CHAPTER VIII

BUSINESS CYCLE THEORY FOR AN OPEN COUNTRY APPLICATION TO THE UNITED KINGDOM

VIII 1. THE SETTING OF THE PROBLEM

The economic mechanism of an open country may be described by a system of relations as given in the first seven chapters, the characteristic properties of which are their containing four types of mathematical entities, viz.

(*a*) constants;

(*b*) internal variables;

(*c*) non-systematic external variables and

(*d*) systematic external variables, whose meaning and function were set out in chapter I.

The occurrence of *systematic* external variables is characteristic of an open country. They are partly — or even, as a limiting case, wholly — responsible for the systematic movements of the economy. As an example we may assume exports to show cycles, entailing a cyclic movement in all divisions of economic life. These movements cannot, as in the case of a closed economy, be understood by some endogenous mechanism alone, but only by the interaction of the systematic external variables and such a mechanism. As to this interaction, there are different problems which should be clearly distinguished:

(1) what movements will the economy show if *no systematic* changes occur abroad;

(2) what movements will it show if *given systematic* changes do occur abroad;

(3) what movements will it show if systematic changes occur abroad which are in some way parallel to those occurring in the economy studied, and therefore are *not given a priori*.

As a rule the solution of the first problem will not give the full explanation of the business cycle phenomenon, since part of the real movements must be attributed to movements abroad. Solving the second problem means explaining the cycles in a country by the cycles in e.g. its exports which themselves are assumed to be given. The complete solution of the business cycle problem would only be obtained if the cycles themselves in these exports were explained. In principle this would mean considering the world as a whole. It goes without saying that this is impossible for lack of adequate statistics in many countries.

A good approximation can only be obtained in those cases where a country with good statistics is almost self-sufficient, as e.g. in the U.S.A. That is why this country was first analysed by the author [1]).

Since, however, the United Kingdom might show and presumably does show a structure differing from that of the United States, it seems desirable to study the United Kingdom separately. This country, however, is far from self-sufficient and the full explanation of its cycles therefore is a problem of type 3 above. In order to give an explanation, some approximative assumptions about the causes of the systematic movements of the external variables (exports, raw material prices, interest rates in other countries, capital exports and income from abroad) must be made. These assumptions may be said to represent a very rough analysis of the rest of the world, as far as it is significant for British cycles. And only if the influence of these assumptions on the results obtained is not great can one speak of a specific British type of business cycles.

From the purely logical viewpoint these assumptions reduce to internal variables the systematic external ones in our system and this is the only way to obtain a business cycle theory which does not partly explain the cycles by external cycles. Only such a theory is, however, a genuine business cycle theory. Calling a "complete system of equations" a system without systematic external variables, we may call our enlarged system a *quasi-complete system.*

We will now discuss the supplementary assumptions made for this purpose.

VIII 2. SUPPLEMENTARY RELATIONS INCLUDED IN ORDER TO OBTAIN A QUASI-COMPLETE SYSTEM

(i) The relations added to those concerning the internal structure of the United Kingdom are meant — as has been said before — to express some sort of *parallelism* between the course of economic events abroad and at home. This procedure has been chosen since the general experience is that in fact cyclic movements have, in the period considered, been largely parallel in the various countries, and since our aim is to describe the actual process as accurately as possible. What is somewhat loosely called "parallelism" is, however, not so well-defined a notion as it would seem at first sight. Since an exact statement of our relations, also of the supplementary ones, is an essential condition for the solution of our problems, we have to go into some detail here.

Parallelism of cyclic movements may mean that the movements are at every moment in the same phase, i.e. that the curves have their heights and troughs at the same time. The term may, however, involve that, in addition, the amplitudes of the curves are the same — in whatever units they are measured. We shall not use this interpretation; in particular since this more restricted form of parallelism is not always compatible with other assumptions concerning the economic mechanism involved.

[1]) Business Cycles in the U.S., Geneva 1939 (League of Nations).

In some cases, differences in structure between two countries make it impossible for these countries to show parallel movements in the more restricted sense of the word. An example may illustrate this. The United Kingdom is, and was in particular in the period studied, a far more "capitalistic" nation than the rest of the world, taken as a whole. The proportion of its production devoted to investment goods was far greater than for the rest of the world. Now if there were parallelism in the narrow sense between the production of consumers' goods in the United Kingdom and abroad, and also between the production of investment goods in the United Kingdom and abroad, then there would not be, in all probability, parallelism in that narrow sense, between incomes in the United Kingdom and abroad. Fluctuations in investment goods industries are always comparatively more intense than those in consumers' goods industries, and given their greater weight within the English economy, incomes would fluctuate more in this country. *The assumption of parallelism in the narrower sense may therefore be in contradiction to some of the fundamental structural data on the economies concerned.* Structural differences may even be such as sometimes to prevent parallelism from existing at all. An interesting case is that of the gold stock. The fluctuations in the British gold stock can hardly be parallel to those in the gold stock of the rest of the world. (Cf. section (iii) *c*, below). So the best thing we can do is to consider each variable separately.

(ii) *External prices.*

(*a*) The hypothesis made on the short-term interest rate abroad (m^{sw}), viz. that it is equal to m^s, is based on the assumption that this market shows a high degree of international competition. It is in rough agreement only with the figures. Since, however, the co-efficient of $m^s - m^{sw}$ in the equation V 2 is very small, the influence of any hypothesis on m^{sw} on our system is small and so the question need not occupy us any further.

(*b*) The parallelism between British prices and competing foreign prices appeared to be far from narrow in various cases. It even seemed that the differences in their movements were of a systematic nature. Since the causes of these differences may be complex and their analysis would lead us too far, we have only replaced those differences by a function of Z, Z_{-1} and Z_{-2} found by correlation analysis. This means that they depend somehow on the economic situation. The following relations are found:

(VI 5) [1] $$p^e - p^w = -\,0.038\,Z - 0.013\,Z_{-1} + 0.053\,Z_{-2}$$

(VI 7) $$-\,0.54\,p^i + 0.48\,p' = -\,0.025\,Z_{-1} + 0.025\,Z_{-2}$$

(VI 8) $$q - q^w = -\,0.16\,Z - 0.13\,Z_{-1} + 0.13\,Z_{-2}$$

[1]) The relations discussed in this chapter have been numbered in accordance with the order of the relations of chapter VI; they have been given alphabetical order.

The coefficients before p^i and p' have been chosen in such a way as to correspond to those obtained in equation (III 3); the left-hand side of (VI 7) therefore roughly represents one-half of the price difference for consumers' goods ready for use. It is clear from (VI 6) and (VI 7) that the price differences in these two groups of consumers' goods are about equal; from (VI 8) we see that the price differences for investment goods are far greater.

(*c*) There remains the necessity to make an assumption on the course of p^i, the price level of imported consumers' goods, itself. Like all prices, this price will depend on demand and supply factors. The chief demand factor probably is world income, which we assume to move parallel to British incomes represented by Z. The possibility of a lag, together with the uncertainty about the timing of Z, justifies the introduction of Z_{-1} as a second variable. The relation found is:

(VI 6) $$p^i = 0.034\,Z + 0.084\,Z_{-1}$$

(iii) *External value figures.*

(*a*) The *gold stock abroad* Au^w can only be assumed to move parallel to the British gold stock, if there were reasons to believe that the production or dishoarding and the hoarding of gold would move exactly parallel to the imports of gold into and the exports of gold from the United Kingdom. Gold production, however, has a tendency to move inversely to the general price level and there is hardly any correlation between the price level and the imports of gold into the United Kingdom. If the total world gold stock were constant, or developing slowly, there would be a tendency toward a negative correlation between Au^w and Au'. Statistically not very much evidence of this tendency either can be detected. Since, in addition, the coefficient before Au^w in equation V 6 is only one tenth of that before Au', it does not make a great difference as to what we exactly suppose to be the case. For greater simplicity, Au^w has been put equal to zero, i.e. is considered as non-systematic.

(*b*) About capital exports, K^e several assumptions seem plausible *a priori*. Capital exports may be considered as an indication that capital can be used better in other countries than in the United Kingdom itself. Now in this country, as well as elsewhere, the "need" of capital will be greater in boom periods than in depressions. It will depend on the degree in which the "boom need" and the "depression need" differ in both areas, whether capital exports will be greater in boom than in depression periods. Therefore it is impossible to give an *a priori* theory here.

The figures show that capital exports are highest in boom periods or a short time before; anyhow, the relation with the cycle seems fairly evident (cf. graph III 5). We might, therefore, suppose that K^e is a function of the central variable Z (profits). This should not be interpreted in such a way as to make us assume that capital exports depend on profits in the United Kingdom only; here Z stands for profits abroad also. A further hypothesis on parallelism is thus implied: parallelism between profits abroad and at home. This parallelism may even be a lagged one: profits abroad may precede, or follow those at home. This possibility is introduced by assuming K^e equal to a function of Z, Z_{-1} and Z_{-2}. The relation found is:

$$K^e = 0.26\,Z + 0.20\,Z_{-1} - 0.39\,Z_{-2}$$

which yields a fairly high correlation.

(*c*) World trade, T^w is assumed to move parallel to British trade. In the strict sense this would come to equalising T^w to an expression like:

$$\tau(X^i + Y^i + U^i + U^e + V^e)$$

where the expression in brackets represents the total of British imports and exports. Since it is our purpose — in what follows — to express this sum in the central variable Z (profits) of our system, we may shorten this way by expressing at once — by means of correlation calculus — T^w in terms of Z and Z_{-1}. This procedure may also be interpreted as the combination of two other steps, viz.

(i) assuming parallelism between "world" trade and "world" profits,

(ii) assuming parallelism between "world" profits and a combination of Z and Z_{-1}.

The relation found by correlation calculus is:

(VI 9) $$T^w = 0.06\,Z + 0.04\,Z_{-1}$$

VIII 3. THE ELIMINATION PROCESS

(i) *Introductory.* The addition of the relations enumerated in the previous section has now provided us with a quasi-complete system of relations (summarised in table VIII 31), i.e. a system without systematic external variables, enabling us to give an explanation of cyclic movements without recurring to external cycles. In order to find the chief features of this explanation, we have to perform — as was set out elsewhere — the elimination process, in order to find the final equation — if one exists. This elimination process is given in tables VIII 32 and VIII 33. These tables are self-explanatory, if only account is taken of certain techniques of an approximative character. The provisional line of attack is to express all variables in three of them, viz. Z, m^s and s, neglecting the variable m^L, the fluctuations of which are very weak and not clearly cyclical. Although this process is to some extent one of trial and error, and not uniquely determined, leaving freedom to the investigator, there is some system in it, which is more or less prescribed by the structure of the equations (i.e. by the occurrence and the absence of variables in each of the equations). This is equivalent to the structure of our model and, in as far as this is a true picture of the real economy, to the structure of the economy; it is but natural that the course of the elimination process, which, as we

TABLE VIII 31
United Kingdom, 1870—1914
Summary of elementary equations describing business cycle mechanism

II 1	$A\dot{u} + \Delta M^c = U^e + V^e + I^A - (U^i + X^i + Y^i + K^e)$
II 2	$a = b + c$
II 3	$b = 0.20\, v$
II 4	$c = 0.14\, u$
II 5	$u = u' + u^e - u^i$
II 6	$v = v' + v^e + \dot{w}^I$
II 7	$U' = 0.99\, u' + 15.3\, p$
II 8	$U^e = 0.90\, u^e + 1.80\, p^e$
II 9	$U^i = 1.10\, u^i + 1.65\, p^i$
II 10	$V' = 0.87\, v' + 1.49\, q$
II 11	$V^e = 0.86\, v^e + 1.41\, q$
II 12	$X^i = 0.95\, x^i + 2.01\, r^i$
II 13	$Y^i = 0.77\, y^i + 0.59\, s$
III 1	$U' = 0.80\, L + 0.50\, Z_{-1} + 0.42\, Z_{-2} + 11.7\, p + 0.45\, \Delta u'$
III 2	$u^e = -\,0.7\, (p^e - p^w) + 0.94\, T^w + 0.18\, K^e$
III 3	$u^i = 0.10\, u' - 0.54\, p^i + 0.48\, p'$
III 4	$v' = 0.45\, Z_{-1} + 0.31\, Z_{-2} - 0.73\, q_{-1} - 6.1\, m^s_{-1} - 39\, m^L_{-1}$
III 5	$v^e = 0.25\, K^e + 2.1\, T^w - 0.88\, (q - q^w) - 0.27\, q$
III 6	$x^i = 0.081\, u + 0.256\, u^e + 0.44\, \Delta x^i$
III 7	$y^i = 0.14\, v + 0.15\, \Delta y^i$
IV 1	$l = 0.20\, q$
IV 2	$p = 0.23\, l_{-\frac{1}{2}} + 0.31\, r$
IV 3	$p^e = 0.30\, l_{-\frac{1}{2}} + 0.38\, r^i_{-\frac{1}{2}} + 0.13\, u^e_{-1} + 0.04\, q_{+1/3}$
IV 4	$q = 0.20\, l_{-\frac{1}{2}} + 0.76\, s_{-\frac{1}{2}} + 0.03\, v_{-1}$
IV 5	$r^i = 0.121\, Z + 0.033\, Z_{-1}$
IV 6	$r = 0.068\, Z + 0.070\, Z_{-1}$
IV 7	$s = 0.45\, w' + 0.30\, v + l - 0.18\, \Sigma v_{-\frac{1}{2}}\, (\overline{w}' + w' = 36/\overline{\overline{w}},\ \overline{w}' = 14)$
IV 8	$\dot{w} = -\,0.029\, (v' + v^e)$
V 1	$B^s = 9.7\, m^s - 34\, m^L + 0.65\, M''$
V 2	$B^s = 0.47\, V' + 3.0\, (m^{Sw} - m^S) + 0.28\, (U^i + X^i + Y^i)$
V 3	$M^C = 0.053\, L + 0.056\, \Sigma Z_{-1} + 0.030\, \Sigma Z - 2.3\, m^s_{-1}$
V 4	$M''^b = -\,3.9\, m^s + 17\, m^L + 0.26\, M''$
V 5	$M'' - M''^b = 0.18\, (U' + V' + K^e) - 4.7\, m^s - 33\, m^L$
V 6	$m^s = -\,0.129\, Au' - 0.013\, Au^w + 0.017\, K^e$
V 7	$n = 0.27\, Z_{-\frac{1}{2}} + 0.33\, \dot{n}_{-1} - 28\, m^L$
VI 1	$I^A = 0.49\, (X^i + Y^i)$
VI 2	$L = 0.89\, a + 6.62\, l$
VI 3	$3Z_{-0.6} = U' + V' + K^e - L - 0.45\, \Delta u + 4.5\dot{r}^i$
VI 4	$K^e = 0.26\, Z - 0.20\, Z_{-1} - 0.39\, Z_{-2}$
VI 5	$p^e - p^w = -\,0.038\, Z - 0.013\, Z_{-1} + 0.053\, Z_{-2}$
VI 6	$p^i = 0.034\, Z + 0.084\, Z_{-1}$
VI 7	$-\,0.54\, p^i + 0.48\, p' = -\,0.025\, Z_{-1} + 0.025\, Z_{-2}$
VI 8	$q - q^w = -\,0.16\, Z - 0.13\, (Z_{-1} - Z_{-2})$
VI 9	$T^w = 0.06\, Z + 0.04\, Z_{-1}$
VI 10	$w' = -\,5.5\, w$

are aware, means deriving deduced economic laws from the elementary laws, should reflect the economic structure.

The object of this section is to describe in plain terms the chief results of the elimination process and the meaning of the deduced laws found. It is hoped that these laws appeal to non-mathematical readers too; but if so, it should not be forgotten that the plain-term theory could not have been given without the stern guidance of an exact elimination process.

(ii) *The explanation of price fluctuations.*

A. The fluctuations in pig iron prices, taken as the chief representative of mineral prices, depend on (cf. eq. IV 7):

(*a*) wage rates [1]); (*b*) pig iron stocks; (*c*) productive capacity of blast furnaces and (*d*) demand factors.

(*a*) Wage rates themselves depend on mineral prices (cf. eq. IV 1) and their influence can therefore be subtracted immediately [2]).

(*b*) Stocks are cumulated annual additions to stock and these run parallel to demand; hence cumulated demand comes in as a determining factor (equations 320 to 325).

(*c*) Productive capacity is the cumulation of annual additions to capacity and these run also parallel with investment activity, i.e. the demand for investment goods. Therefore, again cumulated demand enters into the explanation of pig iron prices (equation 318).

(*d*) Hence pig iron prices depend on demand factors and their cumulation. Now the factors determining demand for investment goods (cf. eq. III 4 and III 5 for home and foreign demand respectively) are profits, iron prices and interest rates, all of them with a lag. It follows that pig iron prices are a function of profits, the interest rate, cumulated profits, the cumulated interest rate and the cumulated iron price, all of them with some lag. Equation 328 gives the exact coefficients. The term with Σm^s (cumulated interest rates) has been neglected because of its smallness. Of the other terms the one with $Z_{-0.6}$ is by far the largest. This means that pig iron prices are chiefly governed by general profits and that the other forces, embodied in the influence of the interest rate and the cumulated Z-term, are only weak.

Putting, as a first approximation,

$$s = 0.32\, Z_{-0.6}$$

[1]) For the sake of brevity the phrase "fluctuations in", which should precede the name of every variable, as we are only considering deviations from equilibrium values, is not always repeated.

[2]) Disregarding lags for a moment, the situation is that $s = l +$ other terms and $l = 0.15\, s$, from which l may be eliminated immediately: $s = 0.15\, s +$ other terms or $0.85\, s =$ other terms and $s =$ (other terms/0.85).

we may replace $\Sigma s_{-1.5}$ in equation (328) by $0.32\ \Sigma Z_{-2.1}$; hence we get

$$s = +0.06\ \Sigma Z_{-1.5} - 0.01\ \Sigma Z_{-2.4} + 0.32\ Z_{-0.6},$$

neglecting the m^s-term. Since

$$+0.01\ \Sigma Z_{-1.1} - 0.01\ \Sigma Z_{-2.1} = 0.01\ Z_{-1.5},$$

approximately, we have [1]):

(VIII 32′) $$s = +0.05\ \Sigma Z_{-1.1} + 0.33\ Z_{-0.6}$$

As a second approximation this may be simplified into:

(VIII 32) $$s = 0.33\ Z_{-0.6}$$

This is only true for moderate deviations of the stock from its trend. If more extreme positions are reached, in particular after a boom period, the influence of stocks, which is curvilinear, increases: eq. 320 is only a linear approximation, losing its validity for low values of the stock. This may also be true for the influence of (*c*) and (*d*) above, which reflect the influence of unused capacity (Cf. section 4, (iii) below).

B. Prices for retail consumers' goods (cost of living) chiefly depend — because of the high elasticity of supply — on cost fluctuations (cf. eq. IV 2). These lead us back to wage rates and raw material prices. Wage rates are intimately connected with mineral prices (cf. eq. IV 1). Variations in raw material prices for consumers' goods depend on crop fluctuations, which are non-systematic and therefore neglected, and on general incomes, for which profits are representative (cf. eq. IV 6). Hence, cost of living fluctuations are connected with mineral prices and profits, as is expressed in equation 105 (table VIII 32).

Mineral prices run chiefly parallel to profits, as just seen, and thus cost of living is reduced to profits only.

(iii) *The balance of payments.* Both income (cf. eq. VI 3) and the short-term interest rate (eq. V 6) depend on a number of items of the balance of payments. Table VIII 33 shows how these items may be brought back to fluctuations in profits, pig iron prices, wage-bill, its rate of increase and the rate of increase in the rate of interest.

As to exports, these are largely determined by our supplementary relations. Exports depend on world trade, capital exports and prices, foreign and home (Eq. III 2 and III 5, together with II 8 and II 11). World trade, capital exports and price margins have been supposed to depend on profits by our "supplementary relations" (this chapter, section 2). Home prices of export products may be shown to depend on profits and mineral prices altogether like other home price (cf. this section, under ii).

[1]) From the equation it would follow that of the two opposite influences exerted by stocks and by productive capacity (both of a cumulative nature) the first is the more important one.

Imports of consumers' goods ready for use depend on (*a*) total consumption and (*b*) the price margin between imported and home products. The latter are immediately brought back — by one of the supplementary relations (VI 7) — to profits. Total consumption depends on (III 1 and II 7): wage bill, lagged profits and cost of living. Cost of living has already been reduced to profits. Hence imports of finished consumers' goods depend on lagged profits and the wage bill, this latter influence being rather weak (eq. 215).

Imports of raw materials depend on total volume of production (consumers' goods plus investment goods) (III 6 and 7); the dependency on Δx^i and Δy^i, representing stocking of raw materials, may be transformed into a lagged relation by using the approximation discussed in chapter VIII 3, under (I). The value of raw material imports depends, in addition, on their prices (II 12 and 13). As to raw materials for investment goods, their prices are parallel with pig iron prices. The prices of raw materials for consumers' goods are brought back to lagged profits (cf. above, ii, B). The total volume of production consists of:

(*a*) production of consumers' goods, being consumption minus imports of consumers' ready goods, plus exports; and

(*b*) production of investment goods, being home demand plus exports. Calculations similar to those already exposed and given in detail in equations 221—230, show that the value of imports of raw materials depends on profits (lagged), pig iron prices and, to a small extent, on the total wage bill. The second term shows a small lead which is a consequence of our approximation; the third term even a rather important lead, for the same reason. But this term is small in itself and therefore there is not much harm in the application of the approximation [1]).

Income from abroad, while chiefly coming from countries producing raw materials has already been connected with raw material imports (eq. VI 1), and therefore depends on the same factors as these.

Capital exports have been related already — again by one of our supplementary relations — to profits (eq. VI 4).

The rate of increase in gold coin in the country (but outside the B.o.E.) depends on profits (industrial use), on the rate of increase in the wage bill and (negatively) on the rate of increase in the rate of interest, these latter two factors reflecting the rate of increase for circulation purposes.

Having thus reviewed all the items of the balance of payments except the inflow of gold into the B.o.E., we are now in a position to use these

[1]) One might ask what is the advantage of applying a procedure which leads to illogical results and whether it would not be better to leave out the leading terms entirely. The answer is that these terms will be combined further on with more important lagged terms and that the procedure followed enables us — still rather accurately — to estimate their influence vis-à-vis other terms.

TABLE VIII 32

The Elimination process (1) [1])

Nr.	Ref.	Equations	Negl.
101	IV 2	$p = 0.23\,l_{-\frac14} + 0.31\,r$	
102	IV 6	$0.31\,r = 0.021\,Z + 0.022\,Z_{-1}$	
103	IV 1	$0.23\,l_{-\frac14} = 0.17\,l + 0.06\,l_{-1} = 0.034\,q + 0.012\,q_{-1}$	
104	101/2/3	$p = 0.021\,Z + 0.022\,Z_{-1} + 0.034\,q + 0.012\,q_{-1}$	
105	104	$11.7\,p = 0.25\,Z + 0.26\,Z_{-1} + 0.40\,q + 0.14\,q_{-1}$	
106	II 6	$v = v' + v^e + \dot{w}^I$	
107	III 4	$v' = 0.45\,Z_{-1} + 0.31\,Z_{-2} - 0.73\,q_{-1} - 6.1\,m^s_{-1} - 39\,m^L_{-1}$	
108	III 5, VI 4	$v^e = 0.07\,Z + 0.05\,Z_{-1} - 0.10\,Z_{-2} + 0.13\,Z + 0.08\,Z_{-1} + 0.14\,Z +$	
	VI 9	$+ 0.11\,Z_{-1} - 0.11\,Z_{-2} - 0.27\,q = 0.34\,Z + 0.24\,Z_{-1} - 0.21\,Z_{-2} - 0.27\,q$	
109	106/7/8	$v = 0.34\,Z + 0.69\,Z_{-1} + 0.10\,Z_{-2} - 0.27\,q - 0.73\,q_{-1} - 6.1\,m^s_{-1}$	m^L, $\dot{w}^I$
110	109	$0.11\,v = 0.12\,Z_{-0.79} - 0.11\,q_{-0.73} - 0.67\,m^s_{-1}$	
111	110	$0.11\,v_{+0.15} = 0.12\,Z_{-0.64} - 0.11\,q_{-0.58} - 0.7\,m^s_{-0.85}$	
112	IV 4	$q = 0.76\,s_{-\frac12} + 0.20\,l_{-\frac14} + 0.03\,v_{-1}$	
113	IV 1	$l = 0.20\,q$	
114	113	$0.20\,l_{-\frac14} = 0.03\,q + 0.01\,q_{-1}$	
115	110	$0.03\,v_{-1} = 0.01\,Z_{-1} + 0.02\,Z_{-2} - 0.01\,q_{-1} - 0.02\,q_{-2}$	m^s_{-1}, Z t.
116	112/4	$q = 0.076\,s_{-\frac12}$	q terms
117	105	$11.7\,p = 0.25\,Z + 0.26\,Z_{-1} + 0.54\,q_{-0.26}$	
118	117/6	$= 0.25\,Z + 0.26\,Z_{-1} + 0.41\,s_{-0.76}$	
119	118	$= 0.51\,Z_{-0.51} + 0.41\,s_{-0.76}$	
120	II 10, III 4	$V^1 = 0.39\,Z_{-1} + 0.27\,Z_{-2} - 0.64\,q_{-1} - 5.3\,m^s_{-1} + 1.49\,q$	
121	116	$- 0.64\,q_{-1} = - 0.49\,s_{-1\frac12}$	
122	116	$1.49\,q = 1.13\,s_{-\frac12}$	
123	120/2	$V' = 0.39\,Z_{-1} + 0.27\,Z_{-2} - 5.3\,m^s_{-1} + 1.13\,s_{-\frac12} - 0.49\,s_{-1\frac12}$	
301	VI 2, II 2/3/4	$L = 0.18\,v + 0.13\,u + 1.32\,q$	
302	109, 116	$0.18\,v = 0.061\,Z + 0.124\,Z_{-1} + 0.018\,Z_{-2} - 0.14\,s_{-1.23} - 1.1\,m^s_{-1}$	m^L
303	219	$0.13\,u = 0.004\,Z + 0.125\,Z_{-1} - 0.017\,Z_{-2} - 0.02\,s_{-0.31} + 0.10\,L_{+0.45}$	
304	116	$1.32\,q = \qquad 1.00\,s_{-\frac12}$	
305	301/4	$L - 0.1\,X_{-0.45} = 0.057\,Z + 0.249\,Z_{-1} + 0.84\,s_{-0.38} - 1.1\,m^s_{-1}$	
306	305	$0.90\,L_{+0.05} = 0.31\,Z_{-0.82} + 0.84\,s_{-0.38} - 1.1\,m^s_{-1}$	
307	306	$L = 0.34\,Z_{-0.87} + 0.93\,s_{-0.43} - 1.22\,m^s_{-1}$	
308	235	"Total I" $= 0.27\,Z + 0.29\,Z_{-1} - 0.27\,Z_{-2} + 0.79\,s_{-0.7} - 0.03\,L_{-0.9}$	
309	237	$- \quad U^i = - 0.06\,Z - 0.17\,Z_{-1} - 0.04\,Z_{-2} + 0.01\,s_{-0.31} - 0.09\,L_{+0.45}$	
310	308/9	$U^e + V^e + I^A - X^i - Y^i - U^i = 0.21Z + 0.12Z_{-1} - 0.31Z_{-2} + 0.80s_{-0.7} - 0.12L_{+0.6}$	
311	307	$- 0.12\,L_{+0.6} = - 0.04\,Z_{-0.27} - 0.11\,s_{0.2} + 0.15\,m^s_{-0.4}$	
312	311	$- 0.12\,L_{+0.6} = - 0.03\,Z - 0.03\,Z_{-1} - 0.11\,s_{0.2}$	m^s
313	310/2	$U^e + V^e + I^A - X^i - Y^i - U^i = 0.18\,Z + 0.11\,Z_{-1} - 0.31\,Z_{-2} + 0.69\,s_{-0.8}$	
314	IV 7	$s = 0.45\,w' + 0.20\,q + 0.30\,v - 0.018\,\Sigma v_{-\frac12}$	
315	116	$0.20\,q = 0.15\,s_{-\frac12}$	
316	109	$0.30\,v = 0.10\,Z + 0.21\,Z_{-1} + 0.03\,Z_{-2} - 0.30\,q_{-0.73} - 1.8\,m_{-1}$	
317	116	$- 0.30\,q_{-0.73} = - 0.23\,s_{-1.23}$	
318	110/6	$- 0.018\,\Sigma v_{-\frac12} = - 0.020\,\Sigma Z_{-1.3} + 0.014\,\Sigma s_{-1.73}$	
319	314/8	$s = 0.45w' + 0.34Z_{-0.79} + 0.15s_{-0.5} - 0.23s_{-1.23} - 0.020\Sigma Z_{-1.3} + 0.014\Sigma s_{-1.73} -$	
		$- 1.8m^s_{-1}$	
320	VI 10, IV 8	$w' = - 5.5\,w = + 0.16 \int (v' + v^e)$	
321	107/8	$v' + v^e = 0.34\,Z + 0.69\,Z_{-1} + 0.10\,Z_{-2} - 0.27\,q - 0.73\,q_{-1} - 6.1\,m^s_{-1}$	m^L
322	321	$= 1.13\,Z_{-0.79} - 1.00\,q_{-0.73} - 6.1\,m^s_{-1}$	
323	322	$0.16\,(v' + v^e) = 0.18\,Z_{-7.79} - 0.16\,q_{-0.73} - 1.0\,m^s_{-1}$	
324	323, 116	$w' = + \text{p}.18\,\Sigma Z_{-1.3} - \text{p}.12\,\Sigma s_{-1.7} - 1.0\,\Sigma m^s_{-1.5}$	
325	324	$0.45\,w' = + 0.08\,\Sigma Z_{-1.3} - 0.05\,\Sigma s_{-1.7} - 0.5\,\Sigma m^s_{-1.5}$	
326	319, 325	$s = + 0.06\,\Sigma Z_{-1.5} - 0.04\,\Sigma s_{-1.7} + 0.34\,Z_{-0.79} + 0.15s_{-0.5} - 0.23s_{-1.2} - 1.8m^s_{-1}$	Σm^s
327	326	$1.08\,s_{-0.2} = + 0.06\,\Sigma Z_{-1.3} - 0.06\,\Sigma s_{-1.7} + 0.34\,Z_{-0.79} - 1{,}8\,m^s_{-1}$	
328	327	$s = + 0.06\,Z_{-1.1} - 0.04\,\Sigma s_{-1.5} + 0.32\,Z_{-0.6} - 1.7\,m^s_{-0.8}$	

[1]) First col.: number of equation.
Second col. ("Ref."): number(s) of equation(s), from which deduced and to which reference is made.
Third col. ("Equations"): new equation obtained.
Fourth col. ("Negl."): terms neglected.

TABLE VIII 33

The Elimination Process (2) [1])

Nr.	Ref.	Equation	Negl.
201	II 8	$U^e = 0.90\,u^e + 1.80\,p^e$	
202	III 2	$0.9\,u^e = -0.59\,(p^e - p^w) + 0.85\,T^w + 0.16\,K^e$	
203	202, VI 6,	$= 0.022\,Z + 0.008\,Z_{-1} - 0.031\,Z_{-2}$	
	VI 9, VI 4	$+ 0.051\,Z + 0.034\,Z_{-1}$	
		$+ 0.042\,Z + 0.033\,Z_{-1} - 0.062\,Z_{-2}$	
204	203	$= 0.12\,Z + 0.07\,Z_{-1} - 0.09\,Z_{-2}$	
205	IV 3, IV 5	$p^e = 0.069_{-\frac{1}{4}} + 0.04\,Z_{-\frac{1}{2}} + 0.01\,Z_{-\frac{1}{2}} + 0.02\,Z_{-1} + \mathrm{p}.01\,Z_{-2} - 0.01\,Z_{-3}$	
	204, IV 1	$+ 0.4\,q_{-\frac{1}{2}}$	
	116	$= 0.10\,q + 0.07\,Z_{-0.6} = 0.08\,s_{-\frac{1}{2}} = 0.03\,Z + 0.04\,Z_{-1}$	
206	II 11, III 5,		
	108	$V^e = 0.215\,K^e + 1.8\,T^w - 0.76\,(q - q^w) - 0.215\,q + 1.41\,q$	
207	206	$= 0.056\,Z + 0.043\,Z_{-1} - 0.084\,Z_{-2}$	
		$0.108\,Z + 0.072\,Z_{-1}$	
		$0.122\,Z + 0.099\,Z_{-1} - 0.099\,Z_{-2} + 0.90\,s_{-\frac{1}{2}}$	
208	207	$= 0.29\,Z + 0.21\,Z_{-1} - 0.18\,Z_{-1} + 0.90\,s_{-\frac{1}{2}}$	
209	II 9, VI 5	$U^i = 1.10\,u^i + 0.056\,Z + 0.138\,Z_{-1}$	
210	III 3	$1.10\,u^i = 0.11\,u' - 0.59\,p^i + 0.53\,p'$	
211	III 1, II 7	$u' = 0.81\,L + 0.51\,Z_{-1} + 0.42\,Z_{-2} - 3.7\,p + 0.45\,(u' - u'_{-1})$	
212	211	$u' = 0.81\,L_{+0.45} + 0.93\,Z_{-1} - 3.7\,p_{+0.45}$	
213	119	$-3.7\,p_{+0.45} = -0.16\,Z_{-0.06} - 0.13\,s_{-0.31}$	
214	·212/13	$0.11\,u' = 0.09\,L_{-0.45} + 0.08\,Z_{-1.2} - 0.01\,s_{-0.31}$	
215	209/10/14	$U^i = 0.09\,L_{+0.45} - 0.01\,s_{-0.31} + 0.06\,Z + 0.17\,Z_{-1} + 0.04\,Z_{-2}$	p^i, p'
	VI 7		
216	II 5	$u = u' + u^e - u^i$	
217	212/3, 203	$= 0.81L_{+0.45} + 0.93Z_{-1} - 0.16Z_{-0.06} - 0.13s_{-0.31} + 0.13Z + 0.08Z_{-1} -$	
		$-0.10Z_{-2} - u^i$	
218	210/2	$u^i = 0.08\,L_{+0.45} + 0.07\,Z_{-1.2} - 0.01\,s_{-0.31} - 0.025\,Z_{-1} + 0.025\,Z_{-2}$	
219	217/8	$u = 0.73\,L_{+0.45} - 0.02\,Z + 0.96\,Z_{-1} - 0.13\,Z_{-2} - 0.12\,s_{-0.31}$	
220	VI 1	$I^A - X^i - Y^i = -0.51\,(X^i + Y^i)$	
221	II 12, IV 5	$X^i = 0.95\,x^i + 0.24\,Z + 0.07\,Z_{-1}$	
222	III 6	$x^i_{-0.44} = 0.081\,u + 0.256\,u^e = 0.081\,u + 0.03\,Z + 0.02\,Z_{-1} - 0.02\,Z_{-2}$	
223	222/19	$x^i_{-0.44} = 0.06\,L_{+0.45} + 0.03\,Z + 0.10\,Z_{-1} - 0.03\,Z_{-2} - 0.01\,s_{-0.31}$	
224	223	$0.95\,x^i = 0.06\,L_{+0.89} + 0.10\,Z_{+0.04} - 0.01\,s_{+0.13}$	
225	224	$= 0.06\,L_{+0.89} + 0.10\,Z - 0.01\,s_{+0.13}$	
226	221/5	$X^i = 0.34\,Z + 0.07\,Z_{-1} - 0.01\,s_{+0.13} + 0.06\,L_{+0.89}$	
227	II 13, III 7	$Y^i = 0.59\,s + 0.11\,v_{+0.15}$	
228	111, 116	$0.11\,v_{+0.15} = 0.12\,Z_{-0.64} - 0.11\,q_{-0.58} = 0.04\,Z + 0.08\,Z_{-1} - 0.08\,s_{-1.08}$	m^s
229	227/8	$Y^i = 0.04\,Z + 0.08\,Z_{-1} + 0.51\,s_{-0.17}$	
230	229, 26	$X^i + Y^i = 0.38\,Z + 0.15\,Z_{-2} + 0.06\,L_{+0.89} + 0.50\,s_{0.17}$	
231	220/6/30	$I^A - X^i - Y^i = -0.19\,Z - 0.08\,Z_{-1} - 0.03\,L_{+0.89} - 0.25\,s_{+0.17}$	
232	201/4/5	$U^e = 0.17\,Z + 0.14\,Z_{-1} - 0.09\,Z_{-2} + 0.14\,s_{-\frac{1}{2}}$	
233	208	$V^e = 0.29\,Z + 0.21\,Z_{-1} - 0.18\,Z_{-2} + 0.90\,s_{-\frac{1}{2}}$	
234	231	$I^A - X^i - Y^i = -0.19\,Z - 0.08\,z_{-1} - 0.25\,s_{+0.17} - 0.03\,L_{+0.89}$	
235	232/3/4	Total I $= 0.27\,Z + 0.27\,Z_{-1} - 0.27\,Z_{-1} + 0.79\,s_{-0.7} - 0.03\,L_{+0.9}$	
236	V 3	$\varDelta M^c = 0.03\,Z + 0.06\,Z_{-1} + 0.05\dot{L}_{-0.5} - 2.3\dot{m}s^s_{-1.5}$	
237	215	$U^i = 0.06\,Z + 0.17\,Z_{-1} + 0.04\,Z_{-2} - 0.01s_{-0.31} + 0.09L_{+0.45}$	
238	VI 4	$K^e = 0.26\,Z + 0.20\,Z_{-1} - 0.39\,Z_{-2}$	
239	236/7/8	Total II $= 0.35\,Z + 0.43\,Z_{-1} - 0.35\,Z_{-2} - 0.01s_{-0.31} + 0.09L_{+0.45} +$	
		$+0.05\dot{L}_{-0.5} - 2.3\dot{m}^s_{-1.5}$	
240	II 1	$A\dot{u} =$ Total I $-$ Total II	
241	240	$A\dot{u} = -0.08Z - 0.16Z_{-1} + 0.08Z_{-2} + 0.80s_{-1.7} - 0.12L_{+0.6} - 0.05\dot{L}_{-0.5} + 2.3\dot{m}^s_{-1.5}$	
242	311	$-0.12L_{+0.6} = -0.04Z_{-0.27} - 0.11s_{0.2} = -0.03Z - 0.01Z_{-1} - 0.09s - 0.025_{+1}$	m^s
243	307	$-0.05\dot{L}_{-0.5} = -0.05L + 0.05_{-1} = -0.02Z_{-1} + 0.02Z_{-2} - 0.05s_{-0.4} + 0.05s_{-1.4}$	m^s
		$A\dot{u} = -0.11\,Z - 0.19\,Z_{-1} + 0.10\,Z_{-2} + 0.69\,s_{-0.71} + 2.3\,\dot{m}^s_{-1.5}$	
244	243	$Au = -0.11\int Z - 0.19\int Z_{-1} + 10.0\int Z_{-2} + 0.69\int s_{-0.71} + 2.3\,m^s_{-1.5}$	
245	244	$-0.129\,Au = 0.014\int Z + 0.000\int Z_{-1} - 0.013\int Z_{-2} - 0.089\int s_{-0.71} - 0.30\,m^s_{-1.5}$	
246	VI 4	$0.017\,K^e = 0.004\,Z + 0.003\,Z_{-1} - 0.007\,Z_{-2}$	
247	V 6, 245/6	$m^s + 0.3\,m^s_{-1.5} = 0.025\int Z_{-0.4} + 0.013\,Z_{-1.5} + 0.004\,Z + 0.003\,Z_{-1} -$	
		$-0.007\,Z_{-2} - 0.089\int s_{-0.71}$	
248	247	$1.3\,m^s_{-0.35} = 0.025\int Z_{-0.42} - 0.089\int s_{-0.71} + 0.004\,Z + 0.009\,Z_{-}$	
249	248	$1.3\,m^s = 0.025\int Z - 0.089\int s_{-0.4} + 0.013\,Z_{-0.3}$	
250	249	$6\,m^s = 0.11\int Z - 0.41\int s_{-0.4} + 0.06\,Z_{-0.3}$	
251	250	$6\,m^s = 0.11\,\Sigma Z_{-0.5} - 0.41\,\Sigma s_{-0.9} + 0.06\,Z_{-0.3}$	
252	251	$6\,m^s = 0.11\,\Sigma Z_{-0.5} - 0.13\,\Sigma Z_{-1.5} + 0.06\,Z_{-0.3} = 0.17\,Z_{-0.4}$	

results. Before doing so in the next part of this section, we shall consider the wage bill.

The wage bill depends on wage rates and employment. Wage rates are a function of mineral prices; employment is parallel with the total volume of production. Production consists of production of consumers' goods and production of investment goods, for exports and home demand. By calculation similar to those already indicated, it is found that the total wage bill depends on profits, pig iron prices and the interest rate, all lagged. Pig iron prices come in chiefly through the dependency on mineral prices (in particular in coal mining) and the — very moderate — influence of interest rates *via* the dependency on investment activity.

(iv) *The short-term rate of interest* [1]). From eq. V 6 we learned that the short-term rate of interest chiefly depends on the decisions of the Central Bank; the influence of international competition appeared to be only secondary and the influence of commerical banks almost absent. This has remarkable consequences for the explanation, in terms of more remote causes, of interest rate fluctuations. The decisions of the Central Bank are primarily based on the gold stock and the gold stock is the result of the annual additions or deductions over all past years. These additions (we consider deductions as negative additions) are equal to the annual inflow of gold into the country, minus the rate of increase in gold in the hands of the public.

The annual inflow of gold is the balancing item of the balance of payments; since we showed, at (iii) in this section, that all other items of the balance of payments are, in the end, dependent on profits, pig iron prices, total wages, the rate of increase in total wages and the rate of increase in interest rates, the same is true of the item making the balance. As profits and iron prices have the greatest influence, one could say that roughly the systematic factors making for the gold inflow are cyclical without lag. The important mathematical feature in the explanation, by these items, of the rate of interest is that it is the cumulation over time (the time integral) of the inflow that makes for the fluctuations in the interest rate. Cumulation over time of a cyclic curve means, however, shifting it backwards. This in itself would already explain the tendency to a lag in the movements of interest rates. From the cumulated inflow we still have to deduct gold in the hands of the public, depending on (*a*) cumulated profits, (*b*) the wage income and (*c*) the rate of interest

[1]) The fluctuations of the long-term rate of interest have been very weak, in the period considered, and not systematically related to the business cycle. In addition, their influence on investment activity has been found to be of subordinate importance only. It is probable therefore that they have not actually influenced the genesis of the cycles very much. They have been neglected altogether in what follows. This means that the equations in the monetary sphere explaining those fluctuations are superfluous. Hence they have been left out of consideration, although they might have led to some interesting conclusions.

(cf. eq. V 3). Total wages may, according to the last section, again be expressed in the other variables. It follows that the Banks' gold stock depends on: cumulated profits with various lags, cumulated iron prices and the rate of interest, both also lagged (cf. eq. 244). Taking account of the influence of capital exports on the rate of interest (eq. V 6) and performing some further minor transformations (eq. 245—251), we find that the rate of interest depends on profits, cumulated profits and cumulated iron prices (eq. 251). Using equation (VIII 32) for s, this may be simplified further still:

$$\begin{aligned} 6\, m^s &= 0.11\, \Sigma Z_{-0.5} - 0.13\, \Sigma Z_{-1.5} + 0.06\, Z_{-0.3} = \\ &= 0.11\, Z_{-0.5} - 0.02\, \Sigma Z_{-1.5} + 0.06\, Z_{-0.3} \end{aligned}$$

or, neglecting the ΣZ-term:

$$6\, m^s = 0.17\, Z_{-0.4} \qquad \text{(VIII 34)}$$

(v) *The income equation and the final equation.*

We have now obtained a number of deduced ecomomic laws for cyclic movements, which enable us to give a fairly clear picture of the mechanism. We have expressed all other variables in terms of profits, iron prices and interest rates, and, in addition, iron prices and interest rates in terms of the other two of these three "strategic" variables. The mathematical form of the functions is held linear as a first approximation, but there are lags, caused by the length of reaction periods of various kinds and sometimes the variables appear in cumulated form. The natural reason for this is that in various places there is an influence of certain stocks on the other variables — either gold stocks or pig iron stocks or stocks of investment goods — and stocks are cumulations of production minus consumption.

The only remaining equation, which we need in order to complete the system of relations, may usefully be taken as the central point for our further considerations. It is the income equation (VI 3), telling how profits are calculated. It may be formulated (cf. chapter VI, section 3):

profits = value of ultimate consumption and home investment,
minus wages, plus correction for foreign trade,
plus correction for historical costs.

The correction for foreign trade represents the consequences of the country being an open country, for which there is a deviation between value of production and value of consumption and investment taken together. It may be expressed, with the help of the preceding part of this section, in terms of profits, with lags.

The value of ultimate consumption equals quantity times price; the price level depends on profits, as we saw in this section at (ii). The other factors determining consumption are total wages and lagged profits (equation III 1, omitting the stock term, which does not represent ultimate consumption). This influence of total wages combines with the term "minus wages" and may be reduced to profits and cumulated profits, according to the end of part (iii) of this section.

The value of home investment depends on lagged profits, iron prices and interest rates, the latter two items again on profits and cumulated profits.

The correction for historical costs (cf. section VI 3) depends on the rate of increase in raw material prices and hence, indirectly, on the rate of increase in profits (cf. IV 5).

Given the importance for our further investigations of this part of the elimination process, it is worth while to comment in detail on the transformation of this equation. In section VI 3 it was given the form:

$$\text{(VIII 35)} \qquad 3\,Z_{-0.6} = (U' - 0.45\Delta u) + V' - L + \\ + (U^e + V^e + I^A - X^i - Y^i - U^i) + 4.5\,\dot{r}^i$$

The left-hand side represents profits; the reason why profits are not simply equal to Z is twofold:

(i) There is a lag between profits and Z which we did not know exactly beforehand;

(ii) Profits probably show larger fluctuations than Z, but we do not know in exactly what proportion. In section VI 3 we found the coefficient to be 1.70, when using the second regression equation — which enabled us to find the lag — but 3.0 when using the first regression equation which logically would be the best, if the lag were known. We have chosen the last coefficient.

For the logic of our further considerations it is more elegant to introduce a symbol $z = 3\ Z_{-0.6}$ showing no lag and no difference of amplitude compared with profits. The income equation now becomes:

$$z = (U' - 0.45\Delta u) + V' + (U^e + V^e + I^A - X^i - Y^i - L) + 4.5\,\dot{r}^i$$

The items occurring on the right-hand side may now also be expressed in terms of z. For these transformations we need, first of all, the two equations (VIII 32) and (VIII 34), expressed in z:

$$\text{(VIII 35 1')} \qquad s = 0.33\,Z_{-0.6} = 0.11\,z$$

$$\text{(VIII 35 2)} \qquad m^s = 0.03\,Z_{-0.4} = 0.01\,z$$

We may replace the first equation by a more accurate one, if we substitute (VIII 351') and (VIII 352) into equation (326 of table VIII 32), where the negative and the cumulated terms are still present. This yields:

$$\text{(VIII 351)} \quad s = +\,0.02\,\Sigma z_{-0.7} - 0.004\,\Sigma Z_{-1.7} + 0.11 z_{-0.2} + 0.017 z_{-0.5} - \\ - 0.025 z_{-1.2} - 0.02 z_{-1} \\ = +\,0.016\,\Sigma z_{-0.7} - 0.004 z_{-0.7} + 0.127 z_{-0.2} - 0.045 z_{-1.1} \\ = +\,0.016\,\Sigma z_{-0.7} + 0.13 z_{-0.2} - 0.05 z_{-1.1},$$

where we may neglect the first term.

Next we want the equation obtained for p (equation 105, table VIII 32).

$$\text{(VIII 353)} \quad 11.7p = 0.51Z_{-0.5} + 0.41s_{-0.8} = 0.17z + 0.05z_{-1.0} - 0.02z_{-1.9}$$
$$= 0.22z_{-0.2} - 0.02z_{-1.9}\ ^{1)}$$

We are now able to transform each of the right-hand side items of the profits equation. The value of ultimate consumption:

$$\text{(VIII 354)} \quad U' - 0.45\Delta u = 0.80L + 0.50Z_{-1} + 0.42Z_{-2} + 11.7p \quad \text{(III 1)}\ ^{2)}$$
$$= 0.80L + 0.31z_{-0.9} + 0.22z_{-0.2} - 0.02z_{-1.9}$$
$$= 0.80L + 0.53z_{-0.6} - 0.02z_{-1.9}$$

The value of home investment:

(VIII 355)

$$V' = 0.39Z_{-1} + 0.27Z_{-2} - 5.3m^s_{-1} + 1.13s_{-0.5} - 0.49s_{-1.5}$$
$$\text{(123, table VIII 32)}$$
$$= 0.22Z_{-0.8} - 0.05z_{-1} + 0.14z_{-0.7} - 0.06z_{-1.6} - 0.06z_{-1.7} + 00.2z_{-2.6}$$
$$= 0.38z_{-0.9} - 0.17z_{-1.5}$$

We do not combine these two terms since in the expression for this item the approximative formula would be too rough.
Total wages

$$\text{(VIII 356)} \quad L = 0.34\,Z_{-0.9} + 0.93s_{-0.4} - 1.2m^s_{-1} \qquad \text{(307, table VIII 32)}$$
$$-0.20L = 0.07Z_{-0.9} - 0.19s_{-0.4} + 0.24m^s_{-1}$$
$$= -0.02z_{-0.3} - 0.02z_{-0.6} + 0.01z_{-1.5}$$
$$= 0.04z_{-0.5} + 0.01z_{-1.5}$$

The correction for foreign trade:

(VIII 357)

$$U^e + V^e + L^A - X^i - Y^i - U^i = 0.18Z + 0.11Z_{-1} - 0.31Z_{-2} + 0.69s_{-0.8}$$
$$\text{(313, table VIII 32)}$$
$$= 0.06z_{+0.6} + 0.04z_{-0.4} - 0.10z_{-1.4} + 0.09z_{-1.0} - 0.03z_{-1.9}$$
$$= 0.19z_{-0.4} - 0.13z_{-1.5}$$

$$\text{(VIII 358)} \quad 4.5\dot{r}^i = 4.5(0.121\dot{Z} + 0.033\dot{Z}_{-1}) \qquad \text{(IV 5)}$$
$$= 4.5 \times 0.154\dot{Z}_{-0.2} = 0.23\dot{z}$$

Here again, a lead is neglected, since it has no logic meaning and may be assumed to fall within the margin of error.
Combining (VIII 354)—(VIII 358) with (VIII 35), we have:

$$\text{(VIII 359)} \quad z = 1.11\,z_{-0.7} - 0.36\,\dot{z}_{-1.4} + 0.23\,\dot{z}$$

1) Here we neglected a lead $+0.1$ in the term $0.51\,z$, since it would have no logical sense and since the difference is only small.

2) The difference between $0.43\,\Delta u$ and $0.45\,\Delta u'$ may again be disregarded.

According to the approximations referred to before, we shall transform the derivative into a difference. Since the interval of this difference may be chosen freely, but has a considerable influence on the results, we have to ask whether some natural interval exists over which the difference has to be taken. In view of the bookkeeping habits, the year seems the most natural interval. At the same time, for reasons of logic, it is desirable to introduce a small lag into the z-term, since, owing to the theory of this equation, z is caused by $\dot{z}$ and not vice versa. We might therefore replace $\dot{z}$ by $z - z_{-1}$ or by $z_{-\theta} - z_{-1-\theta}$, where θ represents a small lag. Finally it is desirable to have lags that are multiples of one and the same unit lag. One simple way (I) in which all this can be performed is to transform $\dot{z}$ into $z_{-0.4} - z_{-1.4}$ and to replace $z_{-0.4}$ by $(3/7)z + (4/7)z_{-0.7}$; another is to replace $\dot{z}$ by $z - z_{-1}$ and to replace z_{-1} by $(4/7)z_{-0.7} + (3/7)z_{-1.4}$. The results are, using 0.7 years as a new time unit, and reducing the first term to z:

(VIII 3591) I: $z = 1.38\, z_{-1} - 0.65\, z_{-2}$

(VIII 3592) II: $z = 1.27\, z_{-1} - 0.60\, z_{-2}$

Since II is more accurate than I, we shall use II as our "standard case" in what follows.

4. SOLUTION OF THE FINAL EQUATION CHIEF FEATURES OF THE MODEL

(i) *The mathematical aspects of the solution.*

The solution of the characteristic equations of I and II are easily found to be:

$$0.69 \pm 0.425\, i \text{ and } 0.635 \pm 0.45\, i,$$

corresponding with periods of 8 and 7 years respectively and $r = 0.81$ and 0.78 respectively, i.e. the movement is considerably damped.

These results are not bad, since the uncertainty in the period and the damping degree are fairly great. Small changes in the co-efficients suffice to cause considerable changes in the period and damping. It is, moreover, probable that the echo principle also works to some extent, tending to fix the period somewhat better and that, in addition, curvilinear relations — which will be discussed later in this section — hold the system within certain limits.

First, the main features of the mechanism without these complications may be described. An outstanding feature of this mechanism is that it explains the cumulative process as well as the turning points. It is sometimes believed — in particular by non-mathematical students of cycle problems — that there must be separate theories for these four phases of the cycle. The theory given here is one for all phases at the same time.

If we assume any initial disturbance of equilibrium — i.e. values of z_1 and z_2 different from zero — then a (rather much damped) cyclic movement. develops. An arbitrary numerical example is given in the following table:

TABLE VIII 4 1

Numerical extrapolation (case II) with given values z_1 and z_2 (— 10 and 0, resp.)

t	z_{t-1}	z_{t-2}	$1.27\ z_{t-1}$	$-0.60\ z_{t-2}$	z_t
3	0	— 10	0	+ 6.0	+ 6.0
4	6.0	0	7.6	0	+ 7.6
5	7.6	6.0	9.7	— 3.6	+ 6.1
6	6.1	7.6	7.7	— 4.6	+ 3.1
7	3.1	6.1	3.9	— 3.7	+ 0.2
8	0.2	3.1	0.3	— 1.9	— 1.6
9	— 1.6	0.2	— 2.0	— 0.1	— 2.1
10	— 2.1	— 1.6	— 2.7	+ 1.0	— 1.7
11	— 1.7	— 2.1	— 2.2	+ 1.3	— 0.9
12	— 0.9	— 1.7	— 1.1	+ 1.0	— 0.1

At the outset, the second term $-0.60\ z_{t-2}$ is positive or, if negative, only small; this means that z is larger than or about equal to $1.27\ z_{-1}$, or that z is larger than z_{-1} and each following z is larger than the preceding one. This is an upwards cumulative process (in the table, z_1 up to z_4). But there are tendencies to disturb this movement. If z_{-2} grows, there comes a point where $0.60\ z_{-2}$ surpasses $0.27\ z_{-1}$ and this means that $z < z_{-1}$: a turning point is reached (at $t = 5$). Then a cumulative process downwards develops, up to $t = 9$, where again a turning point is attained.

(ii) *The economic aspects of the model.*

What in your theory, the economist will ask, is the cause of the cumulative process and what is the cause of the turning point? This question cannot be answered in a qualitative way only. It is largely a question of figures. Of course, in extreme cases, qualitative answers are possible. If, e.g. only the first term on the right-hand side should be present, the process would always remain a cumulative one. But this first term — and the economic phenomenon it represents — only yields a cumulative process in such a case, if its coefficient surpasses one. If not, a process of adaptation would develop. The first term being given as it is, it may be said that the second term is the cause of the turning point. But also this is only conditionally true: it also depends on the numerical strength of that term — i.e. of the economic phenomena behind it. A quantitative problem cannot be formulated qualitatively only.

With these important reservations in mind, we may, nevertheless, give some satisfaction to the economist by stating what economic phenomena are chiefly responsible for the various terms. This can easily be seen if we only go back to the origin of each term.

The first term is chiefly the expression of *the propensity to consume and to invest*: there will be the more consumed and invested the higher incomes one time unit before are and this tends to make incomes high again in the next time period, even somewhat higher. This tendency is helped by the tendency of *higher prices*, as a consequence of higher demand. It is also helped by the influence which *rising prices* have on the profit calculations. In this sense there is a sort of "*accelaration principle*" working: profits depend on the rate of increase in prices.

The negative term $- 0.60\, Z_{-2}$ is to some extent a consequence too of this acceleration principle: the rate of increase in some variable x always means a difference between two values of x, the negative one preceding the positive. In addition, this negative term originates from the negative influence of *interest rates* and *iron prices* on investment activity. These influences may therefore to some extent be said to cause the turning point, but *only in co-operation with the influence of historical cost calculations*. The negative term arising from interest rates alone is only a very small part of the total negative term and would not at all be sufficient to cause a turn. Neither would the combination of interest rates and iron prices. The combination of these two with historical cost calculation does, however.

Another important feature in the explanation of cyclic movements is that the negative term has a greater lag than the positive one. If it should be otherwise, no cycles would occur, except cycles of two time units, i.e. 1.4 years. The reader who is not convinced should only try it out by recalculating table VIII 41 with the lags interchanged or the signs interchanged. The fact that in our model the negative term has the greater lag is a consequence of (i) the above mentioned "*acceleration principle*"; and (ii) the fact that the iron price (cf. equation 326, table VIII 32) by depending on investment activity, depends on earlier iron prices with a negative sign. This generates the negative term in equation (VIII 351), which is responsible for various negative terms in the equations (VIII 354), (VIII 355). As a third reason why the negative term has a greater lag than the positive one we have (iii) the occurrence of the negative s-term in equation (123, table VIII 32) [1] which has a greater lag than the positive s-term. The negative term is due to the influence of prices on the quantity of demand, which is lagged, whereas the positive term reflects the influence of prices on the value of demand, which is not lagged. The fourth reason why the negative term has a greater lag than the positive one is that this is so in the "correction for foreign trade" (eq. 313, table VIII 32). At closer examination, this appears to originate from three elementary equations, viz. those for K^e, $p^e - p^w$ and $q - q^w$ (VI 4, VI 5 and VI 8). Since these equations are all "supplementary equations" (cf. this chapter, sections 1 and 2), their exact economic

[1] Cf. the preceding section.

significance is unknown to us; but the negative sign of Z_{-2} in the equation for K^e may be interpreted as caused by a lead which K^e shows with respect to Z, i.e. with respect to British cycles. The same may be said of $p^w - p^e$ and $q^w - q$, meaning equally that world prices were, generally speaking, in advance of British prices.

It will be clear that most of the elements of our theory have been taken from existing theories. This is but natural since our chief purpose was only to find the relative importance of these theories and, wherever possible, to give quantitative expression to them. Let us see which theories are found again with particular strength in our scheme. For a discussion of the relation of other theories to our scheme we may refer the reader to another essay [4]).

The most important elements in the model are:

(*a*) SCHMIDT's theory on the influence of *historical cost accounting* or, in other words, of the paper profits caused by price rises and corresponding losses caused by price falls [1]);

(*b*) AFTALION's theory on the influence of a *construction period* [2]), accounting, at least partly, for the influence of lagged incomes on incomes themselves.

(*c*) elements of KEYNES' theory on the *marginal propensity to consume*, but also with a *lag*, accounting for the rest of the influence of lagged incomes on incomes themselves;

(*d*) many elements common to many theories and more or less generally accepted, such as the *influence of prices on demand*, of *interest rates on investment activity* and on *money in circulation*, the influence of gold stock on interest rates, etc. accounting for smaller contributions to the coefficients used.

To all this we should add, as we have pointed out elsewhere the probable influence of

(*e*) the *echo principle* [3]).

(iii) *Changes for greater amplitudes.*

The amplitude of the cyclic movements depends on the initial values of z, which, in turn, for a damped system subject to random shocks depend on the disturbances in the last few years. If these disturbances are small, the system will not move away far from the equilibrium development. If they happen to be great, situations far from the equilibrium may be reached and the system may come outside the region where

1) Cf. G. HABERLER, Prosperity and Depression, Geneva 1939, p. 50.

2) Cf. HABERLER, op. cit., p. 134/135.

3) Cf. HABERLER, op. cit., p. 84/85; the author believes that only the echo principle is able to explain the relative stability of the period of cycles observed between 1850 and 1914.

4) Cf. J. TINBERGEN, "Critical Remarks on Some Business Cycle Theories," Econometrica 10 (1942), p. 129.

the linear approximations for the elementary relations are valid. Some of the relations must then be changed. There are three categories of them:
1) the relations which we *found to be curvilinear* but which we replaced by linear approximations: in this class belongs eq. VI 10 for w';
2) the relations for which we did not find statistical evidence of their being non-linear, but that must, *by principle*, be so: this applies to all definition equations for variables representing money amounts (II 7—13 and VI 2) and to those equations in which variables with some maximum or minimum value occur.

Of the latter two categories we should only take the relations with variables that have rather wide fluctuations, for these probably will reach their maximum or minimum values before the other variables. As an example, take production. Production cannot surpass very much normal capacity of production. This is true for the production of investment goods as well as for the production of consumers' goods. But the former shows much wider fluctuations than the latter and will, therefore, usually first reach its limit. Hence we need not bother about the limit of consumers' goods production. For similar reasons we need not occupy ourselves with the non-linearity of the equation $\overline{\overline{U}}' = \overline{\overline{u}}' \overline{\overline{p}}$, for which II 7 stands; but we might do so with the non-linearity of the equations $\overline{\overline{V}}' = \overline{\overline{v}}' \overline{\overline{q}}$ and $\overline{\overline{V}}^e = \overline{\overline{v}}^e \overline{\overline{q}}$. On closer examination, however, the influence of the non-linearity of these equations appears to be small too in comparison with what will now be discussed.

The chief variables to which some *maximum or minimum is set*, seem to be variables representing the absolute size of:

(i) stock quantities, which cannot be negative;

(ii) supply quantities, which cannot surpass the capacity to produce.

The first condition applies, strictly speaking, to all economic variables, but the great majority of them usually does not come near this limiting value, or has not done so in the period studied [1]). Both conditions will be applied, for the reasons just given, to investment goods only, i.e. to pig iron.

(i) As soon as pig iron stocks should be exhausted, the equation (IV 8) for $\dot{w}$ could no longer be satisfied; it would be:

$$\dot{w} = 0$$

as long as $v' + v^e$ are positive, i.e. during the whole boom period and some time after.

(ii) As soon as v, according to equation (II 6), should reach the value

[1]) Some of them did in the period after 1929, e.g. the short-term interest rate. Correspondingly, a curvilinear relation (4.420) had to be introduced in our study of the United States 1919—1932 (loc. cit., p. 85).

of capacity v^c, it would have to stay on that level as long as (II 6) should lead to a higher figure. As long as iron stocks should permit a quicker depletion in order to satisfy the high level of demand, equation (II 6) could be replaced by

$$v^c = v' + v^e + \dot{w}$$

determining $\dot{w}$ instead of, formerly, v; and this would imply the non-validity of the equation for $\dot{w}$ (IV 8). Hence there would be two stages to be distinguished. First, if v reaches v^c, production remains on that level and stocks are depleted more rapidly than would otherwise have been the case. Secondly, if stocks fall to zero, production still remains v^c, but $\dot{w}$ becomes zero, which makes it necessary for v' or v^e to be determined in deviation from what it otherwise would have been: it must be rationed. Long before this stage has been reached, however, another change will come into play. If stocks are getting small and capacity is nearly fully occupied, prices s will rise more than they did before for the same increases in these independent variables. This tendency has already been expressed in the relation between stocks and iron prices that we introduced; according to this relation s would even become infinite, if stocks should fall down to zero. Probably also a reduction to very low levels of unused capacity would have a greater effect on s than would an equal reduction at higher levels. The statistical data on unused capacity are not accurate enough to rely upon our equation for the details of these reactions, in particular for the answer to the question as to which of these two tendencies would be more pronounced. Hence we may assume — since the only remaining term (the l-term) in equation (IV 7) is but small — that the coefficients in the equation for s all increase in the same proportion as stocks and unused capacity get smaller and smaller; and this increase has no clear limit.

Turning to the question as to what statistical evidence there is for the various effects under discussion, we must stress the fact that iron stocks have never been exactly at zero. Hence the second stage would not have come into existence. Moreover, stocks have always shown their minimum some time after the crisis and hence there evidently was still some elasticity in them at the top of the cycle. It is less easy to find whether or not the first stage indicated above has ever been reached. Judging from the statistics of blast furnaces in existence and in use they have never been working all of them, not even in 1873. But, as has already been observed, the figure of the furnaces in existence sometimes included furnaces incapable of production. Another, somewhat more indirect way of testing the presence of stage 1 is whether or not production has been at one and the same high level for some time past. In the annual figures such a persistence cannot be discerned. Unfortunately I have not been able to discover monthly figures for pig iron production.

The one thing which is statistically clear is the curvilinear influence of stocks on iron prices. Iron prices up to levels much higher than a linear relation would be able to explain. Therefore the chief alteration we have to introduce, if the system goes outside the linearity region, is to change the equation for s. Let us calculate the consequences of a proportionate change in all coefficients of the s-equation for the movements of the system as a whole. In order to fix the ideas, let us take instead of (VIII 351):

(VIII 351″) $$s = 0.26\, z_{-0.2} - 0.10\, z_{-1.1}$$

For the sake of simplicity let us neglect the (much smaller) repercussions this assumption has on equation (VIII 352) for m^s and only compute the other effects. Equations (VIII 354) to (VIII 359) become:

(VIII 354′) $$U' - 0.45\, u = 0.80\, L + 0.59\, z_{-0.6} - 0.04\, z_{-1.9}$$

(VIII 355′) $$V' = 0.56\, z_{-0.9} - 0.30\, z_{-1.5}$$

(VIII 356′) $$-L = 0.80\, L + 0.02\, z_{-1.5} - 0.07\, z_{-0.5}$$

(VIII 357′) $$U^e + \ldots - U^i = 0.28\, z_{-0.6} - 0.17\, z_{-1.6}$$

(VIII 358′) $$4.5\, \dot{r}^i = 0.23\, \dot{z}$$

(VIII 359′) $$z = 0.23\, \dot{z} + 1.45\, z_{-0.7} - 0.58\, z_{-1.4}$$

When treating this equation in the same way as (VIII 359) case II, we obtain a final equation:

$$z = 1.72\, z_{-0.7} - 0.88\, z_{-1.4}$$

showing movements of a period of 9.5 years, with $r = 0.94$, i.e. with only slight damping.

This clearly means that *when the system comes into regions farther away from equilibrium, it will perform movements of a somewhat different type*, but *still periodic*; a turning point will come, although *somewhat later*.

VIII 5. EFFECTS OF STRUCTURAL CHANGES ON CYCLES

The nature of the mechanism may further be investigated by studying the effect of various structural changes on the nature of the cyclic movement. These studies have two different scopes. First we may detect how alternative decisions as to some *dubious points in the analysis* influence the system: it is to be hoped that there are no great differences from the standard model chosen. Secondly we may attempt to detect how certain *variations in structure influence the movements of the system*. Here the result may be either a small or a big difference: in the first case the structural variation considered has no great influence on the business cycle mechanism, in the second case it has. Sometimes one and the same variation may be instructive for both purposes.

(i) *Complete parallelism between British and foreign prices.*

Concerning the competing prices p^e and p^w we have assumed that there was a systematic difference which we expressed, by equation (VI 5), as a function of Z, Z_{-1} and Z_{-2}. The same was done with respect to q and q^w and to p and p^i (equations VI 8 and VI 7). It may be, however, that this assumption is not correct and that the differences observed are of a random character. This would mean that their systematic part is zero and we shall therefore enquire how the movements of our standard model are changed by putting

$$p^e - p^w = q - q^w = p - p^i = 0$$

This comes down to recalculating the elimination process, which leads to the final equation:

$$z = 1.16\, z_{-1} - 0.64\, z_{-2}$$

as against $z = 1.27\, z_{-1} - 0.60\, z_{-2}$.
The intensity of the first term appears to be smaller. The corresponding movement shows a period of 6 years, and $r = 0.78$. This result evidently fits in well with our observation that foreign prices preceded British prices, as a rule. Should British prices immediately have adapted themselves to the foreign price level, the cycle would probably be shorter. The deviation from the standard case is not, however, of very much importance, and this in itself is satisfactory.

(ii) *Fluctuations of external raw material prices at half their former amplitude.*

External raw material prices were assumed to depend on Z and Z_{-1}, since the United Kingdom was one of the most important buyers. It was not, however, the only buyer; and our hypothesis was therefore based too on the principle that, in order to obtain a complete theory, assumptions have to be added on a parallelism between the economic fluctuations in the United Kingdom and other buyers. The smaller a country, the less probable this parallelism. By assuming therefore that only half of the raw material price fluctuations are due to home demand fluctuations, we get a situation corresponding to smaller countries. At the same time we get a situation corresponding to a half-way stabilisation of raw material prices. It will be interesting to compare this with the effect of full stabilisation of raw material prices and with the effect of stabilisation of all prices.

The original system of equations is only changed in three places: equations IV 5, IV 6 and VI 5 are changed into:

$$r^i = 0.061\, Z + 0.017\, Z_{-1}; \qquad \text{(IV 5')}$$

$$r = 0.034\, Z + 0.035\, Z_{-1} \qquad \text{(IV 6')}$$

$$p^i = 0.017\, Z + 0.042\, Z_{-1} \qquad \text{(VI 5')}$$

The result of the elimination process is now:

$$z = 0.12\,\dot{z} + 1.06\,z_{-0.7} - 0.35\,z_{-1.4}$$

or choosing the new time unit $= 0.7$ years, and $\dot{z} = z - z_{-1}$ as in standard case II:

$$z = 1.12\,z_{-1} - 0.45\,z_{-2}$$

The corresponding movements show a period of 7.5 years and $r = 0.67$. The figure for the length of the period should not be taken too literally, since the longer the period, the more uncertain the figure (it represents a farther extrapolation than for smaller periods). The period will be somewhat lengthened; more cannot be concluded. More important, however, is the reduction in r. This is an indication that *stabler prices will lead to a stabler cyclic movement.*

(iii) *Capital exports considered as a random variable.*

In our analysis we assumed capital exports to depend systematically on the economic situation. This induced us to introduce equation VI 4. It might be held, however, that investments in new countries — for which capital exports are largely destined — are autonomous events that do not depend very much on the economic situation (of course they influence, of themselves, the economic situation, but that is another question). In the language of our analysis we must suppose then that capital exports are a random variable, and the systematic part $= 0$. Carrying through the elimination process with the assumption yields the final equation:

$$0.77\,z = 1.01\,z_{-0.7} - 0.42\,z_{-1.4}$$

the solutions of which correspond with a movement of a 9 year period and $r = 0.74$. In comparison with standard case II (period 7 years and $r = 0.78$) this is not a very great difference, and perhaps even an improvement, since the period is somewhat longer. The conclusion is important since the meaning of the relation for K^e, if any, is not very clear and the probability is great that K^e is in fact a random variable. If so, it has been one of the most important impulses to British cycles.

(iv) *No historical cost accounting.*

Up to now we have considered the consequences of a change in some of the external variables. We shall now discuss some changes in the equations determining internal variables. We have already discussed the influence of *bottle necks*, i.e. of a change in the price formation for investment goods and of a change in their effective demand as a consequence of full utilisation of capacity (cf. the preceding section). We may now consider the influence of *historical cost calculation* as assumed in the income equation. The most radical alternative assumption on this subject would be that no term with $\dot{r}^i$ occurs, i.e. that replacement cost calculation had been used everywhere. This reduces the final equation (case II) to:

$$z = 1.09\, z_{-0} - 0.33\, z_{-1.4}$$

The corresponding movements are strongly damped ($r = 0.57$) and show a period of 13 years. The influence of this assumption on our results appears therefore to be somewhat more important.

We must not overlook the fact, however, that such a change in our assumptions means that at the same time one of our correlations is more or less severely impaired; in this case, the correlation obtained for the income equation. The alternative assumption now made — viz. that costs are not accounted for by the "historical" method — is therefore less probable [1]) unless we succeed in finding an additional change in our hypotheses, which restores the value of the correlation coefficient. In such a case we shall speak of two *compensating changes*. The second change may take the special form of a change only in the numerical values of the regression coefficients in the equation in which the first change in hypotheses bears, viz. such a change as to give, in the new circumstances, the highest correlation coefficient. If we change the income equation in this twofold way, the correlation coefficient falls from 0.69 to 0.64 and the regression equation becomes:

$$3.9\, Z_{-0.6} = U' - 0.45\, \Delta u + V' + U^e + \ldots\ldots - U^i,$$

the final equation is

$$z = 0.84\, z_{-0.7} - 0.25\, z_{-1.4},$$

corresponding to cycles of 8 years and $r = 0.50$. Whereas the change in period is thus compensated, the change in damping degree is not. The lower correlation coefficient seems to affirm that our original assump tion on historical cost accounting is rather essential for our model. On the other hand it suggests that *replacement cost accounting will be favourable to — although not sufficient for — a stabilisation of the business cycle.*

(v) *Greater elasticity of demand for consumer goods.*

Another uncertain point in our analysis is the extremely low elasticity of the home demand for consumer goods (cf. chapter III, section 1). We may change this hypothesis in a simple way by omitting, in equation III 1, the term with p. This means that the elasticity of demand is taken to be equal to one, representing the other extreme. By so doing, however, we get a particularly bad fit for the consumption equation. It would therefore be more appropriate to re-calculate equation III 1, omitting p as an explanatory variable. This yields other values of the coefficients

[1]) For this same reason it is not quite permissible to reject our model because of the sensitiveness of the results to a change in any single regression coefficient; only sensitivity to compensatory changes in more than one coefficient would be an argument.

for the other variables; meaning, in particular, that the propensity to consume is greater. We get:

$$U' = 0.80\,L + 0.60\,Z_{-1} + 0.83\,Z_{-2} + 0.45\,\Delta u$$

Using this equation, the final equation is changed into:

$$z = 1.17\,z_{-1} - 0.53\,z_{-2} \qquad \text{(time unit 0.8 years),}$$

of which the solutions correspond to a movement of an 8 year period and $r = 0.73$. Evidently this change does not do great harm to our theory. Of course, quite another result (much more harmful) would have been obtained, if only the elasticity of demand had been changed and not the propensity to consume. The two changes are, to some extent, compensatory changes. If, in other words, our first determination of the elasticity of demand is erroneous, the propensity to consume is too, and these two errors are largely compensatory as to their influence on cycles. It follows that the uncertainty in some of our coefficients has not such a great influence on our results as the effect of the variation of one single coefficient on the final equation would make us believe.

(vi) *Greater elasticity of demand for exports of consumer goods.*

The same type of variation may be applied to the export equation. It is to be expected that this variation will give smaller deviations, since exports are not so important as the internal market. Instead of equation III 2 we now find, if the coefficient for $p^e - p^w$ be given the *a priori* value 2.0 (corresponding to unit elasticity for exports):

$$u^e = -\,2.0\,(p^e - p^w) + 0.92\,T^w + 0.09\,K^e$$

and performing, with this equation, the elimination process, we find for the final equation:

$$0.77\,z = 0.98\,z_{-0.7} - 0.48\,z_{-1.4}$$

leading to cylces of 6.9 years and $r = 0.79$. Thus we find a slight change only, as could be expected [1]).

(vii) *Greater elasticity of demand for investment goods.*

As a third example let us consider the consequences of an alternative assumption on the elasticity of home demand for investment goods. This means that equation III 4 is replaced by:

$$v' = 0.94\,Z_{-1.5} - 1.49\,q_{-1} \qquad \text{(III 4')}$$

where q_{-1} has a coefficient equal to the coefficient before q in the equation

[1]) Substituting equation (III 2′) for equation (III 2), i.e. assuming that exports are determined not only by the price difference between English and "world" prices, but also by the absolute level of English prices, we find only slight differences in the equations immediately preceding the final equation.

II 10 for V'. The coefficient for Z has been found by correlation analysis; the coefficient for m^s is found to be small and has been neglected.

The most important changes are in the equation for s and, of course, for V'. The s-equation is found to be
$s = 0.15\, z_{-0.4} - 0.04\, z_{-1.4}$ instead of (VIII 351). The final equation becomes:

$$0.77\, z = 1.10\, z_{-0.8} - 0.47\, z_{-1.6}$$

whose solutions correspond to a movement with a period of 10 years and $r = 0.78$. Again the change, although not negligible, is not revolutionary either.

(viii) *Summary.*

The calculations of the effect of various structural changes on the movements of the model show that the chief characteristics of these movements do not depend very much on

(i) whether British and foreign prices are exactly parallel or whether there is a systematic lead in foreign prices, as we found existing;

(ii) whether or not the amplitude of foreign prices is reduced to one half;

(iii) whether or not capital exports are considered as a random variable;

(iv) whether or not the demand equation for home consumption, home investment or for exports of consumers' goods is replaced by an alternative one, showing a unit elasticity of demand, but with compensating changes at the same time.

The character of the movements is, however, changed somewhat more if:

(*a*) the hypothesis of historical cost accounting is dropped, or

(*b*) the elasticity of home demand is taken as 1, without a compensating change in the other coefficients in its equation.

Generally speaking, non-compensated changes in the elementary equations have a great influence on period and damping, especially changes in equations with a great influence on the final equation, such as the income equation, the home consumption and the home investment equation. Changes in the propensity to consume, the inducement to invest and the elasticities of consumption and investment with respect to prices are examples. But non-compensated changes have the disadvantage of being in direct contradiction to facts, in so far as they usually invalidate one or more of the correlations obtained.