CHAPTER IV

SUPPLY, PRICE OR PRICE FIXATION EQUATIONS FOR LABOUR AND COMMODITIES

IV 0. SUPPLY EQUATIONS, PRICE FIXATION EQUATIONS AND PRICE EQUATIONS

In order to make it clear what is the theoretical background of the relations to be discussed in this chapter, some general remarks may be given first, partially taken from "Business Cycles in the United States"[1]). Using the language of static economic theory, the relations to be considered here may be either of two types: in the first place they may be supply relations which connect up price, quantity sold and certain supply factors, the characteristic of which is that they act only on the supply side, all demand factors being excluded.

Let p be the price, u^s the quantity supplied, F^s a supply factor (e.g. unit cost), then a *supply relation* will be of the form:

$$u^s = f_1(p, F^s)$$

a linear approximation will be of the form:

$$u^s = \omega_{1p}\, p + \omega_{1s}\, F^s \qquad \text{(IV 01)}$$

The well-known concept of *elasticity of supply* may easily be applied to this relation. Since elasticity is defined as the relation between the relative increase in quantity supplied, divided by the relative increase in prices, it will take the form:

$$\varepsilon^s = (du^s/dp) \cdot (p/u^s)$$

for any given pair of values p and u, and for the average values of these variables — which, in our context, will usually be taken —:

$$\varepsilon^s = (du^s/dp)\,(\bar{p}/\bar{u}^s) = \omega_{1p}\,(\bar{p}/\bar{u}^s) \qquad \text{(IV 02)}$$

The relations to be considered here, however, may also result from the combination of the demand and the supply relation, which is obtained by putting equal to each other the quantity demanded and the quantity supplied and then eliminating this quantity. Calling F^D any demand factor (e.g. income), the *demand relation* will be of the form:

$$u^D = f_2\,(p, F^D)$$

with a linear approximation:

$$u^D = \omega_{2p}\, p + \omega_{2D}\, F^D \qquad \text{(IV 03)}$$

[1]) Cf. chapter III.

Putting quantity demanded equal to quantity supplied, we get, in the case of the linear approximations:

$$\omega_{1p}\, p + \omega_{1s}\, F^s = \omega_{2p}\, p + \omega_{2D}\, F^D$$

which may be written:

$$(\omega_{1p} - \omega_{2p})\, p + \omega_{1s}\, F^s - \omega_{2D}\, F^D = 0$$

or:

(IV 04) $$p = \frac{\omega_{2D}\, F^D - \omega_{1s}\, F^s}{\omega_{1p} - \omega_{2p}} \quad (\text{if } \omega_{1p} \neq \omega_{2p})$$

In a more general form:

(IV 05) $$p = f_3\,(F^D, F^s)$$

Such a relation, which for the sake of brevity, may be called a *price equation*, connects the price with demand and supply factors, *but does not contain the quantity sold*. It will be clear that a system of two relations, consisting of a demand equation and a supply equation, is equivalent to a system of two relations, consisting of a price equation and either the demand or the supply equation, since the third equation in each case may be deduced from the others. So the choice of these three systems will simply depend on circumstances. Generally the demand equations will be included as such; these we already discussed in chapter III. In this chapter, either the supply or the price equations will be added.

This Walrasian description of market equilibrium, however, is only valid for some special types of market, where supply is determined independently of quantities demanded and where there is — as a necessary counterpart — absolute adaptability of prices. In many modern markets this is no longer the case. Prices are sticky and supply is based upon quantities ordered. In such markets the demand relation generally remains valid, although it may not, for psychological reasons, immediately react to prices, but only with a lag.

The supply relation rather takes the form of a *price fixation relation* — i.e. of a relation telling on what factors producers or sellers base themselves when fixing a price. This relation contains the same variables as the supply relations, but price is effect rather than cause and may therefore be lagging behind quantities and supply factors. Sometimes we shall refer to this relation as a "*quasi-supply relation*". Using the symbols just introduced, it may be written in the form:

$$p = f_1'\,(F^s, u^s) \text{ or}$$

(IV 06) $$p = \pi_F\, F^s + \pi_u\, u^s$$

which replaces (IV 01).

In this case it is logically preferable not to use the *elasticity* concept — which might then be used to calculate a "quasi-elasticity" — but to use the concept of *price flexibility* which, in our case, comes down to:

$$\varphi = (dp/du^s)\ (u^s/p)$$

or for the average values:

(IV 07) $$\varphi = (dp/du^s) \cdot (\bar{u}^s/\bar{p}) = \pi_u\,(\bar{u}^s/\bar{p})$$

The fixing of prices may also be affected by negotiations with the demand side (e.g. in the labour market) and thus also depend on a demand factor. But in this case the demand factor can always be eliminated again by using the demand equation, and therefore the price fixation equation in its first form may still be used.

The same type of analysis may be used for markets where stocks of unsold goods exist. Strictly speaking, there are usually two markets in such a case, e.g. one where producers sell to dealers, and one where dealers sell to consumers, but a slightly different description is also possible and sometimes easier. The two markets are considered together and demand is supposed to be exerted by consumers, supply by producers. The following differences with the standard case occur. No longer is u^s equal to u^D, but, indicating stocks by w and their rate of increase by $\dot{w}$, we have:

(IV 08) $$u^s = u^D + \dot{w}$$

At the same time, since a new variable comes in, one further relation is needed. This has something to say about the principles governing the accumulation of stocks. For an example we may refer to section IV 8.

Finally it may be observed that in cases of imperfect competition the number of competitors may be one of the factors in the supply or demand equation.

IV 1. PRICE EQUATION FOR WAGE RATES

(i) *Material, Theory.* The index of wage rates used is the one constructed by Mr Wood and is an average for a number of important industries. The fluctuations in the average seem to be caused chiefly by the fluctuations in wages for coal mining, as is very clearly shown in a graph in Rowe's "Wages in Practice and Theory", London 1928, p. 9. The wages in other industries only show very small cyclical fluctuations.

As to the *theory* to be used in the explanation of wage rates, we distinguish between supply factors and demand factors.

A first supply factor will be the level of the cost of living, since the higher this level is, the lower the satisfaction obtained from a given wage.

A second supply factor might be "previous employment"; which may even be the cumulation of all previous employment. The meaning of this theory can be understood best in the following way. If one supposes that the state of employment determines whether, and if so by how much wages will rise or fall, then

(IV 1,1) $$(dl/dt) = \dot{l} = f(a) = \lambda a$$

It then follows that the wage rate itself depends on the cumulation of all previous employment ($\int a$). The theory expressed in formula (IV 1,1) may be given the well-known formulation that a high unemployment figure "exerts a pressure on" the wage rate and that, on the other hand, a small unemployment figure causes wages to go up.

A third supply factor may be the degree of organisation (o) of workers. A highly organised group of workers will approach a monopoly situation and will be able to get higher wages than a group of unorganised workers. The degree of organisation may be approximated by the membership of trade unions, which is known for the years from 1892 onwards and for the period before that time by the members represented at the annual Congress of Trade Unions. In fact, it is emphasized by ROWLEY [1]) that between 1888 and 1890 there was a great spread of organisation among the labourers, which was partly responsible for the rise in wages at that time.

As to the demand factors, the most important will be the profitability of applying labour by entrepreneurs. This profitability may either be indicated by the course of profits or by the fluctuations in the prices of products, particularly export products. The weighted average of prices of export products in our model is $1{,}2\, p^e + q$, which was therefore introduced in a number of calculations. In a number of cases the influence of the prices of products on wages was even recognized very explicitly by the application of *sliding scales*. In the period we are concerned with sliding scales did not, however, play a very important rôle. The number of people under sliding scale conditions was estimated in 1910 to be only about 50.000[2]). As to wages in the coal industry ROWE [3]) observes (p. 35):

> "The extreme depression ....at the end of the seventies .... was the period during which wages were automatically regulated by prices according to sliding scales.......The general opposition to sliding scales gave life to reorganisation in the trade-union world. In 1881 the Yorkshire miners united and terminated their scale: Lancashire followed suit and in 1885 the Midland Federation ....was formed with the double object of abolishing sliding scales ....and of introducing the eight-hour day".

Nevertheless it seems that coal wage rates were still largely determined by the fluctuations in coal prices (p^C):

> "......but it shows how closely wages are in fact regulated by prices whether under the conciliation board system or under sliding scales". (ROWE, loc. cit. p. 42).
>
> "It will be observed how much more violently wages fluctuated in the two export coal fields than in the Federated Area". (ibidem, p. 47).

[1]) Wages in the United Kingdom in the 19th century, Cambridge 1900, p. 138.

[2]) A Report on Collective Agreements (Board of Trade, Labour Department), 1910 (Cd. 5366), p. 429.

[3]) Wages in the Coal Industry, London, 1923.

The reason for this sensitivety of wages to coal prices is to be found (Rowe, p. 120) in the small amount of capital invested:

> ".....a small increase in price means a very large increase in profits. Wages have in fact been formally regulated almost solely by the selling price of coal, either directly, as under the sliding scale, or indirectly, as by joint negotiations conducted on wellestablished principles".

(ii) *Statistical.* The result of a number of calculations, using the suggestions given at (i), are given in table IV 1. The influence of employment has, in some cases, been supposed to be curvilinear, since the graphs showed that this was necessary in order to get a good correlation. This is why, instead of employment, the inverted figure has been taken: $\bar{d} = 1$: unemployment percentage.

TABLE IV 1
"Explanation" of wage rate l

Nr.	Period ↓ / Variable →	Regression coefficients obtained for: p	p_{-1}	d_{-1}	$\int a$	$\int a_{-1}$	o	$1.2\ p^e+q$	$1.2\ p^e_{-1}+ q_{-1}$	p^C	p^C_{-1}	q	R
	Stand. dev. →	2.1	2.1	20.8	0.9	0.9	34.2	12.7	12.9	15.9	16.5	8.4	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1871–1910		0.091	0.019				0.097					0.910
2	1871–1910		0.096	0.059	0.51								0.864
3	1871–1910		0.104	0.058	0.54	0.054							0.864
4	1871–1910	0.126	0.082	0.053	0.42								0.870
5	1871–1910			0.015	0.36			0.084	0.031				0.945
6	1871–1910			0.011	0.34		0.0067					0.15	0.944
7	1871–1910									0.073	0.037		0.884
8	1871–1910				0.60					0.068	0.030		0.929
9	1871–1910											0.20	0.920

On the basis of these results the following factors have been rejected: cost of living, cumulated previous employment and degree of organisation, since their influence is found to be very small;
the variable $1.2\ p^e + q$ has been rejected, since, from direct analysis of the statistical series it seemed more natural to take coal prices only [1]).

The correlation chosen shows only a small deviation in the neighbourhood of the year 1907. Exactly at that time there was a discrepancy between the coal price quotation as chosen here and the other quotations given by statistics. It may therefore be that, if a better quotation had been taken, the deviations would have been eliminated. For the deter-

[1]) It may be observed that some of these calculations were made before it was known to the author that coal mining wages show by far the most important fluctuations; this experience is a remarkable example of how multiple correlation analysis may show one the solution of a problem even without theoretical preparation. The results given in table IV 1 are only really acceptable in those cases where, in some form or other, the price of coal or another variable showing a high correlation with it, i.e. $1.2\ p^e + q$, or q, is introduced.

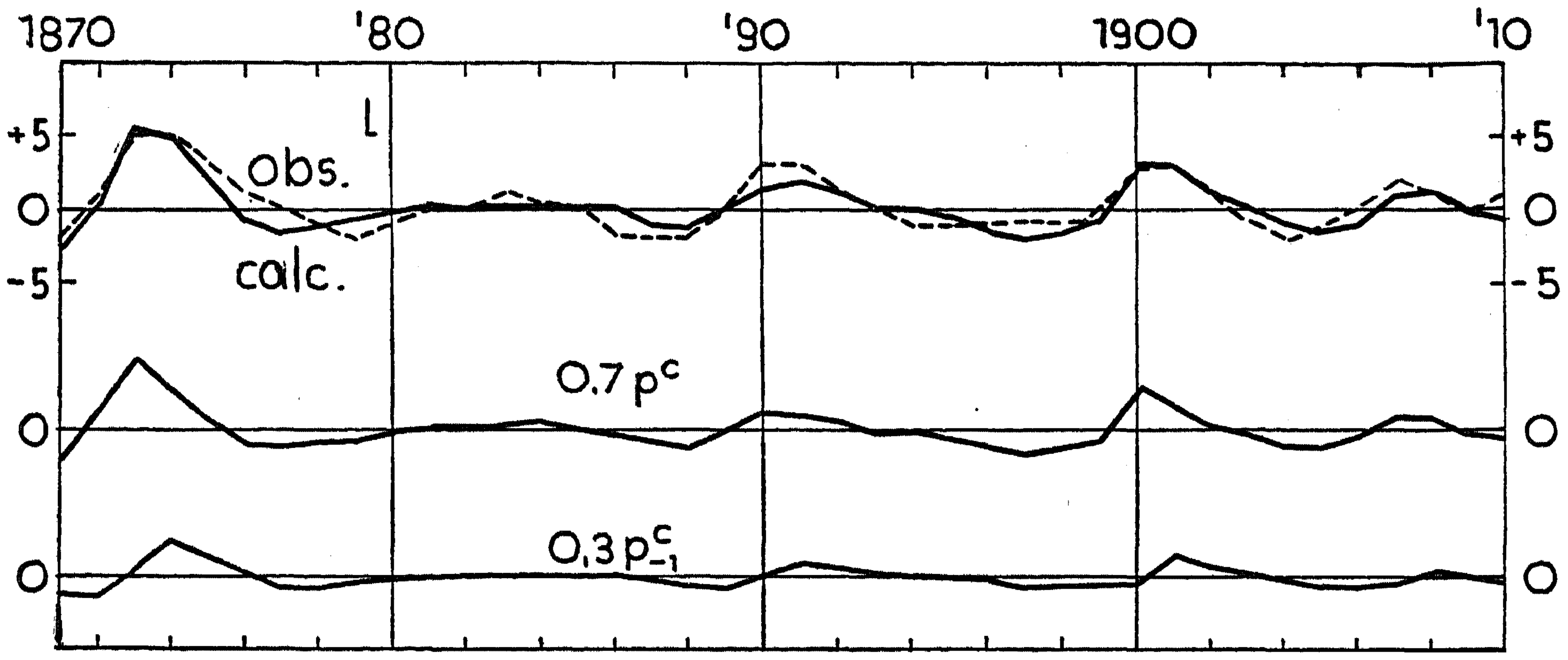
mination of the regression coefficient this does not, however, matter very much. The equation chosen is: $l = 0.11\, p^c_{-\frac{1}{3}}$. which may, however, in view of the very high correlation between $p^c_{-\frac{1}{3}}$ and q, immediately be replaced by

(IV 1) $$l = 0.20q$$

since, in the elimination process (chapter VIII), this is more convenient.

(iii) *Conclusions.* First of all, there appears to be little evidence of a change in the flexibility of wages. The fact, often emphasized, that there would be an increasing lack of adaptation of wages to the economic situation is therefore not supported by our calculations.

The circumstance that our price equation for wages only shows a demand factor means that the supply factors which did influence it were constant through time or at least did not show any cyclical movements.



IV 1. "Explanation" of wage rates.

IV 2. PRICE FIXATION EQUATION FOR THE COST OF LIVING

(i) *Test of index used.* The index of cost of living is composed of (*a*) a price index of services and (*b*) an index of retail prices. The weights in 1907 are 430 and 1430 respectively and are based on figures given in the General Report of the Census of 1907. The index has been compared with an index given by the Second Series of Memoranda, prepared by the Board of Trade. The composition of the second index is: food 7, rent 2, clothing 2, fuel and lighting 1. The agreement is fairly good.

(ii) *Theoretical.* We consider the vertical amalgamation of all branches engaged in some way or other in the production of consumers' goods and services, including also part of coal mining. In order to study the price formation of the corresponding total of consumers' goods and services, we start from the proposition that price equals marginal cost. Let us first consider the simple but not unrealistic case where total costs are a linear function of quantity produced, whereas variable cost consists of raw material and labour cost and fixed cost is supposed to be depreciation allowances, salaries, interest and taxes. All these latter items may be considered to be constant or, at least, to show a trend movement only. Since depreciation and interest e.g. depend on moving averages of prices with various lags, marginal cost may then with great approximation be taken equal to:

$$m.c. = \pi_1 l + \pi_2 r$$

The coefficients π_1 and π_2 will be equal to labour quota and raw material quota respectively. In reality conditions will not be as simple as supposed for a moment, and the following complications may occur:

(1) Fixed cost may be charged by the entrepreneur in the form of either (i) a fixed amount or (ii) a fixed proportion of labour and raw material costs. In this latter case the coefficients π_1 and π_2 will be higher than labour quota and raw material quota;

(2) Next, part of the labour cost may be of an overhead character which would lower the coefficient π_1;

(3) If it were strictly true that all wages are trends except coal mining wages (Cf. IV, 1), our coefficient π_1 would be equal to:

$$\pi_3 \, \pi_1^c / \lambda^c,$$

where π_3 is the coal quota in consumers' goods,

π_1^c is the wage quota in coal,

λ^c is the proportion of coal wages to all wages.

If coal were consumed proportionally to labour in consumers' goods, investment goods and export goods, this would not change the coefficient π_1, but as soon as these proportionalities in coal consumption exist, there will be a difference;

(4) Finally there may be somewhere in the "business column" (i.e. between the market of imported raw materials and retail markets) a market under world competition (e.g. flour, yarn, refined sugar). This would mean that prices would go up, if, for instance, such competitive prices rose in the world market, even if British cost of production should not. This state of affairs which is equivalent to the necessity of considering marginal costs of foreign producers, will enlarge the coefficient π_2 at the expense of π_1.

As a consequence of these four restrictions to the validity of the simple hypothesis originally made, the coefficients π_1 and π_2 may be different from the labour quota and raw material quota respectively.

Further complications may arise from:

(*a*) A curvilinear relation between volume of production and total cost of production. In the latter case marginal cost will also depend on the volume of production. Distinction may be made here between the case where marginal cost depends linearly on the volume of production and the case where even marginal costs themselves depend in a curvilinear way on this volume. This latter case seems to be present, if the volume of production approaches the capacity of production; then prices will go up more rapidly than with ample reserve capacity. This phenomenon is known as the bottle-neck phenomenon.

(*b*) Dynamic features. The word dynamic will be used throughout this study in the sense introduced by FRISCH. A relation is said to be dynamic, if variables relating to different time-points occur in it. One of the simplest ways in which dynamic features may present themselves is the lagged influence of some variable. In price formation there may be a lag between cost and price because of the habit to base prices on the actual (historical) costs that have been incurred during the production of the goods sold. Raw material prices may then be calculated at the value they had at the beginning of the production process or even at the moment they were bought, which may be somewhat longer ago in view of the existence of raw material stocks. Similarly wage rates may be taken at a moment when the production process is going on, which, on an average, will be half the production period before the moment of selling. For this reason lags have been supposed to exist of half a year for raw material prices and of a quarter of a year for wage rates. These figures are possibly not quite correct, but anyhow they are in rough agreement with the statistical observations and a further refinement seems hardly necessary.

Another dynamic feature would show itself if stocks should influence the course of prices. Stocks are the cumulation of unbought goods, produced in the past, and therefore depend on variables at earlier time points. The existence of stocks will in some cases be the reason of a divergency between prices and marginal costs and, in such a case, stocks would have to be added as an explanatory variable in our price fixation equation. In the case now under consideration they have not, however, been included. Stocks of ready consumers' goods do not seem to fluctuate very violently and therefore do not seem to influence prices very much. Stocks of raw materials and of semi-finished articles presumably have a great influence on prices and their influence on raw material prices will not be neglected (cf. section IV 5/6).

(iii) *Statistical.* The statistical work consisted of:

(*a*) the construction of the right raw material price index number;

(*b*) the determination by correlation analysis of:

(1) the regression coefficient and lag to be applied for u;

(2) the regression coefficient and best lag to be applied for r;

(3) the influence of l.

Ad (*a*):

A raw material price index has been constructed, whose weights correspond as exactly as possible to the relative importance which imported raw materials and home produced agricultural products have for total home consumption. In this price index the prices of non-perishable products had to be lagged over half a year. Since the chief non-perishable goods passing through a longer industrial process are textiles, the prices for textiles only were lagged. A summary of the various price index numbers and their components used in our calculations is given in table IV 20.

TABLE IV 20

Summary of price indices calculated (consumers' goods)

Symbol	Brief characteristic	Formula or Composition (weights [1]) in brackets)
r^i	imported raw materials	$(1/28)\ (16r^t + 12\,'r^i)$
r	all raw mat. — textiles lagged ½ year	$(1/63)\ (10r^t_{-\frac{1}{2}} + 12\,'r^i + 30r^F + 11p^i)$
r'	all raw mat. — text. ½, other non-per. ¼ year	$(1/63)\ (10r^t_{-\frac{1}{2}} + 33r^P + 20r^N_{-\frac{1}{4}})$
r''	all raw mat. — text. & other non-per. ¼ year	$(1/63)\ (10r^t_{-\frac{1}{4}} + 33r^P + 20r^N_{-\frac{1}{4}})$
r^t	textile raw materials	cotton, Am. (10), wool, Merino (5), jute (1).
$'r^i$	other imported raw materials	wheat, Am. (5), maize (2), beef, middling (1), petroleum (1), leather (1), barley (1), flour of wheat (1).
r^F	home produced farm products	butter [2]) (10), beef (5), mutton (3), potatoes (3), oats (3), pork (3), wheat (2), barley (1).
p^i	imported consumers' goods, ready for use	butter (3), bacon (3), sugar (3), beef (2).
r^P	perishable food & raw materials	butter (13), beef (8), mutton (3), potatoes (3), pork (3), bacon (3).
r^N	non-perishable food & raw materials, except textiles	wheat (7), maize (2), petroleum (1), leather (1), barley (2), flour of wheat (1), oats (3), sugar (3).

[1]) The weights indicated in brackets for the separate commodities have been applied to the index numbers (1867—1887 = 100) given by the editor of the Statist, in his annual articles in the Journal of the Royal Statistical Society. All composite index numbers, indicated by symbols, are on the basis of 1907 = 100. The error made by not converting the index numbers for the separate commodities into index numbers on the basis 1907 is only slight. The weights are roughly equal to the value of imports in 1907 or the value of production in 1909—1913 (as given by LEO DRESCHER, Die Entwicklung der Agrarproduktion Grossbritanniens und Irlands seit Beginn des 19. Jahrhunderts, Weltwirtschaftliches Archiv 41 (1935), p. 270), expressed in units of £ 7 mln. Commodities for which no quotations where included in the Statist index number have been omitted. Some commodities have been replaced by related goods (cf. foot note 2).

[2]) Standing for milk.

It was found that the influence of the type of index number chosen for the explanation turned out to be considerable, since the course of the prices for different raw materials was very different in the period studied. Both the correlation coefficients and the regression coefficients and lags were satisfactory for those cases where the above mentioned index number was used.

Ad (*b*):

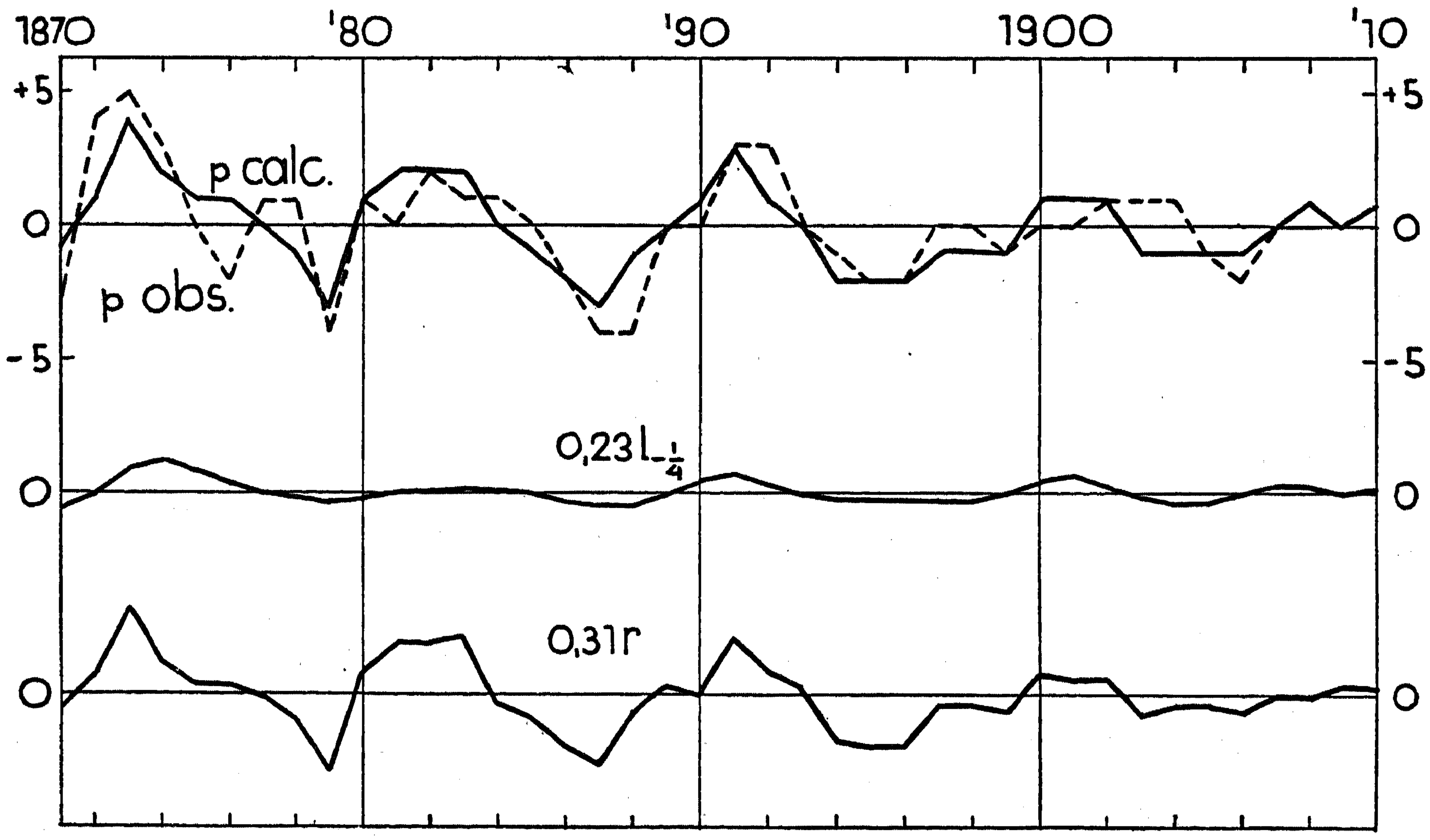
Table IV, 2 gives the result of various calculations made. Of these, formula 4 was chosen:

(IV 2) $$p = 0.31\ r + 0.23\ l_{-\frac{1}{4}}$$

TABLE IV 2
"Explanation" of cost of living p

Nr.	Period ↓ / Variable → / Stand. dev. →	Regression coefficients obtained for: r 4.0	r' 3.9	r'' 4.0	$l_{-\frac{1}{4}}$ 1.75	$L_{-\frac{1}{4}}$ 1.71	u 34.7	u_{-1} 34.6	R
1	2	3	4	5	6	7	8	9	10
1	1871–1910			0.33	0.19			0.00	0.780
2	1871–1910		0.32		0.23			0.00	0.765
3	1871–1910	0.34			0.28		−0.01	0.00	0.792
4	1871–1910	0.31			0.23			0.00	0.780
5	1871–1910	0.32				0.23		0.00	0.777

From this table one draws the conclusion that the influence of u seems to be negligible, meaning that the flexibility of prices was very small. A fortiori, no bottle-necks were found to have existed. The coefficient found for l corresponds fairly well to labour quota and the coefficient found for r to the part played by raw materials with an international market, although this latter part cannot, of course, be determined very exactly. The fact that the regression coefficient for r is well below 1 causes the well-known phenomenon that percentage fluctuations in cost



IV 2. "Explanation" of cost of living.

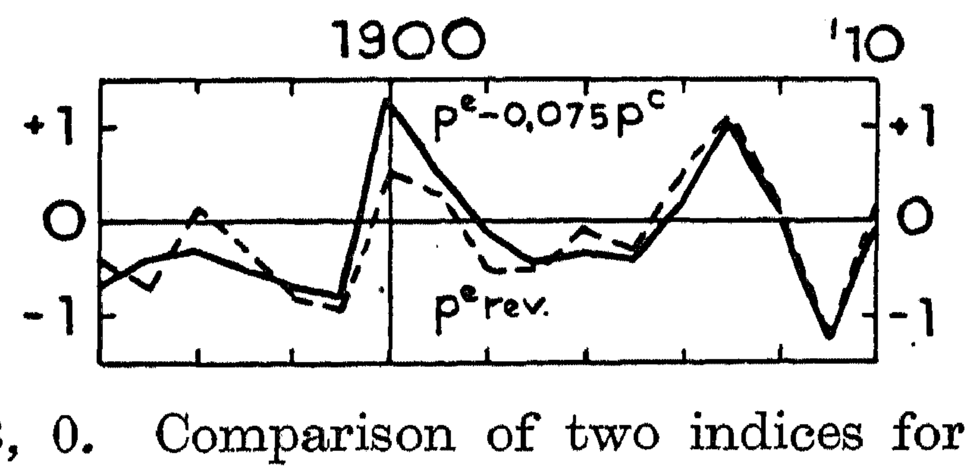
of living are far less than percentage fluctuations in raw material prices, and the lag found between raw material price and cost of living, also reflects a well-known experience.

(iv) *Comparison with other countries.* Both in the U.S.A. and in Holland a very small flexibility is found to have existed in post-war times. The coefficients found for raw material prices and wage rates vary according to the structure of the country. Since imports in the U.S. may be neglected, no corresponding term occurs in the explanation of the cost of living fluctuations for that country.

(v) *Results for separate commodities.* As a test some results for separate commodities were calculated. They relate to (*a*) bread, where no lag was found, (*b*) food in general, showing the same feature, (*c*) coal, also showing the same feature and (*d*) cotton piece goods. In the latter case a clear lag exists (cf. section IV 3) and an influence of the volume of production seems perceptible. This may be attributed to the far more elastic character of the demand for these goods, causing heavier fluctuations in the volume of production and therefore the possibility that they influence to a greater extent the price level of these goods.

IV 3. PRICE FIXATION EQUATION FOR CONSUMERS' EXPORT GOODS

(i) *Material and tests.* The price index for exported consumers' goods has been found as the ratio between value of export of consumers' goods and a quantity index. It has been compared in graph IV 30 with another price index, calculated directly by means of some prices. The agreement is very satisfactory. Since export consists to a very considerable part of coal and textiles and since the price formation of coal is rather different from that of textiles, coal has been taken separately and the statistical explanation was given for the index corrected for coal prices.



IV 3, 0. Comparison of two indices for p^e.

(ii) *Theoretical.* The same theory may be applied as in section IV 2 (ii) and therefore the variables introduced in the explanation are r^i, l and u^e. The lag for r^i was taken at half a year, one quarter of a year and three quaters of a year respectively, in order to find out, whether the lag for export prices was different from that of the cost of living.

(iii) *Statistical.* The results of the calculations are given in table IV 3, from which the following conclusions may be drawn. The cases with a lag of three quarters of a year in r^i show coefficients for this variable far below the raw material quota; they have to be rejected. The remaining cases do not differ very much from each other. The negative coefficient for u^e would seem somewhat strange, although not impossible. Case 1 has been chosen:

$$p^e = 0.30\, l_{-\frac{1}{4}} + 0.28\, r^i_{-\frac{1}{2}} + 0.13\, u^e_{-1} + 0.075\ p^c$$

In this equation we already substitute our variable q for p^c; this yields:

$$\text{(IV 3)} \qquad p^e = 0.30\, l_{-\frac{1}{4}} + 0.28\, r^i_{-\frac{1}{2}} + 0.13\, u^e_{-1} + 0.04\, q_{+\frac{1}{3}}$$

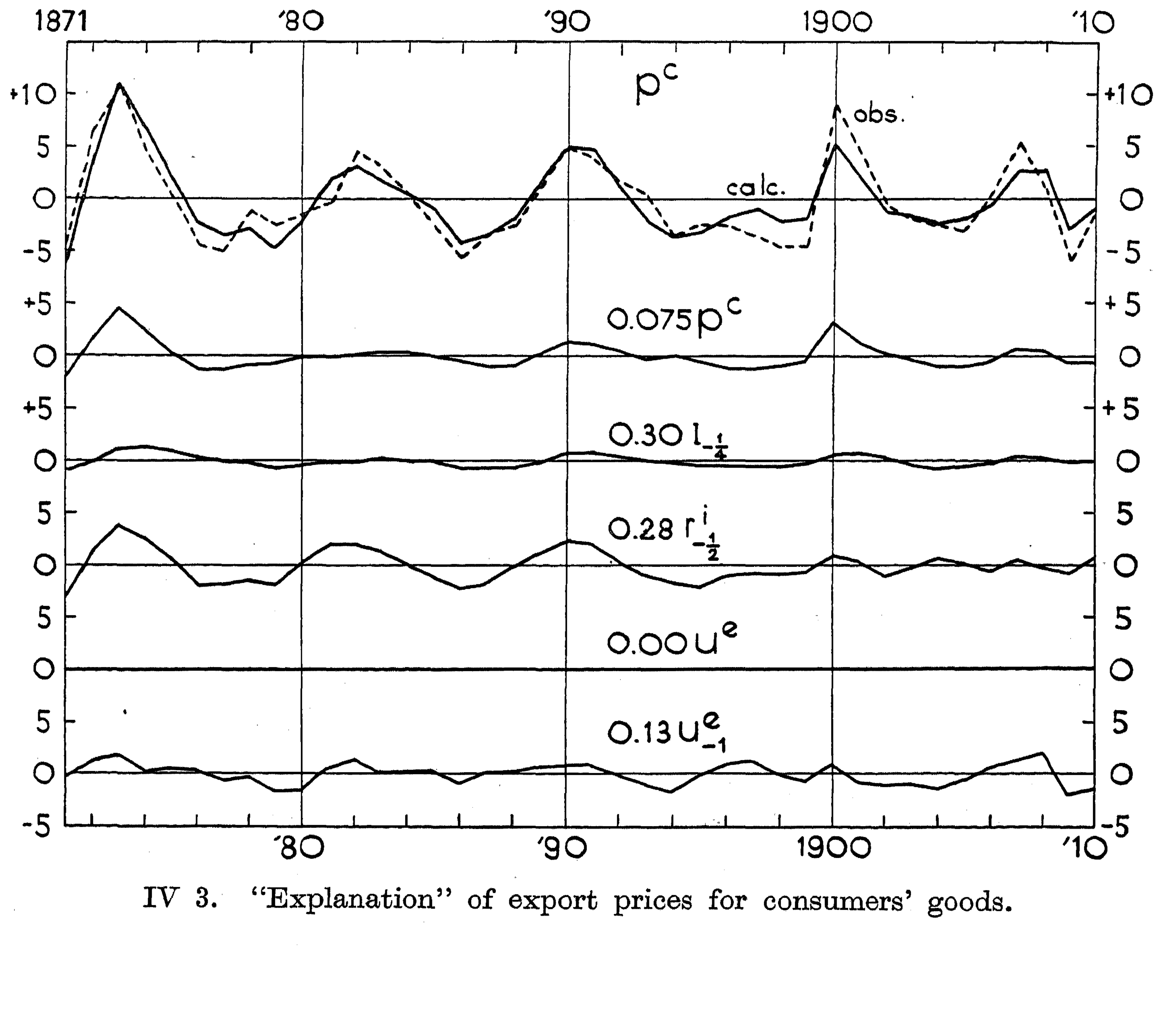
TABLE IV 3

"Explanation" of prices of export goods p^e

Nr.	Period ↓	Regression coefficients 1) for:							R
	Variable →	$l_{-\frac{1}{4}}$	$r^i_{-\frac{1}{4}}$	$r^i_{-\frac{1}{2}}$	$r^i_{-\frac{3}{4}}$	u^e	u^e_{-1}	p^C	
	Stand. dev. →	1.75	4.05	3.95	4.23	7.5	7.5	15.9	
1	1871 – 1910	0.30		0.28		0.00	0.13	*0.075*	0.83 2)
2	1971 – 1910	0.35	0.21			–0.04	0.14	*0.075*	0.85 2)
3	1871 – 1910	0.40			0.07	0.00	0.11	*0.075*	0.80 2)
4	1871 – 1910	0.42	0.20			.	0.13	*0.075*	0.85 2)
5	1871 – 1910	0.36			0.07	.	0.12	*0.075*	0.79 2)
								0.075	

1) Figures in italics represent regression coefficients chosen a priori.

2) Correlation coefficient for "explanation" of $p^e - 0.075p^c$.



IV 3. "Explanation" of export prices for consumers' goods.

It is interesting to note that the coefficient obtained for u^e_{-1} in table IV 3 is very stable. Interpreting equation (IV 3) as a quasi-supply equation (cf. section IV 0), we may calculate the elasticity of supply from this coefficient. Considering export goods as a separate type of good, this elasticity will be:

$$\frac{1}{0.13}\,\frac{\bar{p}^e}{\bar{u}^e} = \frac{1}{0.13} \cdot \frac{90}{180} = 3.8$$

(iv) *Results for separate commodities.* Results, quite similar to those obtained for the group as a whole, may also be obtained for cotton yarn and cotton piece goods separately. Since, however, the bulk of textile exports consists of these goods, they hardly represent any new case and may therefore be left out of consideration here.

IV 4. PRICE FIXATION EQUATION FOR INVESTMENT GOODS.

(i) *Material.* The index chosen is an average of prices of iron and steel products and of machines, the weights of which have been taken as variable, in agreement with the relative importance these products had in British exports. This index is therefore only accurate for exports. For home consumption of investment goods it has been assumed that the same prices prevailed. It is not certain whether this also holds good for building. The index given by JONES in his book "Increasing Returns" shows fluctuations of much smaller amplitude than our index q. This fact has been discounted for in our equation for V' (section II 10).

(ii) *Theoretical.* The theory applied is the same as that used in section IV 2. For the volume of production European production of iron has now been chosen, since the market for the products considered is decidedly international [1]). Stocks have not been included here, since finished investment goods are usually only produced to order. In the explanation of raw material to investment goods, however, the stocks of these goods have been included (cf. section IV 7).

(iii) *Statistical.* The results of some calculations are given in table IV 4. The use of a free coefficient for l is rather dangerous in this

TABLE IV 4
"Explanation" of prices of investment goods q

Nr.	Period ↓	Regr. coeff. for:			R
	Variable →	$l_{-\frac{1}{4}}$	$s_{-\frac{1}{4}}$	${}_Rv^E_{-1}$	
	Stand. dev. →	1.75	9.24	6.60	
1	1871—1910	1.93	0.50	0.11	0.952
2	1871—1910	*0.20*	0.76	0.11	0.952 *)

*) Correlation coefficient obtained for "explanation" of $q - 0.20\ l_{-\frac{1}{4}}$.

[1]) It has been expressed in percentage deviations from its own trend and is written ${}_Rv^E$.

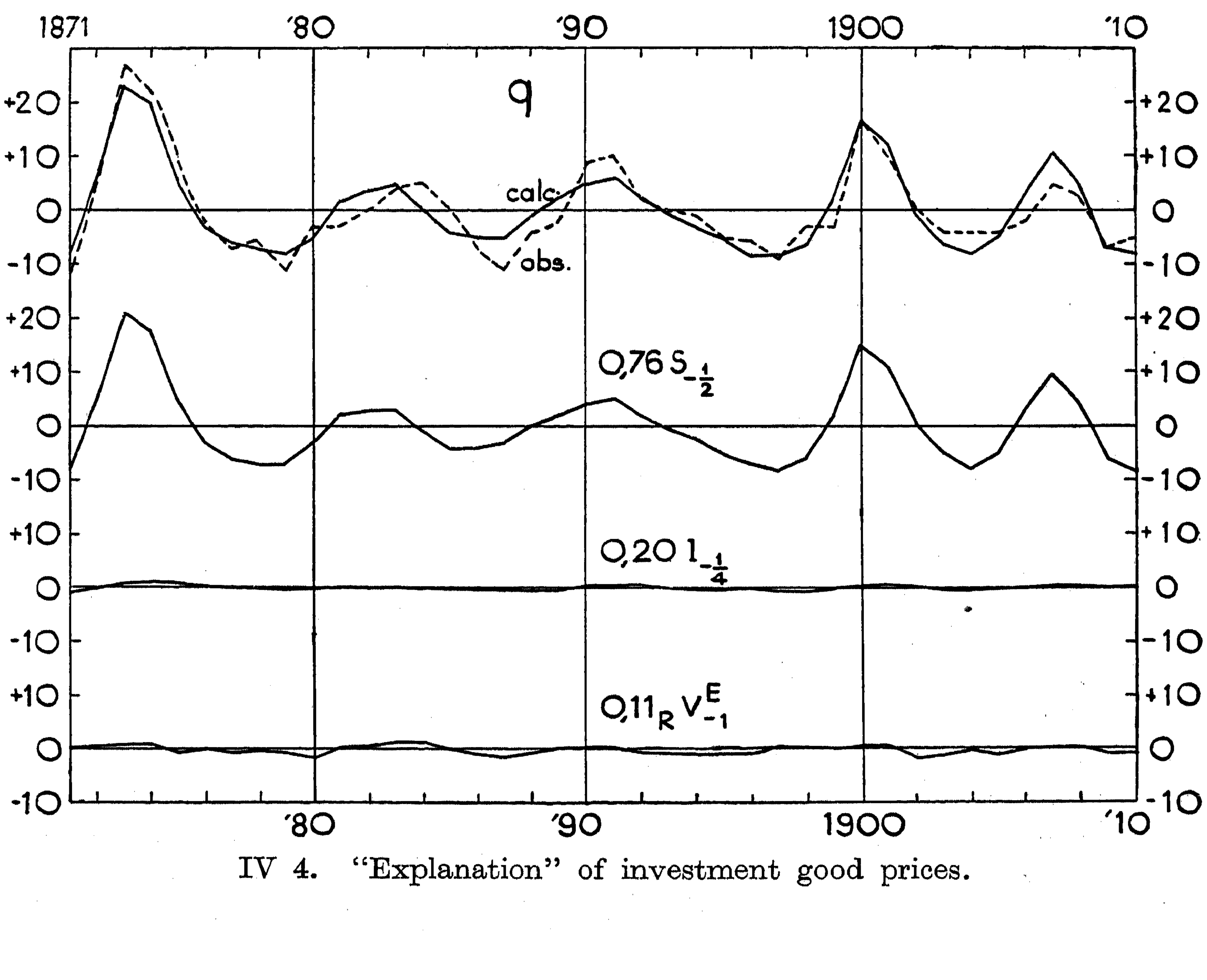
case, since there is a high correlation between q and l and therefore there will be a tendency for the regression coefficient to l to be overestimated. As shown in relation II 3, there is some evidence that the labour quota is about 0.20. Therefore a calculation with an a priori coefficient of 0.20 for l has also been included and has finally been chosen. The regression coefficient found for the volume of production does not change very much in the various cases considered and therefore seems to be rather certain. The high coefficient found for the raw material price seems to reflect the situation that the semi-finished products of the iron and steel industry have a competitive international market (cf. section IV 2 (ii) (2)).

The formula chosen runs:

(IV 4) $$q = 0.76\, s_{-\frac{1}{2}} + 0.20\, l_{-\frac{1}{4}} + 0.03\, v_{-1}$$

This formula has been obtained by replacing the external variable ${}_R v^E_{-1}$ by an internal one, viz. v_{-1}; the regression coefficient was found by multiplication by σ'/σ'', where σ' is the standard deviation of ${}_R v^E$ and σ'' that of v.

(iv) *Comparison with other countries.* For the U.S. (post-war) a considerable influence of the volume of production was found, whereas, on the other hand, almost no such influence was found for Holland. The United Kingdom seems to hold an intermediate position. No doubt this depends on the degree of foreign competition to which each of these countries is exposed. In the U.S. this competition is practically

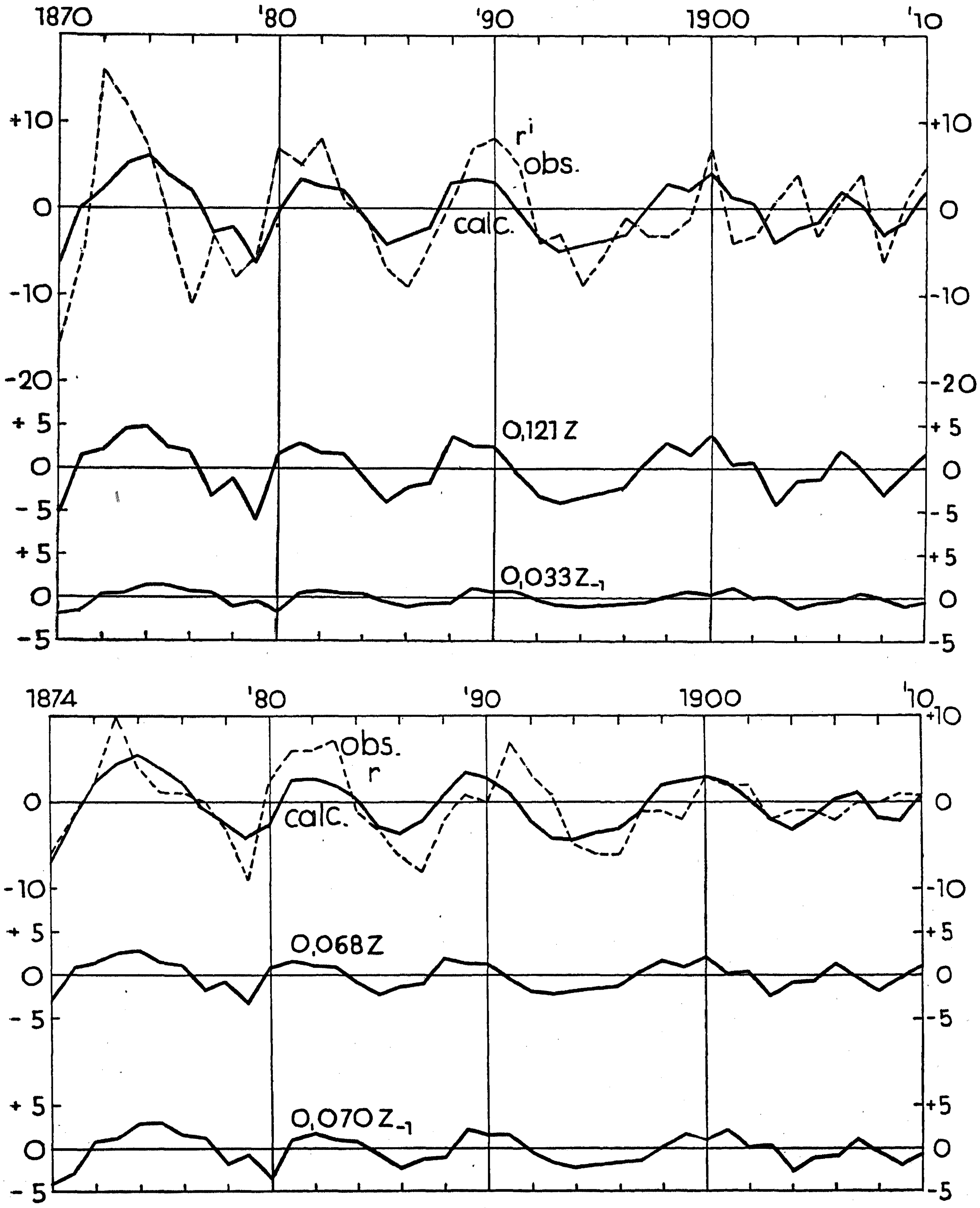


IV 4. "Explanation" of investment good prices.

absent, whereas in Holland it is very strong and prices in the latter country are dictated by foreign competition. Before the war England had such a predominant position in international competition concerning investment goods that this may explain the presence of a moderate term of the type now under consideration.

IV 5/6. PRICE EQUATIONS FOR CONSUMERS' GOODS RAW MATERIALS

(i) *Theoretical.* In section IV 2, two price indices of consumers' goods raw materials have finally been retained in our formulae. They



IV 5/6. "Explanation" of prices of raw materials for consumers' goods.

need further "explanation". The form of a price equation (cf. IV 0) has been chosen, i.e. "explanation" by supply and demand factors. The markets concerned may be considered as competitive and are international. The chief supply factors are the crops, plus carry-over of agricultural raw materials. These figures are, to a high extent, fluctuating at random. Since, in addition, they are almost entirely of an external character, their influence does not interest us and may be considered as residual in the correlation calculation.

The chief demand factor is the income of buyers in the markets concerned. This may be represented by Z, non-labour income. In order to account for the possibility of lags and of expectations, based on the rate of increase in Z, both Z and Z_{-1} have been included as explanatory factors.

(ii) *Statistical.* The results obtained are:

(IV 5) $$r^i = 0.121\,Z + 0.033\,Z_{-1} \qquad R = 0.575$$

(IV 6) $$r = 0.068\,Z + 0.070\,Z_{-1} \qquad R = 0.655$$

The correlation obtained is not very high. But this must be attributed to the rather great influence of crop fluctuations.

IV 7. PRICE FIXATION EQUATION FOR RAW MATERIALS FOR INVESTMENT GOODS

(i) *Theoretical.* The variable to be "explained" is SAUERBECK's index for minerals. It is composed of iron (2 quotations), copper, tin (1 quotation each) and coal (2 quotations), the prices of which are highly parallel. Therefore, although we do not include coal in the class of investment goods, it does no harm to leave it in. It shows a very high correlation with iron prices which, for a large part, must be attributed to the parallelism of demand factors and the similarity of supply conditions for the goods included. Since, moreover, this price index will chiefly be used as a basis for the price of finished investment goods, in which iron prices play a predominant rôle, the "explanation" will be given in the hypothesis that it bears on the iron price. It will be given the theoretical shape of a price fixation equation.

The price will tend to equal the cost of production of the marginal producer, which changes if

(*a*) cost factors change, of which labour cost, to be represented by wage rates l, is the most important (other costs fluctuate but little) and

(*b*) the margin of production moves: if a larger part of productive capacity must be used, a higher price must be earned even without any change in cost factors.

Apart from these factors there may be an influence of (*c*) stocks of pig iron. These may be said to represent the strength of competitive force: high stocks mean a permanent threat of competition; low stocks mean a "producers' market".

Factor (b) will be represented by the ratio of production to productive capacity. The best series available is $\overline{\overline{v}}^R$, the ratio between furnaces in blast and total number of furnaces present. This will be used in the correlation calculations. In order, however, to obtain an expression in variables already introduced, we shall transform this $\overline{\overline{v}}^R$. It is almost equal to

$$\frac{\overline{\overline{v}}}{\overline{\overline{v}}^C},$$

where $\overline{\overline{v}}$ is production of pig iron and $\overline{\overline{v}}^C$ is productive capacity, both expressed in tons; the unit is indifferent and we take it as usual (millions of 1907 pounds sterling). Our variable $\overline{\overline{v}}^R$ would be exactly equal to this ratio, if all furnaces were of the same size. Since, as a rule, the better furnaces are the larger ones, the ratio will be somewhat larger than $\overline{\overline{v}}^C$. According to our habit to establish linear equations, the ratio will be expressed as a linear function of v and v^C. Productive capacity v^C itself depends, however, also on v; roughly it may be said to be the cumulation of v, since investment activity in blast furnaces will, as a rule, fluctuate parallel to general investment activity:

$$v^C = \frac{1}{e}\overline{\overline{I}} = \frac{1}{e}\int_o^t \overline{\overline{v}}dt = \frac{1}{e}\sum \overline{\overline{v}}_{-\frac{1}{2}}$$

Hence we have:

$$\overline{\overline{v}}^R = \frac{e\overline{\overline{v}}}{\overline{\overline{I}}} = \frac{e}{\bar{I}}\left(\bar{v} + v\right)\left(1 - \frac{I}{\bar{I}}\right) = \frac{e}{\bar{I}}\left(\bar{v} + v - \frac{\bar{v}\,I}{\bar{I}}\right)$$

It follows that

$$v^R = \frac{e}{\bar{I}}v - \frac{e\,\bar{v}}{\bar{I}^2}I$$

Now the second term shows considerably smaller fluctuations than the first one. The fluctuations of v^R are, therefore, almost parallel to those of v, and the unknown proportionality factor $(e/\bar{I})$ may simpy be taken equal to the ratio of the standard deviations of v^R and v respectively, which equals 4.7 : 22.1 = 0.21. In order to find the coefficient for I, we have to evaluate $\bar{I}$. A minimum value for $\bar{I}$ is 6500 (it is the cumulation of $\overline{\overline{v}}$ over some 20 years and taken for the middle of the period). A maximum value is thus obtained for $e\bar{v}/\bar{I}^2$ at $0.21 \times (380/6500) = 0.012$; since it will be shown that this coefficient has a very small influence, it is sufficient to have a maximum value and we need not bother about a more exact determination. Hence we shall replace v^R by $0.21\ v - 0.012\ \Sigma\, v_{-\frac{1}{2}}$.

(ii) *Statistical.*

1. In accordance with the above theory, an effort has been made to approximate the fluctuations in s by a function of l, w and v^R.

2. Correlation calculations with a "free" coefficient for l yield too high values for this coefficient. It has therefore been chosen *a priori.* Since the fluctuations of l are not very important, it is not necessary to determine its coefficient very exactly. It has been taken equal to 1, which is the maximum value it could ever attain.

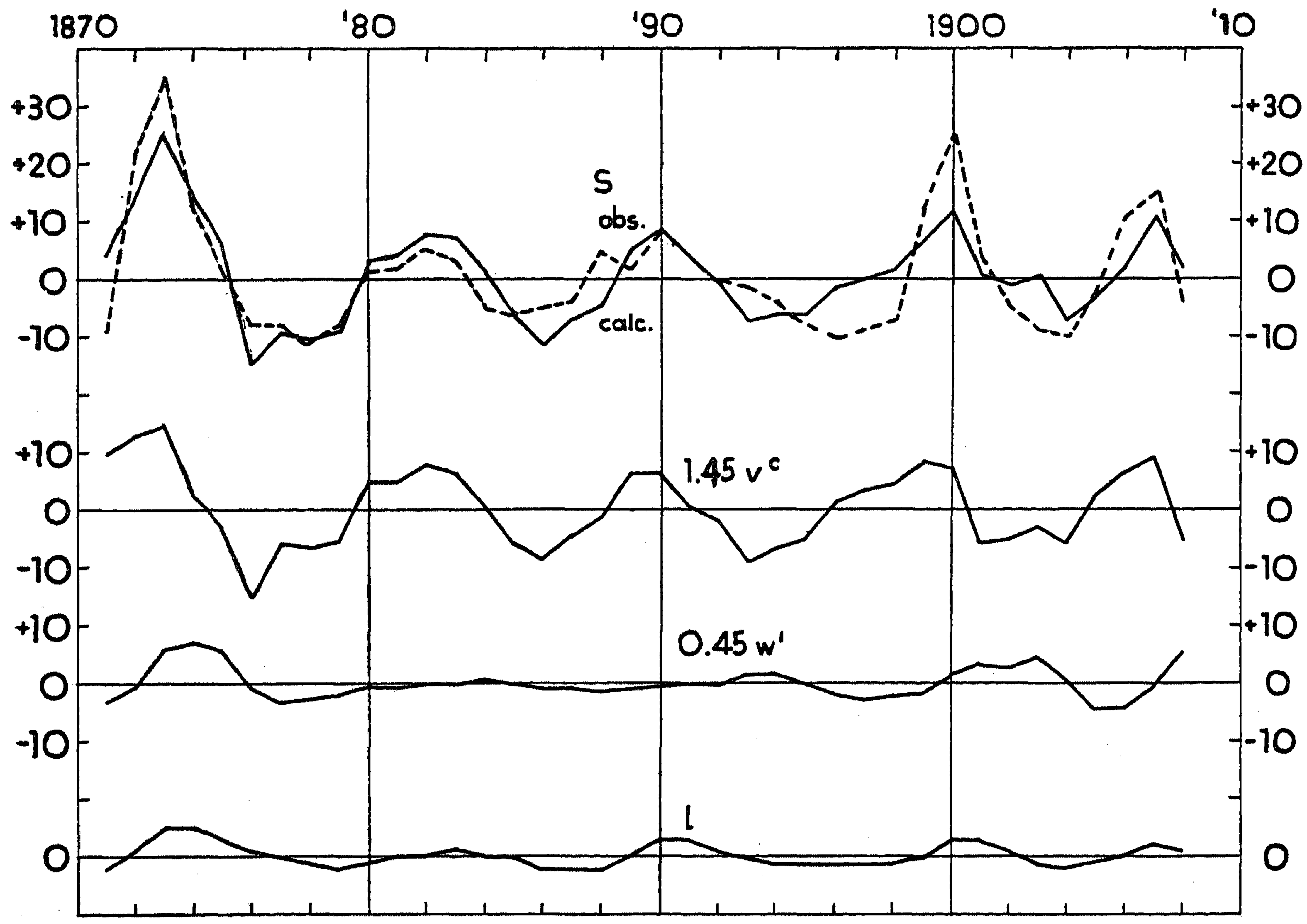
3. From a priori reasoning as well as from a graphical survey it appears that the influence of stocks is curvilinear: with small stocks left, a given further decrease has a stronger influence than with ample stocks. In principle this influence must be about hyperbolic, i.e. for stocks near zero it must be very great, for very large stocks it must be near zero. Therefore inverted stocks $\overline{\overline{w}}' = 36/\overline{\overline{w}}$ have been used as an explanatory variable, the figure 36 being, of course, arbitrary.

The formula obtained was:

$$s = l + 0.45\, w' + 1.45\, v^R$$

which was transformed, owing to the remarks made at (i), into:

$$\text{(IV 7)} \qquad s = l + 0.45\, w' + 0.30\, v - 0.018\, \Sigma\, v_{-\frac{1}{2}}$$



IV 7. "Explanation" of prices of raw materials for investment goods.

The elasticity of supply may be calculated from this formula to be

$$\frac{1}{0.30} \cdot \frac{\bar{s}}{\bar{v}} = \frac{1}{0.30} \frac{77}{417} = 0.6,$$

which is considerably lower than the elasticity found for consumers' goods.

IV 8. THE SUPPLY OF INVESTMENT GOODS

Since our figures do not permit of very detailed calculations on this subject, we shall restrict ourselves to a very rough approach. The chief reason why stocks of pig iron are periodically accumulated and exhausted again seems to be the resistance against changes in production, as a consequence of the high costs necessary for the stopping and the starting of production by blast furnaces. This leads to production lagging perhaps somewhat behind and showing smaller fluctuations than demand. The general formula expressing this state of affairs would be $v_t = a(v' + v^e)_{t-\theta}$, where v is production, $v' + v^e$ is demand, a and θ are constants and $a < 1$. Since $\dot{w} = v - (v' + v^e)$, it would follow that

$$\dot{w}_t = a\,(v' + v^e)_{t-\theta} - (v' + v^e)_t,$$

which may be transformed into:

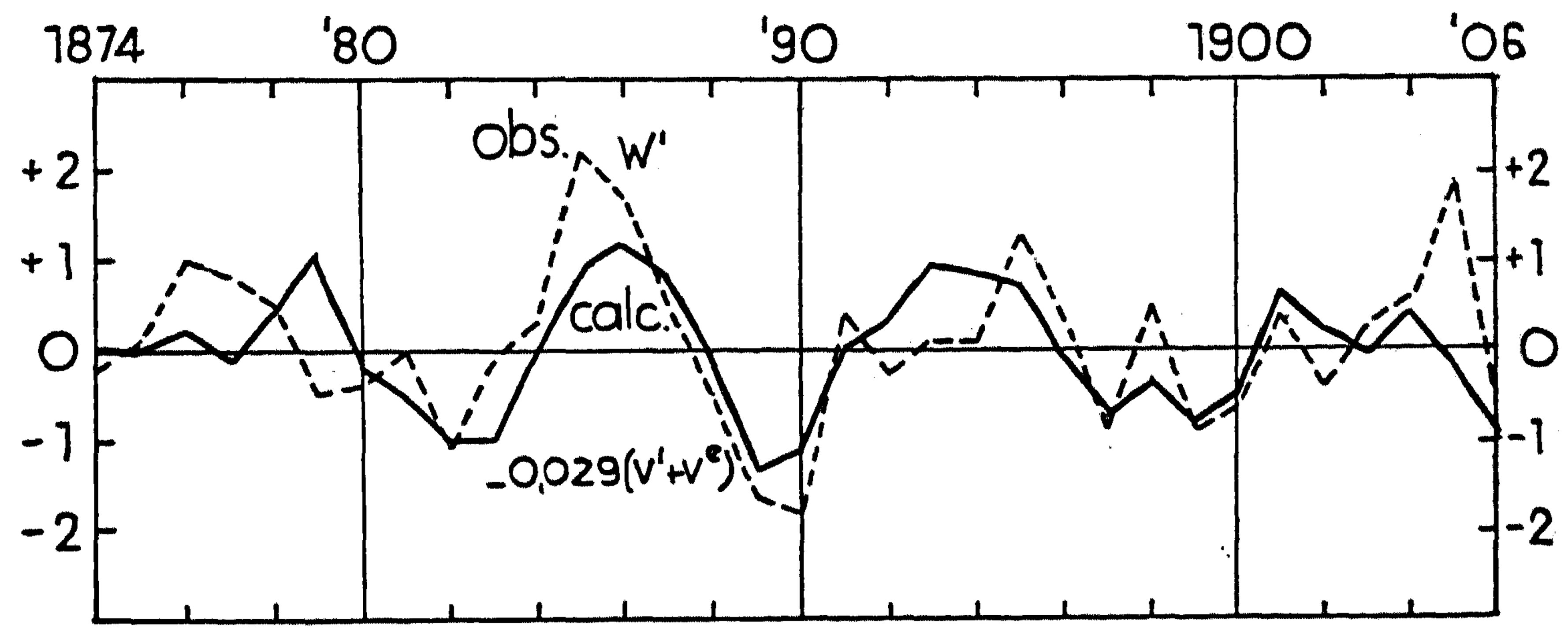
$$\dot{w}_t = -\,\omega_1(v' + v^e)_t + \omega_2(v' + v^e)_{t-1}$$

since for any variable x, $x_{t-\theta} = x_t - \theta(x_t - x_{t-1})$, approximately.

Inspection of the graphs and correlation calculations showed that the term with $(v' + v^e)_{t-1}$ may be neglected and that a reasonably good fit is obtained by taking

$$\text{(IV 8)} \qquad \dot{w} \approx -\,0.03\,(v' + v^e)$$

One of the consequences of this relation — confirmed by the figures — is that stocks themselves show their minima some time after the top of the cycle and their maxima some time after the depression. They are moving in negative correlation with productive capacity which shows its maximum shortly after the top and its minimum shortly after the trough of depression. Since both have a negative influence on iron prices, these influences are of opposite direction. The one exercised by iron stocks appears, according to our analysis of the preceding section, to be the stronger.



IV 8. "Explanation" of increase in pig iron stocks.