I. INTRODUCTORY

The subject of this volume is to be considered as an amplification and a generalization of the "theory of quantitative economic policy" as given in a previous volume. In that volume the main problem considered was: given an economy described by a number of economic variables and an equal number of structural equations; given, also, certain quantitative "targets of economic policy" and an equal number of "instruments of economic policy" or "political parameters"; at what level have these latter to be fixed in order that the targets be attained? What conditions have to be fulfilled in order that a given policy is consistent—i.e. that its targets are mutually compatible and can be attained with the help of the specified instruments? What complications arise if the instrument variables are subject to certain "boundary conditions"? A number of numerical examples were offered, taken from the investigations of the Netherlands Central Planning Bureau.

This subject matter needs amplification in various respects. A first generalization consists of the introduction of what will be called "flexible targets". Not always will the targets of economic policy be set in a numerically fixed way as is the case with such targets as: a given level of employment, a balanced budget or an even balance of payments. There may be others which are given only the form of the maximum attainable under the given circumstances, e.g. the maximum real income per head. Such a target will be called a flexible one in contradistinction to "fixed targets".

This resulting type of policy problems will be discussed in chapter II.

Another generalization and in fact the main one to be studied in this volume will be the introduction of the concept of "decentralized economic policy". In fact, in the previous volume it was tacitly assumed that there was one "policy-maker"—i.e. only one agency handling instruments of economic policy—or at least that there was sufficient contact and understanding between the various policy-makers as to let them act as one. Such cases will be referred to as "centralized policy" cases; if, however, there are various centers of economic policy, i.e. if the number of policy-makers exceeds one, new aspects come in which are of great importance in practice. Such cases will be called cases of decentralized policy; some of their characteristics will be considered in chapter III. The main object of this book is to investigate the differences between centralized and decentralized policies and to make, as far as possible, a choice between them. Comparing situations of a more or less centralized character constitutes, in the language of the first volume, a study of qualitative changes in policy. The general aspects of qualitative changes will be studied in chapter IV; and the problem of centralization versus decentralization in chapter V.

The case of a decentralized economic policy presents itself in various practical forms. On the one hand, there may be, within one country, various centers of political action; apart from the central government, there are the central bank, the trade unions and other organized groups; in addition there are local governments; and, in a sense even central government sometimes is not one single policy-maker: its various departments are to some extent autonomous. With respect to this situation the question has to be put: does it make any difference whether there is more or less centralization in economic policy and is it possible to make a choice? On the other hand, each problem of international
economic policy represents an example of a decentralized policy, each country acting as an autonomous center.

The question of the choice between a more or less centralized policy presents itself inside one country as well as in the international field. An important issue of economic policy is to what extent decisions should be left to individuals, individual industries or groups, the alternative being that central agencies make decisions. In the international field this question takes the special form of what is nowadays often called “integration” and the question arises, to what extent should the policies of a number of countries be integrated, to what extent left autonomous? Under what conditions and with the help of what assumptions can we answer this question?

It is the objective of this study to deal with a number of these problems in a somewhat more systematic way than has so far been customary and to illustrate this treatment with numerical examples taken again from the investigations of the Netherlands Central Planning Bureau.

With a decentralized policy, situations present themselves that are similar to the situations known from the theory of oligopolistic competition. Two types of questions arise; one being which parameters are being manipulated by which policymakers and the other, what assumptions the policymakers make concerning the behaviour of the others. This latter question introduces the concept of “conformity”, i.e. identity between such assumptions and real developments. Upon closer examination the concept of conformity also makes sense, however, in the case of only one policy-maker. Thus one further amplification of the theory of economic policy is supplied by the introduction of this concept.
II. CENTRALIZED QUANTITATIVE POLICY

2.1. FIXED TARGETS

With fixed targets the problem of economic policy with only one policy-maker may be stated as follows. There are four types of variables: two of them belonging to the class of data or exogenous variables, two to the class of endogenous variables. They are:

exogenous: (i) instrument variables \( z_k, k = 1 \ldots K \)
(ii) other data \( u_l, l = 1 \ldots L \)
endogenous: (iii) target variables \( y_j, j = 1 \ldots J \)
(iv) irrelevant variables \( x_i, i = 1 \ldots I \)

Among these variables we consider as given the \( u_i \) and the \( y_j \); as unknown the \( z_k \) and the \( x_i \). This is at variance with the problem of economic analysis or "explanation", where the exogenous variables (\( z \) and \( u \)) are given and the endogenous unknown (\( x \) and \( y \)). The essential difference is that targets are now given and instrument variables unknown. In the normal problem of economic analysis the number \( E \) of relations or equations will be just sufficient to determine the unknowns, which implies that

\[
E = I + J
\]

In order that the policy problem with fixed targets is solvable, this number of equations has to be sufficient also to determine the unknowns of that problem, implying that

\[
E = I + K
\]

It follows that as a rule \( K \) should be equal to \( J \), or the-
number of instruments should be equal to the number of targets.

In the particular type of policy problems now considered the targets are fixed numerical values of the target variables. Some of them may also be interpreted as side conditions, e.g. the condition of equilibrium in the balance of payments or the condition of full employment (whatever that may mean).

Also boundary conditions may interfere, i.e. the conditions that some of the variables cannot surpass certain numerical values. Boundary conditions are essentially inequalities, and do not therefore always become active. If one of them expresses the impossibility for the number of workers employed $a$ to surpass the number of workers available $b$, that is, if it runs: $a \leq b$, there may be situations where the solution found is simply in accordance with this inequality. Then it does not interfere. If, however, a solution $a_1$ would be found which surpasses $b$, i.e. if $a_1 > b$, that solution would have to be rejected and the problem as originally set would have to be considered as insoluble.

This is where the concept of consistency may be used. A policy, i.e. a set of targets together with a choice of instrument variables, may, in a given economy (i.e. an economy described by a given set of structural equations) be called inconsistent if it requires values of the instrument variables or the irrelevant variables which are declared inadmissible by certain boundary conditions.

For purposes of practical analysis we will sometimes use a set of structural equations in which no $x_i$ occur, i.e. a set of derived equations, obtained by eliminating the $x_i$. This set of equations will be referred to as the "simplified version" of the model. Moreover, the equations will often—though not always—be assumed to be linear. They will then be written in matrix form as:

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1 The mathematically less advanced reader may be referred to par. 2.2 for an example in more elementary mathematical symbols.
\[ Ay = Bz + \Gamma u \]

or even as:

\[ Bz = \gamma_0 \]

The solution

\[ z = B^{-1} \gamma_0 \]

will only be determinate if the bordered matrix \([B \gamma_0]\) has a rank of sufficient height; it has to be \( K \): \( \emptyset [B \gamma_0] = K. \)

Even then it may happen that \(|B| = 0\), implying that all \( z \) will be infinite. In practical terms this means that there is no finite solution, pointing to inconsistency in the targets set. Inconsistency will even exist, as we already stated, if finite values for \( z \) are found of which only one surpasses a boundary condition.

The rank of \([B \gamma_0]\) will be lower than \( K \) as soon as only two of the \( K \) by \( K \) determinants are \( = 0 \); for then it follows that all other \( K \) by \( K \) determinants are also \( = 0 \). In this case the policy problem is indeterminate, i.e. at least one of the instruments may be chosen freely. If the rank of \([B \gamma_0] = K - k_0\), the number of instruments that may be chosen freely amounts to \( k_0 \). A special form in which this situation may present itself may be that our equations contain a certain number of instruments \( z_1, z_2, \ldots, z_{k_0+1} \) only in the form of one expression \( \Sigma c_k z_k \); in this case it will be clear that we shall never be able to determine the separate \( z_k (k = 1 \ldots k_0 + 1) \). An example of such a situation was given elsewhere.

One form in which \(|B| = 0\) is possible would be a situation in which all the coefficients of one single equation would be equal to zero, say \( \beta_{11} = \beta_{12} = \ldots = \beta_{1K} \); it then follows

\[ \begin{vmatrix} \beta_{1} & \beta_{2} & \ldots & \beta_{1K} & \gamma_{1} \\ \beta_{2} & \beta_{2} & \ldots & \beta_{2K} & \gamma_{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta_{K} & \beta_{E} & \ldots & \beta_{EK} & \gamma_{E} \end{vmatrix} \]

has to be different from zero.

\[ J. \text{ Tinbergen, Econometrics, Philadelphia 1951, p. 183.} \]
easily that the targets, in order not to be inconsistent, have to obey the relation

\[ \gamma_0 = 0. \]

From this summary of the theory of quantitative centralized economic policy with fixed targets it follows that the logical structure of these problems is rather rigid.

2.2. FLEXIBLE TARGET POLICY; CONSISTENCY

This rigidity is loosened in the case of a flexible target policy; since then the number of relations adapts itself to the number of unknowns. The logical structure is the following: if again we eliminate the \( x_i \), we are left with a set of derived equations, \( N = I \) in number, containing \( y \) and \( z \). Again we write them in the form of the simplified version:

\[ Ay = Bz + Tu \]  \hspace{1cm} (22.1)

In stead of aiming at given values of \( y \) a flexible target policy wants to maximize a "social utility function" assumed to depend not only on the target variables but also on the instrument variables: \(^1\)

\[ \omega (y, z) \equiv \omega (y_1 \ldots y_n, z_1 \ldots z_K) \]

The nature of this social utility function will be discussed in greater detail in section 4.3.

In order to find the maximum we have to express the \( y \) in terms of the \( z \) and to substitute these expressions in \( \omega \); if the resulting function of \( z \) is written as \( \bar{\omega} (z) \), the maximum has to obey the relations:

\[ \frac{\partial \bar{\omega}}{\partial z_k} = 0 \quad k = 1 \ldots K \]  \hspace{1cm} (22.2)

\(^1\) This somewhat more general treatment was suggested to me by Prof. Dr J. B. D. Schouten en Prof. Dr H. Theil.
Using the original function $\omega$ we may write this

$$\sum_{1}^{J} \frac{\partial \omega}{\partial y_{k}} \frac{\partial y_{k}}{\partial x_{a}} + \frac{\partial \omega}{\partial x_{a}} = 0 \quad k = 1 \ldots K$$  \hspace{1cm} (22.3)

There is no difficulty now about the number of equations not being sufficient or being too large; it adapts itself to the number of unknowns. The number of maximum conditions (22.2) will simply be equal to the number of $z$'s. If there are few $z$'s only, the consequence will be that a less "advantageous" situation only can be reached. This may be illustrated by figure 1.

Here it is assumed that there are two $y$'s and that $\omega$ depends on the $y$'s only; if there were also two $z$'s, it would, generally speaking, be possible to attain any combination
(y₁, y₂) we want; if the absolute maximum of ω is reached for (y₁, y₂) the two z's can be chosen such as to obtain these y's. If, however, only one z—say, z₁—is admitted, this means that our freedom of movement is restricted; one might interpret this by saying that the other z (i.e. z₂) cannot be chosen freely and has to have a fixed value. According to the equations (22.1), which in this simple case take the form:

\[
\begin{align*}
\alpha_{11}y_1 + \alpha_{12}y_2 &= \beta_{11}z_1 + \beta_{12}z_2 + y_1u_1 \\
\alpha_{21}y_1 + \alpha_{22}y_2 &= \beta_{21}z_1 + \beta_{22}z_2 + y_2u_2
\end{align*}
\]

(22.4)

this implies that we are bound to a relation between the y's. Solving for z₂ we will find, from (22.4):

\[z_2 = \zeta_{21}y_1 + \zeta_{22}y_2 + f(u_1, u_2)\]  \hspace{1cm} (22.5)

and if z₂ is not allowed to move, the y's have to obey this relation. This means, in the geometric language of fig. 1, that we have to seek our maximum of ω in the vertical plane through the line 1:

\[\zeta_{21}y_1 + \zeta_{22}y_2 = \text{const.}\]

The maximum now obtained will be P₂, which is necessarily lower than P₀. It may be even much lower in cases where ω is heavily dependent on certain y's.

It is easily seen that our argument does not change if ω also depends on z.

In the preceding reasoning it was tacitly assumed that it will be possible to express the y's in terms of z's. This will only be so if equations (22.1) fulfil certain conditions; these, however, will be identical to the conditions the structural equations have to fulfil in order that the economic system considered be determinate.

We may conclude, therefore, that no difficulties of inconsistency arise if the number of z's is smaller than the
the number of $y$'s. If the number of $z$'s exceeds that of the $y$'s the situation depends on the way in which the $z$'s occur in $\omega$. In the extreme case where the $z$'s do not influence $\omega$, meaning that the relevant authorities are indifferent as to what instruments are used, there will again be indeterminacy: the same optimum combination $(y_1^* \ldots y_J^*)$ may then be obtained by an infinity of combinations of $z$'s. In fact, in this case the problem may be solved by two consecutive steps: first, the optimum combination of $y$'s may be determined by

$$\frac{\partial \omega}{\partial y_j} = 0 \quad j = 1 \ldots J$$  \hspace{1cm} (22.6)

and, next, the $z$'s would have to be found in the same way as in the case of fixed targets. Also there, the problem is indeterminate if $K > J$; $K - J$ of the $z$'s may, normally, be chosen freely.

If, on the other hand, the instrument variables do influence $\omega$, the problem may be—and generally will be—determinate. It will be made so by the relative preferences for certain instruments above others.

Targets of economic policy need not all be either fixed or flexible; there may be some fixed targets and some other target variables entering into a utility function, contributing to a flexible target. These mixed problems, it will be understood, are of the same nature as the flexible-target problem.

The flexible-target problem will reduce to a—consistent—fixed-target problem as soon as the "social utility function" is a linear function of its arguments. In this case, as will be clear, the optimum cannot be found by differentiation of $\omega$; we have to proceed as follows. Let $\omega$ be equal to:

$$\sum_j \omega_{1j} y_j + \sum_k \omega_{2k} z_k$$ \hspace{1cm} (22.7)

Some of the $\omega_{1j}$ and $\omega_{2k}$ will be positive; some negative. The $y$'s and $z$'s corresponding with positive coefficients will
have to be chosen as large as possible, the ones corresponding with negative coefficients as small as possible. As long as all $y$'s and $z$'s can be chosen independently, their optimum values will be governed by the boundary conditions imposed upon them; the complications arise from the relations connecting them. As with all boundary condition problems there is little scope in a general treatment. The complications may be shown by some examples.

As a first example we take a mixed fixed-flexible-target problem: let the government of a country use two instruments of policy, namely, total government expenditure $\xi_0$ and the price level $p$. Let there be two targets, one a fixed target: balance of payments equilibrium, and the other flexible, a maximum volume of production $y$. The fixed target may be described as $D = 0$, where $D$ is the balance of payments deficit. The structural equations connecting these four variables are assumed to be, in terms of deviations from an initial situation:

$$
\begin{align*}
    y &= 1.5\xi_0 + 0.4 \ p \\
    D &= 0.6 \xi_0 + 0.56 \ p
\end{align*}
$$  \hfill (22.8)

These equations are similar to the equations used in our examples in vol. I; for a somewhat more exact description the reader may also be referred to example II of chapter III of this study which upon linearisation will appear to be equivalent to equations (22.8). The only difference is that we now use deviations from the initial state, whereas in example II absolute values will be used.

The condition set by the fixed target simply runs

$$0.6 \xi_0 + 0.56 \ p = 0$$

or $\xi_0 = -0.93 \ p$; if this be substituted in the equation for $y$, we obtain

$$y = -p$$

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A maximum volume of production would, according to this equation, be obtained by choosing \( p \) as low as possible; the range of validity of this equation, is, however, only limited and "hurts itself" at the boundary conditions that govern either \( y \) or \( p \); \( y \) will be limited by a situation of full employment which may occur, say, at a value 0.08 (8 \% above the initial state):

\[
y \leq 0.08
\]

And it may be that a fall of \( p \) by more than 5 \% is considered practically impossible; if so, the corresponding boundary condition would be:

\[
p \geq -0.05
\]

It is clear that in this case the boundary condition on \( y \) would become active before the one on \( p \) could come into operation. With other figures it could have been different.

As a second example we use the same structural equations (22.8) but we now assume that the government's policy is directed towards the maximization of a utility function depending positively on the volume of production and negatively on the balance of payments; we choose:

\[
\omega = y - D,
\]

i.e. a linear form, the weights of which (+1 and −1, respectively) are such that with the initial prices \( \omega \) represents the value of production minus capital imports. With the aid of (22.8) we find:

\[
\omega = 0.9 \xi_0 - 0.16 p \quad (22.9)
\]

If no boundary conditions would have to be regarded, this expression would become the larger, the higher \( \xi_0 \) and the lower \( p \). We will now investigate, how boundary conditions on the targets will work out. We assume that, again, \( y \) cannot surpass the value 0.08; in addition we assume that
$D$ cannot surpass the value 0.02 — maybe because no more foreign balances or credits are available. Our boundary conditions are therefore:

$$y = 1.5 \xi_0 + 0.4 \quad p \leq 0.03$$
$$D = 0.6 \xi_0 + 0.56 p \leq 0.02$$

They may be formulated as conditions on $\xi_0$ in terms of $p$:

$$\xi_0 \leq 0.02 - 0.27 p \quad (22.10)$$
$$\xi_0 \leq 0.038 - 0.98 p \quad (22.11)$$

These two conditions may be interpreted graphically in fig. 2; (22.10) meaning that only points in the ($\xi_0$, $p$)-plane are permitted below line I and (22.11) that only points below line II are permitted. With these restrictions a maximum value of $\omega$ has to be found. The loci of constant $\omega$ may be written

$$0.9 \xi_0 = 0.16 p + c'$$

or

$$\xi_0 = 0.18 p + c \quad (22.12)$$

and these are lines with a positive slope of 0.18.
Evidently even our two boundary conditions (22.10) and (22.11) are not sufficient to limit the values of $\xi_0$ and $p$; a maximum value of $\omega$ will be obtained by moving along I to the left-hand side as far as possible. If now, for instance, a third boundary condition would be imposed on $p$, saying that $p$ cannot fall (i.e. $p \geq 0$), a sufficient limitation would be obtained; point $P$ ($\xi_0 = 0.02, p = 0$) would, among the points permitted, be the one with the highest value for $\omega$; it would correspond to values for $y$ and $D$ of 0.08 and 0.012, respectively.

On visual grounds it will be clear to the reader that such a “sufficient limitation” can only be obtained if there are boundary-condition lines for which the slopes are on either side of the slope of the lines of constant $\omega$.

### 2.3. Efficiency

In the case of flexible targets it is also possible to dig somewhat deeper into the concept of efficiency. In principle the efficiency of a certain instrument of economic policy has to be measured by a comparison between advantages obtained and disadvantages suffered; and in the language of utility or opheimitu functions this may be done by calculating the net gain in utility obtained. Such a net gain may now be calculated for a given unit change in any of the instruments $s_k$, without restricting ourselves to the consequences of this change for one of the target variables only. It amounts to:

$$\frac{d\omega}{ds_k} = \sum_{i=1}^{J} \frac{\partial\omega}{\partial y_i} \frac{\partial y_i}{\partial s_k} + \frac{\partial\omega}{\partial s_k} \tag{28.1}$$

where, in a sense, the last term may be said to represent the “disadvantages” and the first group of terms the “advantages.”
2.4. Conformity and Non-Conformity; Trial-and-Error Method

The decisions of any policy-maker will always depend on what he thinks the influences of the changes in instruments on the target variables to be. The real influences may be different from what they are thought to be or expected to be. There is scope therefore for introducing the concept of conformity, if the expected influences are identical to the real ones; and if they are not we may speak of non-conformity. There are notorious examples of non-conformity to be found in the history of economic policy. Surely the governments which in the thirties restricted their expenditures or increased their taxes in order to restore confidence and hence investments had not thought of the effects of their acts on general demand; they may be quoted as examples of non-conformity therefore. Also monopoloid changes in wages or prices have several times been introduced by trade unions or producers without their being aware of the consequences on the demand for either labour or products. The Amsterdam diamond industry in the thirties experienced this just as well as the American cotton farmers. In the language of our theory of economic policy we may briefly state the resulting discrepancies in the following way:

Let the real relations in the economy studied be, disregarding changes in "other data" \( u \):

\[
A' y = B' z
\]  
(24.1)

Let the policy-maker think, however, that they are

\[
A y = B z
\]  
(24.2)

Now if he is aiming at a target \( y_0 \) he will fix his instruments \( z \) according to the equation

\[
z_1 = [B']^{-1}[A'] \{y_0\}
\]  
(24.3)

\(^1\) Cf. also Kaldor's "imagined demand curve"; Sweezy's "subjective demand curve" and Frisch's "conjectural relations".
The result will, however, be:

\[ y_1 = [A^{-1}B]z_1 = [A^{-1}B][A']^{-1}y_0 \]  \hspace{1cm} (24.4)

Conformity will hardly ever exactly be realised in practice, if only because of our imperfect knowledge of the structural constants or because of changes in such "constants". This circumstance makes it necessary and more or less usual for policy-makers to follow a trial-and-error policy method. They will try a certain value of their instrument and observe the result. Dependent on this result they will adjust the instrument variable. In view of this practice it is sometimes doubted whether it is any use to make econometric estimates for the values of the instrument variables. The answer is that although it is possible by trial error to find the correct values this method nevertheless implies some waste, since unnecessary changes always mean losses and trouble. In more scientific terms: social utility is usually unfavourably influenced by the frequency of changes in political instrument values.

A very simple example of non-conformity may be given. Consider a nation with foreign-trade relations, for which the marginal propensity to spend equals 0.9. Calling national income \( Y \), national expenditure \( X \), exports \( E \) and imports \( M \), we have

\[ Y = X + E - M \]  \hspace{1cm} (24.6)

and

\[ X = 0.9Y + X_0 \]  \hspace{1cm} (24.6)

Let there be a deficit \( D = 0.1 \) in the balance of payments which the government wants to eliminate by restricting its expenditure. Let prices be invariable and let imports depend on national income by the relation

\[ M = 0.4Y. \]  \hspace{1cm} (24.7)

The policy required may be calculated by calling the
change in government expenditure $\Delta X_0$; the resulting change in $Y$ will be:

$$\Delta Y = 0.9 \Delta Y + \Delta X_0 - 0.4 \Delta Y$$

$$\Delta Y = 2 \Delta X_0$$

And the corresponding change in the deficit $D$:

$$\Delta D = \Delta M - \Delta E = 0.4 \Delta Y = 0.8 \Delta X_0$$

It follows that, in order to attain a $\Delta D = \Delta D_0 = -0.1$ we have to make

$$\Delta X_0 = 1.25 \Delta D_0 = -0.125$$

If now the government assumes that the reduction in its expenditure has to be equal to the balance of payments deficit—a prevalent opinion, only valid, however, if the marginal propensity to spend $= 1.0$—she will aim at a value $\Delta X'_0 = -0.1$. She will therefore underestimate her task.
III. DECENTRALIZED QUANTITATIVE ECONOMIC POLICY

3.1. NATURE OF PROBLEM; APPLICATIONS

The essential feature of decentralized economic policy is that there are several policy-makers. Each of these policymakers has his own target variables and instruments; in the case where flexible targets exist, their utility or obsolescence functions will also matter. This gives rise to more complicated situations, comparable, in many respects, to oligopolistic situations in ordinary economic analysis. In very simple cases, the theory of centralized economic policy will still apply; for instance when targets are fixed ones, each instrument is in the hands of one policy-maker and all of them show conformity in their expectations. In general, however, a new treatment will be necessary.

Practical applications may be found in national as well as in international economic policy. Within one country economic policy will usually be in the hands of several “centers”; apart from government, there is the central bank, and there are organized groups which play their role. In modern times economic policy often is a “game” or a play between these various policy-makers, each of them pursuing different ends, which only partly coincide. The trade unions will be particularly interested in wages: their instrument being the nominal wage rate and their target perhaps total real income of workers; sometimes they will not be quite clear even about their target: is it nominal or real wage rates or nominal or real total wages? The organizations of employers, as are employers individually, will be interested mostly in profits; their instruments being prices in a number of cases, but
partly also the volume of production or the method of production. A special place within this group is taken by farmers' organizations. The central bank authorities will see to monetary stability in some form or another; their instrument par excellence being the rate of discount, to which further instruments have been added. Other examples may be given but no attempt at stating a complete list is made here. The point to be made is that the targets are different, but will gradually, by education, tend to overlap more and more. Some sense of what is fundamentally necessary for a sound economy has been developed and a greater sense may be achieved.

Upon closer examination one will find that such complicated agencies as "the government" are in fact composed of several units, acting more or less independently. Although in important matters the various ministers will have to act in mutual agreement, the ministerial responsibility often is individual which may account for autonomous acts, sometimes not even consistent with acts of other ministers. Therefore, to some extent they also represent cases of a decentralized policy.

In international affairs it is abundantly clear that we have to cope with decentralized political action. Only in the rarest cases will a common policy be followed. Usually the targets pursued by the governments of different countries will be different; they will refer to the welfare of their own country in the first place and only in an indirect way and often rather weakly be influenced by the welfare of other countries. In addition there may be differences of opinion as to what represents the best policy in given conditions; i.e. there may be differences in political conviction of the governments in office. Gradually it begins to be understood that in certain fields of economic policy it may be wise to act together and thus to introduce an element of centralized economic policy; but surely the main features of the picture are very much those of a decentralized one.
3.2. TYPES OF EQUATIONS; CONFORMITY

In order to give a clear-cut picture of the logical structure of problems of decentralized economic policy we have to distinguish between various types of equations. We will distinguish between objective and subjective equations; the subjective equations referring to the decisions of policy makers. They determine the instrument variables under the control of each policy-maker: they may be the price setting equation of a monopolistic group (in the widest sense, including the setting of the wage rate by trade unions, of the discount rate by the Central Bank, or even of the tax rates by government), or they may be the spending equation of a government. They represent the action by organized groups or the authorities in contradistinction to the action by unorganized masses. The behaviour equations of the latter are among the "objective" equations, together with all other types of equations: balance equations, technical and definition equations. In other words, all non-behaviour equations are among the "objective" group and the behaviour equations are subdivided into "passive" and "active" ones; the passive ones referring to non-policy-makers and the active ones to policy-makers. Passive behaviour equations will be aggregated individual equations; active behaviour equations will not be aggregated in the sense of added up but they will indicate the attitude of representatives of organized groups or authorities. They may be said to form "organic" aggregations.

The concept just introduced may be summarized by the following scheme of the types of equations (see page 21):

The reader will observe the analogy between the types of equations used in describing a monopolistic or oligopolistic market: the demand equation and the balance equation are the objective equations; the supply equation is the
I. Subjective (or active
behaviour) equations:
(a) in conformity, or
(b) in non-conformity
with actual reactions
of outside world

II. Objective equations:
(a) passive behaviour equations
    indicating decisions of non-
policy-makers
(b) balance equations
(c) definition equations
(d) technical equations

subjective one, and at the same time the active behaviour
equation 1.

The active behaviour or subjective equations are derived
from some target—in the case of fixed targets—or from
some utility maximization—in the case of flexible targets.
In this they are analogous to micro-behaviour equations,
which are at the basis of passive behaviour equations in
our macrosystems. One could indeed state that by increasing
the number of policy-makers we gradually transform our
decentralized policy problem into the ordinary problem of
analysis, where the objective equations are only the balance,
technical and definition equations.

The active behaviour equations also depend on certain
hypotheses made, by the policy-maker concerned, on the
behaviour of other policy-makers. They may, in addition,
be based on a certain strategy, including even camouflage
of own intentions. In dynamic models there may be question

1 The difference with problems of monopolistic or oligopolistic
markets is that policy problems are macro-economic, which implies
the advantage that more of the relevant relations have been the subject
of statistical estimation.
of a succession of steps by the various participants in the “game”, such as a succession of wage increases and price increases in a process of inflation (cf. par. 3.4). As far as the hypotheses on the other policy-makers’ and the unorganized groups’ behaviour are concerned there is again the possibility of conformity and the one of non-conformity with the real behaviour. In cases of decentralized economic policy there is even more reason to expect non-conformity and the corresponding complications: policy-makers who only have to do with part of a society are more likely to make erroneous assumptions about other parts of society.

3.3. EXAMPLES: I. GOVERNMENT AND TRADE UNIONS

In order to illustrate the concepts just introduced and to clear the ground for their application, we will discuss some simple examples taken from realistic models of the Netherlands economy around 1950.

Example 1.

Two policy-makers are assumed to act, viz. the government and the trade unions; their targets and instruments are given below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Targets ($y_i$)</td>
<td>Balance of payments</td>
<td>A given real wage</td>
</tr>
<tr>
<td></td>
<td>equilibrium ($D = 0$)</td>
<td>rate ($l^F$)</td>
</tr>
<tr>
<td>Instruments ($z_k$)</td>
<td>Total government expenditure ($\xi_0$)</td>
<td>Nominal wage rate ($l$)</td>
</tr>
</tbody>
</table>

The model to be used will be the one described in our previous volume; in its most simplified version there are only two equations which may be written:
\[ \xi_1 \frac{D - \pi t}{\pi_2} + D + \xi_2 p = \xi_3 l + \xi_0 \quad (38.1) \]
\[ -\mu \frac{D - \pi t}{\pi_2} + D - \delta p = 0 \quad (38.2) \]

- \( P \) internal price level
- \( l \) nominal wage rate
- \( D \) balance of payments deficit
- \( \xi_0 \) government expenditure
- \( \xi_1, \xi_2, \pi_1, \pi_2, \mu, \delta \) structural constants, explained more fully in the volume referred to. The reader should be reminded that the variables are measured as deviations from an initial state.

These two equations have been obtained from the equations (1 I) and (1 II) shown at page 28 of "On the Theory of Economic Policy" by the elimination of \( y \) from the price fixation equation (1 III); (38.1) is the Keynesian multiplier equation summarizing the income formation and income spending process inside the country and (38.2) may be seen as a definition equation for the balance of payments deficit \( D \); \( l' \) has been written as \( l \), since no variations in \( h \) have been considered. By the introduction of the symbol \( l^R \) for the real wage rate, we may replace \( p \) in these equations by \( l^R \):

\[ l^R = l - p \quad (38.3) \]

and by so doing we are left with two equations of the type always considered in our text: two equations containing as variables only the two target variables and the two instrument variables:

\[ a_{11}y_1 + a_{12}y_2 = \beta_{11}z_1 + \beta_{12}z_2 \quad (38.4) \]
\[ a_{21}y_1 + a_{22}y_2 = \beta_{21}z_1 + \beta_{22}z_2 \quad (38.5) \]

where:
- \( y_1 = D \)
- \( y_2 = l^R \)
- \( z_1 = \xi_0 \)
- \( z_2 = l \)

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As in our previous calculations six sets of numerical estimates of the structural coefficients have been made; the first being given for illustrative purposes:

\[ 0.125 D - 0.162 l^R = 0.125 \xi_0 - 0.101 l \tag{33.6} \]

\[ 0.125 D + 0.506 l^R = 0.374 l \tag{33.7} \]

The policy followed by each of the two policy-makers consists of fixing their instrument variable in order to attain a desired target which in our case is: \( D = 0 \) for the government and \( l^R = 0 \), say, for trade unions. Given the values of the instrument variables, equations (33.4) and (33.5) or, specifically, (33.6) and (33.7) yield us the resulting values of target variables. These two equations are the objective ones, in our terminology. The equations telling us how the instrument variables are chosen are the subjective ones, or the active behaviour equations.

In principle these latter ones may be of different types; the simplest situation being considered in this example: the one of conformity. By it we mean that the policy-makers choose their instrument variables on the basis of the objective equations, which they are supposed to know. In addition we assume that each policy-maker chooses his instrument under the hypothesis that the other policy-maker will not change his.

At any moment where, in our example, the government wants to check, or revise, its expenditure \( \xi_0 \) (the government's instrument) it assumes that the trade union instrument \( l \) will remain unchanged and is therefore given; for its own target variable \( D \) the desired value \( \tilde{D} \) is substituted; and the two equations yield us the values of the two unknowns: (i) the value of \( \xi_0 \) and (ii) the value of the realised target variable of the other policy-maker, namely, \( l^R \).

By the conformity assumption we rule out, beforehand, the possibility that there is a discrepancy between the desired

---

1 Equations (33.1) and (33.2) have been multiplied by \( \pi_0 \).
value \( \hat{D} \) and the realised value \( D \) at the moment of action of the policy-maker concerned. Such an active step by policy-maker 1 may be followed by an active step by number 2, the trade unions in our example. Such a step will consist in determining, on the basis of a desired target variable value \( \hat{t}^R \), and a given value of the instrument variable \( \xi_0 \) of the other party, the new value of the instrument variable \( l \). By a succession of such steps an equilibrium situation may—but need not—be reached. If it is reached—a question to which we come back at the end of par. 3.6—it will have to fulfil the set of our two equations (33.6) and (33.7) for the desired values of both targets \( D \) and \( l^R \). In this simple case there will be no difference in result between the centralized and the decentralized case.

3.4. EXAMPLES: II. EMPLOYEES AND TRADE UNIONS

Our second example will also be one of two policy-makers, each of them manipulating one instrument and having one target variable. The two policy-makers are now (1) organized entrepreneurs and (2) organized workers or trade unions; their instruments and targets:

<table>
<thead>
<tr>
<th>Policy-makers:</th>
<th>1. Employers</th>
<th>2. Trade unions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Targets</td>
<td>Real profits ((Z'))</td>
<td>Real wages bill ((L'))</td>
</tr>
<tr>
<td>Instruments</td>
<td>Price of products ((p))</td>
<td>Nominal wage rate ((l))</td>
</tr>
</tbody>
</table>

The structural equations describing the economy will again be taken from the models presented in volume I of this series. We will describe the state of the economy by its two main variables volume of production \( y \) and price level \( p \).

\(^1\) Of course it is somewhat a simplification to assume that trade unions are completely autonomous in fixing wages.
Both will be measured as index numbers with an initial state as their basis, i.e. \( \bar{y} = \bar{p} = 1 \).

Our first equation will be a definition equation for real profits \( Z' \). In order to calculate this we start by calculating nominal national income \( Y \), which equals gross value of production minus imports. In the base period gross value of production was 140% of net national income, and imports were 40%. Accordingly we have for any other situation:

\[
Y = 1.4 \bar{y} \bar{p} - 0.4 \bar{y}
\]

if it is assumed that imports are proportional to volume of production and that import prices are given and equal to 1. In order to obtain profits we have to deduct total wages \( L \); given their value 0.55 in the base period, they will amount to

\[
L = 0.55 \bar{y} \bar{l}
\]

assuming proportionality between employment and production. It follows that profits

\[
Z = Y - L = 1.4 \bar{y} \bar{p} - 0.4 \bar{y} - 0.55 \bar{y} \bar{l}
\]

and real profits:

\[
Z' = 1.4 \bar{y} - 0.4 \frac{\bar{y}}{\bar{p}} - 0.55 \frac{\bar{y}l}{\bar{p}} \quad (34.1)
\]

Our second equation will be a definition of the real wages bill, running:

\[
L' = 0.55 \frac{\bar{y}l}{\bar{p}} \quad (34.2)
\]

As a third equation we will add the demand equation to be based on the following assumptions: demand is composed of home demand and foreign demand. Home demand will be equal to 1 in the base period and vary proportionally with real income changes but the variation of demand will only be 70% of those of real income, since the other 30% are assumed to be taxed away. Foreign demand is equal to
0.4 in the base period, and shows an elasticity of 2 with regard to price changes. Foreign incomes are assumed not to vary. The resulting demand equation runs:

\[ 1.4 \, y = 0.7 \left( 1.4 \, y - 0.4 \frac{y}{p} \right) + 0.8 + 1.2 - 0.8 \, p. \]

where the two last terms refer to foreign demand.
Solution for \( y \) yields:

\[ y = \frac{3.7 \, p - 2 \, p^2}{p + 0.7} \] \hspace{1cm} (34.3)

and it is easily checked that for \( p = 1, y = 1. \)

These three equations (34.1), (34.2) and (34.3) constitute the set of *objective equations*. They will again, for given values of the instrument variables \( p \) and \( l \), as it should be, yield us the economic variables \( Z', L' \) and \( y \).

In addition to these objective equations there will be the *subjective* ones, telling us how the employers fix their price \( p \) and how the trade unions fix the wage rate \( l \). In this example we assume that these latter equations will be based, not on fixed values of the respective target variables, but on certain attempts to *maximize these target variables*. Such maximization processes are only determinate if certain assumptions are specified regarding the behaviour of the other party expected by each policy-maker. From the equations (34.1) and (34.2) we see that we also have to know what our policy-makers think about the influence of prices on demand. We will make the *assumptions* that:

(i) both employers and workers are aware of the influence prices exert on demand and even know the exact extent of it; meaning that in this respect there is “conformity”.

(ii) workers take account of the direct influence of wages on prices, meaning that they assume \( \frac{dp}{dl} = 0.3. \)

The subjective equations then will be:
(a) for employers: \( \frac{dZ'}{dp} = \frac{\delta Z'}{dp} = 0 \) or:
\[-2.8 \ p^2 - 3.9 \ p + 2.8 \ l + 5.7 = 0 \quad (84.4)\]

(b) for workers: \( \frac{dL'}{dt} = \frac{\delta L'}{dt} + \frac{\delta L'}{dp} \ \frac{dp}{dt} = 0 \) or:
\[-1.1 \ p^2 + 1.3 \ p - 0.87 \ l + 1.4 = 0 \quad (84.5)\]

Together with equations (84.1), (84.2) and (84.3) they will determine the economic variables (target variables \( L', Z' \) and the irrelevant variable \( y \)) as well as the instruments \( l \) and \( p \).

It goes without saying that many alternative assumptions could have been made as to the subjective equations. First, the target variables might even have been different; it is well-known that only the most advanced employers’ and workers’ organizations will take total real income of their whole group as their directive for action. Very often more primitive attitudes will prevail; employers’ organizations often are only interested in separate industries which they represent. Trade unions often only think of wage rates and even nominal wage rates as their target variable.

Next, it is dubious whether they are aware of the true relation between price level as a whole and demand as a whole. Also, it will often happen that workers’ organizations do not pay attention to the repercussions of a wage increase on the price level. It has now become customary to do so in the Netherlands, but only recently. The reader may find it useful and attractive to investigate a number of alternative cases.

3.5. EXAMPLES: III. TEN COUNTRIES

As a third example we will consider at some detail an international model to be used especially in chapter V. In
order to avoid complications that would lead us too far, this model will show a high degree of "symmetry" between the countries concerned, i.e. it will be assumed that the countries are of equal size and behave, as far as the non-organized groups are concerned, identically. This will simplify the calculations to a considerable extent and, on the other hand, it does not seem to invalidate the essential features of the argument.

The policy-makers are assumed to be the governments of the countries, numbered by an index $h$, running from 1 to $H$, the total number of countries. In some of our numerical illustrations $H$ will be taken equal to 10.

The economies of each of the countries will be described by the following variables:

- $x^{kh}$: volume of goods supplied by country $k$ to country $h$, measured at base-period $\$ prices; implying that $x^{kh}$ is the volume of goods sold at home. All goods traded between countries are considered as raw materials to the production of the importing country, which does not represent a serious lack of generality. It follows that $x^{kh}$ represents the total volume of goods bought by the consumers and the investors of country $h$.

- $x^h$: total production of country $h$, measured in base-period $\$ prices.

- $p^h$: price of products of country $h$, in $\$ per unit.

- $D^h$: deficit on balance of payments, current items, in $\$.

- $x_0^h$: autonomous demand exerted in country $h$, i.e. demand as far as not depending on real income, represented in this example by $x^h$, volume of production; $x_0^h$ will be considered as one of the instruments of the country's economic policy.

- $p_0^h$: autonomous component in price level $p^h$, apart from price of imports contained in unit of product; also $p_0^h$ will be considered as an instrument of policy; it
may be said to represent, in a simplified way, the exchange rate of the currency of the country in terms of dollars.

The *objective relations* in this model are the following:

**Definition equations:**

\[ x^h = \sum_{h'} x^{h'h'} (h = 1 \ldots H) \quad (35.1) \]

\[ D^h = \sum_{h'} x^{h'h'} p^{h'} - p^h \sum_{h'} x^{h'h'} (h = 1 \ldots H) \quad (35.2) \]

**Demand equations:**

\[ x^{h'h'} = \mu x^{h'} + \epsilon (p^h - p^{h'}) \quad (h, h' = 1 \ldots H) \quad (35.3) \]

\[ x^{hh} = \xi x^h + x^h_0 \quad (h = 1 \ldots H) \quad (35.4) \]

**Supply or price setting equations:**

\[ p^h = \mu \sum_{h'} p^{h'} + p^h_0 \quad (h = 1 \ldots H) \quad (35.5) \]

Summation signs with a prime refer to summation excluding the value \( h \). The coefficients \( \mu, \epsilon \) and \( \xi \), taken equal for all countries (this being the “symmetry” assumed), represent import quota (of a country with respect to imports from each other country), price sensitivity of imports and a marginal propensity to spend (with reference to the volume of production—a simplification which is not exact).

It will appear useful to solve these objective equations for the target variables; as such we consider \( x^h \) and \( D^h \). With this choice, it may be stressed, we follow the standard scheme also defended by Professor Meade: instruments are “financial policy” and “price or exchange rate policy”; and targets are certain values of production and the balance of payments deficit—e.g. “full employment” and “balance of payments equilibrium”.

In a *first version* we will even simplify our equations
further by taking $\varepsilon = 0$. It follows from (35.1), (35.8) and (35.4) that

$$(1 - \xi) x^h - \mu \sum_{h'} x^{h'} = x_0^h \tag{35.6}$$

and from (35.5):

$$p^h - \mu \sum_{h'} p^{h'} = p_0^h \tag{35.7}$$

The solution seems easiest by the introduction of "total" $x$ and $p$:

$$x = \sum_h x^h$$

$$p = \sum_h p^h$$

and writing (35.6) and (35.7) as:

$$(1 - \xi + \mu) x^h - \mu x = x_0^h \tag{35.6'}$$

$$(1 + \mu) p^h - \mu p = p_0^h \tag{35.7'}$$

Adding the equations for all values of $h$ and introducing $x_0$ and $p_0$ as the totals of all $x_0^h$ and all $p_0^h$, respectively, we find:

$$x = \frac{x_0}{1 - \frac{1}{\xi} - (H-1)\mu} \tag{35.8}$$

$$x^h = \frac{x_0^h + \frac{\mu x_0}{1 - \frac{1}{\xi} - (H-1)\mu}}{1 - \frac{1}{\xi} + \mu} \tag{35.9}$$

$$p = \frac{p_0}{1 - (H-1)\mu} \tag{35.10}$$

$$p^h = \frac{p_0^h + \frac{\mu p_0}{1 - (H-1)\mu}}{1 + \mu} \tag{35.11}$$
From these solutions we calculate:

\[ D^h = \mu \left( x^h \sum_{h'}^{p^h} p^{h'} - p^h \sum_{h'}^{x^{h'}} x^{h'} \right) \quad \text{where} \]

\[ \sum_{h'}^{p^{h'}} = p - p^h \quad \text{and} \quad \sum_{h'}^{x^{h'}} = x - x^h; \]

after some transformations we get:

\[
D^h = \frac{\mu}{\mu' \xi'} \frac{1}{M} \left\{ (1 + \mu) \Xi p_0 x_0^h + \xi' \mu p_0 x_0 \right. \\
\left. - (1 - \xi + \mu) M x_0 p_0^h \right\} \quad (35.12)
\]

Here the significance of the newly introduced coefficients is:

\[ \mu' = 1 + \mu; \quad \xi' = 1 - \xi + \mu; \quad M = 1 - (H - 1)\mu; \]
\[ \Xi = 1 - \xi -(H - 1)\mu. \]

For our purposes it is more convenient to use this formula in differentiated form, and in addition to replace the general symbols \( p_0 \) and \( x_0 \) by symbols referring to the country concerned and the average of all other countries, by writing:

\[ p_0 = p_0^h + (H - 1) p_0^{h'} \]
\[ x_0 = x_0^h + (H - 1) x_0^{h'} \]

The symbols with indices \( h' \) refer to the average of all countries outside \( h \). We will finally, in order to fix the ideas, choose certain numerical values for our coefficients:

\[ \mu = 0.92; \quad \xi = 0.7; \quad H = 10; \quad \text{leading to:} \]
\[ \mu' = 1.02; \quad \xi' = 0.82; \quad M = 0.82; \quad \Xi = 0.12. \]

This choice implies that total imports and exports amount to 18% of gross production. In addition, we have to make a choice as to the values of the instruments in the base period; we choose them all equal to 1, i.e.:

\[ \bar{x}_0^h = \bar{p}_0^h = \bar{x}_0^{h'} = \bar{p}_0^{h'} = 1 \]
The result is:
\[ dD^h = -1.5 (dp_0^h - dp_0^{hr}) + 0.7 (dx_0^h - dx_0^{hr}) \] (85.18)

Substituting also these numerical values in our formula for \( x^h \) and writing this also in differential form, we obtain:
\[ dx^h = 3.7 \, dx_0^h + 4.7 \, dx_0^{hr} \] (85.14)

In order to complete the picture we may state the base-period values for the most important variables:
\[ \bar{x}^h = \bar{x}^{hr} = 8.3 \quad \bar{p}^h = \bar{p}^{hr} = 1.23 \]

Exports for each country amount to 1.50 in volume.

These formulae, it will be remembered, are valid for the case only where the price sensitivities for imports are zero; this corresponds to certain very much simplified models of the Keynesian school. The negative influence of \( p_0^h \) on \( D^h \) is unrealistic, however. This we get rid of if we take the second version, where we no longer neglect the terms with \( \varepsilon \) in (85.8). In this version our formulae for \( x, p \) and \( p^h \) will be the same as before. The equation for \( x^h \) is affected by the change, however, and now reads:

\[ x^h = \frac{x_0^h + \frac{1 - \xi - (H - 1) \mu}{1 - \xi + \mu} + \varepsilon (H - 1) (p_0^h - p_0^{hr})}{(1 + \mu) (1 - \xi + \mu)} \text{ or} \]

\[ x^h = \frac{(1 - \xi - (H - 2) \mu) x_0^h + (H - 1) \mu x_0^{hr}}{(1 - \xi + \mu) (1 - \xi - (H - 1) \mu)} + \varepsilon \frac{(H - 1) (p_0^h - p_0^{hr})}{(1 + \mu) (1 - \xi + \mu)} \] (85.15)

Assuming \( \varepsilon \) to be such as to yield an elasticity of \(-2\), we find that it amounts to \(-0.27\). Filling in this value and the values already assumed for the other coefficients we get:

\[ x^h = 3.7 \, x_0^h + 4.7 \, x_0^{hr} - 7.5 \, (p_0^h - p_0^{hr}) \] (35.16)
For \( dD^h \) we find, after a number of substitutions,

\[
dD^h = 2.5 (dp_0^h - dp_0^{h*}) + 0.7 (dx_0^h - dx_0^{h*}) \quad (85.17)
\]

Here the sign of the terms with \( dp_0 \) has changed, as it should with an elasticity of \(-2\) in international trade, so as to yield the "normal" reaction of \( D^h \) on changes in \( p_0^h \).

The objective equations now obtained may be used for the description of a policy of fixed targets as well as for one of flexible targets. Equations (85.17), however, form a set of dependent equations; they add up, for all the countries together, to the identity \( \Sigma dD^h \equiv 0 = 0 \). One equation more is needed, therefore, for which we may take an equation fixing the price level in one of the countries. That being given we may, for the case of fixed targets, think of all \( x^h \) and \( D_t^h \) being the target variables; the equations would yield us the values of the instrument variables, if conformity is assumed. In order that the policy be consistent the targets set by the various governments for their balance of payments deficit should be such as to obey the identity just quoted.

If no conformity is assumed in the case of fixed targets or if a case of flexible targets is considered, subjective equations have to be added. We will assume that they are based on the maximization of some national utility function which for lack of better information we assume to be a linear function of the relevant variables. As relevant we will consider (i) the target variables \( x^h \) and \( D^h \) of the country concerned, and (ii) the instrument variables or certain other indications of the "cost" or disadvantages connected with any policy. Writing, in a general way, the utility function of country \( h \) as \( \omega^h \), the target variables as \( y_j^h \) (where \( j \) is the number of the target variable) and the instrument variables as \( z_k^h \) (\( k \) the instrument's number), we may have

\[
\omega^h = \sum_j \psi_j^h y_j^h + \sum_k \psi_k^h z_k^h \quad (85.18)
\]
In this utility function we will have to substitute, for the values of the target variables \( y \), their expressions in terms of the \( z \) — their objective expressions in the case of conformity, their supposed expressions in case of non-conformity. This makes the utility functions pure functions of the \( z_k^h \), say with capital \( \Psi_k^h \):

\[
\bar{\omega}^h = \sum \Psi_k^h z_k^h
\]  

(85.19)

A linear function has no ordinary maxima, and the maximum will only exist therefore as far as there are boundary conditions imposed on the instrument variables.

Suppose there are upper and lower limits prescribed for every instrument variable \( z \), either directly, derived from a-priori considerations about these instruments, or indirectly, from the boundary conditions for the target variables. We will have to specify these boundary conditions anew in each problem and will not dwell upon this specification now. For a first simple exposition of our problem we assume the upper and lower limits for each \( z_k^h \) to be known. It will be clear then that the maximum obtainable for \( \omega^h \) will be reached if for each \( Z_k^h \) which has a positive \( \Psi_k^h \) the upper and for each \( z_k^h \) with a negative \( \Psi_k^h \) the lower limit is taken. In symbols the solution will be

\[
\begin{align*}
  z_k^h &= z_{k_u}^h \quad \text{for all } k \text{'s for which } \Psi_k^h > 0 \\
  z_k^h &= z_{k_l}^h \quad \text{, } \quad k \text{'s for which } \Psi_k^h < 0
\end{align*}
\]

(85.20)

where \( z_{k_u}^h \) and \( z_{k_l}^h \) indicate upper and lower limit of \( z_k^h \).

The problems become somewhat more complicated, possibly very complicated, if boundary conditions have also to be imposed on the target variables. The character of these problems is the same as of those discussed at the end of section 2.2. We will exemplify both our utility functions and these problems of boundary conditions by returning to
our specific variables; to this effect we may put \( \omega^h \) equal to \( x^h - D^h \) or

\[
\omega^h = 3 \, x_0^h + 5.4 \, x_0^{h^*} - 10 \, (p_0^h - p_0^{h^*}) \tag{35.21}
\]

where the constant term has been omitted—since it is irrelevant—and a linear approximation has been used, which means that only moderate variations in the instruments are permitted. As long as we stick to this linear utility function, again there will be no set of finite values of \( x_0^h \) and \( p_0^h \) which, for given values of \( x_0^{h^*} \) and \( p_0^{h^*} \), make \( \omega^h \) a maximum. Suppose now that the boundary condition is so set that the volume of production cannot surpass full-employment production which we suppose to be \( x_0^h = 9 \) (cf. figures given after equation (35.14)). This yields us:

\[
3.7 \, x_0^h + 4.7 \, x_0^{h^*} - 7.5 \, (p_0^h - p_0^{h^*}) \leq 9
\]

i.e.

\[
x_0^h \leq c + 2 \, p_0^h \tag{35.22}
\]

Since the lines of constant utility, according to (35.21) have the direction:

\[
x_0^h = \frac{10}{3} \, p_0^h + c'
\]

it follows (cf. fig. 8) that the government of country \( h \) will tend to move along the line

\[
x_0^h = c + 2 \, p_0^h \text{ (line F)}
\]

corresponding to full employment. Without further boundary conditions there will, however, still not be a set of finite values of \( x_0^h \) and \( p_0^h \) constituting the maximum-utility point. This may be interpreted as a tendency, in a state of decentralized policy, towards unlimited devaluation.

If, however, a further boundary condition would be set by the fact, say, that public expenditure will not be cut
below a certain figure $x_{oo}^h$, a definite solution will result in point $E$, with coordinates $x_{oo}^h$ and $p_{oo}^h = \frac{x_{oo}^h - c}{2}$

A similar conclusion would be reached if instead of some boundary condition for $x_0^h$ such a condition would be imposed for $p_0^h$, say by some agreement like the Bretton-Woods agreement.

As another example of the use of our utility functions (85.18) we may consider a problem between the ten countries where the $p_0^{h'}$ are ruled out as instruments and hence only the $x_0^{h'}$ remain. If the $p_0^{h'}$ are chosen such as to realise equilibrium in all balances of payments (i.e. $D' = 0$, $h' =$
= 1\ldots H), the flexible target left may be, for each country,

\[ x^h = 3.7 \, x_0^h + 4.7 \, x_0^{h'} \]

We will now consider the policy of all countries simultaneously, but, for the sake of further simplification assume that all countries but $h$ are following identical policies.

(This means the use of a restricted symmetry assumption for policy, together with a complete symmetry assumption for the objective equations.) For the other countries we find:

\[ x^{h'} = 3.7 \, x_0^{h'} + 4.7 \, \frac{8 \, x_0^{h'} + x_0^h}{9} = 7.9 \, x_0^{h'} + 0.5 \, x_0^h. \]

Our unknowns are $x_0^h$ and $x_0^{h'}$; they will be determined by

---

Fig. 4

38
the conditions that the \( \omega \)'s are maxima, together with possible boundary conditions. To the \( \omega \)'s we will now give the form:

\[
\omega^{h'} = ax^{h'} - \beta x_0^{h'} \quad (h' = 1 \ldots H)
\]

As boundary conditions we introduce the conditions of full employment which we in this case give the form:

\[
x^{h'} \leq 8.3 \quad (h' = 1 \ldots H)
\]

Expressing them in terms of \( x_0^{h'} \) yields:

\[
x_0^{h} \leq 2.2 - 1.3 x_0^{h'} \quad \text{for country } h
\]

and

\[
x_0^{h} \leq 16.6 - 15.8 x_0^{h'} \quad \text{for countries } h'.
\]

The corresponding boundaries I and II are drawn in figure 4. They intersect in point \( P \). Since it can easily be shown that the directions of the lines of constant \( \omega^h \) and \( \omega^{h'} \) are between the directions of the two boundaries, point \( P \) will be the maximum point for both country \( h \) and countries \( h' \). This coincidence will not necessarily occur always. In the present case there is a stable equilibrium at \( P \).

8.6. Result of decentralized policy;

Stability of equilibrium

We will now discuss the solutions of our sets of equations and by so doing the nature of the results of decentralized policy. It is necessary also here to distinguish between the cases of fixed and flexible targets.

With fixed targets there is scope to make a distinction between the desired and the realised values of the target variables. Indicating, as before, the desired values by \( \tilde{y}_t \), \( \tilde{y}_2 \ldots \), or, in vector form, by \( \tilde{y} \), we may state the logic of our problem in the following way (where linearity has been assumed throughout):
Objective equations: \( A y = B z \) or \( z = Z y \) \hspace{1cm} (36.1)

Subjective equations: \hspace{1cm} z = Z' \tilde{y}' \hspace{1cm} (36.2)

From the complete set we can determine \( y \) and \( z \) as functions of \( \tilde{y} \); the \( z \) are already shown by (36.2), whereas

\[ y = Z^{-1} Z' \tilde{y} \] \hspace{1cm} (36.3)

There need not be identity of \( y \) and \( \tilde{y} \); there will be identity, however, if \( Z \equiv Z' \), i.e. in the case of conformity. In the latter case the system of equations reduces to the objective equations only, as in the theory of economic policy presented in volume I of this series.

In the case of flexible targets there is no scope in distinguishing between desired and realised values of target variables, since there are no fixed, beforehand determined values aimed at. Again we have the objective equations:

\[ A y = B z \] \hspace{1cm} (36.1)

and, in addition to these, the subjective ones:

\[ \frac{\partial \omega_k}{\partial h_k} = 0 \hspace{1cm} k = 1 \ldots K \] \hspace{1cm} (36.4)

where \( h \) indicates the number of the policy-maker, \( k \) the number of the instrument; it is assumed that each instrument is manipulated by only one policy-maker, i.e. \( h = h(k) \).

The procedure may be illustrated by its application on our two examples.

**Example I.** This is a case of fixed targets and where conformity has been assumed. Because of the choice of the target values \( D = l^R = 0 \) and of the neglection of any changes in data, the result will be \( \xi_0 = l = 0 \), i.e. neither \( \xi_0 \) nor \( l \) should be changed. Had we admitted any changes in data, our equations would have supplied us the corresponding new values of \( \xi_0 \) and \( l \).
Example II. Here we have to do with flexible targets; equations (34.1) — (34.5) may be solved for \( y, p, l, Z' \) and \( L' \). The results are:

\[
\begin{align*}
y &= 0.78 \\
p &= 1.29 \\
l &= 1.44 \\
Z' &= 0.85 \\
L' &= 0.45, \\
\end{align*}
\]

showing that the sort of monopolistic attitude here supposed to exist would have reduced the volume of production to 78 % only of the volume in the actual situation (which is certainly more competitive); it would also have been fairly inflationary. The result would have been even worse if no account would have been taken of the influence of \( p \) on \( y \).

We come back to the interpretation of our result in Ch. V.

The equilibrium situation thus calculated need not be stable. Its stability depends, by the very nature of that concept, on the dynamics of the system. If we think of the policy-making process as a succession of steps as described in example I, stability will be identical with convergency of the series of successive values for each of the variables. The determination of the convergency ratio is illustrated by the following computation:

As was stated in section 3.4, policy-maker 1 fixes the value of his instrument value \( z'_1 \) at time \( t \) on the basis of a desired value \( \tilde{y}_t \) and a previously fixed instrument value \( z_{2t-1} \). At the same time this \( \tilde{y}_t \) and \( z_{2t-1} \) determine the value of policy-maker 2's target \( y'_2 \) that will be realised as a consequence. Accordingly \( z'_1 \) and \( y'_2 \) will follow from the equations:

\[
\begin{align*}
y'_2 &= \delta_{11}\tilde{y}_t + \delta_{12}z_{2t-1} \\
z'_1 &= \delta_{21}\tilde{y}_t + \delta_{22}z_{2t-1} \\
\end{align*}
\]
which, in the case of conformity, are to be considered as the solution of the structural equations with respect to \( y_2 \) and \( z_1 \). Similarly policy-maker 2's action may be described by equations

\[
\begin{align*}
    y_1^t &= \epsilon_{11} \hat{y}_2 + \epsilon_{12} \hat{z}_1^{t-1} \\
    z_2^t &= \epsilon_{21} \hat{y}_2 + \epsilon_{22} \hat{z}_1^{t-1}
\end{align*}
\]

which are also to be considered as solutions from the structural equations. It follows that

\[
\begin{align*}
    z_1^t &= \delta_{21} \hat{y}_1 + \delta_{22} (\epsilon_{21} \hat{y}_2 + \epsilon_{22} z_1^{t-2}) \\
    z_2^t &= \epsilon_{21} \hat{y}_2 + \epsilon_{22} (\delta_{21} \hat{y}_1 + \delta_{22} z_2^{t-2})
\end{align*}
\]

showing that the convergency of the process is conditioned by

\[ |\delta_{22} z_2| < 1 \]

In the case of conformity this condition may be translated into

\[
\begin{vmatrix}
\beta_{12} a_{12} & \beta_{11} a_{11} \\
\beta_{22} a_{22} & \beta_{21} a_{21}
\end{vmatrix} \times
\begin{vmatrix}
-\beta_{11} a_{12} & -\beta_{12} a_{11} \\
-\beta_{21} a_{22} & -\beta_{22} a_{21}
\end{vmatrix} < 1
\]

With the numerical values of example I we find

\[ |\delta_{22} z_2| = 0.091 \]

pointing to a very rapid convergency. The other sets of numerical values of the coefficients appear all of them to yield even lower values.
3.7. Consistency

There may be scope for somewhat widening the concept of consistency and thinking of two forms of, in particular, inconsistency. The first, most strict, sense, which we might call *unconditional* inconsistency, would then be the impossibility to attain all the targets set. With fixed targets this is the type of inconsistency we discussed in volume I; it may be due to a too large number of fixed targets or to straight incompatibility—if e.g. all the variables of one equation would have been chosen as targets, whereas their numerical values do not obey that equation. These cases of inconsistency were already discussed in volume I; the conditions of consistency in this sense are the same as in the case of centralized quantitative policy with fixed targets: the set of equations has to be solvable with respect to the instruments. With flexible targets this might be the case if they are interpreted to be the absolute maxima (considering all the instruments admitted) of several target variables at one time. As an example we may quote the case of example II where the two targets are total real profits and total real wages; the absolute maximum of profits, admitting the wage rate and the price level as instruments, would be obtained at another wage and price level than the absolute maximum of total wages. This latter type of unconditional inconsistency will usually be avoided by the circumstance that each instrument is at the command of only one policy maker. We may also say that this represents the condition that it be avoided.

It may be useful, now, to speak of inconsistency also if by a certain policy, i.e. the choice of certain instrument variables, the targets set are not reached, although it is possible to reach them. We will call this *conditional inconsistency*. With fixed targets this is a clear-cut definition; with flexible targets it needs some further interpretation.
The one we propose is that the values of the target variables attained are not the maximum values possible. In both cases the discrepancy between target values realised and target values aimed at or possible may be due to different causes. It may be due to non-conformity—meaning a lack of insight or knowledge—or it may be due to the fact that qualitatively different policies are possible. If in particular a higher degree of centralization would have been possible and would have led to target variable values with a higher utility, the reason of the discrepancy may be said to be the very fact of decentralization. In the case of non-conformity it is by the wrong insight into or knowledge of the objective equations that wrong instrument variables are chosen. Simple though this phenomenon and its remedy may seem, it is not so simple in practice. One reason is that exact knowledge on the mechanism of an economy does not exist and that there can, therefore, only be question of better or worse approximations. Another reason is that even experts in this field are not unanimous and that it will be very difficult, then, for non-experts to be. Even if there is unanimity among experts in some cases it is not easy to convince the rank and file of large organisations of this expert opinion. Typical recent examples of lack of agreement between experts are the discussions about foreign-trade elasticities; a typical example of the difficulty of convincing non-expert policymakers of certain fundamental economic laws is the resistance economists experience when emphasizing the influence of wage rates on employment.

We shall not now discuss in what ways decentralization in economic policy may contribute to inconsistency in this sense: it will be discussed in chapter V.
IV. QUALITATIVE ECONOMIC POLICY:
GENERALITIES

4.1. DEFINITION

Whereas quantitative economic policy consists of manipulating certain quantitative parameters (called instruments in this text) appearing in the equations describing the economic structure, qualitative economic policy consists of admitting changes in the nature of the equations. In non-mathematical language this means admitting changes in the “organization” of economic life. The easiest definition of this latter term may be in the enumeration of examples. These may be listed under two headings, namely, the introduction, by existing institutions, of new policies and the establishment of new institutions.

The introduction of a new policy may refer to new taxes, open-market policy, a new monetary standard, a new price policy, the introduction (or abolition) of import duties or quantitative restrictions—the latter in international trade or in national distribution (i.e. rationing)—, the introduction of some new form of social insurance, or of convertibility of certain currencies.

The establishment of new institutions may be exemplified by the formation or dissolution of a monopolistic organisation, an international or a supranational agency, as e.g. a customs union.

Each of these examples—and the other that could be added—in fact corresponds to a change in the nature of the structural equations of the economy considered. In the case of rationing, for instance, the usual demand equation will be replaced by an equation telling that the quantity
bought of the rationed commodities equals a given constant per head; or, as the case may be, that the quantity bought cannot surpass that constant. The introduction of an international employment policy might consist of the establishment of a supra-national agency fixing the budget surpluses or deficits of the countries concerned; and the autonomous behaviour equations of the various governments would have to be replaced by others indicating the decisions of the new agency. Here we have to do with an example of centralization, which leads us back to the main topic of this study.

4.2. DETERMINATION OF CONSEQUENCES

Acts of qualitative economic policy will influence the economic situation and the consequences may be investigated by comparing the equilibrium position before and after the change in organization they bring. Less so than with problems of quantitative policy will it be possible, however, to investigate the consequences of a number of alternative policies in a systematic way. Such a systematic study consists, in the case of quantitative problems, of comparing the outcome of different numerical choices for the instrument variables and this our formulae more or less automatically yield. In the case of qualitative changes all we can do, as a rule, is just to calculate the outcome for the alternative cases and to compare them. In rare cases only can this be done in a more systematic way; e.g. when different forms of a tax rate curve have to be compared, or when the influence of increasing the number of competitors has to be ascertained and we would adhere to a market description like Cournot's, where that number occurs explicitly in the formulae.

Nevertheless, the consequences of various alternative policies can be compared and certain questions can be answered regarding the extent to which certain targets can be approached by these various alternatives. Some examples
will be worked out in the next chapter on centralization. Here it may be stressed already that the problem of comparing alternative forms of organisation of economic life constitutes the problem *par excellence* of economic science; its real *raison d'être*.

Apart from the examples to be worked out in Chapter V we may illustrate the procedure by two incidental examples. In the case of inconvertibility between, say, currencies $a$ and $b$, it is not the total deficit $D^h$ on any country $h$'s balance of payments which matters to that country but rather the partial deficits (positive or negative) in the currencies considered, say $D^h_a$ and $D^h_b$. Depending on their values, reserves in currencies $a$ and $b$ will deplete and accumulate and accordingly determine the country's policy as to trade with the various currency areas. At the moment where convertibility between $a$ and $b$ would be reestablished no such distinction between $D^h_a$ and $D^h_b$ would be necessary or even make sense. Accordingly trade policy would have no more reason to discriminate between areas $a$ and $b$.

Customs unions problems may be studied, as may some other problems of international trade, with the help of a very simple model, in which a number of "labour places" is distinguished in each industry in each country. These places represent the opportunity for one man to produce, usually with the aid of some equipment. The places may be given a certain order according to their net productivity, i.e. in this context, the value added if prices are equilibrium prices prevailing with completely free trade. According to this measure of productivity, in each country the most productive places will be occupied; with full use of all labour, just as many of them as the number of workers. Protection, say of product $b$ in country 2 and product $c$ in country 3, will lead to a situation where more labour places (say just one) in these countries and in the industries specified are occupied; as a consequence, less places in other industries...
(to be indicated, as one group, by the letter a) will be used. It will be assumed that in country 2 no change in production of c and that in country 3 no change in production of b takes place. The places occupied and left may symbolically be indicated by:

<table>
<thead>
<tr>
<th>Country</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Products protected</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>Products</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Changes in labour units applied</td>
<td>-1</td>
<td>+1</td>
</tr>
</tbody>
</table>

The total product of the two countries, measured at free-trade equilibrium prices, will now be lower than with free competition. This is the well-known central thesis of free-trade theory. The situation will lead to a fall in the prices for b and c in the remaining part of the world, to be indicated as country 1. Not only will demand in countries 2 and 3, because of the higher (protected) prices, fall, which is the cause of the price fall just mentioned, but production has increased; hence production will decrease in country 1. Because of the assumed policy of full use of all labour, production in other industries (a) will increase there. The full situation may therefore be symbolized as follows:

<table>
<thead>
<tr>
<th>Country</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Products protected</td>
<td>b</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>Products considered</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>Changes in labour units applied (compared with free trade)</td>
<td>+2</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Here the number of labour places involved in country 1 is not necessarily equal to the numbers involved in countries 2 and 3.

If now a customs union be concluded between countries 2 and 3, whereby their tariffs will be unified, and no tariffs
will be levied on trade between them, it may happen that a situation develops as symbolized below:

<table>
<thead>
<tr>
<th>Country</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Products protected</td>
<td>b,c</td>
<td>b,c</td>
<td></td>
</tr>
<tr>
<td>Products considered</td>
<td>a b c</td>
<td>a b c</td>
<td>a b c</td>
</tr>
<tr>
<td>Changes in labour units applied (compared with free trade)</td>
<td>+2</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

This will only happen if country 2 happens to be more productive in industry c than country 3, and country 3 more productive in industry b than country 2. In this case the union will be a clear advantage to the partners. The question may be put whether it cannot, at the same time, be a disadvantage to others. It clearly may be to producers of the goods b and c in country 1, since more is now being produced of these goods in countries 2 and 3 whereas total demand will not increase or may even decrease.

4.3. CHOICE OF OPTIMUM POLICY

Next to a comparison of the different outcomes is the choice of the “best” policy. This presupposes the existence, or acceptance, of a criterion for our choice. In the case of a decentralized policy there is more reason than in the case of a centralized policy to consider this choice as a separate problem. In the case of a centralized policy, in particular when a social utility function is introduced as the basis for a flexible-target policy, it is natural to use that same utility function as the criterion, i.e. to prefer the policy which yields the maximum utility. It is not necessary, though one may wish so, to distinguish between the preference of some government and those of some observer or some political movement.
In the case of a decentralized policy there is much more probability that the various policy-makers are not led by the same utility functions and hence it will be difficult to consider one of the utility functions as the natural criterion. Since, on the other hand, the choice depends on what criterion we accept, it seems appropriate to consider, in some more detail, the nature and the elements of some possible criteria. Before doing so we want to add that the elements of the social utility function are not the only determinants of our choice; it will, in addition, depend on the structure of the economy or economies studied.

The first approximation to a social utility function and hence its main element may be said to be real income per head, whatever its exact definition. In certain circumstances this may be taken equivalent to production per head.

Material and immaterial refinements or physical and psychological ones may be made. The physical ones will refer to the distribution of income or production over time, groups of persons and types of goods. Distribution over time may be considered in the short or in the long run. When speaking of short-run distribution over time we will usually think of problems of cyclic stability; when speaking of long-term distribution we think of problems of development, which will come down first of all to the distribution of income over consumption and investment. In order to do justice to the aspect of distribution over time, it will often—although not always—be necessary to include several time units in the analysis.

Distribution over groups of persons may, first of all, take the form, already used in some of our examples, of distinguishing between labour and non-labour income—however defined. As the case may be, other distributions may be relevant, e.g. the one between farmers and non-farmers, the one between a larger number of industries, etc. For short-term problems of employment policy this is particu-
larly important. Going into extreme detail we may want to introduce income distribution over a large number of occupational groups, or personal income distribution.

Distribution of production or rather consumption over types of goods will come in as soon as the complementarity of goods comes into play and the obstacles which sometimes prevent the adaptation of the structure of production to that complementarity.

For all these types of distribution certain numerical distributions will be preferred over others and for each policymaker and each observer there will be an optimum. The question what distribution exactly has to be preferred is a very complicated and at the same time a very important one; it is not, however, the topic of this study.

In addition to these material refinements there are psychological refinements to be introduced into the social utility function. Their measurement will be difficult in many cases or at least open to discussion. They may be listed as the degree of freedom left to individuals; the degree of justice, including as one element the degree of discrimination between whatever groups or persons we may want to consider; the degree of security in various senses and the level of education and of culture.

Only in rare cases will it be possible to indicate generally accepted standards of measurement for both the more complicated material elements and the psychological elements just mentioned. This state of affairs reflects the still rather primitive state of welfare economics generally and will only gradually—if ever—be overcome. It is not the object of this study to contribute to this development; some attempts will be made elsewhere. It follows that the applications of the general approach just outlined to the problem of centralization in economic policy, to be given in chapter V, will have to recur to the simpler criteria only or have to be given in the form of arbitrary examples.
4.4. SOME PARTICULAR FORMS OF UTILITY FUNCTIONS

For practical purposes it will often be desirable and sometimes possible to specify the mathematical form of the utility functions in such a way as to make them workable. For modest ranges of the relevant variables simple mathematical forms will always be acceptable as approximations, which could be adapted to such observations as can be made on the indifference functions. For these same practical purposes it seems useful to make a distinction between variables for which a finite optimum value is relevant and variables for which such a value does not come into play. An example of the latter will be the central variable real income per head: the higher this is the higher utility will be; there is no finite saturation point beyond which a further increase would not be interesting to present-day populations. Such a variable may be taken to influence the utility function just linearly. It is different from the former type of variables which may be exemplified by (i) the ratios between certain group incomes or between the production of specified goods; (ii) the deficit on the balance of payments; (iii) the volume of employment. Here generally there is a finite optimum. For the deficit on the balance of payments it is the value "zero" or some value not far from it which in most cases will be preferable to both higher and lower values; for the volume of employment it may be the figure of 97 or 98 % which is the optimum; and there will be similar figures for a number of distribution ratios. Utility could by no means be even approximated as a linear function in these variables; the simplest mathematical form acceptable here will be a quadratic function with a maximum at the optimum value. Calling the variable in question \( y \) and its optimal value \( c \), such a function would be \(-\beta(y - c)^2\), which for \( c = 0 \) becomes \(-\beta y^2\).

Similar remarks may be made with regard to the negative
influence of the instrument values on the utility functions. As far as an intensification of the application of a certain instrument would meet with increasing feelings of concern, aversion or more generally disutility, we may approximate its influence by a linear function.
V. ON THE THEORY OF CENTRALIZATION

5.1. OBJECT OF A THEORY OF CENTRALIZATION AND ESSENCE OF A CENTRALIZED ECONOMIC POLICY

In this chapter the main subject of this publication will be discussed, namely a comparison between the results of a decentralized and a centralized economic policy, or rather a comparison between the results of more and less centralized forms of economic policy. At the same time this subject represents the most important example to be given of the subject dealt with, in a more general way, in chapter IV: the effects of qualitative changes in economic policy.

More precisely, not only a comparison will be made between what happens in a more and what happens in a less centralized state of economic policy. Also, the way in which a choice could be made will be discussed, i.e. the principles upon which such a choice should be based and the actual outcome of the choice.

As a first step it will be necessary to state more precisely the problem to be studied. There is no scope in studying it for a fixed-targets policy: in such cases there are only objective equations. With fixed targets these equations will supply us the values of the instrument variables; and these values will be the same whether the instruments are supposed to be manipulated in a decentralized or in a centralized way. The problem will have vanished; and it actually only makes sense if we treat it in the more general way implied in the problems of flexible-targets policy, where the choice of the targets itself is included in some optimum problem.

If we want, however, to make a fair comparison between situations which are only different as far as their degree of
centralization is concerned, we will have to be precise in the stating of the problem. The essential elements are the following:

In the decentralized state there are a number of $H$ policy-makers, each of them acting on the basis of a certain social utility function $\omega^h$, which is a function of certain target variables $y_j^h$ and of certain instrument variables $z_k^h$. As before, we assume that a set of objective equations relates the $y$’s to the $z$’s of the whole group of policy-makers; we will sometimes take these equations to be linear and may then write them in a general form:

$$y_j^h = \sum_{h'} \sum_{z} \eta_j^{h'} z_k^{h'}$$  \hspace{1cm} (51.1)

If the objective equations are used to substitute the $y$’s in the $\omega^h$, these latter may be expressed as functions of the $z$’s only:

$$\omega^h (y_j^h, z_k^h) = \overline{\omega}^h (z_k^{h'})$$  \hspace{1cm} (51.2)

Not only the $z$’s of policy-maker $h$ himself but also those of the other policy-makers will then, however, occur in the $\overline{\omega}^h$. The influence exerted by the $z_k^h$ on the $\overline{\omega}$ through the $y$’s, according to equations (51.1), will be called their indirect influence; the influence exerted because of their presence in the functions $\omega^h$, to be called direct influence, represents the “disutility” of using an instrument at all, showing itself in costs to the community and interference with the freedom of some members.

The policies pursued by each policy-maker will be directed towards maximization of their $\overline{\omega}^h$ by a proper choice of their own instruments $z_k^h$; in general symbols:

$$\overline{\omega}^h (z_k^{h'}) \text{ max. with resp. to } z_k^h$$  \hspace{1cm} (51.3)

This process will be effectuated under certain assumptions as to what other policy-makers will do with their instruments. In the case where
(i) the assumption is made that other policy-makers will not change their instrument variables,
(ii) the $\omega^h$ are supposed to be continuous functions, and
(iii) no boundary conditions have to be imposed on the $z_k^h$ or the $y_j^h$.

the policies may be expressed by the equations:

$$
\frac{\partial \omega^h(z_k^h)}{\partial z_k^h} = 0 \quad \text{if} \quad \left\{ \begin{array}{l} h = 1 \ldots H \\ k = 1 \ldots K_h \end{array} \right. \quad (51.4)
$$

In the case of a centralized policy, on the other hand, there will be only one policy-maker, trying to maximize one function $\omega(y_j^h, z_k^h)$ ($h = 1 \ldots H, \; j = 1 \ldots J, \; k = 1 \ldots K_h$) which, after substitution of equations (51.1) into it, will yield a function $\omega(z_k^h)$; the policy-maker’s action will in a general way be indicated by

$$
\omega(z_k^h) \max \quad (51.5)
$$

and under the assumption that $\omega$ is continuous and no boundary conditions have to be imposed on the $y$’s and the $z$’s, the maximum conditions read:

$$
\frac{\partial \omega(z_k^h)}{\partial z_k^h} = 0 \quad \text{if} \quad \left\{ \begin{array}{l} h = 1 \ldots H \\ k = 1 \ldots K_h \end{array} \right. \quad (51.6)
$$

The cases of decentralization and centralization of all instruments are the extremes; in between there are cases where some instruments are centralized and others not. We will come back to their description at the end of section 5.2.

The cases of decentralization and centralization would not be strictly comparable, however, if not something more would be assumed about the relations between $\omega$ and $\omega^h$: these must be supposed to be “similar”, i.e. $\omega$ depends “in the same way” on the targets and instruments relevant to the greater area it covers as $\omega^h$ depends on its targets and instruments. It is necessary to specify what this means. We will do so first with regard to the target variables.
We have gone, in some more detail, into the nature of target variables and the way in which they occur in the $\omega^h$, in sections 4.3 and 4.4; ponderable target variables will occur as “total” quantities available (real income, consumption or a comparable variable) and as ratios pertinent to some distribution. In order to rule out differences of the size of the area covered by a policy-maker—the problem now before us—it seems advisable to bring the “total” quantities on a per capita basis. It is true that by so doing we rule out certain policies aiming themselves at a larger population; these policies can as a rule hardly be called economic policies at all and may deliberately be left out. Indicating, then, an arbitrary target variable by $y^h$ and assuming it to be a per capita figure, we may state that the analogous target variable for the group of $H$ policy-making areas as a whole will be

$$y = \frac{1}{H} \sum_k y^h$$  \hspace{1cm} (51.7)

and a first characteristic of similarity between $\omega^h$ and $\omega$ will be that $\omega$ depends in the same way on $y$ as $\omega^h$ on $y^h$. This similarity should be typical for “total” quantities supposed to be pertinent to the social utility of the policy-makers. We have argued in section 4.4 that as a good approximation we may sometimes assume $\omega^h$ to be linearly dependent on the various “total” quantities among its arguments. As far as there would only be such total quantities in a linear function, say:

$$\omega^h = \sum_k a_k y_k^h$$

there would be similarity between $\omega$ and $\omega^h$ if:

$$\omega = \sum_k a_k y_k$$

meaning at the same time that, then (cf. 51.7):

$$\omega = \frac{1}{H} \sum_h \omega^h$$  \hspace{1cm} (51.8)
It is interesting to note that our definition of similarity implies that a certain evaluation of the interests of area $h$ and area $h'$, say, is established, namely according to the number of people present in $h$ and $h'$. One could, of course, think of alternative treatments, i.e. giving weights to different populations; this suggestion will not, however, be worked out here.

For the target variables taking the form of ratios it is not to be expected that they are replaced by other targets, but there will be more of them for a greater area. Suppose that the target variable $y^h$ is, for each policy-maker, composed of a number of $y^h_k$ pertaining to groups of persons or commodities, and that policy-maker $h$ is striving to maintain ratios $c^h_k$, which may be expressed by a utility function (cf. section 4.4):

$$\omega^h = -\sum_k \beta^h_k \left( \frac{y^h_k}{y^h} - c^h_k \right)^2$$  \hspace{1cm} (51.9)

then we may assume, in order to define similarity as to these targets, that the central policy-maker in the centralized case also strives for the maintenance of such ratios between $y^h_k$ and $y^h$ and that, in addition, he strives for ratios $c^h$ between the $y^h$ and $y$. This we could express by a utility function

$$\omega = -\sum_h \sum_k \beta^h_k \left( \frac{y^h_k}{y} - c^h_k c^h \right)^2$$  \hspace{1cm} (51.10)

which, as to its structure, is analogous indeed to (51.9). It has, in addition, the property that, if the $y^h = c^h y$, \[\omega = \sum_h \omega^h\]  \hspace{1cm} (51.11)

which will only be approximately so if the $y^h$ are not equal, but only near to $c^h y$.

We have now to consider what similarity can or cannot mean with regard to the direct occurrence, in the utility functions, of the instrument variables; the occurrence of which,
as we saw, represents the fact that the application of instruments of economic policy involves costs and, more generally, disutilities. Here it would have to mean that the way in which these disutilities are valued is not different for the separate policy-makers under decentralization on the one hand and the central policy-maker under centralization on the other hand. Similarity could not, however, mean that the application of anyone individual instrument to a given level $z^k$ would, under decentralization, cause the same disutility as under centralization. For it is in the nature of centralization that it will be more costly, as soon as purely technical economies of scale are out of the question. It generally requires a bigger administrative machinery and, before all, more interference with private freedom. It will therefore be realistic to assume that “per unit of $z^k$” the discounting terms in $\omega$ will be higher than the corresponding ones in $\omega^k$.

It will have been observed and understood that strict similarity between $\omega^k$ and $\omega$ requires, by its nature, similarity between the various $\omega^k$s themselves; for practical problems, however, it will be very necessary also to consider cases in which this strict similarity does not exist. It will be possible to distinguish also cases of a smaller degree of similarity; it may happen, e.g., that two countries have $\omega^k$s in which the same variables occur, but with different coefficients $\alpha_k$. Some such cases will also be discussed.

The special cases mentioned in equations (51.8) and (51.11) will not always apply. A more general situation may be:

$$\bar{\omega} = F(\bar{\omega}^1, \bar{\omega}^2 \ldots \bar{\omega}^k \ldots \bar{\omega}^H) \quad (51.12)$$

but this is not the most general case either; the most general situation being the one where $\bar{\omega}$ depends on the same variables $y^k$, $z^k$ as occur in the $\omega^k$, without being a function of the $\omega^k$ only.
So far we only dealt with the extremes of decentralizing the use of all instruments $z_k^h$ and centralizing that use. There may be scope for *decentralizing some, and centralizing other instruments*; and the setting of the problem has also to be given for these intermediary policies. Let the decentralized instruments be $z_k^h$, and the centralized ones $z_k^h$. The former are being manipulated by the area policy-makers $h = 1 \ldots H$ and the latter by the central policy-maker. Again, the $\omega^h$ and the $\omega$ will be functions of the target variables $y_i^h$ and a number of instruments, namely the ones manipulated by the corresponding policy-makers:

$$
\omega^h (y_i^h, z_k^h) \quad h = 1 \ldots H
$$

$$
\omega (y_i^h, z_k^h) \quad h = 1 \ldots H
$$

and they will have to be maximized, after the objective equations (51.1), expressing the $y_i^h$'s in terms of the $z_k^h$'s, have been used to eliminate the former:

$$
\omega^h (z_k^h) \max \text{ with resp. to } z_k^h
$$

$$
\bar{\omega} (z_k^h) \max \text{ with resp. to } z_k^h
$$

It should then be borne in mind that the variables present in $\omega^h$ are, in principle, all the $z$'s: the symbol $h$ indicating that not only the special $h$ occurs to which $\omega^h$ refers, and the $k$ indicating that not only the decentralized instruments occur; and that the variables occurring in $\bar{\omega}$ are also all $z$'s. The maximum has, however, under our assumption as to the conduct of the policy-makers to be determined with respect to the instruments handled by each policy-maker.

For making a choice between centralized and decentralized policy we have to have, as in any political problem, and as was already pointed out in section 4.3, a criterion or a utility function "of our own". We will indicate this by $\Omega$, in order to distinguish it from $\omega$, the central policy-maker's utility function. It need not, however, be different from $\omega$ and as a rule will even be taken to coincide with it.

60
5.2. COMPARISON BETWEEN RESULTS OF CENTRALIZED AND DECENTRALIZED ECONOMIC POLICY

In a general way our problem has already been set in section 5.1: under a decentralized policy, the situation will be defined by equations (51.3) or (51.4), and under a centralized policy by equations (51.5) or (51.6). The answer to the question in what respect the two situations are different has, in principle, to be derived from those equations and can be given in this way. But it is not very illuminating. It will be useful to clarify it by the introduction of a few concepts which may visualize the nature of the problem somewhat better. What we want to know is whether certain instruments will be used to a higher degree and others to a lower, if we pass from a centralized to a decentralized state of affairs. A convenient starting point for the clarification needed will be found in the situation where, as in equation (51.12), \( \omega \) depends on the \( \omega^k \)'s only and positively on each, with the special case included where \( \omega \) is a linear combination of the \( \omega^k \).  

We propose to define four classes of instruments according to the influences they exert on the \( \omega^k \)'s and will call an instrument \( z^k \)

(a) **supporting** if a change in it brings forth changes in each of the various \( \omega^k \)'s which are of the same sign;
(b) **conflicting** if a change in it acts on \( \omega^k \) in the opposite way as on all the other \( \omega^k \)'s;
(c) **neutral** if a change in it does not affect the \( \omega^k \)'s of all the other policy-makers, and finally
(d) **mixed** if the effects it exerts on the various \( \omega^k \)'s other than its own \( \omega^k \) are of different sign among themselves.

\[ \text{It does not seem to the author to be of great practical importance to consider the cases—if existant—where there would be similarity between } \omega \text{ and } \omega^k \text{ and nonetheless } \omega \text{ would not be considered approximately to be a function of the } \omega^k \text{'s.} \]
It should be immediately stressed that the property class to which an instrument variable belongs need not under all circumstances be the same. It may very well be that, dependent on its own value or on other factors, it changes its class. If we are working with linear functions, this will not be the case, except possibly where boundary conditions come in.

The following statements may now be made.

(i) The same classification as just proposed may be made with respect either to the total influence exerted by any instrument variable on the $\bar{A}^n$'s or to its direct influence or its indirect influence only.

(ii) Since it is assumed that there exists only a direct influence of any instrument variable on its "own" $\bar{A}^n$, all instruments, according to their direct influences are of the neutral type: a change in them never affects the "other" $\bar{A}^n$.

(iii) This implies that their class is the same whether judged according to their total or to their indirect influence.

(iv) As already stated in section 5.1, any application of an instrument in a centralized way will imply higher disutilities, or will lower utility more than the same application in a decentralized situation would. There is, therefore, a general tendency in a centralized situation to make less use of all the instruments, which will be the stronger the higher the disutilities are.

(v) We will call positive changes in instrument variables the changes that increase the corresponding $\bar{A}^n$ values. Apart, then, from the tendency mentioned under (iv), there will be the tendency:

- to bring *supporting instruments at higher levels* under a centralized régime than under a decentralized one;
- to bring *conflicting instruments at lower levels* under a centralized régime, and
to bring neutral instruments at the same levels in both situations.

No general answer can be given as to mixed instruments, but clearly it follows from the foregoing that they will be brought, under a centralized régime, at higher levels than under a decentralized régime if they are nearly supporting, i.e. if for most of the "other" $\omega'$ the effect is of the same sign as on $\omega$, whereas they will be brought at lower levels if they are nearly conflicting. In between therefore they will probably not be brought on levels differing much between a centralized and a decentralized state.

Statements (v) may be proved as follows. For a supporting instrument a given positive change will affect favourably not only $\omega$ but also the other $\omega'$. In a decentralized situation these latter contributions will be irrelevant to policy-maker $h$; in a centralized state they will, however, be relevant to the central policy-maker. In the decentralized state the advantages of increasing the instrument value will therefore be underestimated as compared to the centralized state; hence these instruments will be less increased than under centralization, with the possible boundary case where there will be no difference.

A given positive change in a conflicting instrument variable will affect the other $\omega'$ unfavourably and therefore lead to a relative overestimation of the advantages to be obtained, in a situation of decentralization. This type of variable will therefore tend to be used more intensively, or more often, under decentralization than under centralization.

No such "errors" of estimation will be involved in the use of neutral instruments.

These proofs—admittedly of a sketchy character and subject to refinement—might be ineffective in the cases excluded in the beginning of this section, namely where $\omega$ could not be considered a positively monotonous function
of the $\omega^s$. The author believes, however, that these cases will probably be of little practical importance. The relevant point in the proof is that for clearly supporting instruments (the corresponding argument for conflicting instruments will be easily found by the reader) the advantages brought to other countries are significant and nevertheless not taken into account by the policy-makers in a decentralized situation. Even if $\omega$ is not only a function of the $\omega^s$, and the total effect of a given change in such an instrument is therefore not equal to the weighted sum of the effects on the individual $\omega^s$ (weighted by the expressions $\frac{\partial \omega}{\partial \omega^s}$) it seems improbable that the presumably small additional influence on $\omega$ would compensate these significant non-accounted-for advantages. The case remains open, in principle, for further investigation.

5.8. EXAMPLES II, III A AND III B; EXAMPLES OF TYPES OF INSTRUMENTS

The argument may first be illustrated by applying it to examples II and III.

Example II, it will be remembered (cf. section 3.4), considers the decentralized action by employers and workers to improve their position. Target variables are $Z'$ and $L'$, real profits and real wages bill and no distinction is being made here between utility functions and these target variables. It was assumed, further, that both parties are aware of the influence of prices on demand and that the workers are also aware of the direct influence of wage rates on prices. Under these circumstances it was found that a situation would result in which nominal wages and prices would be fairly high, but real incomes to both groups rather low in comparison to the situation actually prevailing in the Netherlands in 1949.
The question of the influence of centralization on the outcome of this process will now be studied. In this case we assume that the one authority taking the place of the two policy-making groups takes as its target total real national income \( Y' = Z' + L' = \frac{-2.8 \cdot p^3 + 6 \cdot p - 1.5}{p + 0.7} \). It cannot surprise us that the wage rate does no longer enter into this expression: it only determines the distribution of national income between the two groups. This may be different if the wage rate would surpass a certain level; this situation we will, for the moment, disregard. In other words; in this simple model one instrument—the price level—entirely determines the external position of the country; the other—the wage rate—is only relevant to internal distribution.

It is possible, to start with, to calculate which price level will yield the maximum real national income; it has to obey the equation

\[
\frac{dY'}{dp} = \frac{-2.8 \cdot p^3 - 3.9 \cdot p + 5.7}{(p + 0.7)^2} = 0
\]

(53.1)

There is one root which is economically significant, namely \( p = 0.9 \), meaning that the maximum will be attained at a price level (especially an export price level, evidently) of 90 % of the level that actually prevailed. With this price level real national income would be at 1.02, which compares very favourably with the situation under the decentralization formerly considered (with \( Y' = 0.80 \)), and even slightly favourably with the actual position of the country in 1949.

It is interesting to place this result against the background of some further figures. In order to judge whether the instruments used have parallel or opposed influence on the targets of the two policy-makers we calculate, under the assumptions made as to the behaviour of the other party, the four relevant differential coefficients (for the values of \( p \) and \( l \) in the base period, i.e. \( p = l = 1 \)):
Influence on target of

1. Employers
Exerted by instrument 1 (price) \( \frac{dZ'}{dp} = +0.62 \quad \frac{dL'}{dp} = -0.29 \)

" 2 (wage) \( \frac{dZ'}{dl} = -0.55 \quad \frac{dL'}{dl} = +0.26 \)

With other assumptions as to the behaviour of the policymakers we would of course have obtained other results. For the simplest hypotheses, namely, that only the direct influence of \( p \) and \( l \) on \( Z' \) and \( L' \) is calculated we find e.g.

\[
\frac{\varepsilon Z'}{\varepsilon p} = +0.6 \quad \frac{\varepsilon L'}{\varepsilon p} = -1.0 \\
\frac{\varepsilon Z'}{\varepsilon l} = -0.55 \quad \frac{\varepsilon L'}{\varepsilon l} = +0.55
\]

From these figures it is clear that the interests of the two policymakers with respect to both instruments are opposed; they are examples of conflicting instruments, making it probable that they will be pushed further under decentralization than with centralization. This actually appears to be the case, as our foregoing calculations showed.

Example III will be applied in two different versions.

III A. A simple application will first be given, where the two instruments \( x_0^h \) and \( p_0^h \) will be used in order to pursue one fixed target or rather side condition and one flexible target: the fixed one being balance of payments equilibrium \( D^h = 0 \ (h = 1 \ldots 10) \) and the flexible one a maximum well-being \( \omega^h \), considered to be a function of the volume of production \( x^h \) and of the instrument \( x_0^h \); we assume

\[ \omega^h = ax^h - ax_0^h \quad (53.2) \]

From the side condition a relation between the two instruments follows leaving only one degree of freedom; we assume that instrument \( x_0^h \) will be used to attain the flexible target. Since

\[ D^h = 2.5 (p^h - p_0^h) + 0.7 (x_0^h - x_0^{h'}) \]
for values in the neighbourhood of \( p_0^h = x_0^h = 1 \), it follows that

\[-7.5(p_0^h - p_0^{h'}) = + 2.1(x_0^h - x_0^{h'})\]  \hspace{1cm} \text{(53.3)}

From this we deduce that

\[ x^h = 3.7 x_0^h + 4.7 x_0^{h'} - 7.5 (p_0^h - p_0^{h'}) \]
\[ = 5.8 x_0^h + 2.6 x_0^{h'} \]  \hspace{1cm} \text{(53.4)}

We will now compare the effects of centralization on the value chosen for the instrument variable \( x_0^h \). With a decentralized policy the policy-makers will aim at a maximum of:

\[ \omega^h = (5.8 \sigma - \sigma) x_0^h + 2.6 x_0^{h'} \]

If their valuation of the “cost” of using a unit increase in the instrument variable \( x_0^h \) (i.e. \( \sigma \)) surpasses 5.8 \( \sigma \) they will not use the instrument, i.e. they will fix \( x_0^h \) at the lowest value possible \( x_0^{h'} \), say at the indispensable minimum of government expenditure. If their valuation of this cost is less, they will apply the instrument at its maximum value \( x_0^h \), say the value corresponding with “full” employment. The critical point 5.8 \( \sigma \) evidently corresponds to a valuation \( \sigma \) which shows a ratio to the valuation \( \alpha \) of the effects of \( x_0^h \) on \( x^h \) equal to the national multiplier 5.8.

With a centralized policy the central policy maker will be guided, we assume, by a \( \omega \) equal to the sum of all national \( \omega^h \), or

\[ \omega = \omega^h + 9\omega^{h''} = \alpha \Sigma x_0^{h'} - \alpha \Sigma x_0^{h'} \]
\[ = 5.8 \alpha \Sigma x_0^{h'} + \frac{10 \times 2.6}{9} \Sigma x_0^{h'} - \sigma \Sigma x_0^{h'} \]
\[ = 8.7 \alpha \Sigma x_0^{h'} - \sigma \Sigma x_0^{h'} \]  \hspace{1cm} \text{(58.3)}

From this it follows that here again there is a critical value of \( \sigma \) above which the policy of public expenditure will be at
its maximum $x_0^*$, and below which it will be at its minimum $x_0^*$. The critical value now corresponds, however, to a ratio of 8.7 between $\sigma$ and $a$, being the international multiplier. There is a greater probability therefore that the policy will be followed under a centralized than under a decentralized regime. This conclusion is in agreement with the fact that the instrument is of the supporting type, expressing itself in the equal signs of the terms with $x_0^a$ and $x_0^b$ in (58.4); the conclusion would be inverted if the coefficient of $x_0^b$ in (58.4) would have been negative, i.e. if the instrument influences the target variable of the other countries in an opposite way.

III B. A more complicated application of example III may now be given where both target variables enter into the utility function and where we assume this to be quadratic \(^1\). Since in our example not all countries are able to determine $D^a$ freely (as $\Sigma D^a = 0$), we will consider a problem of centralization for two countries only (leaving the other eight out of the picture) but assuming that these two countries are free to choose the $D^a$ they want. In order to make our example easily comparable to the general set-up of our study we switch from the special notation to the general one; instead of $x^a$ and $D^a$ we write $y_1^a$ and $y_2^a$. Instead of $x_0^a$ and $x_0^b$, $z_1^a$ and $z_2^a$. The objective equations will then run for our two countries:

$$
\begin{align*}
y_1^1 &= 3.7z_1^2 + 0.5 \ z_1^1 + 4.2 \ z_1^3 - 7.5z_2^1 - 0.8z_2^2 + 6.7z_2^3 \\
y_1^2 &= 3.7z_1^2 + 0.5 \ z_1^3 + 4.2 \ z_1^2 - 7.5z_2^2 + 0.8z_2^3 - 2.2z_2^3 \\
y_2^1 &= 0.7z_1^2 - 0.08z_1^1 - 0.62z_2^1 + 2.5z_2^1 - 0.6z_2^2 - 2.2z_2^3 \\
y_2^2 &= 0.7z_2^2 - 0.08z_2^1 - 0.62z_1^1 + 2.5z_1^1 - 0.6z_1^2 - 2.2z_1^3
\end{align*}
$$

(58.6)

\(^1\) It will even be assumed that it is quadratic with regard to the “total-quantity target” $x$, contrary to what was suggested in section 4.4, in order to avoid the necessity of introducing boundary conditions in the maximizing process.
Here, $z_1^3$ and $z_2^3$ are the average values for $z_1$ and $z_2$ applied in the eight other countries, supposed to be constant — which evidently is an approximation only — under the process of centralization. They will be disregarded in what follows. The utility function $\omega^1$ and $\omega^2$ are assumed to be:

\[
\begin{align*}
\omega^1 &= ay_1^1 + \gamma(y_1^1)^2 - \beta(y_1^3)^2 - \delta z_1^1 - \varepsilon z_2^1 \\
\omega^2 &= ay_2^2 + \gamma(y_2^2)^2 - \beta(y_2^3)^2 - \varepsilon z_2^2
\end{align*}
\] (53.6)

From these equations we deduce, for the case of decentralization, the following subjective equations:

\[
\begin{align*}
\frac{\partial \omega^1}{\partial z_1^1} &= 3.7(a + 2\gamma y_1^1) - 1.4 \beta y_2^3 = 3.7 u - 1.4 v = \delta \\
\frac{\partial \omega^1}{\partial z_1^2} &= -7.5 u - 5.0 v = \varepsilon \\
\frac{\partial \omega^2}{\partial z_1^2} &= 3.7 u' - 1.4 v' = \delta \\
\frac{\partial \omega^2}{\partial z_2^2} &= -7.5 u' - 5.0 v' = \varepsilon
\end{align*}
\] (53.8)

Here we have written $u$ for $a + 2\gamma y_1^1$, $u'$ for $a + 2\gamma y_1^2$, $v$ for $\beta y_2^3$ and $v'$ for $\beta y_2^3$. Since the second pair of equations is identical with the first, except that it runs in terms of $u'$ and $v'$ instead of $u$ and $v$, $u' = u$ and $v' = v$. Solution of the first couple yields:

\[
\begin{align*}
u &= 0.17\delta - 0.22\varepsilon \\
v &= -0.26\delta + 0.13\varepsilon
\end{align*}
\]

With centralization, the subjective equations will be, in the abbreviated notation, assuming that $\omega = \omega^1 + \omega^2$:

\[
\begin{align*}
\frac{\partial \omega}{\partial z_1^1} &= 3.7u - 1.4 v + 0.5u' + 0.16v' = \delta \\
\frac{\partial \omega}{\partial z_1^2} &= -7.5u - 5.0 v + 0.8u' + 0.6 v' = \varepsilon \\
\frac{\partial \omega}{\partial z_1^3} &= 0.5u + 0.16v + 3.7u' - 1.4 v' = \delta \\
\frac{\partial \omega}{\partial z_1^4} &= 0.8u + 0.6 v - 7.5u' - 5.0 v' = \varepsilon
\end{align*}
\] (53.9)
Again we find \( u' = u \) and \( v' = v \) and:

\[
\begin{align*}
u &= 0.17\delta - 0.04\varepsilon \\
v &= -0.25\delta - 0.15\varepsilon
\end{align*}
\]

It is clear, from these results, that in the case of centralization \( u \)—for almost every set of values of \( \delta \) and \( \varepsilon \)—will be larger and \( v \) smaller than in the case of decentralization. Transposed to \( y_1^1 \) and \( y_1^2 \) this means that the volume of production tends to be larger and the balance of payments deficit smaller. In plain words, in the case of centralization the fact that the policy-maker will consider the total area as a whole will lead him to choose a higher level of activity and this fact in itself—although it positively influences the balance of payments deficit of the countries themselves—will at the same time influence negatively the deficit of the other country. In the case of decentralization it is the fear for a balance of payments deficit which keeps down the level of government expenditure in each country and thereby influences positively the balance of payments deficit of the other.

To avoid misunderstandings it may be stated that the setting of the problem is characteristic to the problems of countries in a situation of underemployment, as will be clear from the positive influence of the volume of production on utility.

The results are what one could expect after our general statements. From equations (85.13) and (85.14) it is clear that the instrument \( x_0 \) is a strongly supporting instrument as far as the target \( x \) is concerned and a weakly conflicting instrument as far as \( D \) is concerned; \( p_0 \) is conflicting as far as \( D \) is concerned and neutral as far as \( x \) is concerned. For utility as a whole—although no definite judgment can be given as long as the relative weights given in it to \( x \) and \( D \) are not known—there is a great chance for \( x_0 \) to be supporting and \( p_0 \) to be conflicting. One could expect therefore that
under a centralized régime $w_0$ will be used more intensively than under a decentralized régime, and that the inverse is true for the use made of $p_0$. This would lead to higher volumes of production, as was found. The consequences for the balance of payments deficit are less easy to determine with this general type of reasoning and here the full calculations as given above are the only way to get exact answers.

A second set of examples is meant to exemplify the various types of instruments that have been introduced with concrete types of measures.

The most pronounced example of a supporting instrument is probably supplied by the instrument of government—or more generally, autonomous national—expenditures, for a group of countries which are all either in a state of depression or all in a state of inflation. During a depression an increase in autonomous national expenditures will raise the volume of production and hence, according to almost all politicians, the level of wellbeing, not only in the country concerned but also in the other countries. During a period of inflation a lowering of autonomous expenditures would equally improve the situation not only in the country concerned but also in the other ones.

Good examples of conflicting instruments are mainly to be found in the field of pricing. Any form of price cutting applied by a country as a whole—be it devaluation or a general lowering of incomes—will lead to an improvement in the situation of that country, judged by its volume of production and employment, and its balance of payments situation—usually at some detriment to other countries. This applies especially in the case where, again, the countries considered are in roughly similar conditions, i.e. either all of them in a state of underemployment or all of them in a state of overemployment.

If, on the other hand, the situation of the countries in
a group is different, some of them being in a state of under-
employment (subgroup I) and others in one of inflation
(subgroup II), the two classes of instruments interchange
their role as far as their influence on the subgroups as a
whole is concerned (not, however, as far as the members
of the subgroup among themselves are concerned). Expansion
of autonomous national expenditures then improves the
situation in the depressed countries and deteriorates them
in the inflated ones; this instrument is, then, a conflicting
one, as far intersubgroup relations are concerned, and a cut
in prices in the depressed countries would represent, as far
as intersubgroup relations are concerned, a supporting
instrument. Looked at for all the countries of the original
group, both instruments would have the characteristic of
mixed ones, as may often be the case.

Examples of neutral instruments will have to be found
before all in instruments which aim at internal redistribution
of income and do not therefore, as a first approximation,
affect the total demand of a country and hence not the
influence on other countries. This need not be always so,
however; a change in the wage rate of a country, which
sometimes represents an internal change in distribution only,
also will affect the price level at which the country concerned
is able to export and hence also influence the situation of
other countries; and even if this would not be so, there
might be a shift in the demand exerted by the country which
would be to the advantage of some and to the disadvantage
of other foreign countries.

Other examples of instruments that may be almost neutral
are measures that refer to typically local markets, i.e.
markets which because of the high costs of transportation
involved (heavy or perishable products) do not influence
their foreign counterparts very much.

This latter point may also be formulated in another way:
it may be said that for all instruments of economic policy there is a tendency to become more and more neutral to the outside world, the larger the geographic area is taken. The point of neutrality will not always, however, be reached before the area considered has become the world as a whole. If the point of neutrality has been reached the corresponding area may be called the “area of neutrality”. It is evidently the area for which the effects of the instrument considered on the outside world may be neglected.

From the foregoing it will be clear that in principle most instruments will be of the mixed type; but that with a certain approximation certain instruments are clearly of one of the other types. In doubtful cases all the refinement given in our calculation schemes will be needed and our general statements will be of little help. But in the most pronounced cases they will be helpful, the author believes, to the imagination of the reader.

5.4. THE CHOICE BETWEEN CENTRALIZATION AND DECENTRALIZATION OF INSTRUMENTS

After having studied the consequences of an increase in the degree of centralization of economic policy we are going to answer the question which of the various possibilities is optimal. In principle, the answer, which presupposes the acceptance of a criterion function \( \Omega \), has to be given in two stages: first, for each possible situation the outcome of the policy has to be calculated, and second, the value which each of these “outcomes”, i.e. sets of figures for all the \( y \)'s and \( z \)'s, yields to \( \Omega \) has to be determined. Among the sets \( (y, z) \) the one will be preferred which yields the highest \( \Omega \).

To begin with, we assume \( \Omega \) to be identical to \( \omega \). In the case of similarity between \( \omega^2 \) and \( \omega \) there is more to be said for this procedure than where this would not be so.
As a second initial restriction of the generality of our argument we assume that the direct influence of the values of the instrument variables on \( \omega \) is absent. Our question then reduces to a tautology: it is self-evident that \( \omega \) is maximized under a centralized régime.

What is maximized, however, is the function \( \omega \) under the assumption \textit{a priori} that all instruments are administered by the central policy-maker. It may be that \( \omega \) may assume still higher values if certain of the instruments are used in a decentralized way.

From the analysis given in section 5.2 it follows that this will be so if the neutral instruments are decentralized. This does not change the values at which they are fixed, and hence not the \( y_k \), but it will reduce their direct influence which for all instruments is always assumed to be negative. In plain language, the cost and trouble will be diminished, without changing the situation any further.

A further gain may be obtained if some of the other instruments are decentralized. Each decentralization means reduction of costs and disutility generally. However, for instruments which are outspoken supporting or conflicting, there would be considerable loss via the indirect influences they have on \( \omega \): the changes in \( y \)'s would be considerably disadvantageous. Therefore the argument applies to the groups of instruments—if any—which are “near to” neutral ones, whether supporting, conflicting or mixed. What the term “near” has to mean can of course only be ascertained if the whole of the function \( \omega \) is numerically known. The margins of uncertainty in our knowledge about it imply differences of opinion according to personal guesses or feelings.

The remaining instruments should therefore be centralized. It is useful to note that this centralization will lead to opposite effects as far as the use of the instruments is concerned: the supporting ones will be brought at higher
levels, whereas the conflicting ones will be fixed at lower levels than under a decentralized régime. One might speak of positive and negative centralization, respectively. The latter may take the special form of the ruling out of their use.

Decentralization may sometimes be advocated only to a limited degree, if the “area of neutrality” is fairly large; then decentralization may take the form of entrusting a certain instrument to regional authorities (such as the European Payments Union); inside the regions there will be centralization.

Summarizing, and using some of the examples discussed in section 5.8, we may say that the optimum policy, in the case of identity between $\omega$ and $\Omega$, and of similarity between $\omega$ and the $\omega^p$'s, is constituted by decentralization of neutral and near-neutral instruments (such as instruments mainly tending to change the internal distribution inside each area or instruments bearing on local markets), positive centralization of supporting instruments (such as government expenditure policy in a situation of general depression or general inflation) and negative centralization of conflicting instruments (such as devaluation and other price adjustments, in a situation of general depression or general inflation). Negative centralization may mean, in certain cases, the prohibition of certain instruments at all, as e.g. the application of monopolistic pricing, as far as the conditions are fulfilled to make this instrument detrimental to general welfare.\footnote{These conditions are the well-known conditions that income distribution is considered “acceptable” and that there are decreasing returns at least at the margin of each industry.}

Some considerations may be added, in this connection, on the practical applications of the foregoing principles on the problem of international integration. Activities have been directed primarily towards the elimination of trade barriers, in the form either of tariffs or of quota systems. This clearly
represents a case of "negative centralization". The argument in favour of this procedure is certainly partly that trade barriers to some extent are conflicting instruments of economic policy. To the fullfledged freetrader they are not, however; trade barriers according to him are not even in the interest of the countries applying them. In his view therefore, their elimination is an even more elementary act of economic wisdom: it is the elimination of instruments which are just stupid, or in our wordings, rest on non-conformity between what the policy-makers expect them to do and what they really do to the country. There are some other examples of such elimination—under certain conditions the just-mentioned elimination of monopolies inside a community or the elimination of erroneous wage policies—which have nothing to do with centralization as such and could just as well be tackled inside any separate country.

The conditions under which the freetrader's view is correct are not always, however, fulfilled. Under certain conditions—the existence of unemployment, the existence of a temporary surplus of capital goods and skilled workers in a certain industry, the fact that for an infant industry productivity can be increased by a process of education or training, the absence of certainty of a continuous overseas supply of food or the possibility to influence the terms of trade—import duties may be an appropriate policy \(^1\) for a certain country and then will be an example of a conflicting instrument of economic policy. The arguments in favour of their elimination in an integrated area are then the ones presented in this study; they may, however, not be really arguments in favour of their complete elimination, but only

\(^1\) It could almost always also be replaced by a policy of subventions; only not where the costs involved in controls would be much higher than the costs of import duties. The argument presented would, however, also apply to subventions.
in favour of their reduction and their use in rare and well-described situations, mostly temporary in nature.

Any attempt at integration would not, however, be complete if it would not contain at the same time elements of "positive centralization". The most conspicuous example is the one already quoted in our previous examples, namely, the centralization of anti-deflationary and anti-inflationary policies.

In practical applications the problem of centralization will almost always be combined with another element, namely, that of *dissimilarity between the* \( \omega^a \)'s, implying, as a necessary consequence dissimilarity between \( \omega \) and some of the \( \omega^b \)'s. This element will, at the same time, make more ambiguous the judgment of the outcome of a possible process of centralization, since it is less natural now to use as the criterion an \( \Theta \) which is identical to \( \omega \). For the areas that have \( \omega^b \)'s, distinctly different from the central \( \omega \), there will be a good reason also to judge the result by a criterion \( \Theta \) similar to \( \omega^a \). And the judgment may then be different. It does not follow that centralization should not be preferred; it may well be that part of the apparent argument against centralization could be eliminated by a special construction of the central utility function \( \omega \), and that the other part upon closer examination does not make sense. This statement has to be clarified.

As to the special construction of \( \omega \): if certain areas (say the Northern-European countries, in a possible integration of Europe) would value high employment \( y_1 \) relatively higher as compared with the cost of public-expenditure policy \( z_1 \), whereas other areas (say the Southern European countries) would value it relatively lower, then, in a central \( \omega \), one could imagine that this would be expressed by giving \( y_1^a \) for the former countries a higher \( a_1^b \), and \( z_1^b \) a lower \( \beta_1^a \) than for the latter group, leading to an \( \omega \), in which employment in the first group of countries were given a higher
weight in “total employment” and government expenditure a lower weight in “total government expenditure”. By doing so it would be possible to take account of differences in “taste” between the areas. A further possibility to do so would already be present in the manipulation of the neutral and near-neutral instruments which would be decentralized anyhow.

It would seem that this is not only about all that could be done but that no more could reasonably be asked for. It cannot be the intention that any area would impose its own preferences on others. Preference for decentralization just because the local authorities want to exert power for its own sake would seem a weak argument if centralization would, on the other hand, yield important advantages as to the manipulation of both supporting and conflicting instruments.

Apart from the problem of international integration there is also, as observed already in chapter I, the internal problem to each country to which our considerations apply. Questions of centralization present themselves with regard to the activity of entrepreneurs and industries as well as to that of social groups and of local authorities and the author believes that the general principles and the subdivision of instruments of policy on which they rest may prove useful for these internal problems as well. It is, he believes, in rough accordance with these principles, that the Dutch attempt at giving a certain autonomous task to business even in matters of a public interest has been conceived. Certain tasks, mostly concerning matters of the distribution of an industry’s product between employers and employees, have been delegated to business organisations, whereas others, clearly of significant interest to the outside world—price setting, investment decisions—have not been entrusted to them. It would lead too far to go into the details of these questions in this study.