CHAPTER 8

THE INTRODUCTION OF INDIVISIBILITIES; NO FULL SPECIFICATION OF TOPOGRAPHY

8.1. The Hierarchy Model; No Foreign Trade; Agriculture Evenly Spread over the Country; Vertical Integration

So far we have assumed linear production processes, that is processes for which the increase in inputs is proportional to the increase in outputs. For large geographical areas this assumption is realistic. The smaller the areas become, the more we shall be faced with the existence of *indivisibilities* or *increasing returns* to scale, which imply non-proportionality between inputs and outputs.

If we were to continue to apply the type of models dealt with so far to ever smaller spatial units, we would finally have a situation where in some sectors the increases in production to be allocated to the spatial units would be so small that we would be confronted with decreasing average production costs when output increases because of the indivisibilities of the production processes. We could take account of these indivisibilities in our models in two ways. First, by determining a minimum level of output increase above which the marginal costs equal the average costs but below which production increases are not feasible at all unless having the value zero. Mathematically our models would then become mixed integer linear programming models for which no completely satisfactory methods of solution are available, as yet. Secondly we could allow the cost coefficients to decrease when output increases, which would also represent the phenomenon of economies of scale. A linear approximation would lead to a representation where, at any level of production above zero, fixed costs constitute a part of total costs. This type of problem and methods to solve it has been dealt with recently in the literature in various ways (See e.g. BALINSKI, 1965; GRAY, 1967; KENDRICK, 1967; MANNE, 1967 and HAMMER *et al.*, 1968). However, we will

not attempt to incorporate these features into the models presented so far in order to make them applicable to planning for spatial units of lower level. Instead we will present here a different approach based on the work of one of the authors (TINBERGEN, 1964) and Bos (cf. BOS, 1965) which does take account of the existence of indivisibilities and which later on will be integrated with the methods dealt with in the preceding chapters. This approach is a highly simplified one.

In Section 1.7 we indicated briefly the type of problem and the type of solution we have in mind for it. The problem, of the planning type and formulated as an optimization problem, is to find the optimal dispersion of economic activities over a spatial unit, taking into account indivisibilities in the production structure, and optimal in the sense of minimizing total production and transportation costs, while the total income of the spatial unit is given. The spatial unit will be called a country from now on.

In order to deal with this complex problem we will make a number of simplifying assumptions. The introduction of these assumptions has two important consequences which suggest a possible link between this new approach and the preceding models. They imply that we may consider, in each sector, total production and total costs of production as given. Then one could consider this approach e.g. as a first step towards a final stage after the application of the models assuming homogenous production processes with the reservation that this approach works with variables representing total production instead of increases in production. In this final stage transportation costs *within* a spatial unit are minimized taking account of indivisibilities, once the distribution of production over different spatial units has been determined by means of minimizing total production costs and transportation costs between the spatial units.

The *first* assumption is that for each economic activity there exists a minimum level at which it is at all feasible. It the end this is due to the existence of indivisibilities in some of the capital goods used[1]). Even if above this level unit production costs remain by and large constant, one would expect, if demand is evenly spread, the plants to operate at or near to this minimum efficient size since the costs of transportation increase with an increase in the geographical size of the market. We will therefore assume that production takes place solely in technical units of minimum efficient

[1]) For a more precise treatment see (Bos, 1965).

size which is assumed to be given. We believe that this phenomenon of indivisibilities is of essential importance for the explanation of the existence of "*centres*" of different size, where by "centre" we mean cities, towns, villages, etc. The type of product will be indicated by a suffix

$$h \ (h = 0, 1, \ldots, H),$$

in particular agriculture by $h = 0$.

The *second* assumption we make relates to the exceptional position of agriculture. We will assume that it is evenly spread over the country's surface which, strictly speaking, implies that its minimum plant size is zero.

The *third* assumption is that transportation costs for each product do exist. We will, however, not specify them, as it is not not necessary in the presentation in this chapter.

The *fourth* assumption, which we make only in the first three sections of this chapter in order to simplify the treatment, is that the country's economy is closed.

The *fifth* assumption, which we make only in the first four sections of this chapter, is that production is *vertically integrated*, meaning that each industry supplies only finished products, uniting within it all stages of a "production column". This means that we do not take explicit account of interindustry deliveries. Of course this cannot be a realistic assumption, but we could also say that we assume that such a simplification might still represent a useful first approximation to the problem, even in combination with the first assumption.

The *sixth* assumption is that, at constant prices, final demand for product h is equal to $\alpha_h Y$, where Y is the country's income. It follows naturally that

$$\sum_{0}^{H} \alpha_h = 1 \tag{8.1.1}$$

The same assumption is made for any part of the country's income, in other words, tastes are assumed to be the same throughout the country.

An immediate and important consequence of the first, fourth, fifth and sixth assumption is that we know the number n_h of production units in each sector, since we know its total production and its plant size. Therefore, we can now rank the industries so as to let the h go up with decreasing n_h. For example $h = 1$, the lowest ranking industry, might represent bakeries,

and $h = H$, the highest ranking industry, the steel industry or the central government.

The *seventh* assumption is that the income earned at some place is spent at the same place. This implies that for each place or area imports must equal exports.

The *eighth* assumption is made for normalization purposes: $n_H = 1$. Before we proceed and suggest a hypothetical solution for the problem, so narrowed down by the assumptions made, we will indicate some differences between the problems and methods in this chapter and those in the earlier Chapters 3-7.

While in the former problems the subspaces, e.g. the regions, were given in advance, in this problem nothing as regards the topography is assumed to be given in advance, not even places where cities might be located. Production costs are assumed to be the same in all parts (except in Section 8.3); there are no income targets for subspaces in addition to the given national income. Therefore, only a national optimum is sought for, without any regional policies. Agriculture is assumed to be evenly spread over the country, which implies that, if in the solution "centres" arise, which can be conceived of as points, then these "centres" do not contain agricultural activities. Another difference with the former models is that variables which indicate production levels cannot have values between the minimum plant size and zero. This presents an essential mathematical difficulty. A more formal difference with the former models has already been mentioned before: all variables represent total amounts instead of increases.

The complexity of the original problem has been reduced considerably by the assumptions. In later sections we will change the assumptions somewhat and so we will get different reduced problems. A common feature that will remain however, is that in particular the costs of production are already determined, and only the costs of transportation in the widest sense remain to be minimized.

From now on we are going to discuss hypothetical solutions to the problems. By this we mean that the solutions are proposed as a hypothesis, which we know, however, is not the general solution, since we can construct counterexamples, but which would be correct under a number of further assumptions, not exactly known to us. For a few simple versions of the models proofs of optimality have been given (cf. TINBERGEN, 1964 and BOS, 1965, CHAPTER 5). It is believed, however, that the hypothetical solutions represent

at least some of the main features of the general solution. This amounts to admitting that we have not yet really solved the problem but are only offering hints for its solution.

The hypothetical solutions have another shortcoming. They only specify the value of some of the variables of the problem and not of all. In particular they deal rather with *groups* of centres of the same type than with individual centres. Therefore they do not specify the precise location of each of the centres; in other words, this geometrical dimension of the problem is almost completely disregarded. It will not be disregarded completely, as will become clear later on.

The models to be discussed, which specify the hypothetical solutions, will be called *hierarchy models*. They start from the notion that all non-agricultural production takes place in centres which can be conceived of as points in the model. Two centres will only be said to be of the same type, if they contain industries of the same rank. Although from this definition one can conceive of a great variety of possible types of centres the hypothesis will imply that only a very restricted number of different types of centres appears in the solution. By definition agricultural production takes only place outside these centres and can be considered as one area of a special type, producing only agricultural products.

The hypothesis consists of three elements:

(1) If the highest ranking sector in a centre is of type h', then this centre also produces all products of types $1 \leq h < h'$; the demand for these products within the centre is fully satisfied by production within the centre.

Therefore, each type of centre can be characterized by its highest ranking sector, and we will rank the types of centres correspondingly.

(2) A centre exports only the products of its highest ranking sector.

This is the central element of the hypothesis and it has, in combination with the first element, some important implications. It implies that of all centres of rank h' the sectors of rank $1 \leq h < h'$ produce exactly enough for the local market. Therefore, these sectors play the role often indicated as secondary or servicing industries (for that type of centre). It implies further that all centres export only to centres of lower rank and to the agricultural area, which in its turn exports to all centres. All centres import agricultural products and products of higher rank than their highest ranking sectors, while the agricultural area imports from all centre types.

(3) Each centre contains only one enterprise of the highest ranking sector.

This third element of the hypothesis determines the number of centres of each type.

These three elements of the hypothesis together determine some main elements of a solution to the problem mentioned above. It might occur, however, that this solution indicates production levels below the minimum plant size for certain centres. The interpretation of such a result will be discussed later on.

In particular the hypothesis suffices to calculate the size distribution expressed by the total income $Y^{h'}$ of each type h' of centres and the number of centre of each type h' : $n^{h'}$. This can easily be derived from Table 8.1, which shows the size, composition and direction of the trade flows as specified by the hypothesis.

Equating, for each type of centre, the value of imports to that of exports we find a set of equations from which we can derive the total income for each type of centre:

$$(\alpha_1+\alpha_2+\ldots\alpha_H)\,Y^0 = \alpha_0(Y-Y^0) \tag{8.1.2}$$

With (8.1.1) we find:

$$Y^0 = \alpha_0\,Y \tag{8.1.3}$$

Similarly we find:

$$Y^1 = \frac{\alpha_0\alpha_1\,Y}{1-\alpha_1} \tag{8.1.4}$$

and consequently:

$$Y^0+Y^1 = \frac{\alpha_0\,Y}{1-\alpha_1} \tag{8.1.5}$$

We can easily verify that the results can be generalized to

$$Y^0+\ldots+Y^{h'} = \frac{\alpha_0\,Y}{1-\alpha_1\ldots-\alpha_{h'}} \qquad h' = 1,\ldots,H \tag{8.1.6}$$

from which we can check that

$$\sum_0^H Y^{h'} = Y$$

TABLE 8.1

Survey of types of centres and their imports and exports

Rank h'	Income	Imported products	Value of imports	Exported products	Destination of exports	Value of exports
0	Y^0	$h = 1, \ldots, H$	$(\alpha_1 + \ldots \alpha_H)\ Y^0$	$h = 0$	$h' = 1, \ldots, H$	$\alpha_0(Y - Y^0)$
1	Y^1	$h = 0, 2, \ldots, H$	$(\alpha_0 + \alpha_2 + \ldots \alpha_H)\ Y^1$	$h = 1$	$h' = 0$	$\alpha_1 Y^0$
2	Y^2	$h = 0, 3, \ldots, H$	$(\alpha_0 + \alpha_3 + \ldots \alpha_H)\ Y^2$	$h = 2$	$h' = 0, 1$	$\alpha_2(Y^0 + Y^1)$
3	Y^3	$h = 0, 4, \ldots, H$	$(\alpha_0 + \alpha_4 + \ldots \alpha_H)\ Y^3$	$h = 3$	$h' = 0, \ldots, 2$	$\alpha_3(Y^0 + Y^1 + Y^2)$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$H-1$	Y^{H-1}	$h = 0, H$	$(\alpha_0 + \alpha_H)\ Y^{H-1}$	$h = H-1$	$h' = 0, \ldots, H-2$	$\alpha_{H-1}(Y^0 + \ldots Y^{H-2})$
H	Y^H	$h = 0$	$\alpha_0 Y^H$	$h = H$	$h' = 0, \ldots, H-1$	$\alpha_H(Y^0 + \ldots Y^{H-1})$

We can also write (8.1.6) in the alternative form:

$$Y^0 + \ldots + Y^{h'} = \frac{\alpha_0 Y}{\alpha_0 + \alpha_{h'+1} + \ldots + \alpha_H} \qquad h' = 1, \ldots, H \qquad (8.1.6')$$

For $Y^{h'}$ we find:

$$Y^{h'} = \frac{\alpha_{h'} \alpha_0 Y}{(\alpha_0 + \alpha_{h'+1} + \ldots + \alpha_H)(\alpha_0 + \alpha_{h'} + \ldots + \alpha_H)} \qquad h' = 1, \ldots, H \qquad (8.1.7)$$

The $Y^{h'}$ refer to the total income of all centres of type h'. In order to determine the income per centre we must know the number of centres of type h'. This number $n^{h'}$ is equal to the total number of production units $n_{h'}$, minus the numbers of such units needed for the consumption of good h' in all centres of higher rank:

$$n^{h'} = n_{h'} - n_{h'} \frac{Y^{h'+1} + \ldots + Y^H}{Y} \qquad h' = 1, \ldots, H \qquad (8.1.8)$$

since the ratio of consumption of centres $h'+1, \ldots, H$ to total consumption is equal to the fraction in this last equation. It follows that:

$$n^{h'} = n_{h'} \frac{\alpha_0}{1 - \alpha_1 \ldots - \alpha_{h'}} = n_{h'} \frac{\alpha_0}{\alpha_0 + \alpha_{h'+1} + \ldots \alpha_H} \qquad h' = 1, \ldots, H \quad (8.1.9)$$

For the income $\bar{Y}^{h'}$ per centre of rank h' we then find:

$$\bar{Y}^{h'} = \frac{Y^{h'}}{n^{h'}} = \frac{\alpha_{h'} Y}{(\alpha_0 + \alpha_{h'} + \ldots + \alpha_H) n_{h'}} \qquad h' = 1, \ldots, H \qquad (8.1.10)$$

These formulae indicate a frequency distribution of the centres according to size, where size is measured by the income of the centres of different types.

The composition of the centres of different type is described by the following formulas. The number of production units of industry h in all centres of type h', $n_h^{h'}$, is:

$$
\left.
\begin{aligned}
n_h^{h'} &= n_h \frac{Y^{h'}}{Y} \\
&= \frac{\alpha_0 \alpha_{h'} n_h}{(\alpha_0 + \alpha_{h'} + \ldots + \alpha_H)(\alpha_0 + \alpha_{h'+1} + \ldots + \alpha_H)} && \text{for } h < h' \\
&= \frac{\alpha_0 n_h}{\alpha_0 + \alpha_{h'+1} + \ldots + \alpha_H} && \text{for } h = h' \\
&= 0 && \text{for } h > h'
\end{aligned}
\right\} \quad h, h' = 1, \ldots, H \qquad (8.1.11)
$$

The number of production units of industry h in each centre of type h' separately, $\bar{n}_h^{h'}$, follows from (8.1.11) and (8.1.19)

$$
\left.
\begin{aligned}
\bar{n}_h^{h'} &= \frac{n_h}{n_{h'}} \cdot \frac{\alpha_{h'}}{\alpha_0 + \alpha_{h'} + \ldots + \alpha_H} && \text{for } h < h' \\
&= 1 && \text{for } h = h' \\
&= 0 && \text{for } h > h'
\end{aligned}
\right\} \quad h, h' = 1, \ldots, H \qquad (8.1.12)
$$

8.2. Appropriate Choice of Groups of Industries

Before discussing some principles to be applied to the grouping of industries the simplest version of the hierarchy model may be illustrated by a simple numerical example. We assume $H = 4$ and $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.2$. Further, we assume $Y = 1200$ and $n_1 = 288$, $n_2 = 36$, $n_3 = 6$ and $n_4 = 1$. Applying the formulas derived in the previous section we then find the following composition of centres: (Table 8.2)

The example has been chosen so as to yield integer figures not only for the number of centres, but also for the number of production units of each industry in each centre. The probability that the model shows such elegant results for any concrete case where the values of the α's and those of the n_h are given is very small. It can be seen from (8.1.9) and (8.1.12) that as a rule, the number of centres of each type as well as the number of production units of each industry in each centre will not be integers. This does not mean that the model is meaningless. It is probable that the optimum grouping of production units will then be close to the theoretical figures found, but only

TABLE 8.2

Distribution of all production units over the centres and number of units per centre

Total number of units						Number of units per centre					
Centres	$h' = 1$	2	3	4	All centres	Centres	$h' = 1$	2	3	4
Ind. $h = 1$	72	24	48	144	288		1	2	16	144
2	.	12	6	18	36		.	1	2	18
3	.	.	3	3	6		.	.	1	3
4	.	.	.	1	1		.	.	.	1
Number of centres	72	12	3	1			72	12	3	1

integer numbers are allowed to appear of course. If one finds a number of 3.1 centres of a certain composition this will have to be interpreted to mean that three centres should be established and the remaining production units corresponding to the 0.1 be distributed either over these three centres or over centres of neighbouring types. Of course this implies that the elements of the hypothesis with regard to imports and exports will not then be obeyed in all strictness. If the number of production units per centre of any type appears to be non-integer, one may either think of units deviating from the pre-assumed size, or of slight differences in the composition of some of the centres of the type considered. If this number is close to zero, it might be an indication that this sector should not be located in these centres but only in centres of higher rank.

Since in reality the number of industries is very large the question arises as to whether or not groups of industries must be formed in order to apply the model. It is already in the spirit of the principles behind the model that industries appearing in exactly the same number should be combined and treated as one industry. Thus, if the number of bakeries and the number of groceries were the same, say, 100,000, we should simply think of the presence of 100,000 combined "units" consisting of one bakery and one grocery each. The centres of the rank of that industry will consist of one such combined "unit" and of whatever lower-ranked industries there might be. If the number of some industries of consecutive ranks do not differ very

much, the same principle seems in place; suppose the number of printing and the number of building enterprises does not differ much and is, for instance 10,000 and 9,000 then we may also combine them. This implies that in the larger centres where these industries occur they may occur in the proportion 10 : 9 or even 10 : 8, while in the centres of the rank of this combined industry there will either be a somewhat smaller building enterprise or, in one out of each ten similar centres the building enterprise will be lacking. Thus the original model will still yield first approximations to the best dispersion of economic activity over centres but must afterwards be adapted to reality.

This poses the important question of how far we should or can go with this grouping process; will we not change the structure of centres completely by the operation of grouping?

We will try and give a tentative answer to this question by discussing a numerical example. In this example $H = 8$, $\alpha_0 = 0.2$ and all other $\alpha_h = 0.1$. The number of units n_h of each of the 8 (H) industries is indicated in Table 8.3. The number of centres of rank h' is calculated from (8.1.9).

$$n^{h'} = n_{h'} \cdot \frac{\alpha_0}{1 - \alpha_1 - \ldots - \alpha_{h'}} = n_{h'} \Phi_{h'} \qquad h' = 1, \ldots, H.$$

When grouping the sectors, one has first to choose which sectors to take together. Secondly one has to decide on the number of technical units n_h which one will assume to exist in the new aggregated industry after the operation of grouping. Evidently the highest and lowest possible number of technical units in this new aggregated sector are determined by the number of units of the sector with lowest and highest rank respectively of the sectors which are grouped together. These two extreme possibilities have been dealt with in our example and can also be found in Table 8.3. It is a matter of course that one can also choose any number of units n_h between these two limits.

One can easily see that when grouping the sectors, the number of centres greatly depends on the number of units n_h that is chosen. If this number is chosen equal to the n_h of the lowest ranking industry before grouping, then the total number of centres in a group does not differ very much from the sum of centres of the original type, unless the numbers of centres in these original types of centre do not differ much from each other (e.g. n^3 and n^4). In the case when the number of units after grouping is equal to the

TABLE 8.3

Examples of the effect of grouping of industries on the number of centres

1. Without grouping				2. Grouping two by two with maximum n_h				3. Grouping two by two with minimum n_h			
h'	$n_{h'}$	$\Phi_{h'}$	$n^{h'}$	h'	$n_{h'}$	$\Phi_{h'}$	$n^{h'}$	h'	$n_{h'}$	$\Phi_{h'}$	$n^{h'}$
1	40,000	0.22	8,888	1	40,000	0.25	10,000	1	8,000	0.25	2,000
2	8,000	0.25	2,000								
3	1,500	0.28	420	2	1,500	0.33	500	2	1,200	0.33	400
4	1,200	0.33	400								
5	100	0.40	40	3	100	0.50	50	3	20	0.50	10
6	20	0.50	10								
7	5	0.67	3.4	4	5	1.00	5	4	1	1.00	1
8	1	1.00	1								

n_h of the highest ranking industry before grouping, the number of centres is exactly equal to the smallest number of centres of the original types of centre. Both possibilities have some advantages. Choosing the highest possible n_h leads to more centres which means that transportation costs between this type of centre and the agricultural sector will be smaller. On the other hand, choosing the smallest possible n_h, which leads to fewer centres, implies that transportation costs within each type of centre, due to a somewhat different composition of centres of the same rank, will be smaller.

We can easily generalize our formulas for the case of grouping. Assume that we group m sectors of neighbouring rank $h_1, \ldots, h_m$

$$(h_1 < \ldots < h_m \quad \text{and} \quad n_{h_1} > \ldots > n_{h_m})$$

constituting a new sector $(h_1, \ldots, h_m)$. We choose the number of technical units to be equal to n_{h_k} where $h_1 \leq h_k \leq h_m$.

Then our formulas become:

$$Y^{(h_1,\ldots,h_m)} = \frac{(\alpha_{h_1} + \ldots + \alpha_{h_m})\,\alpha_0\,Y}{(\alpha_0 + \alpha_{h_1} + \ldots + \alpha_H)\,(\alpha_0 + \alpha_{h_m+1} + \ldots + \alpha_H)} = Y^{h_1} + \ldots + Y^{h_m} \tag{8.2.1}$$

$$n^{(h_1,\ldots,h_m)} = n_{h_k} \cdot \frac{\alpha_0}{\alpha_0 + \alpha_{h_m+1} + \ldots + \alpha_H} = n_{h_k} \cdot \frac{n^{h_m}}{n_{h_m}} \tag{8.2.2}$$

$$\left.\begin{aligned} \bar{n}_h^{(h_1,\ldots,h_m)} &= \frac{n_h}{n_{h_k}} \cdot \frac{(\alpha_{h_1} + \ldots + \alpha_{h_m})}{(\alpha_0 + \alpha_{h_1} + \ldots + \alpha_H)} && \text{for } h < h_1 \\ &= \frac{n_h}{n_{h_k}} && \text{for } h_1 \leqslant h \leqslant h_m \\ &= 0 && \text{for } h_m < h \end{aligned}\right\} \quad h, h_1, h_m = 1, \ldots, H \tag{8.2.3}$$

The example of Table 8.3 shows clearly that grouping makes the application of the hierarchy hypothesis rather flexible. Given this flexibility we must ask the question whether there exists some optimal way of grouping. Clearly the ratio of consecutive $n^{h'}$ will be higher, the larger groups we form. With hundreds of groups, this ratio will come close to 1. In our example without grouping it varied between 1 and 10; with grouping it varied between 10 and 20 in one case and between 10 and 40 in the other case. Obviously

there is not much point in having the ratio close to 1. This implies that there is practically no need for separate centres of consecutive rank. Hence it should be at least 2. The ratio has something to do with the geometric aspect of the problem. For single commodities or for single centre type this problem has been considered by Lösch (1944) in particular (See also CHRISTALLER, 1933).

The number of centres of one rank lower around a given centre is dependent on the way in which a plane can be subdivided into regular polygons. As is well known, this is only possible with the aid of triangles, squares and hexagons. Of these, the hexagon is closest to the circle and is to be preferred therefore for some maximization problems. However, it lacks two properties which may be of importance for our problem. The hexagon cannot be subdivided into smaller hexagons and cannot therefore be used for the creation of a hierarchy of regular spaces, where each higher-ranked space (or market area) consists of a number of next lower-ranked spaces. In contrast, the square allows such a hierarchical subdivision. It should be recognized that the precise relevance of this feature is not yet clear. As already said, our remarks are tentative answers only. Another property of squares is that, for the microstructure, they fit in better with a rectangular road network, which for many purposes of transportation efficiency is superior to a network with crossings at other angles. Our tentative answer to the question of the optimal ratio of the number of centres of a given rank to the number of centres of the next higher rank is that it should be four, except if the form of a country (for example Norway or Chile) is a long quadrangle; then, in the higher ranks, a ratio of two may be optimal, until the country has been subdivided into smaller squares. From there on, downward, the ratio should again be 4.

From (8.2.2) we can derive

$$\frac{n^{(h_1, \ldots, h_m)}}{n^{(h_m+1, \ldots, h_{m'})}} = \frac{n_{h_k}}{n_{h_{k'}}} \cdot \frac{\alpha_0 + \alpha_{h_{m'}+1} + \ldots \alpha_H}{\alpha_0 + \alpha_{h_m+1} + \ldots + \alpha_H} \tag{8.2.4}$$

where $h_1 \leqslant h_k \leqslant h_m$ and $h_m+1 \leqslant h_{k'} \leqslant h_{m'}$

and hence

$$\frac{n^{(h_1, \ldots, h_m)}}{n^{(h_m+1, \ldots, h_{m'})}} < \frac{n_{h_k}}{n_{h_{k'}}} \tag{8.2.5}$$

It follows that the grouping of industries should be so as to let $n_{h_k} > 4n_{h_{k'}}$, or more precisely

$$n_{h_k} = 4 \cdot \frac{\alpha_0 + \alpha_{h_{m'}+1} + \ldots + \alpha_H}{\alpha_0 + \alpha_{h_m+1} + \ldots + \alpha_H} \cdot n_{h_{k'}} \qquad (8.2.6)$$

with the figure 4 replaced by 2 for the higher-rank centres in a "long" country. There may be other geographical circumstances which may lead to other choices. We will not go into any more detail here.

8.3. The Hierarchy Model: Non-Shiftable Industries

In the simplest version of the hierarchy model so far discussed, no differences in production costs between different parts of the country were assumed. Only agriculture could by definition not take place in the centres while the opposite was assumed to be true for the other sectors.

We are now going to introduce two activities which can only be carried out at one point, say the mining of two different commodities. For short-run problems one may well also consider the central government machine, usually located in the capital of the country, to belong to that type. Later on, when we introduce foreign trade, this may also be assumed to be located in one big port and hence be non-shiftable. Because each of them is located in one centre, we will consider the number of production units of each of these industries as being equal to one, even if there will as a rule be more enterprises. For this reason we give them the two highest ranks, H and $H-1$, and we change our hypothesis of Section 8.1 by supposing that the corresponding centres H and $H-1$ do not produce product $H-1$ and H, respectively, and that $n_H = n_{H-1} = 1$. All other elements of the hypothesis are maintained and Table 8.1 will accordingly change into Table 8.4.

We can derive the values of $Y^{h'}$ for $h' = 0, \ldots, H-2$ in exactly the same way as before and we shall find the same answers (cf. equations (8.1.2)–(8.1.7)). Indicating by Y^* the expression

$$Y^0 + Y^1 + \ldots + Y^{H-2} = \frac{\alpha_0 Y}{\alpha_0 + \alpha_{H-1} + \alpha_H} = Y^* \qquad (8.3.1)$$

TABLE 8.4

Survey of types of centres and their imports and exports, with industries H and $H-1$ each in one centre

Rank h'	Income	Imported products	Value of imports	Exported products	Destination of exports	Value of exports
0	Y^0	$h = 1, \ldots, H$	$(\alpha_1 + \ldots + \alpha_H)\ Y^0$	$h = 0$	$h' = 1, \ldots, H$	$\alpha_0(Y - Y^0)$
1	Y^1	$h = 0, 2, \ldots, H$	$(\alpha_0 + \alpha_2 + \ldots + \alpha_H)\ Y^1$	$h = 1$	$h' = 0$	$\alpha_1 Y^0$
2	Y^2	$h = 0, 3, \ldots, H$	$(\alpha_0 + \alpha_3 + \ldots + \alpha_H)\ Y^2$	$h = 2$	$h' = 0,1$	$\alpha_2(Y^0 + Y^1)$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$H-2$	Y^{H-2}	$h = 0, H-1, H$	$(\alpha_0 + \alpha_{H-1} + \alpha_H)\ Y^{H-2}$	$h = H-2$	$h' = 0, \ldots, H-3$	$\alpha_{H-2}(Y^0 + \ldots + Y^{H-3})$
$H-1$	Y^{H-1}	$h = 0, H$	$(\alpha_0 + \alpha_H)\ Y^{H-1}$	$h = H-1$	$h' = 0, \ldots, H-2, H$	$\alpha_{H-1}(Y^0 + \ldots + Y^{H-2} + Y^H)$
H	Y^H	$h = 0, H-1$	$(\alpha_0 + \alpha_{H-1})\ Y^H$	$h = H$	$h' = 0, \ldots, H-1$	$\alpha_H(Y^0 + \ldots + Y^{H-1})$

we now find two simultaneous equations for Y^{H-1} and Y^H, namely:

$$(\alpha_0+\alpha_H)\,Y^{H-1}-\alpha_{H-1}\,Y^H=\alpha_{H-1}\,Y^* \tag{8.3.2}$$

$$-\alpha_H\,Y^{H-1}+(\alpha_0+\alpha_{H-1})\,Y^H=\alpha_H\,Y^* \tag{8.3.3}$$

from which we can easily obtain

$$\left.\begin{aligned} Y^{H-1} &= \frac{\alpha_{H-1}}{\alpha_0}\,Y^* \\ Y^H &= \frac{\alpha_H}{\alpha_0}\,Y^* \end{aligned}\right\} \tag{8.3.4}$$

and, as it should be,

$$Y^{H-1}+Y^H=\frac{\alpha_{H-1}+\alpha_H}{\alpha_0+\alpha_{H-1}+\alpha_H}\,Y \tag{8.3.5}$$

This model represents an example of the flexibility of the method used to arrive at suggestions for optimal dispersion of economic activity. It will be clear that the number of centres of type $H-1$ and H in this example can be easily increased. All we get is a superstructure of as many simultaneous equations of type (8.3.2, 8.3.3) as we have centres of the types just quoted.

8.4. The Hierarchy Model: Foreign Trade

We will now remove another restriction on the model presented in Section 8.1 and introduce foreign trade into it. To begin with we do this by assuming exports of agricultural products are possible. The goods to be exported are transported to centre H, supposed to be a big port; in contrast this centre supplies imported goods; we now introduce, as before, a demand coefficient α_0 for home consumption of agricultural products; then, α'_H as the ratio of exports to national income and α_H as the demand coefficient for imported goods; $(\alpha_H-\alpha'_H)\,Y$ representing the income derived by centre H from the transformation of exported agricultural products into import products. Or, in other words, the peasants are receiving $\alpha'_H\,Y$ for these export goods,

but foreign countries are paying $\alpha_H Y$ for them. Our usual table, in somewhat abbreviated form, now becomes:

TABLE 8.5

Exports and imports of each type of centre if part of agricultural production is for export by centre H.

Rank h'	Value of exports to other centres, not to abroad	Value of imports from other centres, not from abroad
0	$\alpha_0(Y-Y^0)+\alpha'_H Y$	$(\alpha_1+\alpha_2+\ \dots\ +\alpha_H)\ Y^0$
1	$\alpha_1 Y^0$	$(\alpha_0+\alpha_2+\ \dots\ +\alpha_H)\ Y^1$
2	$\alpha_2(Y^0+Y^1)$	$(\alpha_0+\alpha_3+\ \dots\ +\alpha_H)\ Y^2$
.	.	.
.	.	.
.	.	.
H	$\alpha_H(Y-Y^H)$	$\alpha_0 Y^H+\alpha'_H Y$

As well as these exports and imports a value will be exchanged with foreign countries, amounting to $\alpha_H Y$ and not shown in this table.

From this table we can again find the values of $Y^{h'}$. The results are:

$$Y^0 = (\alpha_0+\alpha'_H)\,Y \tag{8.4.1}$$

$$Y^0+Y^1 = \frac{\alpha_0+\alpha'_H}{1-\alpha_1}\,Y \tag{8.4.2}$$

$$Y^0+Y^1+Y^2 = \frac{\alpha_0+\alpha'_H}{1-\alpha_1-\alpha_2}\,Y \tag{8.4.3}$$

. . . . . .

$$Y^0+Y^1+\dots+Y^{H-1} = \frac{\alpha_0+\alpha'_H}{\alpha_0+\alpha_H}\,Y \tag{8.4.4}$$

$$Y^H = \frac{\alpha_H-\alpha'_H}{\alpha_0+\alpha_H}\,Y \tag{8.4.5}$$

and it will be easily seen that all $Y^{h'}$ add up to Y, as it should be.

It is left to the reader to generalize this version of the model for cases where not only agricultural products are exported. For developing countries, however, precisely agricultural products are a main item in exports. Moreover, we expect the hierarchy hypothesis not to lead to an optimal location for international sectors which export a high percentage of their production.

8.5. Corrections for Industries with Heavy Agricultural Inputs

It has already been stated that the hierarchy model as presented in Section 8.1 does not always constitute the optimal arrangement of industries. A clear exception has been indicated by J. SERCK-HANSSEN (1961), and referred to in personal discussions as the dairy and sugar industry case. The essential feature of these industries is that they do have some economies of scale and at the same time use huge quantities of agricultural products as inputs. They will not appear therefore in the smallest centres, but they may not appear either in all the centres with higher rank than they have themselves, since then their inputs would have to be transported over longer distances than if they are in smaller centres. We may build into the hierarchy model a first approximation to this state of affairs by assuming that such an industry appears in one type of centre only and with higher-ranked centres disappears again. In exactly which rank of centre they must be for the sake of an optimum should be a matter for preliminary study. We assume that study leads to the conclusion that they can best be located in centres of a given rank. In the example below we assume this to be rank 2. The demand for the product of this exceptional industry will be taken equal to βY and now

$$\sum_{0}^{H} \alpha_h + \beta = 1 \tag{8.5.1}$$

The industry and its product will be given the index k (which consequently does not belong to the collection 0, ..., H), allowing for the possibility of having more than one such an industry, a case which we leave to the reader.

Below, we give the usual table and the values of the $Y^{h'}$ resulting from it. Again these can be found.

TABLE 8.6

Survey of types of centres and their imports and exports, if one industry k only occurs in centres of type 2

Rank h'	Imports h =	Values of imports	Exports h =	Destination h' =	Value of exports
0	$1, \ldots, H, k$	$(1-\alpha_0)\ Y^0$	0	$1 \ldots H$	$\alpha_0(Y-Y^0)$
1	$0, 2, \ldots, H, k$	$(\alpha_0+\alpha_2+ \ldots +\alpha_H+\beta)\ Y^1$	1	0	$\alpha_1\ Y^0$
2	$0, 3, \ldots, H$	$(\alpha_0+\alpha_3+ \ldots +\alpha_H)\ Y^2$	$2, k$	0, 1 for $h = 2$ 0, 1, 3, ..., H for k	$\alpha_2(Y^0+Y^1)+\beta(Y-Y^2)$
3	$0, 4, \ldots, H, k$	$(\alpha_0+\alpha_4+ \ldots +\alpha_H+\beta)\ Y^3$	3	0, 1, 2	$\alpha_3(Y^0+Y^1+Y^2)$
4	$0, 5, \ldots, H, k$	$(\alpha_0+\alpha_5+ \ldots +\alpha_H+\beta)\ Y^4$	4	$0, \ldots, 3$	$\alpha_4(Y^0+ \ldots +Y^3)$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$H-1$	$0, H, k$	$(\alpha_0+\alpha_H+\beta)\ Y^{H-1}$	$H-1$	$0, \ldots, H-2$	$\alpha_{H-1}(Y-Y^{H-1}-Y^H)$
H	$0, k$	$(\alpha_0+\beta)\ Y^H$	H	$0, \ldots, H-1$	$\alpha_H(Y-Y^H)$

As before $Y^0 = \alpha_0 Y$; $Y^1 = (\alpha_0 \alpha_1 Y)/(1-\alpha_1)$; $Y^0 + Y^1 = (\alpha_0 Y)/(1-\alpha_1)$, but from centres 2 on, where industry k is located, α_0 is replaced by $\alpha_0 + \beta$ in the expressions for the incomes of the types of centres. Thus we find

$$Y^0 + Y^1 + Y^2 = \frac{\alpha_0 + \beta}{1-\alpha_1-\alpha_2} Y \tag{8.5.2}$$

implying that

$$Y^2 = \left(\frac{(1-\alpha_1)\beta + \alpha_2 \alpha_0}{(1-\alpha_1)(1-\alpha_1-\alpha_2)} \right) Y \tag{8.5.3}$$

Further, more generally

$$Y^0 + Y^1 + \ldots Y^{h'} = \frac{\alpha_0 + \beta}{1-\alpha_1 - \ldots - \alpha_{h'}} Y \qquad h' = 2, \ldots, H \tag{8.5.4}$$

implying, as it should, that:

$$\sum_0^H Y^{h'} = Y$$

The number of production units of industry h in all centres of type h will again be found analogously to (8.1.11) with the aid of

$$n_h^{h'} = n_h \frac{Y^{h'}}{Y} = \varphi^{h'} \cdot n_h \quad \text{for} \quad h < h' \quad \text{and} \quad h, h' = 1, \ldots, H \tag{8.5.5}$$

where now

$$\varphi^0 = \alpha_0; \; \varphi^1 = \frac{\alpha_0 \alpha_1}{1-\alpha_1}; \; \varphi^2 = \frac{\alpha_0 \alpha_2 + \beta(1-\alpha_1)}{(1-\alpha_1)(1-\alpha_1-\alpha_2)}$$

and generally

$$\varphi^{h'} = \frac{\alpha_0 + \beta}{1-\alpha_1 - \ldots - \alpha_{h'}} - \frac{\alpha_0 + \beta}{1-\alpha_1 - \ldots - \alpha_{h'-1}} \tag{8.5.6}$$

for $h' \geq 3$.

8.6. No Vertical Integration; Input-Output Relations

The assumption used in the simple version of the hierarchy model that all industries are vertically integrated is equivalent to assuming that all industries produce finished goods. In a number of cases the model will not be invalidated by the assumption that raw material producing industries also exist separately, supplemented by other industries processing the raw materials and producing only the value added. Demand coefficients α_h could still be used, but for the processing industry they would be equal to the ratio of their value added to national income as a whole and not of the total value of their product to national income; for the raw material producing industry, however, the α_h would be equal to the ratio of the total value of production (now, however, equal to intermediate and final demand together) to national income. The model would implicitly assume that the raw materials and the value added are sent to the consumers separately as if they could be assembled by the consumers. This remains realistic only if the processing industry is of lower rank than the raw material industry. For then the processing units are always between the raw material production and the consumer, and the flow of the raw materials will be correctly indicated by the model, namely from higher-ranked to lower-ranked centers. While such a situation usually prevails if mineral or imported raw materials are used, it is clearly not so in the case of national agricultural raw materials. This is why a corrected model for such industries was presented in Section 8.5.

Raw materials, however, are not the only material inputs of a production process and even though fuel and auxiliary materials often show the same flows from higher-ranked to lower-ranked industries, a more satisfactory treatment is clearly one where input-output relations are introduced explicitly, that is a treatment where the fifth assumption of Section 8.1 has been dropped.

Then we have to see first whether the specification of the size, composition and direction of the trade flows which follows from the simple hierarchy hypothesis is still sufficient to determine the number, size and composition of centres of different types. As can be seen from Bos (1965, p. 82 ff.) who restricts himself in analysing this version of the model to the case $H = 3$, this turns out to be so. However, it is no longer possible to calculate first only the incomes of the centres and next their sectoral composition. They have to be determined simultaneously now, since the demand for all com-

modities – including export products – also includes intermediate demand and therefore also depends on the sectoral composition of the centres.

Now a new problem arises, i.e. whether a system of centres derived from these trade flows is always in conformity with the original hierarchy hypothesis, in particular with its first two elements. This is indeed the case for vertically integrated industries, but Bos shows that this is not so if account is taken of intermediate products. He obtains some new results which we will consider later.

In his analysis Bos starts from the implication of the hierarchy hypothesis that in centres of rank h only the highest ranking sector, h, exports and that it does so only to the lower ranking centres. This leads again, mathematically speaking, to a system of homogeneous equations the number of which is one less than the number of variables i.e., the absolute values of the variables are determined except for a scale factor.

Instead of using Y as the scale factor it is easier now to use V_0, agricultural output value, for that purpose and Bos presents the following system of equations between the $V_h^{h'}$ (standing for the value of output of industry h in centre type h') (cf. Table 8.7 and his Table 7.24.1):

Here the coefficients have the following meaning:

α_h as before, is the coefficient relating final demand for product h to income.

$\omega_{hh'}$ is the input coefficient of product h in the production process of product h'.

$\omega_{\cdot h} = 1 - \sum_{h'} \omega_{h'h}$ is the ratio between value added and output of industry h.

Bos points to the block-triangular structure of the system of equations: the output values of the industries in each type of centre depend only on the output values in the lower-ranked types of centres. The system permits the calculation of the $Y^{h'}$ for the various types of centres, but the relationships have become more complicated and their numerical use requires the knowledge of a complete input-output table. The coefficients appearing in the system can be described, with J. Serck-Hanssen, as generalized input-output coefficients if we also consider as inputs the quantities of products demanded by the consumers corresponding to each type of activity; in a general formula:

$$\omega'_{hh'} = \omega_{hh'} + \alpha_h \omega_{\cdot h'} \tag{8.6.1}$$

TABLE

Equations determining a Tinbergen system[a] of centres

Equations	Variables		
	Centres 1	Centres 2	
	V_1^1	V_2^2	V_2^2
Equation centres 1	$1-(\omega_{11}+\alpha_1\omega_{.1})$	0	0
Equations centres 2	0	$1-(\omega_{11}+\alpha_1\omega_{.1})$	$-(\omega_{12}+\alpha_1\omega_{.2})$
	$-(\omega_{21}+\alpha_2\omega_{.1})$	$-(\omega_{21}+\alpha_2\omega_{.1})$	$1-(\omega_{22}+\alpha_2\omega_{.2})$
Equations centres 3	0	0	0
	0	0	0
	$-(\omega_{31}+\alpha_3\omega_{.1})$	$-(\omega_{31}+\alpha_3\omega_{.1})$	$-(\omega_{32}+\alpha_3\omega_{.2})$

a) Bos defines a Tinbergen system as a system in which the implication of the hierarchy hypothesis mentioned on page 233 is satisfied.

For all $\omega_{hh\cdot} = 0$ the system reduces to the simplest version of the hierarchy model; V's become Y's with the same suffixes and

$$Y^1 = Y_1^1,\ Y^2 = Y_1^2 + Y_2^2,\ Y^3 = Y_1^3 + Y_2^3 + Y_3^3.$$

We then find

$$Y^1 = \frac{\alpha_1}{1-\alpha_1} Y^0 = \frac{\alpha_1\alpha_0}{1-\alpha_1} Y \tag{8.6.2}$$

$$Y^2 = \frac{\alpha_2}{1-\alpha_1-\alpha_2}(Y^0 + Y^1) = \frac{\alpha_2\alpha_0}{(1-\alpha_1)(1-\alpha_1-\alpha_2)} Y \tag{8.6.3}$$

the same results as found previously.

Bos also finds some new results. Here we will formulate them slightly differently from his version. He finds that an industry producing only intermediate products for non-agricultural industries of higher rank can never be an exporting industry in a Tinbergen system, but will always appear

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th industries producing both final and intermediate products

Centres 3			Centre 0
V_1^3	V_2^3	V_3^3	V_0
)	0	0	$= \omega_{10} + \alpha_1 \omega_{.0}$
)	0	0	$= 0$
)	0	0	$= \omega_{20} + \alpha_2 \omega_{.0}$
$-(\omega_{11} + \alpha_1 \omega_{.1})$	$-(\omega_{12} + \alpha_1 \omega_{.2})$	$-(\omega_{13} + \alpha_1 \omega_{.3})$	$= 0$
$-(\omega_{21} + \alpha_2 \omega_{.1})$	$1-(\omega_{22} + \alpha_2 \omega_{.2})$	$-(\omega_{23} + \alpha_2 \omega_{.3})$	$= 0$
$-(\omega_{31} + \alpha_3 \omega_{.1})$	$-(\omega_{32} + \alpha_3 \omega_{.2})$	$1-(\omega_{33} + \alpha_3 \omega_{.3})$	$= \omega_{30} + \alpha_3 \omega_{.0}$

together with only the industries using its product; and an industry producing only intermediate products for agriculture can be an exporting industry of centres of a certain rank in a Tinbergen system, but will not occur in centres of other ranks.

8.7. Agglomeration Effects

By agglomeration effects we indicate the phenomenon that unit production costs of some sectors are influenced by either (i) the size of the centre in which their production units are situated or (ii) the presence of other units of the same sector in the same centre, or, finally (iii) the presence of some "complementary" sector in the same centre. Accurate data on these agglomeration effects are rare and city planners and economists have only started to study this subject. The effects may be negative or positive. Thus, some sectors may become more expensive to run if established in larger centres than in smaller; others may be cheaper to operate in a larger than in a smaller

centre. It is also possible that the effect is different for private than for social costs.

For these reasons different opinions can be found even among experts in the field of city planning about the desirability, from the social point of view, of having some sectors in large centres.

If the agglomeration effects as now defined are non-negligible, the problem of the optimal grouping of a country's production units into centres requires, for its solution, more complicated methods than used in the hierarchy model. In the total cost expression which we want to minimize, these effects must now be included. So far the optimality proof for the hierarchy model has only been given for a few simple cases. In simple cases some of the agglomeration effects may also be included. A more general treatment has not yet been developed.

Within some limits – still to be specified, however – the case dealt with in Section 8.5 may be considered as the simplest approach open to us. For the case where the operation of plants of some sector in larger centres is more expensive than in smaller centres, such a sector may be in the same position as sector k in that example. The choice of the rank of centres in which such a sector should be operated should then be the subject of a preparatory study or of a check afterwards.

We may point out a difference between the results of the hierarchy hypothesis and studies on the optimal size of a city which aim at minimizing the per capita cost of public services or maximizing the benefit-cost ratio for such services. Often the latter studies lead to formulating an optimal size or size range for individual centres, while the hierarchy hypothesis, in whatever improved form, leads to formulating an optimal system of centres of different sizes. In as far as minimizing the per capita cost of public services takes account of relevant agglomeration effects which are not yet taken into account by the hierarchy hypothesis, integration of both approaches is useful and exactly what is called for in the previous paragraph.

8.8. Stepwise Spatial Planning

We consider it a characteristic of our approach to planning that the process is carried out in stages or stepwise. This also applies to the element of space: as already set out in Chapter 1 and illustrated by Chapters 3-7 we deal with

various size categories of space and think that location planning should start with a choice among large areas such as continents, may then be continued, by a choice among countries within a continent, by a further choice of regions within a country and possibly down to the local level.

As with all successive approximations, this stepwise spatial planning is also, on the one hand, prone to some dangers which require checks and revisions or trial and error methods. We have already seen that a second step sometimes shows that the first step was not based on the relevant information and therefore has to be revised. An elementary example – which contains a difficulty of this type which can perhaps be avoided – is that an international industry is chosen for some country and that on considering regional data that first choice appears to have been erroneous. It must then be revised.

On the other hand, the clear advantage of a stepwise approach is its simplicity. The first step is simplified by not considering too large a number of spatial units (say, continents). The second step is simplified because then we consider only one continent – assuming that the solutions for the continents were correct – and start planning for the countries of that continent. The third step again neglects the other countries of the continent and only concentrates on the regions of one country. The volume of work may be reduced considerably. It depends on the number of revisions of previous steps whether an overall saving on work is achieved. While there are some methods which reduce the probability of errors in the first steps, there is no guarantee that errors will be avoided. Some of the methods available may be illustrated by a very simple example again. Suppose we choose the international industries in which some country has to specialize on the basis of capital-output ratios. Now one may either take the average capital-output ratio over the country for each industry under consideration or the capital-output ratio for the cheapest region for that industry. Clearly the second method reduces the probability that our first choice appears to be erroneous on stepping down to the regional level. Once we are planning for the regions it is clear that the regional capital-output ratios are more meaningful than the national averages.

At the beginning of this chapter we indicated that, while for higher spatial units such a stepwise planning might usefully employ models of the type described in Chapters 3-7, one might run into trouble at lower spatial levels because of indivisibilities, since a useful subdivision into spatial units

becomes more difficult and because the cost coefficients become more and more dependent on the existing situation. While this chapter has outlined an approach which might be useful as a framework for further studies which deal with spatial planning in the presence of indivisibilities, it cannot be claimed that the models presented here could be as useful for spatial planning as the former ones. In particular the actual presence of a given spatial dispersion of economic activities might make it questionable to aim at a situation which more or less conforms to the results of the models within not too long a period. In addition, an integration with the previous models should still be attempted. For that reason we present in the next chapter some more practical observations about planning at these lower levels.

8.9. An Empirical Investigation for The Netherlands

In this section we want to report on an empirical study which will be published separately (PARKER, forthcoming) and which examines whether the industrial composition of centres in the Netherlands is in accordance with the hierarchy hypothesis. In particular it investigates whether industries show a hierarchical pattern given a certain hierarchy of centres, i.e. whether an industry of, say, rank h (which corresponds to the lowest ranked centres in which it appears) does regularly appear in types of centres of higher rank and whether there is any evidence for the implication of the hierarchy hypothesis with regard to the trade flows of the groups of centres.

The study deals with the following sectors: mining, manufacturing including public utilities, transport, communication and the hotel- and restaurant sector; it uses census data for 1950.

The census distinguishes 28 activities which are subdivided into 593 industry groups. For each of these 593 industry groups the census gives the number of firms, the number of local establishments and the number of technical units, and the number of people employed in each of them. Moreover, these data are available for each municipality, of which there are about 1000 in the Netherlands.

A sample of these municipalities was taken. Not a random sample in the strict – statistical – sense of the word but rather a selection of municipalities more or less corresponding to what is meant in the hierarchy hypothesis by

the concept of an industrial centre and whose size distribution shows a wide range.

In short: as smaller centres, the municipalities with one nucleus – the central part of the town or village – surrounded by a relatively sparsely inhabited area were selected. Sparsely inhabited here means a maximum of 200 inhabitants per square kilometer. With the larger centres – more than 100,000 inhabitants – the neighbouring municipalities were also considered as belonging to the larger centre. In this way 49 centres were selected with the number of inhabitants varying from 500 to 900,000.

Next, these 49 centres were grouped together in 6 groups; this grouping was done in a more or less arbitrary way since there is no empirically usable criterion for it.

TABLE 8.8

Grouping of centres according to the number of inhabitants

	Number of inhabitants	Number of centres in the sample
1	Less than 4,000	15
2	4,000 - 9,999	8
3	10,000 - 19,999	6
4	20,000 - 49,999	8
5	50,000 -100,000	5
6	More than 100,000	7

For each of these 49 centres and for each of the 593 industry groups the data on the number of local establishments together with those on the number of people working in them were used.

First it was examined as to whether the distribution of these industries over the centres is in conformity with the first element of the hierarchy hypothesis, that is, to its implication that if an industry appears in centres of a certain rank, it appears also in centres of a higher rank. This was done in the following way. Each of the industry groups was examined to see in which of the groups of centres it occurred. The following criterion was used here: an industry group occurs in a group of centres of a certain rank if it occurs in more than half the number of municipalities of the rank concerned.

The result was that the industry groups could be classified in three categories:

(1) The industry groups which, once occurring in a group of centres of a certain rank, also occur in all groups of centres of higher rank.
(2) The industry groups which, once occurring in a group of centres of a certain rank, do not occur in all groups of centres of higher rank.
(3) The industry groups which do not appear at all in any group of centres.

The group of industries mentioned in point 1 are considered to be in conformity with the first element of the hierarchy hypothesis. They are, therefore, provisionally called hierarchical industries; a number of exceptions will be dealt with later on. The industry groups mentioned in points 2 and 3 are called non-hierarchical industries.

The hierarchical industries can now be given a rank which corresponds to the lowest rank of all the groups of centres in which they appear. So industry groups of rank 1 occur in all groups of centres; those of rank 2 occur only in groups of centres of rank 2 and higher etc., etc.

Next, one investigates whether these provisionally hierarchical industries are in conformity with an implication of the first two elements of the hierarchy hypothesis, namely, that the contribution of these industries to the incomes of the centres of different ranks measured as a percentage shows the following pattern. An industry of rank h in centres of rank $> h$ should contribute to the incomes of these centres a percentage – called the centre percentage – equal to its total contribution to the income of the country, measured as a percentage, called the country percentage. In the centres of rank h the centre percentage of industry h should be higher than its country percentage, as it has to export commodity h to centres with a rank $< h$ (including agriculture). According to the hypothesis one would expect to

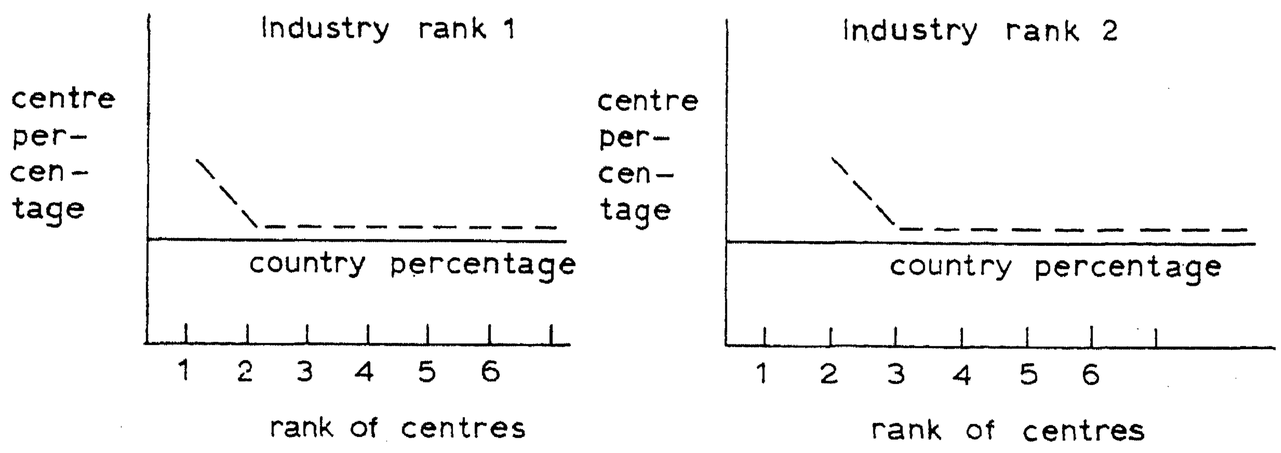

Fig. 8.1. Theoretical centre percentages of industries of rank 1 and 2.

find the following diagrams for e.g. industries of rank 1 and 2, where the solid line represents the country percentage and the dotted line the centre percentage (see Fig. 8.1; horizontal solid and vertical dotted lines coincide).

As proxy variables for the incomes of the industries and the country, the numbers of persons working in the industries and the whole country have been used, and for all provisionally hierarchical industries diagrams of their centre percentages and country percentages were made. When examining these diagrams of the industry groups it turned out that for the hierarchical industries the centre percentages were generally downwards sloping as they should be, but that they were never in complete accordance with the theoretical diagrams above. The diagrams of the hierarchical industries can be divided into three categories showing: (a) declining centre percentages, (b) rising centre percentages and (c) alternating centre percentages.

The study examines a number of possible explanations for these deviations from the theoretical diagrams. For the declining centre percentages, e.g. the following explanatory factors are mentioned: diffusion of the export function i.e. a group of centres of, say, rank *h* also exports commodities of lower rank which is, of course, in contradiction with the elements of the hierarchy hypothesis; economies of scale, leading to a greater labour productivity in larger technical units in higher ranking centres; smaller demand coefficients in larger centres.

For rising centre percentages, specialization is mentioned as an explanatory factor. Specialization in the sense that in centres of higher rank local establishments no longer produce or sell a number of commodities jointly as happens rather often in smaller centres.

The cases mentioned in point *c* of which the centre percentages show an irregular pattern are in general reclassified as non-hierarchical industries for that reason.

The results of the sample are summarized in Table 8.9 where the different industry groups have been combined into three big groups and where their relative importance is represented by the number of persons working.

We see that the data of the census cover about two thirds of the total working population and that according to the sample about 60 per cent of the industry groups can be considered as showing a hierarchical pattern in the sense of being in conformity with some of the implications of the hierarchy hypothesis.

TABLE 8.9

Number of persons working in the sectors covered by the census as a percentage of total working population of the Netherlands in 1950 and their classification as working in hierarchical or non-hierarchical industries

	Total	Hierarchical	Non-hierarchical
Manufacturing	42	21	21
Commerce	13	12	1
Transport	9	6	3
	64	39	25

The reader should be reminded that a comparison of the hierarchy hypothesis with empirical data should be evaluated carefully. Such a comparison can hardly refute the hypothesis, as a number of the assumptions of the model for which the hierarchy hypothesis is formulated are not completely satisfied in reality (e.g. the Dutch economy is far from closed), nor need the actual distribution be optimal. On the other hand neither can it be concluded from a deviation of the empirical data from the hypothetical pattern that the actual distribution of economic activities is not optimal. In addition it should be emphasized that the hypothesis does not give a theory of how actual distributions come into being.