We begin with two alternative assumptions which constitute the essence of the first approximation.

[7]) In this section we will not always repeat that we mean the main body of the model, always supposing that the equations of the General Model remain available to complete the model or to correct its results.

Assumption 7.9a (similar to the second part of Assumption 4.1 or 3.1 for heavy products). The differences between different solutions of the model as regards the costs of heavy transportation are considered to be irrelevant for the regional distribution of the production increases. These costs may be approximated by an independent estimate, possibly as a linear function of the regional income increases.

Assumption 7.9b (similar to the second part of Assumption 4.1 or 3.1 for heavy products; the last part is similar to Assumption 4.6 for heavy transport). The differences between different solutions of the model as regards the costs of heavy transportation are considered to be irrelevant for the regional distribution of the production increases. The variable part of these costs, that is the change in them compared with the existing costs, may be represented in the model by redefining intraregional transport so as to include also heavy transportation. The intermediate demand exerted by heavy transport is then also represented in the model as being included in the intermediate demand exerted by this redefined intraregional transport sector; so Assumptions 7.8a or 7.8b are replaced by this assumption.

With these alternative assumptions transport has lost its special treatment as discussed in Section 7.2. In fact, if one chooses Assumption 7.9a, the estimated value of the heavy transport costs is irrelevant for the solution of the model and could be left out as well. If one estimates them nevertheless, but deducts from them the existing costs of heavy transport, the objective function represents the total net increase in costs of production and transport, net in the sense that the change in costs of transport might be negative.

The reason for presenting the second alternative is that this might be statistically convenient, as one does not need to distinguish intraregional, interregional and international transport in the statistical material anymore. If one chooses this Assumption 7.9b, it would also be consistent to use Assumption 7.6b and Assumption 7.7b, while the increase in intermediate demand by heavy transportation is then accounted for by the intraregional transport sector. Obviously, the mathematical formulations of Assumptions 7.8a or 7.8b have to be changed correspondingly.

Note that due to Assumptions 7.7a (or 7.7b) and 7.8a (or 7.8b) or 7.9b, intermediate demand by transportation is no longer related to any explicit variables for such transportation. In fact, after Assumption 7.9 (a or b) such

variables have vanished from the main body of the model altogether. The variable part of this intermediate demand – that is, its change during the planning period – has been included in intermediate demand by the regional sector k according to Assumptions 7.7b and 7.9b. Assumption 4.6 states that this variable part can be neglected altogether. Whatever assumption is used, its effect on the model is qualitatively the same.

The variables representing heavy transport are eliminated from the model in the same way as those for non-heavy transport in Section 7.3. So all equations for the heavy international sectors also disappear from the model, while for each national sector only one equation remains. In the model of Section 4.8, Assumptions 7.6b, 7.7b and 7.9b (which replaces Assumption 7.8a or 7.8b) are chosen.

* The mathematical formulation of Assumptions 7.9a or7.9b and of the resulting model is as follows:

Assumption 7.9a

$$\sum_{h''=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' {}^{rr'}\tau^{h''}\, {}^{rr'}t^{h''} + \sum_{h''=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^{r}\varepsilon^{h''}\, {}^{r}e^{h''} + {}^{r}\mu^{h''}\, {}^{r}m^{h''}) \approx\!\approx$$

$$\approx\!\approx A_{T''} + \sum_{r=1}^{R} {}^{r}B_{T''}\, {}^{r}y \qquad (7.4.1)$$

(total costs of heavy transport)

$A_{T''}$ and ${}^{r}B_{T''}$: coefficients of the linear expression of the total costs of heavy transport in the variables ${}^{r}y$.

$A_{T''}$ could again be most conveniently estimated as the existing costs of heavy transport.

Assumption 7.9b

$$\sum_{h''=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' {}^{rr'}\tau^{h''}\, {}^{rr'}t^{h''} +$$

$$+ \sum_{h''=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^{r}\varepsilon^{h''}\, {}^{r}e^{h''} + {}^{r}\mu^{h''}\, {}^{r}m^{h''}) \approx \bar{z}_{T''} + \sum_{r=1}^{R} {}^{r}c_k\, {}^{r}\hat{x}^{k} \qquad (7.4.2)$$

k : index of the regional sector of intraregional and non-heavy transport.

$\bar{z}_{T}{}''$: the existing costs of heavy transportation.

${}^{r}c_k$: the costs for heavy transportation per unit increase in intraregional transport (strictly speaking ${}^{r}c_k$ should be $= {}^{r}c^k$).

${}^{r}\hat{x}^k$: the difference between the new and the old variable for intraregional transport, the new one now also including heavy transport.

$$\sum_{h''=H_1+1}^{H} \sum_{r'=1}^{R}{}' ({}^{r}\varphi_{h\,rr't^{h''}}\,{}^{rr'}t^{h''} + {}^{r}\varphi_{h\,r'rt^{h''}}\,{}^{r'r}t^{h''}) +$$

$$+ \sum_{h''=H_1+H_2+1}^{H} ({}^{r}\varphi_{h\,re^{h''}}\,{}^{r}e^{h''} + {}^{r}\varphi_{h\,rm^{h''}}\,{}^{r}m^{h''}) \approx {}^{r}\bar{d}''_h + {}^{r}\varphi_{hk}\,{}^{r}\hat{x}^k \qquad (7.4.3)$$

$$h = 1, \ldots, H; \quad r = 1, \ldots, R$$

${}^{r}\bar{d}''_h$: the existing intermediate demand for product h in region r exerted by heavy transport.

Now equation (7.4.3) replaces equations (7.3.5) or (7.3.6).

In representing the simplified model we will make, as has been done in Section 4.8, the Assumptions 7.6b, 7.7b and 7.9b. We can then formulate the model as follows:

$$\text{Minimize}: \; z_P = \sum_{h=1}^{H} \sum_{r=1}^{R} {}^{r}c^h\,{}^{r}x^h \qquad (7.4.4)$$

subject to:

regional sectors:

$${}^{r}x^{h_1} = \sum_{h=1}^{H} {}^{r}\varphi_{h_1h}\,{}^{r}x^h + {}^{r}\gamma^{h_1}\,{}^{r}y \qquad r = 1, \ldots, R; \quad h_1 = 1, \ldots, H_1 \qquad (7.4.5)$$

Number of equations: (RH_1)

national sectors:

$$\sum_{r=1}^{R} {}^{r}x^{h_2} = \sum_{r=1}^{R} \sum_{h=1}^{H} {}^{r}\varphi_{h_2h}\,{}^{r}x^h + \sum_{r=1}^{R} {}^{r}\gamma^{h_2} \cdot {}^{r}y \qquad (7.4.6)$$

$h_2 = H_1+1, \ldots, H_1+H_2$ Number of equations: (H_2)

regional income increases:

$${}^{r}y = \sum_{h=1}^{H} {}^{r}\varphi_{0h}\,{}^{r}x^h \qquad r = 1, \ldots, R \qquad (7.4.7)$$

Number of equations: (R)

All variables $x \geqslant 0$ (7.4.8)

Explanation of the symbols

z_P : the total costs of the increase in production.

$k : k \in \{H_1\}$: regional sector of intraregional transport, including heavy and non heavy (interregional and intraregional) transport.

Technical details

The model contains $R(H_1+H_2+H_3)$ variables in $R(H_1)+H_2+R$ equations, not counting z_P and the ${}^r y$. No equations are redundant now. This means that in this model there are

$$R(R-1)(H_2''+H_3'')+2RH_3''$$

variables and

$$R(H_2''+H_3'')+H_2'-H_2 = (R-1)H_2''+RH_3''$$

equations less than in the Basic Model.

Again the equations of the former models remain available in order to calculate the values of the other variables. If, however, one is only especially interested in the resulting exports and imports, one may use the following equations, which result from equations (7.2.4) for the international sectors in the general model (obviously, after the introduction of the other simplifying assumptions):

international sectors:

$$\sum_{r=1}^{R} {}^r x^{h_3} = \sum_{r=1}^{R} \sum_{h=1}^{H} {}^r\varphi_{h_3 h}\, {}^r x^h + \sum_{r=1}^{R} ({}^r e^{h_3} - {}^r m^{h_3}) + \sum_{r=1}^{R} {}^r\gamma^{h_3} \cdot {}^r y \tag{7.4.9}$$

$$h_3 = H_1+H_2+1, \ldots, H$$

The e and m refer here to increases (unrestricted in sign).

The model in Section 4.8 represents for $R = 3$, $H_1 = 1$, $H_2 = 1$, and $H_3 = 3$ the main body of this simplified model plus equations (7.4.9). *

The model of Section 4.9

The simplifications made in Section 4.9 aim at the possibility of breaking down the model of Section 4.8, and simplifying the way of solving it. The simplifying assumptions made in Section 4.9 only serve the purpose of ascertaining that the solution found is consistent, but only approximale optimality of the solution can be ascertained (cf. App. III). One may also

characterise the assumptions by saying that they indicate the reasons why the solution, calculated by this decomposition method, may not be consistent or optimal, or that one accepts the inconsistencies or non-optimality due to the fact that these assumptions may not be satisfied in reality. We will begin by introducing two assumptions, which make it possible to formulate the first problem of Section 4.9.

Assumption 7.10 (the same as Assumption 4.7 and implying Assumption 4.3). The differences between regions as regards the increases in intermediate demand for a product, due to differences between regions in the input-output coefficients, can be neglected.

Assumption 7.11 (the same as Assumption 4.9).The differences in total domestic final demand for each regional, national and international product due to different regional income increases, which add up, however, to the same national income increase, can be neglected.

This assumption is made for notational and statistical convenience, but we could easily do without it.

* The mathematical formulation of these assumptions is as follows:

Assumption 7.10

$$\sum_{h=1}^{H} {}^{r}\varphi_{h'''h}\,{}^{r}x^{h} \approx \sum_{h=1}^{H} {}^{r'}\varphi_{h'''h}\,{}^{r'}x^{h} \quad \text{for} \quad ({}^{r}x^{h} = {}^{r'}x^{h},\ h = 1, \ldots, H) \tag{7.4.10}$$

$$r, r' = 1, \ldots, R;\ h''' = 1, \ldots, H$$

${}^{r}x^{h}$: feasible solutions to the model of Section 4.8.

We have observed earlier (Section 7.3) that in fact this assumption needs only to hold for the relevant solutions of the model of Section 4.8 and all solutions which may actually be the result of this and the other simplifying assumptions made after this one. The most convenient expression for this assumption is just to omit the index r of the technical coefficients, although, strictly speaking, this expresses the actual equality of the technical coefficients between regions, which is a stronger assumption than necessary. We will

use this convenient expression from now on:

$$^{r}\varphi_{h'''h} = {}^{r'}\varphi_{h'''h} = \varphi_{h'''h} \tag{7.4.11}$$

Assumption 7.11

$$\sum_{r=1}^{R} {}^{r}\gamma^{h}\, {}^{r}y(1) \approxeq \sum_{r=1}^{R} {}^{r}\gamma^{h}\, {}^{r}y(2) \qquad h = 1, \ldots, H \tag{7.4.12}$$

${}^{r}y(1)$ and ${}^{r}y(2)$: any set of regional income increases, which satisfies the relation $\sum_{r=1}^{R} {}^{r}y = y$, where y is the national income increase target.

A convenient expression for this assumption is:

$$^{r}\gamma^{h} = {}^{r'}\gamma^{h} = \gamma^{h} \qquad h = 1, \ldots, H; \quad r, r' = 1, \ldots, R \tag{7.4.12.a}$$

If one rejects possible changes in the ${}^{r}y$, one can consider this assumption as just a definition:

$$\gamma^{h} = \sum_{r=1}^{R} {}^{r}\gamma^{h} \cdot {}^{r}y \qquad *$$

If we introduce these assumptions into the model of this section and add up the equations for each regional sector – and also add up all equations (7.4.7) – then the result is as follows:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^{r}c^{h}\, {}^{r}x^{h} \tag{7.4.13}$$

subject to:

$$\sum_{r=1}^{R} {}^{r}x^{h_1} = \sum_{h=1}^{H} \varphi_{h_1 h} \sum_{r=1}^{R} {}^{r}x^{h} + \gamma^{h_1} \cdot y \qquad h_1 = 1, \ldots, H_1 \tag{7.4.14}$$

$$\sum_{r=1}^{R} {}^{r}x^{h_2} = \sum_{h=1}^{H} \varphi_{h_2 h} \sum_{r=1}^{R} {}^{r}x^{h} + \gamma^{h_2} \cdot y \qquad h_2 = H_1 + 1, \ldots, H_1 + H_2 \tag{7.4.15}$$

$$\sum_{r=1}^{R} {}^{r}y = \sum_{h=1}^{H} \varphi_{0h} \sum_{r=1}^{R} {}^{r}x^{h} \tag{7.4.16}$$

All variables $x \geqslant 0$ (7.4.17)

From (7.4.14)-(7.4.17) follow (4.9.2)-(4.9.5), if one substitutes:

$$\sum_{r=1}^{R} {}^{r}x^{h} = x^{h} \tag{7.4.18}$$

Next we introduce into the model (7.4.4)–(7.4.8) and possibly (7.4.9) corresponding to Section 4.8 an important *substitution* of the variables, using at the same time Assumptions 7.10 and 7.11. This substitution changes the units in which the variables are measured and gives the model an extremely simple form, as will become clear later on:

$$\boxed{{}^r x^h = \frac{{}^r y^h}{\varphi_{0h}}} \qquad \begin{array}{l} r = 1, \ldots, R; \\ h = 1, \ldots, H \end{array} \tag{7.4.19}$$

${}^r y^h$ is the increase in the value added or in the income derived corresponding to the increase in the production of product h in region r, ${}^r x^h$, as can be seen from ${}^r y^h = \varphi_{0h}\, {}^r x^h$.

Because of the rather fundamental importance of this substitution, and another one to be introduced later on, we will show how the model of Section 4.8, expressed in (7.4.4)-(7.4.8) looks after this substitution (7.4.19), also using Assumption (7.10) and (7.11) but not adding up the equations for each regional sector, nor the equations (7.4.7).

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} \frac{{}^r c^h}{\varphi_{0h}}\, {}^r y^h \tag{7.4.20}$$

subject to:

regional sectors:

$$\frac{{}^r y^{h_1}}{\varphi_{0h_1}} = \sum_{h=1}^{H} \frac{\varphi_{h_1 h}}{\varphi_{0h}}\, {}^r y^h + \gamma^{h_1} \cdot {}^r y \qquad \begin{array}{l} r = 1, \ldots, R; \\ h_1 = 1, \ldots, H_1 \end{array} \tag{7.4.21}$$

national sectors:

$$\frac{1}{\varphi_{0h_2}} \sum_{r=1}^{R} {}^r y^{h_2} = \sum_{h=1}^{H} \frac{\varphi_{h_2 h}}{\varphi_{0h}} \sum_{r=1}^{R} {}^r y^h + \gamma^{h_2} \cdot y \tag{7.4.22}$$

$$h_2 = H_1 + 1, \ldots, H_1 + H_2$$

regional income increases:

$${}^r y = \sum_{h=1}^{H} {}^r y^h \qquad r = 1, \ldots, R \tag{7.4.23}$$

All variables $y \geqslant 0$ (7.4.24)

Notice that the total value of each term is still the same as in (7.4.4)-(7.4.8). Multiplying now both sides of (7.4.21) and (7.4.22) by φ_{0h_1} and φ_{0h_2} respectively, and introducing *new coefficients* defined as follows:

$$\frac{{}^r c^h}{\varphi_{0h}} = {}^r\tilde{c}^h \qquad \begin{array}{l} r = 1, \ldots, R \\ h = 1, \ldots, H \end{array} \tag{7.4.25}$$

$$\frac{\varphi_{0h'''}}{\varphi_{0h}} \varphi_{h'''h} = \tilde{\varphi}_{h'''h} \qquad h, h''' = 1, \ldots, H \tag{7.4.26}$$

$$\varphi_{0h}\gamma^h = \tilde{\gamma}^h \qquad h = 1, \ldots, H \tag{7.4.27}$$

we obtain after substitution:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^r\tilde{c}^h \, {}^r y^h \tag{7.4.28}$$

subject to:

regional sectors:

$${}^r y^{h_1} = \sum_{h=1}^{H} \tilde{\varphi}_{h_1 h} \, {}^r y^h + \tilde{\gamma}^{h_1} \cdot {}^r y \qquad \begin{array}{l} r = 1, \ldots, R; \\ h_1 = 1, \ldots, H_1 \end{array} \tag{7.4.29}$$

national sectors:

$$\sum_{r=1}^{R} {}^r y^{h_2} = \sum_{h=1}^{H} \tilde{\varphi}_{h_2 h} \sum_{r=1}^{R} {}^r y^h + \tilde{\gamma}^{h_2} \cdot y \qquad h_2 = H_1+1, \ldots, H_1+H_2 \tag{7.4.30}$$

regional income increases:

$${}^r y = \sum_{h=1}^{H} {}^r y^h \qquad r = 1, \ldots, R \tag{7.4.31}$$

All variables $y \geqslant 0$ (7.4.32)

The formulation of correspondingly new equations for the international sectors would require, in addition, the substitutions:

$$\varphi_{0h_3} \, {}^r e^{h_3} \ (\text{or } \varphi_{0h_3} \, {}^r m^{h_3}) = {}^r\tilde{e}^{h_3} \ (\text{or } {}^r\tilde{m}^{h_3}) \tag{7.4.33}$$

They yield instead of (7.4.9):

$$\sum_{r=1}^{R} {}^r y^{h_3} = \sum_{h=1}^{H} \sum_{r=1}^{R} \tilde{\varphi}_{h_3 h} \, {}^r y^h + \sum_{r=1}^{R} ({}^r\tilde{e}^{h_3} - {}^r\tilde{m}^{h_3}) + \tilde{\gamma}^{h_3} \cdot y \tag{7.4.34}$$

$$h_3 = H_1+H_2+1, \ldots, H_3$$

All ${}^r y^h$ in (7.4.34) have been determined already by (7.4.28)–(7.4.32).

The main simplification due to this substitution is that the coefficients in (7.4.31) all have become = 1 compared with e.g. (7.4.7). This makes it possible, after introducing another simplifying assumption, to employ the uv-method in order to solve the second problem of Section 4.9 and the problem of Section 4.3. If we did not introduce this substitution, in these sections we would end up with models representing the so-called "weighted distribution problem". For its solution the reader is referred to the literature (DANTZIG, 1963, Chapter 21)

Next we make a rather strong assumption, which makes it possible to easily determine the distribution of total production of a regional sector over the regions.

Assumption 7.12 (the same as Assumption 4.8). The differences between different solutions as regards the increases in the intermediate demand for a regional product in any region can be neglected.

* The mathematical expression of this assumption is:

$$\sum_{h=1}^{H} \tilde{\varphi}_{h_1 h}\, {}^r y^h(1) \approx \sum_{h=1}^{H} \tilde{\varphi}_{h_1 h}\, {}^r y^h(2) \qquad \begin{array}{l} r = 1, \ldots, R; \\ h_1 = 1, \ldots, H_1 \end{array} \tag{7.4.35}$$

${}^r y^h(1)$ and ${}^r y^h(2)$ are any solutions to (7.4.29)-(7.4.32).

With regard to ${}^r y^h(1)$ and ${}^r y^h(2)$ the same observation applies as was made with regard to ${}^r x^h$ in the mathematical formulation of Assumption 7.10. There is again a somewhat stronger, but convenient expression for this assumption.

$$\tilde{\varphi}_{h_1 h(1)} = \tilde{\varphi}_{h_1 h(2)} = \tilde{\varphi}_{h_1} \qquad \begin{array}{l} h(1), h(2) = 1, \ldots, H; \\ h_1 = 1, \ldots, H_1 \end{array} \tag{7.4.36}$$

This expression will be used from now on.

Introducing Assumption 7.12 into the model (7.4.28)-(7.4.32) gives:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^r\tilde{c}^h\, {}^r y^h \tag{7.4.37}$$

subject to:

regional sectors:

$${}^r y^{h_1} = (\tilde{\varphi}_{h_1} + \tilde{\gamma}^{h_1})\, {}^r y \qquad \begin{array}{l} r = 1, \ldots, R; \\ h_1 = 1, \ldots, H_1 \end{array} \tag{7.4.38}$$

national sectors:

$$\sum_{r=1}^{R} {}^{r}y^{h_2} = \sum_{h=1}^{H} \tilde{\varphi}_{h_2 h} \sum_{r=1}^{R} {}^{r}y^{h} + \tilde{\gamma}^{h_2} \cdot y \qquad h_2 = H_1+1, \ldots, H_1+H_2 \qquad (7.4.39)$$

regional income increases:

$$ {}^{r}y = \sum_{h=1}^{H} {}^{r}y^{h} \qquad r = 1, \ldots, R \qquad (7.4.40)$$

All variables $y \geqslant 0$ (7.4.41)

For the international sector, equations (7.4.34) remain the same.

The second model of Section 4.9 is:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^{r}\tilde{c}^{h}\, {}^{r}y^{h} \qquad (7.4.42)$$

subject to:

regional sectors:

$$ {}^{r}y^{h_1} = \frac{{}^{r}y}{y} \cdot y^{h_1} \qquad \begin{array}{l} r = 1, \ldots, R; \\ h_1 = 1, \ldots, H_1 \end{array} \qquad (7.4.43)$$

national sectors:

$$\sum_{r=1}^{R} {}^{r}y^{h_2} = y^{h_2} \qquad h_2 = H_1+1, \ldots, H_1+H_2 \qquad (7.4.44)$$

international sectors:

$$\sum_{r=1}^{R} {}^{r}y^{h_3} = y^{h_3} \qquad h_3 = H_1+H_2+1, \ldots, H \qquad (7.4.45)$$

regional income increases:

$$\sum_{h=1}^{H} {}^{r}y^{h} = {}^{r}y \qquad r = 1, \ldots, R \qquad (7.4.46)$$

All variables $\geqslant 0$ (7.4.47)

If the y^h are derived from (4.9.1)-(4.9.5) after using

$$y^h = \varphi_{0h}\, x^h \qquad (7.4.48)$$

it can be shown that any solution of (7.4.42)-(7.4.47) is also a solution of (7.4.38)-(7.4.41). So in Section 4.9 these assumptions are made indeed.

However, it is not certain that the optimal solution of (7.4.42)-(7.4.47) is also the optimal solution of (7.4.37)-(7.4.41) and possibly (7.4.34).

There is no simple assumption which ascertains the optimal solution of the decomposed problem of Section 4.9 to be equal to the optimal solution of the simplified problem (7.4.37)-(7.4.41). This point is discussed in Appendix III, where it is also stated that in many cases the two optimal solutions will coincide, if the international sector selected for expansion in the first problem of Section 4.9, is also the cheapest international sector within each region. *

The model of Section 4.3

The model of Section 4.3 can be derived from the last simplified model (7.4.37)-(7.4.41) by one more step; an assumption with regard to national sectors, similar to Assumption 7.12 for regional sectors.

Assumption 7.13 (together with the Assumptions 7.7b, 7.9b, 7.10, 7.11, 7.12, this is practically the same as Assumption 4.4 or 3.4). The differences between different solutions as regards the increases in the intermediate demand for each national product in the country can be neglected.

* The mathematical expression of this assumption is:

Assumption 7.13

$$\sum_{h=1}^{H} \tilde{\varphi}_{h_2 h} \sum_{r=1}^{R} {}^r y^h(1) \approx \sum_{h=1}^{H} \tilde{\varphi}_{h_2 h} \sum_{r=1}^{R} {}^r y^h(2) \qquad h_2 = H_1+1, \ldots, H_1+H_2 \quad (7.4.49)$$

${}^r y^h(1)$ and ${}^r y^h(2)$ are any solutions to (7.4.38)-(7.4.41).

With regard to ${}^r y^h(1)$ and ${}^r y^h(2)$ the same observation applies as was made with regard to ${}^r x^h$ in the mathematical formulation of Assumption 7.10. There is again a somewhat stronger, but convenient, expression for this assumption:

$$\tilde{\varphi}_{h_2 h(1)} = \tilde{\varphi}_{h_2 h(2)} = \tilde{\varphi}_{h_2} \qquad \begin{array}{l} h(1), h(2) = 1, \ldots, H; \\ h_2 = H_1+1, \ldots, H_1+H_2 \end{array} \quad (7.4.50)$$

This expression, particularly important for $h(1)$ or $h(2) = H_1+H_2+1, \ldots, H$, will be used from now on.

Introducing Assumption 7.13 into the model (7.4.37)-(7.4.41) gives:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^{r}\tilde{c}^{h}\, {}^{r}y^{h} \tag{7.4.51}$$

subject to:

regional sectors:

$$ {}^{r}y^{h_1} = (\tilde{\varphi}_{h_1} + \tilde{\gamma}^{h_1})\, {}^{r}y \tag{7.4.52}$$

national sectors:

$$\sum_{r=1}^{R} {}^{r}y^{h_2} = (\tilde{\varphi}_{h_2} + \tilde{\gamma}^{h_2})\, y \tag{7.4.53}$$

regional income increases:

$$\sum_{h=1}^{H} {}^{r}y^{h} = {}^{r}y \tag{7.4.54}$$

All variables $y \geqslant 0$ (7.4.55)

Essentially, this is the model of Section 4.3; in addition one has to substitute:

$$ {}^{r}\tilde{\eta}^{h_1} = {}^{r}\tilde{\varphi}_{h_1} + {}^{r}\tilde{\gamma}^{h_1} \quad \text{and} \quad \tilde{\eta}^{h_2} = \tilde{\varphi}_{h_2} + \tilde{\gamma}^{h_2} \tag{7.4.56}$$

If one wants to calculate the resulting total imports or exports of each international sector, one can use (7.4.34) or one can make:

Assumption 7.14 The differences between different solutions as regards the increases in intermediate demand for each international product in the country can be neglected. This results in the following simplified version of (7.4.34):

$$\sum_{r=1}^{R} {}^{r}y^{h_3} = \tilde{\eta}^{h_3} y + \sum_{r=1}^{R} ({}^{r}\tilde{e}^{h_3} - {}^{r}\tilde{m}^{h_3}) \qquad h_3 = H_1 + H_2 + 1, \ldots, H \tag{7.4.57}$$

where $\tilde{\eta}^{h_3} = \tilde{\varphi}_{h_3} + \tilde{\gamma}^{h_3}$.

The extreme simplification reached in the last version of the model, (7.4.51)-(7.4.55) and possibly (7.4.57), is both of a computational and of a statistical nature. With regard to computational aspects it has been shown in Section 4.4 that the relatively simple modified uv-method can be used for solving the model. Statistically it is convenient that the $\tilde{\eta}^{h}$ can be estimated from the elasticities of demand with respect to income, assuming that these elasticities remain fairly constant for some time. One should be careful, however, with

these last assumptions if structural changes occur in the economy. The only additional statistical information needed is the costs per unit increase in value added of each sector in each region. Obviously, it will be desirable also to know the existing production per sector and per region.

The model of Section 6.2.

Essentially in the model of Section 6.2 all former assumptions and substitutions apart from Assumptions 7.9a or 7.9b are employed. We need, however, to be somewhat more specific. The assumptions 7.1-7.5 are made anyhow, of course. In addition Assumption 7.6b and 7.7b are made with respect to non-heavy transport, while Assumption 7.8b is made with respect to the intermediate demand by heavy transport. This last assumption must then be interpreted as either neglecting altogether any change in this intermediate demand like in Assumption 4.6 (assuming, therefore, all coefficients of ${}^r x^k + {}^r \hat{x}^k$ in (7.3.6) as $= 0$), or increasing the ${}^r\varphi_{hk}$ by ${}^r\varphi''_{hk}$. Next Assumptions 7.10 and 7.11 are made, strictly speaking with the exception of intermediate and final demand for regional products.

Then, substitutions (7.4.19) and (7.4.25)-(7.4.27) are made, and in addition the following substitutions.

$$\left.\begin{array}{l}\left\{\begin{array}{l}\varphi_{0h''}\,{}^{rr'}t^{h''} = {}^{rr'}\tilde{t}^{h''} \\[2ex] \varphi_{0h''_3}\,{}^r e^{h''_3}\ (\text{or } \varphi_{0h''_3}\,{}^r m^{h''_3}) = {}^r\tilde{e}^{h''_3}\ (\text{or } {}^r\tilde{m}^{h''_3})\end{array}\right. \\[2ex] \left\{\begin{array}{l}\dfrac{{}^{rr'}\tau^{h''}}{\varphi_{0h''}} = {}^{rr'}\tilde{\tau}^{h''} \\[2ex] \dfrac{{}^r\varepsilon^{h''}}{\varphi_{0h''}}\left(\text{or } \dfrac{{}^r\mu^{h''}}{\varphi_{0h''}}\right) = {}^r\tilde{\varepsilon}^{h''}\ (\text{or } {}^r\tilde{\mu}^{h''})\end{array}\right. \\[2ex] \left\{\begin{array}{l}\varphi_{0h''}\,{}^r\bar{d}^{h''}_{\mathrm{B}} = {}^r\tilde{\bar{d}}^{h''} \\[2ex] \varphi_{0h''}\,{}^r\bar{x}^{h''} = {}^r\tilde{\bar{x}}^{h''}\end{array}\right.\end{array}\right\} \quad \begin{array}{l} r, r' = 1, \ldots, R \\ h'' = h''_2, h''_3 \\ h''_2(h''_3) \in \{H''_2\}\ (\in \{H''_3\}) \end{array} \qquad \begin{array}{r}(7.4.58) \\ \\ (7.4.59) \\ \\ (7.4.60)\end{array}$$

Next, Assumptions (7.12), (7.13) are made simultaneously with Assumption (7.14) but the last one only for heavy international products.

Finally we draw the attention to one consequence of (7.4.50), the somewhat strong, but convenient, expression for Assumption 7.13, and to its counterpart for Assumption 7.14. They imply

$$\tilde{\varphi}_{h''h} = \tilde{\varphi}_{h''} \qquad h'' = h''_2, h''_3; \qquad h''_2 \in \{H''_2\}; \quad h''_3 \in \{H''_3\} \tag{7.4.61}$$

The relations (7.4.61) in their turn imply:

$$\sum_{h=1}^{H} \tilde{\varphi}_{h''h}\, {}^r y^h = \tilde{\varphi}_{h''}\, {}^r y \qquad h'' = h''_2, h''_3 \tag{7.4.62}$$

This expression is used in the following model.

The general mathematical formulation of the resulting model, leaving out the irrelevant part of the objective function, is as follows:

$$\text{Minimize}: \sum_{h=1}^{H} \sum_{r=1}^{R} {}^r\tilde{c}^h\, {}^r x^h + \sum_{h''=H_1+1}^{H} \sum_{r=1}^{R} \sum_{r'=1}^{R}{}' {}^{rr'}\tilde{\tau}^{h''}\, {}^{rr'}\tilde{t}^{h''} + \sum_{h''_3=H_1+H_2+1}^{H} \sum_{r=1}^{R} ({}^r\tilde{\varepsilon}^{h''_3}\, {}^r\tilde{e}^{h''_3} + {}^r\tilde{\mu}^{h''_3}\, {}^r\tilde{m}^{h''_3}) \tag{7.4.63}$$

subject to:

regional sectors:

$${}^r y^{h_1} = (\tilde{\varphi}_{h_1} + \tilde{y}^{h_1})\, {}^r y \qquad r = 1, \ldots, R; \; h_1 = 1, \ldots, H_1 \tag{7.4.64}$$

national non-heavy sectors:

$$\sum_{r=1}^{R} {}^r y^{h'_2} = (\tilde{\varphi}_{h'_2} + \tilde{y}^{h'_1})\, y \qquad h'_2 \in \{H'_2\} \tag{7.4.65}$$

national heavy sectors:

$${}^r\bar{y}^{h''_2} + {}^r y^{h''_2} = \sum_{r'=1}^{R}{}' ({}^{rr'}\tilde{t}^{h''_2} - {}^{r'r}\tilde{t}^{h''_2}) + (\tilde{\varphi}_{h''_2} + \tilde{y}^{h''_2})\, {}^r y + {}^r\tilde{d}^{h''_2} \qquad r = 1, \ldots, R; \quad h''_2 \in \{H''_2\} \tag{7.4.66}$$

international heavy sectors:

$$
{}^r\bar{y}^{h_3''} + {}^r y^{h_3''} = \sum_{r'=1}^{R}{}' ({}^{rr'}\tilde{t}^{h_3''} - {}^{r'r}\tilde{t}^{h_3''}) + {}^r\tilde{e}^{h_3''} - {}^r\tilde{m}^{h_3''} + \\
+ (\tilde{\varphi}_{h_3} + \tilde{\gamma}^{h_2''})\, {}^r y + {}^r\tilde{\bar{d}}^{h_3''} \qquad r = 1, \ldots, R; \qquad (7.4.67)
$$

$$
h_3'' \in \{H_3''\}
$$

All variables y and $\tilde{t} \geqslant 0$ (7.4.68)

In Section 6.2 this model is given for $H_1 = 1$, $H_2 = 2$, $H_3 = 3$, $R = 3$, while in addition $(\tilde{\varphi}_h + \tilde{\gamma}^h) = \tilde{\eta}^h$, $h = 1, \ldots, H$, is substituted.

At the conclusion of the survey of these models we want to point out a few examples of considerations necessary when using these framework models for planning purposes.
a) It should be considered whether not a part of the increase in income from production in a region actually contributes to another region's income increase, e.g. if the remunerations to capital leave the region. b) In setting the regional income targets in order to raise the per capita income of a region, as well as in estimating shadow prices for (skilled) labour, spontaneous and induced population movements should be considered. c) Foreign exchange considerations, while left out of these models as belonging to short-term policies, enter into the models if bounds like discussed in Section 4.11 are introduced into them. d) Considerations concerning the implementation of the plans would add a number of behavioural equations, partly related to the macro-stage (e.g. equations related to savings and taxation).

7.5. Schematic Representation of the Assumptions and Models

In this section we present three schemes. The first one lists the assumptions in this chapter with a short indication of their content or function and their connection with the assumptions made in earlier chapters. The second scheme lists the assumptions in earlier chapters and the connection with the assumptions in this chapter. The third scheme shows the interconnection between the assumptions, the substitutions, and the resulting models, together with their solution methods.

TABLE 7.1

The assumptions in Chapter 7 and their related assumptions in Chapters 3-6

Assumption in Chapter 7	Summary of assumption	Related assumptions in Chapters 3-6
7.1	Distinction of mobility classes	First part of 3.1 or 4.1; 5.2 or 6.1 apart from heavy products
7.2	Linearity of all relations	3.2 or 4.2; comprises 5.1a
7.3	Prices constant and not interregionally different	3.5 or 4.5
7.4	Proportionality of final demand increases and income increases	3.4′ or 4.4′; part of 3.4 or 4.4; part of 5.3 or 6.2
7.5	Nonnegativity of variables	
7.6	Costs of non-heavy transport:	Part of second part of 3.1 or 4.1
7.6a	estimated separately;	
7.6b	included in intraregional transport sector k	
7.7	Intermediate demand by non-heavy transport:	Part of 3.4 or 4.4; part of 5.3 or 6.2 for heavy products; 4.6
7.7a	estimated separately;	
7.7b	included in intraregional transport sector k	
7.8	Intermediate demand by heavy transport:	Part of 3.4 or 4.4; part of 5.3 or 6.2 for heavy products; 4.6
7.8a	estimated separately;	
7.8b	related to intraregional transport sector k	
7.9	Costs of heavy transport:	Part of second part of 3.1 or 4.1
7.9a	estimated separately;	
7.9b	included in intraregional transport sector k; replaces 7.8a or 7.8b	Part of 3.4 or 4.4; part of 5.3 or 6.2
7.10	No interregional differences in technical coefficients	Slightly stronger than 3.3 or 4.3; part of 3.4 or 4.4 for non-regional products; 4.7
7.11	No interregional differences in final demand coefficients	Part of 3.4 or 4.4, but for regional products stronger; 4.9

TABLE 7.1 *(Continued)*

Assumption in Chapter 7	Summary of assumption	Related assumptions in Chapters 3-6
7.12	Increase in intermediate demand for regional products independent of which other sectors are expanded	4.8; part of 3.4 or 4.4
7.13	Increase in intermediate demand for national products independent of which other sectors are expanded	Part of 3.4 or 4.4; comprises a part of 5.3 or of 6.2
7.14	Increase in intermediate demand for international products independent of which other sectors are expanded	Comprises a part of 5.3 or of 6.2

TABLE 7.2

The assumptions in the Chapters 3-6 and their related assumptions in Chapter 7

Assumptions in the Chapters 3-5	Related assumptions in Chapter 7
3.1 or 4.1	7.1 and 7.6 and 7.9
3.2 or 4.2	7.2
3.3 or 4.3	slightly weaker than 7.10
3.4 or 4.4	7.4, 7.7, 7.8 or 7.9b, 7.12, 7.13, and part of 7.10 and 7.11
3.4′ or 4.4′	7.4
3.5 or 4.5	7.3
4.6	7.7, 7.8 or 7.9b
4.7	7.10
4.8	7.12
4.9	7.11
5.1a	7.2
5.2 or 6.1	7.1
5.3 or 6.2	7.4, 7.7, 7.8 or 7.9b, 7.13, 7.14 (only for heavy products) and part of 7.10 and 7.11

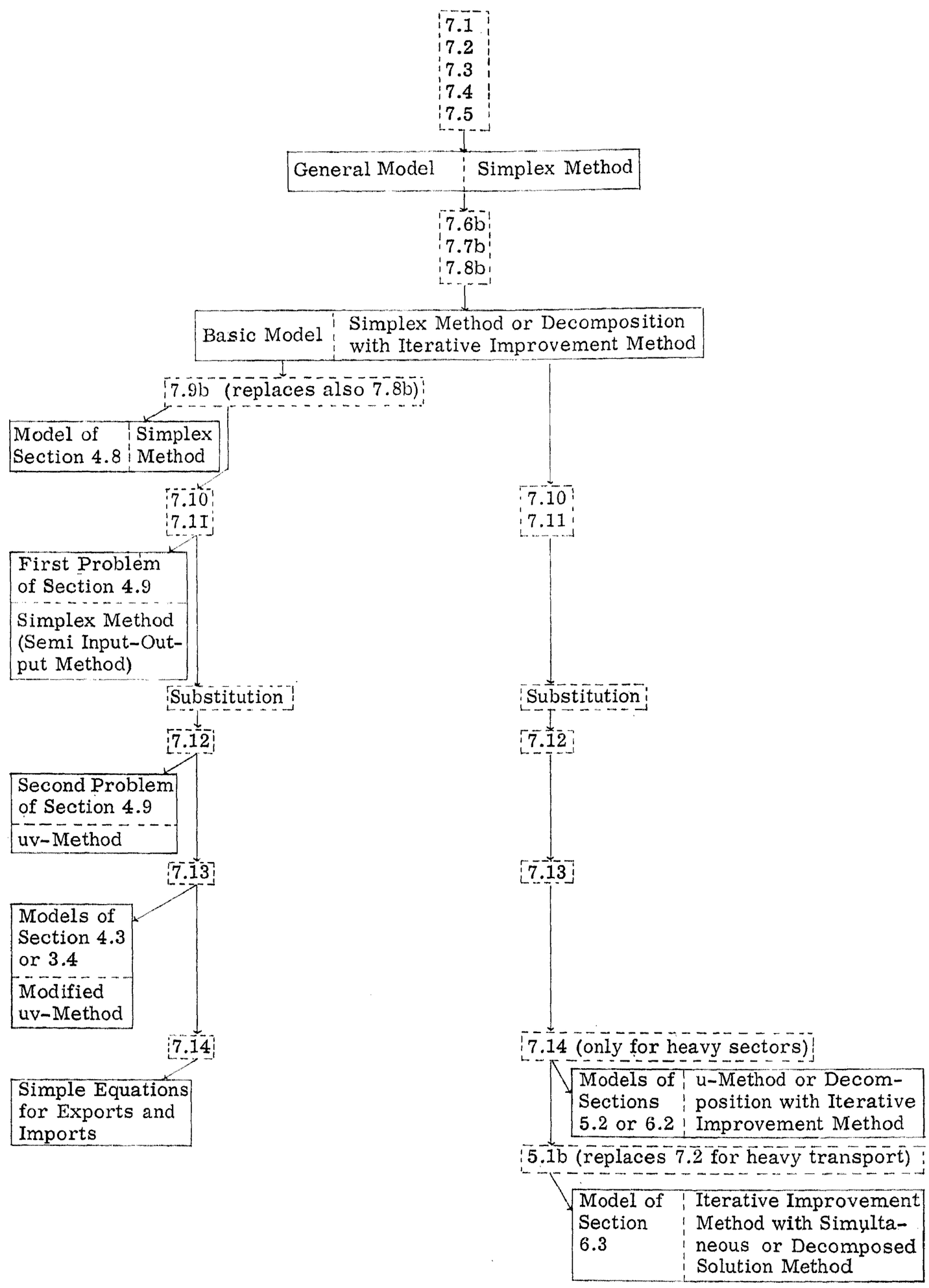

Fig. 7.1. The relationships between the assumptions and substitutions in Chapter 7 and the resulting models, with their possible solution methods.