CHAPTER I

VARIABLES CONSIDERED

1. INTRODUCTORY

The object of this investigation is the construction of an econometric model of the mechanism of British business cycles in the classical period 1870—1914: a period in which free enterprise and free trade predominated and for which a minimum of necessary statistical information is available. The construction of such a model implies, to some extent, the testing of a number of theories on the business cycle and enables us to judge of a number of measures of business cycle policy. Moreover it establishes a basis for a comparison with the business cycle mechanism found from similar investigations for other countries. These were made by the present author for the United States and for Holland in the period around the great crisis [1]).

A model of this sort is described with the aid of various kinds of mathematical concepts representing the phenomena with which the economist deals. From the economic viewpoint, we have to do with "*data*" and "*phenomena to be explained*". The data are phenomena of a natural, technical or institutional nature, and, if we are treating problems of national economics, also extra-national phenomena. We shall call them *external* phenomena. The phenomena to be explained by the economist may be called *internal*. From the mathematical point of view it is interesting to know whether these phenomena are of constant or of variable magnitude and accordingly we distinguish between *constants* and *variables*. In static economics we deal with constants only; in dynamic economics we deal also with variables. In statics, the external constants are given, the internal ones are to be calculated. In complete dynamics, both the internal constants ("equilibrium levels", e.g.) and the internal variables are to be calculated. In our type of analysis, the internal constants are considered as given; and the internal variables to be calculated [2]).

According to the type of variation they show, external variables may

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[1]) J. TINBERGEN, Statistical Testing of Business Cycle Theories, Vol. II: Business Cycles in the United States, 1919—1932. League of Nations, Geneva 1939. This volume will be referred to as "Business Cycles in the United States", etc.

J. TINBERGEN, An Economic Approach to Business Cycle Problems, Paris 1937 (HERMANN).

[2]) Not all values of these variables can be calculated, however. Some initial values must be given. This will be considered later.

be either non-systematic (such as crop variations, e.g.) or systematic (cyclic movements abroad). Since non-systematic variations are due to unknown causes, they can never be calculated and cannot, therefore, be chosen as internal variables. Thus, internal variables are always systematic ones.

Summarising we have to do with:

1. Data, which in our case are: (i) external and internal constants;
   (ii) external variables, systematic and non-systematic.
2. Phenomena to be explained, or internal variables.

As has been pointed out in previous work on this theme, it seems best first to give a "systematic catalogue" of the model constructed. For the reader such a catalogue is not very amusing literature, but this fact is counteracted by the possibility of passing over what he is not interested in. The more systematic treatment of problems of the cycle as a whole starts with chapter VII.

The following system of notations—corresponding to that adopted in the publications quoted—has been used:

Capital latin characters indicate money amounts;

Other latin characters: physical quantities or prices;

Greek letters: coefficients (incl. regression coefficients);

Lower suffices to latin letters: time period to which variable refers, e.g. $t-3$; sometimes t is left out;

Top suffices refer to restrictions in the field to which they apply: i = imports, e = exports, w = foreign countries, b = banks, I = iron, etc.;

One bar ($\bar{u}$) indicates the nine year moving average, two bars ($\bar{\bar{u}}$) the original variable and unbarred symbols (u) relate to the deviations from nine-year moving averages ($u = \bar{\bar{u}} - \bar{u}$).

Further mathematical symbols used:

$$\Delta x_t = x_t - x_{t-1}; \quad \dot{x}_t = \frac{dx_t}{dt} \sim x_{t+\frac{1}{2}} - x_{t-\frac{1}{2}};$$

$\sum x_t$ = cumulated value of x at the end of period $t = x_0 + x_1 + \ldots x_t$, where 0 is some arbitrary time period, independent of t; $\int x_t = \sum x_{t-\frac{1}{2}}$ = cumulated value of x in the middle of period t.

The *constants* introduced are about one hundred equilibrium values, elasticity coefficients, marginal propensities, wage quota, raw material quota, etc. They may be said to represent the natural, psychological and institutional structure of the British economy during the period under consideration. They will be introduced successively in the following chapters, where the chief relations or "laws", constituting the business cycle mechanism, are discussed.

I 2. LIST OF VARIABLES WITH SHORT INDICATION OF STATISTICAL SOURCE OR METHOD OF CALCULATION

	Sources and method of calculation.
Internal variables.	
A. Physical variables.	
a: total employment	Census of occupations, trade union unemployment figures, reweighted [1]) and production figures.
b: employment — investment goods ind.	HOFFMANN, W., Weltw. Archiv. [2]) corrected.
c: employment — consumers' goods ind.	Cf. table I B
u: production of consumers' goods	Trade statistics; index [3]) covering 87 % in 1907
u': consumption of cons. goods	Trade statistics; index [3]) covering 69 % in 1907
u^e: exports of ,, ,,	Cf. equation II 6 and table I C.
u^i: imports of ,, ,,	Cairncross estimate [4]) of pig iron cons. + imports of wood (hewn and sawn or split), average of last two years [3]).
v: prod. of investment goods	Trade Statistics; index [3]) covering 33 % in 1907.
v': cons. ,, ,, ,,	Report on Depression, 2, 1, B.o.T., 1886, p. 325; Statist.
v^e: exports of investment goods	Trade Statistics; index [3]) covering 77 % in 1907.
w^I: stocks of pig iron, average of two consecutive December figures.	Trade Statistics; index [3]) covering 87 % in 1907.
x^i: imports of cons. goods	
y^i: imports of raw mat. [5]) inv. goods	
B. Prices.	
l: wage rate	BOWLEY—WOOD [6])

[1]) Figures for persons occupied in dependent positions in each of the census years have been divided into three groups, viz. (i) trade (excl. retail trade) and transport, (ii) building and metal industries, (iii) all other industries. Group (i) has been distributed over groups (ii) and (iii) in proportion to their relative strength: (ii) represents investment goods industries, (iii) consumers' goods industries. For non-census years these figures have been interpolated. Those of group (ii) have been multiplied by $1 - 0.01 \times$ the unemployment percentage (building: carpenters and joiners; metals: engineering, shipbuilding and metal; 17th Abstract of Labour Statistics, 1915, p. 2). Those of group (iii) have been multiplied by $1 + 0.4\ (u/\bar{u})$, i.e. 1 + 40 % of the relative deviation from nine year moving average for production (cf. section II 3/4). All figures have finally been multiplied by such a figure as to make the 1907 total of both groups together equal to the wage bill (775) for that year.

[2]) W. HOFFMANN, Ein Index der industriellen Produktion für Grossbritannien seit dem 18. Jahrhundert, Weltwirtschaftliches Archiv 40 (1934), p. 383. Coal production has been included in consumers' goods and additions have been made for agricultural production and services (railway, post, housing and domestic services, the latter admittedly in a very rough way). Details are given in table I B.

[3]) Weighted average of quantities; weight: prices for 1907.

[4]) Cf. J. TINBERGEN, A Method and its Application to Investment Activity, League of Nations, Geneva 1939, p. 148.

[5]) Including semi-finished articles.

[6]) Taken from LAYTON, An Introduction to the Study of Prices, London 1920, p. 184.

m^L: long-term interest rate	Yield on consols, Stat. Abstract.
m^s: short-term interest rate	B.o.E. rate, Statistical Abstract of the U.K. (annual fluctuations, almost exactly equal to those in market rate).
n: share price index	London and Cambridge Econ. Service, Special Memorandum Nr. 37.
p: cost of living index	Cyclic fluctuations in WOOD's index of retail prices [6]) combined with trend of cost of living index in C. CLARK, National Income and Outlay, London 1937 [7])
p': home price index for some foods with a high import quota	Average of SAUERBECK prices for beef, bacon, butter and mutton; weights are import values for 1907.
p^c: export price of coal	Trade Statististics
p^e: price index of exported cons. goods	Trade Statistics: ratio of $\overline{\overline{U}}^e$ to $\overline{\overline{u}}^e$
p^i: price index for imports of finished consumers' goods	Weighted average for import prices of sugar, butter, bacon, mutton, eggs and beef.
q: price index of investment goods	Trade Statistics: Weighted index of (i) ratio of $\overline{\overline{V}}^e$ to $\overline{\overline{v}}^e$, and (ii) estimated machine prices [8]).
r: price index of raw materials for cons. goods	SAUERBECK's price indices and Rousseaux, Les Mouvements de fond de l'Economie anglaise 1800—1913, Louvain-Brussels, 1938, p. 264/5.
r^i: price index of imported raw materials for cons. goods	SAUERBECK's price index for all goods except minerals
s: price index of raw materials for investment goods	SAUERBECK's price index for minerals
s^B: price index of imported building materials	Wood import prices, statistical abstract
C. Money amounts.	
Au: Gold Stock, B.o.E.	U.S. National Monetary Commission, Statistics for G.B., Germany and France, Washington 1910. Statistical Abstract for the U.K.
Au': Gold reserve, B.o.E. [9])	
B^s: Discounts and Advances, all banks (incl. B.o.E.), Dec. 31th.	1870—'80: Miscellaneous Statistics of the U.K., B.o.T. 1877—1914: Economist [10])

[7]) Cf. table I D.

[8]) Machine prices have been estimated by a relation found for German export prices for machinery, viz. machine price per ton in shillings = 140 + 20 × pig iron export price, average for year and preceding year. The machine price index for 1907 = 100 has been combined with the index for the other prices of exported investment goods by variable weighting corresponding to the composition of exports:

Weights	1870	1880	1890	1900	1910
Machine prices . . .	0.10	0.15	0.20	0.25	0.25
Other prices	0.90	0.85	0.80	0.75	0.75

For the other years linearly interpolated weights have been taken.

[9]) I.e. in excess of non-fiduciary circulation of notes.

[10]) A number of the Economist figures has been taken from the U.S. National Monetary Commission: Statistics for G.B., etc., since it was the editor of the

I^A:	Income from abroad, freight and interest, etc.	Hobson, The Export of Capital, London 1914, p. 197 col. 5 + p. 204 col. 1.
L:	Wage bill	Bowley, Economic Journal 1904, and Pigou, Industrial Fluctuations, London 1927, p. 356.
M':	Notes in circulation	Cf. Au
M'':	Total deposits, all banks, incl. B.o.E., Dec. 31th	Cf. B^s
M''^b:	Cash on hand, all banks, incl. B.o.E., Dec. 31th	Cf. B^s
M^c:	Gold in country (Dec. 31th), outside B.o.E. in excess of stock at Jan. 1st, 1870	Cf. Au
U':	Value of total consumption	Cf. Table I B
U^e:	Value of exports of consumers' goods	Trade Statistics, complete [11])
U^i:	Value of imports of consumers' goods	Trade Statistics, sample covering 81 % [12]) in 1907
U^{re}:	re-exports	Statistical Abstract
V'	value of home investment	Cf. Table I C.
V^e:	Value of exports of investment goods	Trade Statistics, sample covering 65 % [11]) in 1907
X^i:	Value of imports of cons. goods	Calculated as product of corresponding quantity index and price index.
Y^i:	Value of imports of inv. goods	Calculated as product of corresponding quantity index and price index.
Z:	Non-workers' income (home)	Assessed incomes for years ending June, corrected for change in exemption limit (Bowley, Economic Journal, 1904, continued proportionally to figures given by Stamp); two-year moving average in order to get correct timing; multiplied by 1.5 [13]) and corrected for short fluctuations in agricultural income [14]).

Economist who supplied the figures. For the years 1878—1886 the Economist figures have been corrected according to the percentage coverage indicated in the former publication.

[11]) Of a number of export products, covering, in 1907, 86 % of total exports, value figures were tabulated and brought under one of the two headings — consumers' goods and investment goods. The residue was then "explained" by multiple correlation analysis, by a linear expression of the two provisional group totals. Since, in the corresponding regression formula, consumers' goods got a slightly negative coefficient, it was decided, that the rest of the group was to be considered as investment goods.

[12]) As compared to figure given by Sir Alfred Flux, Census of Production 1907, General Report.

[13]) This factor is obtained in the following way. Non-workers' income below exemption limit is, in the average, estimated by various authors at 25 % of assessed incomes, which requires multiplication by 1.25; the application of two-year moving averages (by our procedure) on figures which are already three-year moving averages (by tax laws) reduces the amplitude to about 0.83, which necessitates a further multiplication by 1.2; $1.2 \times 1.25 = 1.5$. From some results of our correlation calculations it would seem, however, that Z underestimates the true amplitude of non-workers' income still about threefold (Cf. chapter VI, 3).

[14]) Cf. Table I F.

External variables.		
Au^w:	gold stock abroad	Total for U.S.A., Germany and France
K^e:	capital export	HOBSON, The Export of Capital, London, 1914
m^{Sw}:	short term interest rate abroad	Average for U.S.A., France and Germany (I. FISHER, The Theory of Interest, New-York 1930, p. 521 and Annuaire Statistique de la France 1912 (dated 1913, published 1914) p. 70 and 210).
p^w:	"world" price index for British export goods (consumers' goods)	Average price index for cotton goods exported by France (weight: 1) Germany (2), India (1) and the United States (4), trade statistics [15])
q^w:	"world" price index for British export goods (investment goods)	Average price index for iron and steel products exported by France (weight: 1), Germany (5) and the United States (3), trade statistics [16])
T^w:	world trade value	WAGEMANN, Struktur und Rhythmus der Weltwirtschaft, Berlin 1931, p. 389.
v^{Iw}:	"world" production of pig iron	Federation of Iron and Steel Manufacturers.

A number of other variables, both internal and external, was introduced temporarily and then eliminated for various special reasons. Some of them were assumed to be almost trends, others to be narrowly correlated with other variables, still others to be of an erratic nature. This will be discussed in the next chapters. Other variables are considered in applications to special branches serving as illustrations.

The units chosen are, unless stated otherwise:

Physical quantities: in millions of £ of 1907 buying power.

Prices: 1907 = 100, except share prices, where the London and Cambridge Economic Service index was taken unchanged, and interest rates which were measured in per cent.

Money amounts: in millions of £.

[15]) France: Toiles, percales, calicots et coutils écrus et blancs; teints; imprimés; Germany: Baumwollwaren; United States: Cotton manufactures, colored; ditto, uncolored.

[16]) Commodities included (weights in brackets) for each country: France: fonte (10), fer étiré en barres, au coke (3), rails (6), fil de fer recouvert (1), ferrailles (4). Germany: Roheisen (2), Luppeneisen, usw. (2), Platten und Bleche (3), Eisendraht, nicht pol. (2), Eisendraht verzinkt, usw. (2), Röhren (2), Eisenbahnschiene (5), Eisenbahnachsen (2), schmiedbares Eisen (5). United States: bar iron (1), railroad bars, steel (13), plates and sheets, iron (9), plates and sheets, steel (15), wire (11), car wheels (1), nails and spikes (3), pig iron (4).