CHAPTER VII

SURVEY OF CHARACTERISTICS OF OUR MODEL AS SUGGESTED BY STATISTICAL FINDINGS

1. THE PLACE OF STATISTICAL MEASUREMENT IN THIS STUDY

Let us now take a bird's-eye view of the characteristics of the model as described in the preceding chapters, in particular of the quantative findings resulting from our correlation analysis. Before doing so, and, in order to avoid misunderstandings, we must clearly state the rôle that statistical measurement is supposed to play in this study.

The statistical calculations we have made serve two ends, that could be called *testing* and *measurement*. They *test* the theory or theories embodied in our hypothesis, i.e. in the general mathematical form of our equations, since they tell us what degree of correlation is obtained by using these hypotheses. Since, in addition, theories yielding correlations below, say 0.75, have generally been rejected, there is already some guarantee that our hypotheses do not directly contradict the figures. Moreover they perform a *measurement* of the coefficients and lags occurring in the equations, so far as they indicate the values of these coefficients for which the best agreement with observations is obtained. In the most favourable cases a small change in these coefficients will already yield considerable deterioration of the correlation; then, provided that the general form of the equation is right, the calculation yields a fairly exact measurement of the influences of the various causes assumed to co-operate. By the general form of the equation we mean the choice of the variables included, as well as the form of the mathematical functions assumed. Hence, in three respects our measurement is only conditional. Even if we have a good correlation, it may be

(i) that with other variables included an equally good or better correlation could be obtained;

(ii) that this is the case with another mathematical function of the variables included; and

(iii) that rather wide changes in the coefficients and lags chosen do not invalidate the correlation very much.

Point (i) must be judged by *economic considerations*, whereas points (ii) and (iii) may, to some extent, be tested *statistically*. The importance of the restrictions they represent varies from one case to another. There

will be a number of equations where practically no doubt exists as to the variables to be included; there are other ones where various alternative hypotheses could be formulated. Some of there alternatives which seemed to the author the most probable ones have been tried and mentioned in the text. Critical readers will, no doubt, be able to suggest other alternatives. They must not, however, think that it is a very simple matter to obtain a "good fit".

As a rule, the restrictions embodied in point (ii) are not serious; it is rather easy, to a practised eye, to discover whether, in a given correlation, a considerably better test could be obtained by another mathematical form of the formula used. By the method of partial scatter diagrams [1]) this may even be done systematically. Moreover the well-known statement that almost any function can be approximated, for small intervals of the variables included, by a linear formula, is of substantial help. In business cycle analysis only such *small intervals* are important as a rule, more particularly when problems of stabilisation are studied. This is a characteristic difference compared with some other problems of economics, such as, e.g. the interpretation of ENGEL curves. Here, wide intervals of incomes must necessarily be considered. Total incomes of the community, however, only fluctuate moderately as compared with the divergency between high and low incomes, present at one particular moment.

The restrictions of point (iii) are sometimes serious, but they can clearly be recognised, if certain hypotheses be made on the nature of the disturbances in the series or in the relation. Computation of standard deviations in the regression coefficients [1]) or of bunch maps [1]) may help here. In some of the sections of chapters II to VI we have given examples. Sometimes an impression of the stability of the regression coefficients was obtained by not computing the complete bunch map, but some of the more important relations between the variables. Therefore we are fairly well informed about the restrictions of this type.

We will arm ourselves somewhat against misinterpretations by considering a number of alternative models, where either other values for some uncertain coefficients or even other relations are substituted for the coefficients and relations assumed in the original model. The nature of these alternatives and their implications will be treated in chapter VIII.

2. SUMMARY OF STATISTICAL FINDINGS AND THEIR INTERPRETATION

With the reserves formulated in the preceding section in mind, we shall now summarise the results of the analysis of chapters II to VI. This amounts to giving a *picture of the structure of British economic life* during

[1]) For this concept, cf. e.g. A Method and Its Application to Investment Activity, chapter II.

the period studied. This picture shows an "average" structure and does not pay attention to possible changes that occurred during that period.

The structure of an economy is characterised by various types of figures. In our language, these types may be enumerated as follows:

(*a*) the average or trend values of the variables, indicating e.g. the *relative importance of one section of economic life with regard to other sections.* These figures can best be taken from table I H. Here we find what is the comparative importance of imports and exports against home production, what proportion of income is wages, what is the relation of deposits to national income, and so on. Most of these relations are well-known figures, which, moreover, have been taken from various recognised sources. They require no further comment.

(*b*) the *coefficients* and *lags* occurring in our equations, indicating e.g. the *elasticities* of various demand and supply functions and the *lags* involved in certain reactions. These figures are more particularly the result of our analysis and are believed to be additions to our knowledge. We shall therefore dwell somewhat longer on their consideration.

(i) Considering first the *technical relations* between production, employment and imports of raw materials, we find that employment shows a remarkable lack of responsiveness with regard to an increase in production: the elasticity of employment with respect to production being about one half only. Imports of raw materials seem to obey an "*acceleration principle*", tending to make stocks proportional (or at least moving parallel) to imports and therefore tending to make imports vary parallel to the rate of increase in imports about half a year ago. On the other hand not much evidence is found for the acceleration principle to determine the short-run fluctuations in investment in fixed capital goods.

(ii) Turning, secondly, to the *demand functions* for consumers' and investment goods on various markets, we find that *demand is almost everywhere inelastic.* This applies to home as well as to foreign demand for both types of goods. It need not necessarily be true for each good separately as well; and it is found not to be so for the demand for British pig iron, tin plates and railway bars in the United States and for British cotton goods in Egypt. On these markets a severe competition existed already from the start of our period; it may be observed that all these goods are mass-products. The low elasticities found for exports as a whole may be understood by the character of the chief goods exported (cotton and woollen yarns and tissues, coal and machinery) and from the fact that, as a whole, England had not to face serious competition. Of course this changed rapidly after 1900 and the consequences will be discussed later (cf. chapter IX, § 5 (vi)). The following figures have been obtained and are given in order to fix the ideas, without claiming a high degree of accuracy for any of them individually:

Investment goods		*Consumers' goods*		
Elasticity of home demand	0.06	Elasticity of home demand		0.2
Relative elasticity [1]) of exports	0.5	Relative elasticity [1]) of exports		0.3
Total elasticity [2]) of exports	0.7	Total elasticity [2]) of exports		0.3
		Substitution elasticity of imports		0.2
Competition elasticity [3]) of:		Competition elasticity [3]) of:		
Pig iron on American market	3.75	Cotton goods on Indian market	1896–1918	0.9
Tin plates, ditto	4.7		1896–1913	0.6
Railway bars, ditto	4.4	Cotton goods on Egyptian market		4.0

[1]) This term is used to indicate the elasticity derived from the term with the price difference between British and "world" prices in equation III 5 or III 2.

[2]) This term is used to indicate the elasticity derived from both terms containing British prices; in equation III 2, the term with p^e alone is, however, negligible.

[3]) This term indicates the elasticity of demand for British goods as a percentage of total demand with respect to British prices as a percentage of the competitors' prices; it becomes equivalent to the concept of total elasticity, as soon as total demand is assumed to be constant.

(iii) The *supply* of home products is, as a rule, found to be *elastic*; for consumers' goods for home consumption the elasticity to be derived from their price equation (IV 2) would even be infinite; for export goods, if these could be considered as a separate type of goods, it would be 3.8 — and if it should not be a separate type of goods, the figure would be higher still — ; for investment goods, however, it amounts to only 0.6. High elasticity of supply is characteristic of *industrial* output, in particular if capacity is not fully utilised. It may be that the lower figure for investment goods and export goods is to be attributed to the fact that in these branches years of full utilisation of plant are more frequent than in general consumers' goods branches.

The supply of *agricultural* raw materials has not been studied in a direct way, but the equations assumed to be valid for the prices of imported raw materials for consumers' goods imply that this supply is *less elastic*. If it were completely elastic, prices of agricultural raw materials would not depend on demand factors such as British incomes, which they were assumed to do. Looking at the amplitudes of the price and quantity fluctuations and disregarding possible cyclical fluctuations in costs (which may simply mean that we consider all stages of their production taken together) we may roughly derive the elasticity of supply by calculating the proportion between percentage fluctuations in quantities imported and percentage fluctuations in prices. For raw materials for consumers goods we get: (7.4/201) : (5.2/95) = 0.7, which points to a rather inelastic supply.

(iv) *Demand for short credits* and *for money* turns out to be *very inelastic* with respect to the short-term interest rate; for discounts and advances the elasticity is — 0.02, for coin — 0.06, for deposits held by the public — 0.03 and for cash held by banks — 0.065. Of course these figures depend on what other variables are included in the explanation; in the explanation

of short credits the value of investments and that of trade are taken; investment activity itself also depends, however, on interest rates and here a somewhat higher elasticity is found: with respect to short-term rates it is — 0.08 and with respect to long-term rates — 0.5.

The *supply of short credits* is also found to be *inelastic*, (elasticity 0.18), but the supply of money (i.e. Central Bank credit) is found to be elastic (elasticity 1.2).

The question may, however, be put, whether the ordinary concept of elasticity is the proper one to be used for a discrimination, whether or not the supply of credit (or of money) is elastic in the sense of the business cycle theory. In the ordinary concept the rate of interest is considered as the price of credit and hence this concept is based upon the relative changes in that rate. In fact the interest rate is not, however, the price, but only a price margin. The price of borrowed money is, strictly speaking, interest plus capital to be paid back. If this point of view were accepted, the elasticities would be one hundred times as great! All elasticities found would then at once be very great.

There is some reason indeed for looking at these matters in this way. In fact, in comparison with the other costs of production, of trading or of speculation the increases in interest charges, even if these assume the proportions that follow from the elasticity figures we found, are very moderate and therefore do not hinder *the demand for credits very much.* And in this other sense the usual situation in the period studied may indeed be characterised as a situation of elastic supply.

(v) The *marginal propensity to consume* of workers is assumed to be 0.8, from which the marginal propensity to consume of non-workers follows at 0.31; by changing the figure 0.8 into 0.9 or 0.7, we get the marginal propensity to consume of non-workers at 0.28 and 0.34 respectively.

(vi) Little evidence is found — as has already been observed — of the *acceleration principle* being of much value for the explanation of short-run fluctuations in investment in fixed plant; the *profit situation* seems to be much more relevant [1]).

Little evidence is also found for the liquidity of private banks to determine short-term interest rates. These seem to depend primarily on the gold reserves of the B.o.E. and other items of the balance of payments. The gold reserves alone would not be able to explain these rates either.

Most of the fluctuations in *wage rates* have to be attributed to wages in *coal mines* which primarily depend on coal prices.

Incomes cannot be reconstructed by means of production and price data without paying attention to the *lag of prices of raw materials as used in cost items*, behind current prices.

Share prices seem to depend to a considerable degree on expectations

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[1]) Here in particular, Mr PESMAZOGLU comes to different conclusions (loc. cit).

that are based on the *rate of increase in those prices* as observed some time before.

The chief examples of *lags* introduced are:

a lag of about one year of non-workers' consumption outlay behind incomes; one of half a year of investment activity behind incomes; one of half a year of finished goods prices behind non-perishable, raw material prices, and one of one year of finished export and investment goods prices behind the volume of production of these goods.

In some equations there are rather *large residuals.* These may be caused by errors in the statisticsal material, e.g. in banking statistics; of which equations V 1, 2, 4 and 5 are examples. It may also be the effect of the presence of random external influences. Equations IV 5 and 6 are examples: the influence of crop variations — which are not included as an explanatory variable — is a random series. Another possible example is equation III 4, explaining investment activity: here inventions may be the disturbing random influence. A last example is equation III 2, explaining exports of consumers' goods. These exports are dependent on crop fluctuations in colonial areas, which certainly are random variations, and on the opening-up of new markets, which may be a random variable.

All the random variables mentioned have played the role of *external shocks*, bringing the economic system out of equilibrium and hence being stimuli to the fluctuations.

There is one simple way of testing whether the residuals behave as *random variables*, i.e. whether their values for any year are, on an average, independent of the values of the preceding year. This test consists in calculating the serial correlation for the residual, i.e. the correlation between the series of the residuals R_t and the series R_{t-1}, being the same series lagged one year. This correlation r_R should not differ significantly from zero. Since the standard deviation may be shown to be equal to $(1/\sqrt{n})$, where n is the number of years included, the coefficient r_R must be less than say twice $(1/\sqrt{n})$. For a number of the regression equations these values have been calculated:

No.	Explaining fluctuation of:	r_R	$1/\sqrt{n}$
III 1	Consumption outlay	−0.05	0.17
III 2	Exports of consumers' goods	0.14	0.19
III 3	Imports of finished cons. goods	0.07	0.24
III 4	Investment activity	0.21	0.16
III 5	Exports of investment goods	−0.06	0.19
IV 2	Cost of living	0.09	0.16
IV 3	Export price of consumers' goods	0.31	0.16
IV 5	Price index of imp. raw materials	0.26	0.16
IV 5	Price index of raw mat. for cons. goods	0.32	0.16
V 5	Deposits held by the public	0.25	0.17
VI 3	Profits	0.47	0.17

As a rule there is no significant difference from zero, but the fact that most r_R values are positive, means that the residuals of annual figures are not quite independent. This may be interpreted as meaning that the number of independent observations with which we have to deal is less than n and hence the standard deviation of the serial correlation coefficient is greater than even $(1/\sqrt{n})$.

Finally we have tested the significance of our equations in still another way, viz. by calculating, as far as the series were available, the *regression coefficients for the periods* 1870—1895 *and* 1896—1914 *separately.* The aim of this test is to find out whether or not the regression coefficients are constant and, if not, whether there is some plausible reason for an observed change. The results obtained for the most important equations are presented below. Some comments may be made.

A number of coefficients appears to be wholly insignificant, e.g. most coefficients before m^L and those before the quantities in the price equations (IV 2), (IV 3) and (IV 4). Those before m^L terms are uncertain since m^L shows only very small fluctuations; we did not use them in our final equation. The coefficients before the quantities in the price equations all represent rather small flexibilities of those prices and it is this smallness that matters, not the relative deviations of the small coefficients.

The coefficient for y^i in equation III 7 also belongs, more or less, to that class, but does not play an important role either, since y^i itself is only a small quantity.

Both the equation for q and that for s point to a decreased flexibility of prices in the second sub-period as compared to the first. This seems to reflect a well-known fact. If the change in the first two coefficients in equation V 2 were significant, it would mean that home investment activity was becoming decreasingly and foreign trade increasingly important for the demand for short credits. Since we have not used this equation in our standard model, it does not matter, with regard to that model, whether or not this is true. The increased coefficient for Au^w in equation V 6 might be interpreted as an increased international cohesion in the financial sphere. It is remarkable that the slight influence of commercial banks on the rate of interest we found to exist is again found for the sub-periods, if the corresponding term is introduced [1]).

The outstanding fact for our purpose is that *the coefficients in the equations with a great influence on the final equation are pretty stable.* This is more particularly true for the consumption and the investment equations (III 1 and III 4) and the equations for q, s, m^s and K^e.

[1]) For the second sub-period even a positive regression coefficient is found, which is inacceptable from the economic view-point and should therefore be rejected.

TABLE VII 1

Regression equations for period A (1870 – 1914), B (1870 – 1895) and C (1896 – 1914)[1]

(Figures *in italics* have been chosen on *a priori* grounds)

No.	Per.	Equations	R
III 1	A	$U' = \mathit{0.80}\, L + 0.50\, Z_{-1} + 0.42\, Z_{-2} + 11.7\, p + 0.45\, \Delta u'$	0.92
	B	$\mathit{0.80}\, L + 0.35\, Z_{-1} + 0.68\, Z_{-2} + 9.0\, p + 0.55\, \Delta u'$	0.95
	C	$\mathit{0.80}\, L + 0.41\, Z_{-1} + 0.73\, Z_{-2} + 8.4\, p + 0.55\, \Delta u'$	0.83
III 4	A	$v' = 0.45\, Z_{-1} + 0.31\, Z_{-2} - 0.73\, q_{-1} - 6.1\, m^s_{-1} - 39\, m^L_{-1}$	0.77
	B	$0.51\, Z_{-1} + 0.37\, Z_{-2} - 0.81\, q_{-1} - 5.2\, m^s_{-1} + 25\, m^L_{-1}$	0.83
	C	$0.34\, Z_{-1} + 0.24\, Z_{-2} - 1.31\, q_{-1} - 3.9\, m^s_{-1} - 10\, m^L_{-1}$	0.69
III 6	A	$x^i = 0.081\, u + 0.256\, u^e + 0.44\, \Delta x^i$	0.86
	B	$0.084\, u + 0.34\, u^e + 0.42\, \Delta x^i$	0.87
	C	$0.061\, u + 0.21\, u^e + 0.48\, \Delta x^i$	0.89
III 7	A	$y^i = \mathit{0.14}\, v + 0.15\, \Delta y^i$	
	B	$\mathit{0.124}\, v + 0.23\, \Delta y^i$	0.36
	C	$\mathit{0.153}\, v - 0.07\, \Delta y^i$	0.07
IV 2	A	$p = 0.23\, l_{-\frac{1}{2}} + 0.31\, r + 0.00\, u_{-1}$	0.78
	B	$0.41\, l_{-\frac{1}{2}} + 0.28\, r + 0.0\, u_{-1}$	0.81
	C	$0.30\, l_{-\frac{1}{2}} + 0.12\, r - 0.02\, u_{-1}$	0.28
IV 3	A	$p^e = \mathit{0.075}\, p^c + 0.30\, l_{-\frac{1}{2}} + 0.38\, r^i_{-\frac{1}{2}} + 0.0\, u^e + 0.13\, u^e_{-1}$	0.83
	B	$\mathit{0.075}\, p^c + \mathit{0.30}\, l_{-\frac{1}{2}} + 0.28\, r^i_{-\frac{1}{2}} - 0.04\, u^e + 0.11\, u^e_{-1}$	0.85
	C	$\mathit{0.27}\, p^c + \mathit{0.30}\, l_{-\frac{1}{2}} + 0.15\, r^i_{-\frac{1}{2}} + 0.15\, u^e + 0.05\, u^e_{-1}$	0.74
IV 4	A	$q = \mathit{0.20}\, l_{-\frac{1}{2}} + 0.76\, s_{-\frac{1}{2}} + 0.11\, {}_R v^E_{-1}$	0.92
	B	$\mathit{0.20}\, l_{-\frac{1}{2}} + 0.86\, s_{-\frac{1}{2}} + 0.13\, {}_R v^e_{-1}$	0.94
	C	$\mathit{0.20}\, l_{-\frac{1}{2}} + 0.64\, s_{-\frac{1}{2}} - 0.02\, {}_R v^e_{-1}$	0.92
IV 5	A	$r^i = 0.12\, Z + 0.03\, Z_{-1}$	0.58
	B	$0.11\, Z - 0.04\, Z_{-1}$	0.67
	C	$0.04\, Z + 0.02\, Z_{-1}$	0.23
IV 6	A	$r = 0.07\, Z + 0.07\, Z_{-1}$	0.66
	B	$0.08\, Z + 0.07\, Z_{-1}$	0.68
	C	$0.04\, Z + 0.06\, Z_{-1}$	0.61
IV 7	A	$s = 0.45\, w' + 1.45\, v^c + l$	0.76
	B	$0.68\, w' + 2.61\, v^c + l$	0.78
	C	$0.64\, w' + 1.17\, v^c + l$	0.83
V 1	A	$B^s = 9.7\, m^s - 34\, m^L + 0.65\, M''$	0.62[2]
	B	$8.9\, m^s - 13\, m^L + 0.68\, M''$	0.65[2]
	C	$8.6\, m^s - 55\, m^L + 0.65\, M''$	0.89[2]
V 2	A	$B^s = 0.47\, V' + 0.28\, (X^i + Y^i + U^i) + 3.0\, (m^{sw} - m^s)$	0.78
	B	$0.78\, V' + 0.06\, (X^i + Y^i + U^i) + 3.5\, (m^{sw} - m^s)$	0.86
	C	$0.34\, V' + 0.24\, (X^i + Y^i + U^i) + 3.4\, (m^{sw} - m^s)$	0.63
V 3	A	$M^c = 0.053\, L + 0.056\, \Sigma Z_{-1} + 0.030\, \Sigma Z - 2.3\, m^s_{-1}$	0.68
	B [3]	$0.072\, L + 0.059\, \Sigma Z_{-1} - 0.015\, \Sigma Z - 1.2\, m^s_{-1}$	0.76
	C	$0.073\, L + 0.068\, \Sigma Z_{-1} + 0.118\, \Sigma Z - 4.4\, m^s_{-1}$	0.88
V 4	A	$M''^b = -3.9\, m^s + 17\, m^L + 0.26\, M''$	0.52
	B	$-4.3\, m^s - 13\, m^L + 0.21\, M''$	0.60
	C	$-4.5\, m^s + 24\, m^L + 0.28\, M''$	0.54
V 5	A	$M'' - M''^b = 0.18\, (U' + V' + K^e) - 4.7\, m^s - 33\, m^L$	0.74
	B	$0.19\, (U' + V' + K^e) - 6.1\, m^s - 86\, m^L$	0.84
	C	$0.12\, (U' + V' + K^e) - 1.4\, m^s - 22\, m^L$	0.53
V 6	A	$m^s = -0.129\, Au' - 0.013\, Au^w + 0.017\, K^e$	0.78
	B	$-0.146\, Au' - 0.007\, Au^w + 0.016\, K^e$	0.76
	C	$-0.123\, Au' - 0.016\, Au^w + 0.018\, K^e$	0.84
V 7	A	$n = 0.27\, Z_{-\frac{1}{2}} + 0.33\, \dot{n}_{-1} - 28\, m^L$	0.88
	B	$0.33\, Z_{-\frac{1}{2}} + 0.21\, \dot{n}_{-1} - 3\, m^L$	0.89
	C	$0.19\, Z_{-\frac{1}{2}} + 0.53\, \dot{n}_{-1} - 15\, m^L$	0.92
VI 3	A	$3\, Z_{-0.6} = U' + V' + K^e - L - 0.45\, \Delta u + 4.5\, r^i$	0.69
	B	$3.08\, Z_{-0.6} = U' + V' + K^e - L - \mathit{0.45}\, \Delta u + 4.7\, r^i$	0.68
	C	$3.58\, Z_{-0.6} = U' + V' + K^e - L - \mathit{0.45}\, \Delta u + 3.2\, r^i$	0.75

(Notes: see page 100)

As a rule the correlation coefficients obtained are lower for the second sub-period than for the first, the averages being 0.67 and 0.77 respectively. To some extent this may be due to the larger amplitude of the fluctuations in the first sub-period, being that of the falling long wave phase. On the other hand it may be that political disturbances (wars) have influenced the second sub-period more strongly than the first.

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[1]) In a number of cases a few years at the beginning or at the end of the period have not been included, since the corresponding figures were not available.

[2]) These correlation coefficients are those between $\left\{\frac{B^s}{M''}\right\}$ and a linear combination of m^s and m^L; the correlation between B^s and a linear combination of m^s, m^L and M'' is much higher.

[3]) Excl. the year 1878.