APPEN

SURVEY OF MODELS

Number of Model	Brief Characteristic of Model (for fuller description see Appendix 3)	Variables												
		National Income Y	Expenditure X	Tax Receipts T	Total Wages L	Total Non-Labour Inc. Z	Value of Consumption C	Value of Investment J	Value of Exports E	Value of Imports I	Deficit in Bal. of Paym D	Real National Income y	Real Expenditure x	Volume of Gross Prod. v
	Closed, static, macro													
01	Money flow	1	1											
02	Money and product flow	1	1										1	
03	Money, product and factor flow	1	1		1								1	
04	Money flow and public finance	1	1	1										
05	Money flow and assets	1	1	1										
	Closed, static, micro													
06	Horiz. money and prod. flow	1	$H+2$										H	
07	Horiz. money and prod. flow with monopolies					H						1	H	
08	Real income distribution [1]													
	Closed, dynamic, macro													
09	Development											1		1
10	Cycle (money-flow, paper profits)	1	1											
	Open, static, macro													
11	Money flow	1	1							1	1			
12	Money and product flow	1	1						1	1	1		1	1
13	Money, product and factor flow	1	1						1	1	1		1	1
14	Money and prod. flow, exch. rate	1	1						1	1	1		1	1
15	Money flow and assets	1	1							1				
	Open, static, micro													
16	Money, prod., fact. flow, publ. fin.	1			1	1	2	1	1	1	1			1
17	Money flow assets and banking	1								1				
18	Horiz. money, prod. and fact. flow	1	2						2	1	1		2	2
19	Horiz. money and prod. flow	1	H						H	1	1		H	H
	Open, dynamic, micro													
20	Investm. project appraisal	$2H+4$		2		$H+1$					1		1	$H+$
	Group of economies, static													
21	Money and product flow	H									H		H^2	

[1]) The variables of this model are explained in the description of the model itself.

DIX 1

USED AS EXAMPLES

Occurring in Model																						
Volume of Employment	Depreciation	Volume of Consumption	Volume of Investment	Volume of Exports	Volume of Imports	Replacement Investm.	Real Savings	Price of Product	Price of "Other" Prod.	Wage Rate	Interest Rate	Exchange Rate	Money in Circulation	Deposits	Bonds Held	Bills and Advances	Redisc. with C. Bank	Reserves	Gold Stock	Foreign Debt	Stock of Real Cap.'	Stock of Equipment
a	d	c	j	e	i	r	s	p	q	l	m	k	M	M'	B	B^B	B^R	R	Au	K^i	b	w
								1														
1								1														
													1									
								H														
								$H+1$														
	1	1	1				1														1	1
								1														
				1	1			2														
				1	1			2														
				1	1			2														
													6		6							
1		1	1	1	1			1		1												
											2		1	1		1	1	1	1			
2				2	2			2														
$H+1$				H	H			$2H$														
$H+2$	1			1	$H+1$		1	2			1	1								H	$H+1$	
								H														

APPENDIX 2

PROBLEMS OF ECONOMIC POLICY TREATED WITH MODELS

Probl. No.	Nature of economy[1])	Model No.	Targets or other aims	Instruments or other means	Cf. section
011	Closed static macro	01	Full employment	Public expenditure	3.521
021		02	Full employment	Public expenditure	3.522
031		03	Full employment	Public expenditure	3.523
032		03	Full employment	Wage rate	3.523
033		03	Full employment; monetary equilibrium	Public expenditure; wage rate	3.524
041		04	Full employment	Public expenditure	3.525
042		04	Full employment	Taxes	3.525
051		05	Full employment;	Public expenditure;	3.531
052		05	no potential	debt policy	3.532
053		05	inflation		3.533
054		05	Full employment	Money circulation	3.534
061	Closed static micro	06	Full employment in each industry	Public expenditure; indirect tax rates in each industry	3.535
071		07	General aims [2])	Free competition in all industries	5.52
072		07	General aims	A monopoly for one product	5.53
073		07	General aims	Vertic. integrated monopolies for all products	5.54
074		07	General aims	Cumulative monopolies for one or all final products	5.55
081		08	General aims	Tax rates, education and family allowances	6.42
091	Closed dynamic	09	Development	Investment	3.62
101	macro	10	Stabilization	Compensatory public expend.	5.32
102		10	Stabilization	Raw material standard	6.5
111	Open static macro	11	Full employment	Public expenditure	4.121
112		11	Balance of paym. equilibrium	Public expenditure	4.122
113		11	Optimum combination of full employment and balance of payments equil.	Public expenditure	4.123
121		12	Full employment	Public expenditure	4.131
131		13	Full employment and balance of payments equil.	Public expenditure and wage rate	4.141
141		14	Full employment and balance of payments equil.	Public expenditure and wage rate	4.151
142		14	Full employment and balance of payments equil.	Public expenditure and exchange rate	4.152

[1]) For fuller description cf. Appendix 3

[2]) As described in section 1.4; cf. also sections dealing with problems concerned.

Probl. No.	Nature of economy[1])	Model No.	Targets or other aims	Instruments or other means	Cf. section
143		14	Full employment, bal. of paym. eq., monetary eq.	Public expenditure, wage rate, exchange rate	4.156
151		15	Full employment and balance of payments equil.	Public expenditure and debt policy	4.161
152		15	Full employment and balance of payments equil.	Public expenditure and discount rate	4.162
161	Open static micro	16	Full employm., balance of paym. eq., investment volume, distr. of income	Public expenditure, wage rate, indirect and direct tax rate	4.221
161		16	Full employm., balance of paym. eq., investment volume, internal price level	Public expenditure, wage rate, indirect and direct tax rate	4.231
171		17	Full employment, balance of paym. equilibrium	Discount rate, reserve requirements	4.241
181		18	Full employment and bal. of paym. equil.	Productivity in two industries	4.251
191	Open static micro	19	Full employment in each industry	Public expenditure and indirect taxes	4.261
201	Open dynamic micro	20	General aims	Investment projects	5.73
211	Group of economies, static	21	Full employment and bal. of paym. equil.	Public expenditure and price levels in each country	4.43

[1]) For fuller description cf. Appendix 3.

APPENDIX 3

INDIVIDUAL MODELS DEFINED [1])

Symbols for variables: see Appendix 4

MODEL 01. CLOSED, STATIC, MACRO, MONEY-FLOW MODEL

ACTORS	*Number:*	1, economy as a whole.
	Income:	equal to total expenditure.
	Income behaviour:	supply of commodity perfectly elastic (equation 1).
	Spending habits:	see equation 2.
	Wealth:	does not influence current economic process.
MARKETS	*Number:*	1, general commodity.
	Character:	flow.
TECHNICAL RELATIONS		Not explicitly considered.

EQUATIONS

Income formation: $Y = X$ (1)

Income spending: $X = X_0 + \xi_1 Y$ (2)

where X_0 = autonomous expenditure

ξ_1 = marginal propensity to spend

From this equation the elasticity of demand for the general commodity can be derived as follows. Equation (2) may be written as:

$$xp = X_0 + \xi_1 yp$$
$$x = \frac{X_0}{p} + \xi_1 y$$

Elasticity of demand $= -X_0$ for $x = p = 1$

By a proper choice of units we may choose two variables = 1; the other will then follow.

If expenditure X_0 is not autonomous, but corresponding real expenditure

$$x_0 = \frac{X_0}{p}$$

we have instead $\qquad x = x_0 + \xi_1 y$

and the elasticity of demand is zero.

If we choose $x = p = 1$ in the point of equilibrium, we have, in addition, $X = 1$, $Y = 1$, and hence $1 = X_0 + \xi_1$; also $x_0 = X_0 = 1 - \xi_1$.

[1] Cf. § 2.14 for explanation of the terms used.

MODEL 02. CLOSED, STATIC, MACRO, MONEY AND PRODUCT FLOW MODEL

ACTORS *Number:* 1, economy as a whole.
Income: determined by total expenditure (equation 1); supply of product not perfectly elastic.
Spending habits: expenditure dependent on income and price level (see equation 2).
Wealth: does not influence current economic process.
MARKETS *Number:* 1, general commodity.
Character: flow
TECHNICAL RELATIONS Not explicitly considered.
EQUATIONS
Income formation: $Y = X$ (1)
Income spending: $X = x_0 p + \xi_1 Y$ (2)
where x_0 = autonomous real expenditure
ξ_1 = marginal propensity to spend.
Price fixation: $p = p_0 + \pi x$ (3)
where p_0 = autonomous price component
π = demand sensitivity of price level.
Definition: $X = xp$ (4)

Apart from the type of calculations already made with model 02, we may add a calculation of price flexibility in this case. To this end, we have again taken the units of goods and money such as to make, in the equilibrium situation, $p = x = 1$. It follows that also $X = Y = 1$ and hence $1 = \xi_1 + x_0$ and $1 = p_0 + \pi$. And the flexibility of p is $\frac{\partial p}{\partial x}\frac{x}{p} = \pi = 1 - p_0$.

MODEL 03. CLOSED, STATIC, MACRO MONEY, PRODUCT AND FACTOR FLOW

ACTORS *Number:* 2, wage-earners and independents.
Income: wage bill and profits.
Income behaviour: supply of labour perfectly elastic; supply of goods: see equation (3)
Spending habits: wage earners have a marginal propensity to spend of $\xi_1 + \xi_2$, independents of ξ_1.
Wealth: does not influence current economic process.
MARKETS *Number:* 2, for labour and for product.
Character: both are flow markets.

TECHNICAL RELATIONS Linear relation between quantity of product produced and quantity of labour needed.

EQUATIONS

Income formation: $Y = X$ (1)

Income formation, workers: $L = la$ (2)

Income spending: $X = \xi_1 Y + \xi_2 L + X_0$ (3)

where ξ_1 = propensity to spend for independents

$\xi_1 + \xi_2$ = propensity to spend for workers

X_0 = autonomous expenditure. This was not, as in model 02, taken to vary with p, since that would make it dependent on l, which we are going to consider as an instrument competing with X_0 as an instrument [1].

Price fixation: $p = p_0 + \pi_1 l + \pi_2 x$ (4)

where p_0 = autonomous price component

π_1 = wage sensitivity of prices

π_2 = demand sensitivity of prices.

For an appropriate choice of units of goods, money and labour we may have, in the situation of equilibrium, $x = p = l = 1$. It follows that

$1 = p_0 + \pi_1 + \pi_2$ (4')

whereas π_1 may now be called the marginal labour quota, and π_2 is the flexibility of prices (cf. model 02).

Technical: $a = a_0 + \alpha x$ (5)

where a_0 = autonomous demand for labour

$\alpha = \pi_1$ = marginal labour quota.

Assuming that in the situation of equilibrium $a = \frac{1}{2}$, implying that labour income al is one-half of total income; and also assuming that the marginal labour quota equals 0.4 [2], we have:

$\alpha = \pi_1 = 0.4;\ a_0 = 0.1$

A realistic value for π_2 will be 0.1 or 0.2.

Definition: $X = xp$ (6)

[1] Assuming X_0 to vary with p and hence with l would mean that with a wage increase X_0 would also rise; it would overestimate the positive effects of a wage rise on employment.

[2] The coefficient π_1 need not necessarily be equal to the marginal labour quota α; certain (smaller) entrepreneurs calculate a gross profit margin which is proportional to wages per unit of product, a habit which tends to increase π_1. (Cf. also model 16).

MODEL 04. CLOSED, STATIC, MACRO MONEY-FLOW AND PUBLIC FINANCE MODEL

ACTORS *Number:* 2, government and "rest of economy".
Income: taxes [1] and total expenditure.
Income behaviour: inelastic supply of services in both cases.
Spending habits: government, inelastic; rest of economy, see equation (2).

MARKETS *Number:* 1, general commodity.
Character of good: flow.

TECHNICAL RELATIONS Tax revenue assumed to depend on income, for simplicity's sake according to a linear formula.

EQUATIONS

Income formation { *government:* see equation (3)
rest of economy: $Y = X^P + X^G$ (1)

Income spending { *government:* X^G is autonomous
rest of economy: $X^P = X_0^P + \xi (Y - T)$ (2)

Technical: $T = \tau Y + T_0$ (3)

where T_0 represents autonomous additions to tax revenue, and τ represents the marginal rate of taxation. This technical equation makes the simplest possible distinction between an "autonomous" component in taxes T_0 and an "induced" component τY. [2]

MODEL 05. CLOSED, STATIC, MACRO MONEY FLOW, PUBLIC FINANCE AND ASSETS MODEL

ACTORS *Number:* 2, government and "rest of economy".
Income: taxes and total expenditure.
Income behaviour: inelastic supply of services.

[1] Taxes have been treated as direct taxes. It would not have been difficult to bring indirect taxes also into the picture. They are part of government income as well; in order to define income of rest of the economy they have to be deducted from total expenditure.

[2] Various authors speak of taxes T as an instrument of economic policy, a viewpoint criticized by B. Hansen (Finanspolitikens ekonomiska teori, ch. II). In principle we agree with Hansen as our choice of τ and T_0 as instruments shows. It seems permitted, however, in certain cases, and by way of approximation, to handle T, or even the surplus of T over government expenditure, as an instrument.

Spending habits: government, inelastic; rest of economy, see equation (2).

Nature and origin of wealth: wealth of rest of economy consists of accumulated investment goods stock, plus government bonds, plus money; wealth of government does not enter explicitly into model.

Investment habits: rest of economy buys all bonds issued by government for reasons of financial policy, to be discussed in applications.

MARKETS *Number:* 1, general commodity.

Character: flow.

TECHNICAL RELATIONS cash balance relation for rest of economy: see equation (3), tax revenue equation (4).

EQUATIONS

Income formation, rest of economy: $Y = X^P + X^G$ (1)

Income spending, rest of economy: $X^P = \xi_1 Y + X_0{}^P (M,B)$. (2)

where it is assumed that expenditure also depends on the quantity of liquid assets held.

Cash balance for rest of economy: $\Delta M = Y - X^P - T - \Delta B$ (3)

Tax revenue: $T = T_0 + \tau Y$ (4)

MODEL 06. CLOSED, STATIC, MICRO, HORIZONTAL MONEY AND PRODUCT FLOW MODEL WITH TAXES

ACTORS *Number:* $H + 2$, namely H industries, the government and all households together.

Income: *industries*, sales of their products the supply of which is supposed te be inelastic;
government: not considered explicitly,
households: follows from total expenditure; supply of services is supposed to be inelastic.

Spending habits: *industries:* not considered explicitly;
government: expenditure autonomous, to be used as an instrument;
households: see equation (5).

Wealth: not considered explicitly.

MARKETS *Number:* H, for each of the products of the H industries.

Character: flows.

TECHNICAL RELATIONS Supply price of each product assumed to be given. An indirect tax (or subsidy) for each product, to be used as instruments.

EQUATIONS

Income formation: $Y \quad = X$ (1)

Definitions: $X \quad = X^G + X^P$ (2)

$X^P = \sum_1^H{}^h X^h \qquad h = 1 \ldots H$ (3)

$X^h = x^h p^h$ (H eqs) (4)

Spending: $X^h = \xi_1{}^h Y + \xi_2{}^h p^h$ (H eqs) (5)

where $\xi_1{}^h$ are marginal propensities to consume good h and $\xi_2{}^h$ are autonomous demand coefficients for each of the goods. These spending equations are not the most general type possible; in the latter also the prices of the other goods would appear.

Price fixation: $p^h \quad = p_0{}^h + \tau^h$ (H eqs) (6)

where $p_0{}^h$ are the autonomous supply prices of the goods h and τ^h is an indirect tax rate (if negative, a subsidy) for each good.

MODEL 07. CLOSED, STATIC, MICRO, HORIZONTAL MONEY AND PRODUCT FLOW MODEL WITH MONOPOLIES

ACTORS *Number:* $H + 1$, namely H industries and all households together.

Income: *industries,* sales of their products the supply of which is supposed to be manipulated in order to obtain a maximum income, taking account of the influence of prices (monopolistic behaviour) *households:* total expenditure; supply of services is supposed to be inelastic.

Spending habits: *industries* spend their incomes on services; *households* spend their incomes on products.

Wealth: not considered explicitly.

MARKETS *Number:* H, for each of the products of the industries.

Character: flows.

TECHNICAL RELATIONS Unit costs of production for each industry assumed to be given; and units chosen so as to make these units costs = 1.

EQUATIONS

Income formation: *households* $y = \sum_1^H{}^h x^h$ (1)

where x^h represents the values, at constant prices, of the output of each industry and y, therefore, the total value, at constant prices, of all production. This is taken to represent real income.

industries: $Z^h = (p^h - 1)\, x^h \quad h = 1 \ldots H$ (2)

Demand: $x^h = \xi_0{}^h y - \xi_1{}^h (p^h - p) \quad h = 1 \ldots H$ (3)

It is assumed that $\Sigma\xi_0{}^h = 1$, meaning that an increase in real income as defined above will be spent completely.

Supply: expressing that industries manipulate their prices so as to obtain a maximum profit:

$$\frac{dZ^h}{dp^h} = 0 \qquad h = 1 \ldots H \qquad (4)$$

Definition:

$$p = \frac{\Sigma\xi_1{}^h p^h}{\Sigma\xi_1{}^h} \qquad (5)$$

This definition of the general price level, which enters into the demand equations is admittedly intended to simplify the analysis; the weights used coincide with the coefficients in the demand function, which need not always be so. It has the consequences that the addition of all demand equations leads to

$$\Sigma\, x^h = y$$

meaning that one demand equation is dependent on the others.

MODEL 08. CLOSED, STATIC, MICRO MODEL FOR INCOME DISTRIBUTION

ACTORS *Number:* very large, unspecified, equal to total number of employers and employees.

Incomes: employers, product minus real wages
employees, real wage.
Income behaviour determined by supply functions described below.

Spending habits: all entirely spend their income.

Wealth: implicitly considered to influence employers' demand for and employees' supply of labour.

MARKETS *Number:* very large, unspecified, equal to number of "occupations".

Character: flow of services.

TECHNICAL RELATION production function, indicating quantity of product as a function of organization of production, occupation and personal properties.

Variables. Because of the special characteristics of this model, deviating from the usual practical models, it is desirable to describe in some more detail the concepts and variables used. This model concentrates on the labour market in the widest sense, i.e. the market for all occupations, where even being a rentier may be considered an occupation. On the product side it is held as simple as possible, i.e. one general product is supposed to be produced and consumed by all; the quantities, only, are of interest and represent the real income of employers and employees. Each employee is described by a number of personal proporties, being the intensities t_i of certain "abilities" such as force, willpower, intelligence, etc. and including wealth; property number 1 indicates the speed (or the intensity) of his work of which t_1 is the quantitative expression; as another property, not important for his work perhaps, but certainly for his well-being, we consider the (weighted) size of his family t_o. Individuals are not introduced as separate actors, but a description of the total occupied population is given by a multi-dimensional frequency distribution of the number combinations $(t_o, t_1, \ldots, t_I)$ occurring. This frequency distribution will be indicated by $n(t_o, t_1, \ldots, t_I)\, dt_o, dt_1, \ldots, dt_I$.

There will be another frequency distribution of the degrees of the properties, namely the one required by the organizers of production; the properties will be numbered with the same indices; and the degree desired by s_1; there will be some properties in which the organizers are not interested. In our example this will be property 0, family size. Each combination of "desired degrees" $s_1 \ldots s_I$ represents a "job". The frequency distribution of desired properties will be indicated by $m(s_1 \ldots . s_I) ds_1 \ldots . ds_I$. This frequency distribution will depend on, among other things, the productive result to be obtained in each job. The averages and standard deviations of the degrees s_i and t_i will be indicated by $\bar{s}_i$, $\bar{t}_i$, σ_i and τ_i respectively. The task of income formation is to induce each individual to accept the job for which he is most appropriate under the resulting income scale and so to attribute an individual to each job as to let all jobs be filled and all individuals employed. This presupposes equality between the total frequencies M and N of m and n respectively. Indicating the income scale by $l\,(s_1 \ldots s_I)$ or briefly by $l(s)$ we have to find this function, which may depend on more variables than the $s_i (i = 1 \ldots I)$ alone. We assume that it will also depend on the product $p(s, t)$ obtained by a certain individual t in a job s.

The process of choice of the individual is based on certain preferences with regard to income l and job s as well as on the characteristics of the individual t. We assume the utility function ω to be of one and the same type for all individuals; the differences between individuals enter into it only in the form of the parameters t_i [1]. With a given income scale $l(s)$ each individual t chooses a job s and because of the uniqueness of ω a correspondence between s and t is established:

$$s = f(t)$$

This correspondence and hence the income scale $l(s)$ has to be so as to equate, for any set of s and t, the frequency densities of the m- and n-distribution:

$$m(s) = n(t)\,\frac{\partial(t)}{\partial(s)},$$

where $\frac{\partial(t)}{\partial(s)}$ is the Jacobian.

The process and the nature of the resulting income scale can best be clarified by considering a still more special case. To this effect the following further assumptions will be made.

In order to illustrate policy problems it will be assumed that family allowances are paid to the employees, that the income scale is made to depend on the quantity of product the individual puts out, and that taxes are paid out of which training facilities are financed.

1. The *production function* p, indicating a person's product, will be assumed to be:

$$p = \pi_1 t_1 + \sum_{2}^{I}{}_i\, \pi_i^s s_i + \sum_{2}^{I}{}_i\, \pi_i^t t_i + \pi_0\,(\bar{s}_i, \sigma_i) = p_0 + \sum_{2}^{I}{}_i\, \pi_i{}^s s_i \qquad (1)$$

The essence of the assumption is that p is linearly dependent on the s and t.

2. The *utility function* will be supposed to be:

$$\omega = \log \bar{t}_0 - \log t_0 + \log (1 - \gamma) + \log l - \sum_{2}^{I}{}_i\, \lambda_i\,(s_i - t_i)^2 - \varphi\,(t_1) \qquad (2)$$

This implies that utility is assumed to be a rising function of $\frac{(1-\gamma)l}{t_0}$ or the income per (weighted) person after proportional taxes γl lave been paid. $\bar{t}_0$ represents the average family size; the corresponding term is a constant and has been added to facilitate the further analysis. It further implies that utility is negatively dependent on all $s_i - t_i$, i.e. the "tensions"

[1] It would lead us too far here to discuss the mathematical and philosophical implications of this assumption.

between the required and the actual properties—required for the job chosen and actually present with the individual. This dependency is such that both a positive and a negative deviation between required and actual properties evokes an aversion; the simplest way to represent this type of relation is to assume it to be a quadratic function. For small values of the tensions this is a perfectly general relation. The last term in the expression for ω represents an aversion from high speeds of work: φ is assumed to be increasing function of t_1 and even increasing in an accelerated way.

3. It is further assumed that the frequency distributions m and n both show two characteristics. The frequency of any one combination of degrees is the product of the frequencies of each degree separately (the properties are mutually independent):

$$m\,(s_1, s_2, \ldots, s_I) = m_1\,(s_1)\,.\,m_2\,(s_2)\,\ldots\,m_I\,(s_I)\,M \tag{3}$$

$$n\,(t_1, t_2, \ldots, t_I) = n_1\,(t_1)\,.\,n_2\,(t_2)\,\ldots\,n_I\,(t_I)\,N \tag{4}$$

Further it is assumed that all partial frequency distributions are normal:

$$m_i\,(s_i) = \frac{1}{\sigma_i\,\sqrt{2\pi}}\,e^{-\frac{(s_i-\bar{s}_i)^2}{2\sigma_i^2}} \tag{5}$$

$$n_i\,(t_i) = \frac{1}{\tau_i\,\sqrt{2\pi}}\,e^{-\frac{(t_i-\bar{t}_i)^2}{2\tau_i^2}} \tag{6}$$

With these hypotheses the process of income formation may be illustrated in an explicit way. We will not give the full illustration nor will we give the details of the supply of jobs. The supply of jobs will be derived by the organizers of production from a process of profit maximizing; calling the total product of all employees P and their total wages L, the organizers will so choose the frequency distribution of their jobs as to make $P - L$ a maximum with respect to their parameters s_1 and σ_i ($i = 1, 2, \ldots, I$). We will not give the expressions for these parameters, which depend, incidentally, on the form given to the function $\pi(\bar{s}_i, \sigma_i)$ in equation (1).

We will, however, give the full details of the demand side. For each individual t the choice of his job s will derive from the maximization of ω as a function of t_1 (his working speed) and $s_2 \ldots s_I$:

$$\frac{\partial\omega}{\partial t_1} = \frac{\partial\omega}{\partial \log l}\ \frac{\partial \log l}{\partial p}\ \frac{\partial p}{\partial t_1} - \varphi'(t_1) = 0$$

or

$$\frac{\partial \log l}{\partial p}\pi_1 = \varphi'(t_1) \tag{7}$$

and

$$\frac{\partial\omega}{\partial s_i} = \frac{\partial\omega}{\partial \log l}\ \frac{\partial \log l}{\partial s_i} - 2\lambda_i (s_i - t_i) = 0, \quad i = 2 \ldots . I$$

or

$$\frac{\partial \log l}{\partial s_i} = 2\lambda_i (s_i - t_i), \quad i = 2 \ldots . I \tag{8}$$

For any conceivable income scale l these equations will determine t_1 and $s_2 \ldots s_I$. With these values there has now to be equilibrium between supply of and demand for jobs:

$$m (s_2 \ldots . s_I) = n \{t_2 (s) \ldots . t_I (s)\} \frac{\partial (t)}{\partial (s)} \tag{9}$$

It is a well-known method in mathematics to try a certain solution; in the present circumstances it appears to be useful to try an income scale:

$$\log l = \psi (\log t_o - \log {}^t{}_o) + L_o + L_1 p + \sum_2^I {}_i L_i s_i \tag{10}$$

With this scale equations (7) and (8) become:

$$L_1 \pi_1 = \varphi'(t_1) \tag{11}$$

$$L_1 \pi_i^s + L_i = 2 \lambda_i (s_i - t_i), i = 2 \ldots . I \tag{12}$$

where L_1 and L_i are constants for all individuals. Equation (12) enables us to express $t(s)$ occurring in (9) more explicitly:

$$t_i = -\frac{L_1 \pi_i^s + L_i}{2 \lambda_i} + s_i \tag{13}$$

This means that every t_i only depends on the corresponding s_i; implying that

$$\frac{\partial(t)}{\partial(s)} = \frac{\partial t_2}{\partial s_2} \cdots\cdots \frac{\partial t_I}{\partial s_I} = 1 \tag{14}$$

The special form chosen for m and n (cf. (3) and (4)) now enables us to "separate the variables" in (9) and to deduce that

$$m_i (s_i) = n_i \left(s_i - \frac{L_1 \pi_i^s + L_i}{2 \lambda_i} \right) \tag{15}$$

whereas the further specifications (5) and (6) transform this into

$$\frac{(s_i - \bar{s}_i)^2}{2 \sigma_i^2} = \frac{\left(s_i - \dfrac{L_1 \pi_i^s + L_i}{2\lambda_i} - \bar{t}_i \right)^2}{2\tau_i^2} - \log \frac{\sigma_i}{\tau_i} \tag{16}$$

We now make the further—rather stringent—hypothesis that the standard deviations of the s_i- and t_i-distributions are equal to each other: $\sigma_i = \tau_i$. It will then be possible to perform a further considerable simplification: the equality of demand for and supply of jobs now requires

$$s_i - \bar{s}_i = s_i - \frac{L_1 \pi_i^s + L_i}{2\lambda_i} - \bar{t}_i$$

or

$$L_1 \pi_i^s + L_i = 2 \lambda_i (\bar{s}_i - \bar{t}_i),\ i = 2 \ldots . I \tag{17}$$

The mathematical interpretation of this result is that the income scale (10) is, under the various assumptions as to the frequency distributions, the production function and the utility function (equations (1) to (6) inclusive) the correct one provided that its constants L_1 to L_I are not arbitrary but satisfy the relations (17). Since these relations are $I - 1$ in number, one of the L's, say L_1 can be chosen freely; the others cannot, however. The constant L_o in (10) can also be chosen arbitrarily but will be limited by general productivity. Substituting the values from (17) for $L_2 \ldots L_I$ we arrive at the income scale:

$$\log l = \psi (\log t_o - \log \bar{t}_o) + L_o + L_1 p + \sum_2^I {}_i (2\lambda_i (\bar{s}_i - \bar{t}_i) - L_1 \pi_i^s)\, s_i$$

Substituting equation (1) for p, we get:

$$\log l = \psi (\log t_o - \log \bar{t}_o) + L_1 \{\pi_1 t_1 + \sum_2^I \pi_i t_i + \pi (\bar{s}_i, \sigma_i) \} +$$
$$+ 2 \Sigma_i (\bar{s}_i - \bar{t}_i) \lambda_i s_i \tag{18}$$

In this formula t_1 has to be taken from (11).

Because of the complicated nature of this model it may be as well to summarize it by enumerating the types of variables and parameters that together form its logical hierarchy. It is useful to make a distinction between four types of concepts:

1) First, there are the *technical and natural constants* describing the framework in which society works; in this model they are the constants π_1, π_i^s and π_i^t ($i = 2 \ldots I$) of the production function (1) and those hidden in π_0, also part of that function; the coefficients λ_i ($1 = 2 \ldots I$) of the ophelimity function (2) and those hidden in φ, also part of that function. We will also consider as constants the standard deviations τ_i of the frequency distributions of the "degrees" of human properties as present with the population; not, however, all of the $\bar{t}_i$, the average "degrees", since we assume some of them accessible to "education", supposed to be a government activity. We will assume t_0. family size and some of the innate properties to be constant, however.

2) Secondly, there are the political parameters or *instruments, and the coefficients influenced by them*, considered constants by the "organizers" as well as the other citizens, but amenable to change by the government; as such we consider the tax rate γ, determining in its turn, as explained, some of the $\bar{t}_i$ (the "changeable" ones); further the coefficient L_1, to be compared to piece rates and supposed to be supervised by a centralized wage policy. This coefficient, according to equation (11), will determine the intensity t_1 of work, individually or collectively. Finally the level ψ of family allowances may be considered such an instrument.

3) The third category of parameters are those *chosen by the organizers* and considered given by the individual citizen choosing his job; as such we met $\bar{s}_i$ and σ_i, although we made the assumption that by the special form of π_0 (see category 1) each σ_i would happen to be chosen equal to τ_i. Also the coefficients L_i in the income scale may be grouped in this category, but we have found (cf. equations (17)) that these have to be chosen linearly dependent on the deviations $\bar{s}_i - \bar{t}_i$ between the average "degrees required" and "degrees present"; L_0, finally, will have to depend on the average productivity.

4) Finally, the fourth category of entities are those which have to be chosen by the individual citizens in order to maximize their satisfaction, in particular the individual s_i's defining their jobs and consequently their income level. The choice of the level of intensity t_1 is also an individual choice, although for simplicity's sake we have assumed that all individuals would make the same choice.

As a consequence of the process of government policy, organizers' choice and individual's choice a certain production P will be obtained that may be derived from (1):

$$P = M \{\pi_1 \bar{t}_1 + \sum_2^I \pi_i^s \bar{s}_i + \sum_2^I \pi_i^t \bar{t}_i + \pi_0 (\bar{s}_i, \sigma_i)\} \tag{19}$$

and a certain income distribution, that may be derived form (18). Under the assumption of mutual independence of the various required properties s_i and family size the standard deviation of the logarithms of incomes can be calculated:

$$\sigma^2_{\log l} = \psi^2 \tau_0^2 + L_1^2 \pi_1^2 \tau_1^2 + L_1^2 \sum_{i=2}^{I} \pi_i^2 \tau_i^2 + 4 \sum_{i=2}^{I} \lambda_i^2 (\bar{s}_i - \bar{t}_i)^2 \sigma_i^2 -$$

$$- 4 L_1 \sum_{i=2}^{I} \lambda_i \pi_i (\bar{s}_i - \bar{t}_i) \sigma_i \tau_i \tag{20}$$

In this formula τ_1 will be zero if there are no individual differences in φ, and hence in t_1.

MODEL 09. CLOSED, DYNAMIC, MACRO, DEVELOPMENT MODEL

ACTORS *Number:* 1, the national economy as a whole.
Income: net national product; the supply of services is supposed to be inelastic.
Spending habits: a fixed proportion of income is assumed to be spent on consumer goods; the remainder is invested.
Wealth: takes form of equipment and influences current economic process since equipment determines volume of production.

MARKETS *Number:* 2, consumer goods and equipment; prices assumed to be constant and markets not considered explicitly.
Character: flow; but total stock of equipment also plays a role.

TECHNICAL RELATIONS Life time T of equipment assumed to be constant; productivity of equipment φ also constant.

EQUATIONS

Income formation: $y = v - d$ (1)

Income spending: $y = c + s$ (2)

$s = \sigma y$ (3)

Definition of gross product: $v = j + c$ (4)

Accumulation of equipment: $\frac{dw}{dt} = j - r$ (5)

This equation expresses that the stock of equipment available increases to the extent that new equipment is produced (j) after allowance has been made for replacement ($-r$).

Capital formation: $\frac{db}{dt} = s$ (6)

Technical: depreciation $d = \frac{w}{T}$ (7)

replacement $r = j_{-T}$ (8)

Here the symbol j_{-T} stands for j, T time units before; a more exact notation being j_{t-T}.

production $v = \varphi w$ (9)

This equation expresses the technical relation assumed to exist between production volume and stock of equipment (rather than between production volume and capital).

MODEL 10. CLOSED, DYNAMIC, MACRO, CYCLE MODEL (MONEY-FLOW PAPER PROFITS)

ACTORS *Number:* 1, the national economy.

Income: income calculations are based on the assumption that paper profits on stocks are an element of income, together with net income as usually defined by economists.

Supply of the national product is assumed not to be inelastic; supply price is assumed to lag behind money expenditure.

Spending habits: national expenditure depends on income calculation and lags.

Wealth: only element of wealth explicitly considered is commodity stocks, assumed to be equal to half the annual national product.

MARKETS *Number:* 1, the national product.

Character: flow.

TECHNICAL RELATION Stocks are supposed to have been acquired, on the average, one quarter previously.

EQUATIONS — All symbols represent *deviations from equilibrium values*. Time units are quarters.

Income calculation: $Y_t = X_t + \eta\,(p_{t+\frac{1}{2}} - p_{t-\frac{1}{2}})$ (1)

In this equation $p_{t+\frac{1}{2}}$ stands for price level at end of period t and $p_{t-\frac{1}{2}}$ therefore for price level at beginning of period t. The difference indicates the value increase per unit of product since the average moment of acquisition of stocks. Taking the equilibrium value of national product equal to 1, we have an average value of stocks equal to 2 (namely equal to 2 quarters' production). The coefficient η may be different from 2 to the extent that not every producer follows the practice of including "paper profits" in his income calculation.

Demand: $X_{t+1} = \xi_1 Y_t + \xi_2\,(p_{t+\frac{1}{2}} - p_{t-\frac{1}{2}})$ (2)

Here ξ_1 represents the marginal propensity to spend; a term $\xi_2\,(p_{t+\frac{1}{2}} - p_{t-\frac{1}{2}})$ has been added in order to account for the possibility of paper profits being spent in a different manner from normal income.

Supply: $p_t = \pi X_{t-\frac{1}{2}}$ (3)

MODEL 11. OPEN, STATIC, MACRO, MONEY-FLOW MODEL

ACTORS

Number: 2, national economy and rest of the world.

Income: for national economy, net national product, of which supply perfectly elastic, for rest of the world not explicitly considered.

Spending habits: for national economy, see equation 2; for rest of the world: money flow of demand given.

Wealth: does not influence current economic process.

MARKETS

Number: 2, national product and import commodity.

Character: flows.

TECHNICAL RELATIONS — Imports to be fixed proportion of national (gross or net) product.

EQUATIONS

Income formation: $Y = X + E - I$ (1)

Income spending: $X = X_0 + \xi_1 Y$ (2)

where X_0 = autonomous expenditure

ξ_1 = marginal propensity to spend.

The same conclusions as to the elasticity of demand can be drawn as in model 01.

Imports: $I = \iota Y$ (3)

In this simple model it does not make any difference whether we assume I to be a function of Y or of gross national product $X + E$, since either assumption would follow from the other.

Definition of balance of payments deficit:

$D = I - E$ (4)

In the simple model used here, the balance of payments consists only of the items of the balance of trade. In more general cases, other current items will have to be considered, which it will, however, be possible to include in the definitions of I and E; and capital items will have to be added (cf. model 15).

MODEL 12. OPEN, STATIC, MACRO, MONEY-AND-PRODUCT-FLOW MODEL

ACTORS *Number:* 2, national economy and rest of the world.

Income: *national economy*, equal to net national product; supply of product not perfectly elastic;
rest of the world, not explicitly considered; supply of import commodity not perfectly elastic.

Spending habits: expenditure of national economy dependent on income and price level;
expenditures of rest of the world for national (export) product dependent on price level.

Wealth: does not influence current economic process.

MARKETS *Number:* 3, national product sold at home and at foreign market; and import commodity.

Character: flow

TECHNICAL RELATIONS Import volume assumed to be proportional to volume of physical production (may also be considered as a demand equation for market of import commodity).

EQUATIONS All symbols represent deviations from initial situation.

Income formation: $\underline{Y} = X + E - I$ (1)

Demand: spending of national income:

$\bar{X} = X_0 + \xi_1 Y + \xi_2 p^x$ (2)

where X_0 = autonomous expenditure
ξ_1 = marginal propensity to spend
ξ_2 = price coefficient of expenditure, related to elasticity of demand;

for export product: $e = e_0 - \varepsilon_1 (p^e - p^w)$ (3)

where e_0 = autonomous export demand
ε_1 = price coefficient of demand, related to elasticity.

Supply: These will be written in the form of price fixation equations [1]

$p^x = \pi_1^x v + \pi_3^x p^i$? (4)

$p^e = \pi_1^e v + \pi_3^e p^i + \pi_4^e p^w$ (5)

Here the terms with p^i represent the influence of import prices as cost elements and the term with p^w, in the case of p^e, of competing export prices. The coefficients π_1^x and π_1^e are related to the flexibility of prices, i.e. the inverted elasticities of supply.

TECHNICAL

import demand: $i = \iota v$ (6)

volume of production: $v = x + e$ (7)

Definition: value [2]*:* $X = \bar{x} p^x + x$ (8)

$E = \bar{e} p^e + e$ (9)

$I = \bar{\imath} p^i + i$ (10)

balance of payments deficit: $D = I - E$ (11)

For a discussion of the numerical values assumed for the coefficients cf. model 14.

MODEL 13. OPEN, STATIC, MACRO, MONEY, PRODUCT AND FACTOR FLOW MODEL

ACTORS *Number:* 3, wage earners, independents and rest of the world.

Income: for wage earners and independents together equal to net national product; supply of product not perfectly elastic; for rest of the world not

[1] Cf. e.g. J. Tinbergen, Econometrics, Philadelphia 1951, p. 29 and 166.
[2] These equations are linear approximations neglecting products of deviations and assuming that $\bar{p}^x = \bar{p}^e = \bar{p}^i = 1$.

explicitly considered; supply of import commodities not perfectly elastic.

Spending habits: expenditure of wage earners and independents together dependent on income and price level; expenditure of rest of world for national (export) product dependent on price level.

Wealth: does not influence current economic process.

MARKETS *Number:* 4, national product sold at home and at foreign market; import commodity and labour.

Character: flow

TECHNICAL RELATIONS Import volume proportional to volume of physical production.

EQUATIONS All symbols represent deviations from initial situation. For simplicity's sake it has been assumed that $p^i = p^w = 0$.

Income formation: $Y = X + E - I$ (1)

Demand: spending of national income:

$X = X_0 + \xi_1 Y + \xi_2 p^x$ (2)

(for explanation cf. model 12). For simplicity's sake it has been assumed that there exists no difference in marginal propensity to spend between wage earners and independents. [1]

for export product: $e = e_0 - \varepsilon_1 p^e$ (3)

Supply: $p^x = \pi_1{}^x v + \pi_2{}^x l$ (4)

$p^e = \pi_1{}^e v + \pi_2{}^e l$ (5)

Here $\pi_2{}^x$ and $\pi_2{}^e$ indicate the influence on home and export prices respectively, exerted by a unit increase in wage rates. If prices are exactly equal to marginal costs, and if initial prices as well as wage rates are chosen equal to 1, $\pi_2{}^x$ and $\pi_2{}^e$ represent marginal labour quota. If prices are fixed in a different way—possible because of imperfect competition—, e.g. by applying profit margins, themselves proportional to certain primary cost elements, $\pi_2{}^x$ and $\pi_2{}^e$ may surpass marginal labour quota. On the other hand, export prices may, in the short run, be dependent on the price level of competing countries, which

[1] For a model where such a difference has been assumed to exist, cf. Central Economic Plan 1955, Netherlands Central Planning Bureau.

may diminish the influence of wage rates on these prices.

Technical: import demand: $i \quad = \iota v$ (6)

volume of production: $v \quad = x + e$ (7)

Definition: value equations: $X = \bar{x}p^x + x$ (8)

$E = \bar{e}p^e + e$ (9)

$I \quad = i$ (10)

balance of payments: $D = I - E$ (11)

For a discussion of the numerical values given to the coefficients cf. model 14.

MODEL 14. OPEN, STATIC, MACRO MONEY, PRODUCT AND FACTOR FLOW MODEL WITH EXCHANGE RATES

The only difference from the preceding model is the introduction, as a separate instrument of policy, of the exchange rate of the national currency (in terms of gold, say); no general description will be given. It may be said that 5 markets are now considered, the fifth being the market for national currency; but it should be added that this latter market, as well as the labour market, are highly schematized.

Because of the similarity with the previous two models, the equations may be given without further explanation, except for the new terms.

$$Y = X + E - I \tag{1}$$
$$X = X_0 + \xi_1 Y + \xi_2 p^x \tag{2}$$
$$e = e_0 - \varepsilon_1 (p^e - p^w + k) \tag{3}$$
$$p^x = \pi_1{}^x v + \pi_2{}^x l + \pi_3{}^x (p^i - k) \tag{4}$$
$$p^e = \pi_1{}^e v + \pi_2{}^e l + \pi_3{}^e (p^i - k) + \pi_4{}^e (p^w - k) \tag{5}$$
$$i = \iota v \tag{6}$$
$$v = x + e \tag{7}$$
$$X = \bar{x}p^x + x \tag{8}$$
$$E = \bar{e}p^e + e \tag{9}$$
$$I = \bar{i}(p^i - k) + i \tag{10}$$
$$D = I - E \tag{11}$$

This model is to be considered as the general form from which 12 and 13 derive by special assumptions. The foreign price levels p^i and p^w are now being assumed to be quoted in terms of gold; the quotations in national currency will therefore be $p^i - k$ and $p^w - k$, respectively. Model 12 will be obtained by taking l and k equal to 0; indicating that these two instruments are not supposed to be used. Model 13 will be obtained by assuming both the exchange rate and the foreign prices p^i and p^w equal to 0, i.e. non-variable.

The *numerical values of the coefficients and the other constants* will now be discussed for all three models. *Initial values* of all prices have already been chosen equal to 1. [1] As a consequence, for the initial period, volume figures for each flow of goods or factors will be equal to the corresponding value figure, e.g. $\bar{x} = \overline{X}$, $\bar{i} = \bar{I}$ etc. We will assume that there is balance of payments equilibrium in the initial situation, i.e. $\bar{I} = \overline{E}$ and hence $\bar{i} = \bar{e}$. Finally we assume that national income was equal to 1, or $\overline{Y} = 1$. It follows that $\bar{x} = 1$. From the figures of the national accounts we then deduce the values that have to be given to $\bar{I}$ and $\overline{E}$, since these are expressed in the same units as Y and cannot therefore anymore be chosen freely. In the "central case" C we assume that $\bar{I} = \overline{E} = \bar{i} = \bar{e} = 0{,}5$, representing a country with intensive international trade, comparable to the Netherlands. In a few examples we will also consider cases where trade is less intensive (A, where it is completely absent and B, where $\bar{i} = \bar{e} = 0{,}25$) or more intensive (D, where $\bar{i} = \bar{e} = 1{,}0$).

Variations in e_0 are indicative of changes in foreign demand for the economy's products. We will not give specific numerical values to e_0, leaving this to the reader; but we will, in a number of examples where variation in foreign demand is not assumed to occur, take $e_0 = 0$.

Some of the *coefficients* will depend on the intensity of the country's international trade and others not. Apart from these variations we will consider three other types of variation. The most important aspect is the length of the period considered: a distinction will be made between *short-term* (s) and *long-term* (l) values. In a complete dynamic system the short-term values of the coefficient will indicate a variable's reaction to a simultaneous change in another variable whereas the long-term values indicate the total influence exerted by simultaneous and lagged changes in the latter variable. Another aspect to be considered is the *cyclical position:* some coefficients will be higher in boom conditions (b) than in normal (n). This reflects the curvilinearity of certain relations.

Finally we will make some incidental changes in the coefficients of the spending equation, mainly because these coefficients have a considerable influence on some results and are not too well known.

[1] This is only possible if for any one commodity only one price is introduced, since it can then be obtained by a proper choice of the unit of that commodity. Of course, one may, even if a price is not 1, introduce an index number, but this complicates the relations.

Variation in the *intensity of foreign trade* is only assumed in problem 121, using model 12.

The coefficients in this model varying with the intensity of a country's foreign trade are ι, π_3^x and π_3^e, which are assumed to be proportional to $\frac{\bar{\imath}}{1+\bar{\imath}}$, the ratio of imports to G.N.P.; π_1^x and π_1^e which are assumed to be proportional to $\frac{1}{1+\bar{\imath}}$, the ratio of net to gross N.P.; and ε_1, taken proportional to $\bar{e}$. If variations in the intensity of foreign trade were considered in models 13 and 14, the other coefficients appearing in the price equations would also have to be varied. This may be left to the reader.

The marginal propensity to spend ξ_1 has been assumed to be 0.8; a fixed physical expenditure ξ_2 of 0.1 and, in the initial situation, a fixed nominal expenditure X_0 of 0.1 have been assumed in addition.

The following table summarizes the values of the coefficients for the various intensities of foreign trade distinguished in the s–n examples of model 12:

Table 1 (App.) ↔ Values of coefficients in model 12
(short-term reactions with normal cyclical position)

Case	$\bar{\imath} = \bar{e}$	$\frac{\bar{\imath}}{1+\bar{\imath}}$	$\iota = \pi_3^x$	π_3^e	$\frac{1}{1+\bar{\imath}}$	π_1^x	π_1^e	ε_1	π_4^e	ξ_1	$\cancel{\varepsilon_1}$ ξ_2
Asn	0	0	0	0	1	0.15	.	.	.	0.8	0.1
Bsn	0.25	0.20	0.20	0.10	0.80	0.12	0.06	0.5	0.5	0.8	0.1
Csn	0.5	0.33	0.33	0.17	0.67	0.10	0.05	1.0	0.5	0.8	0.1
Dsn	1.0	0.50	0.50	0.25	0.50	0.08	0.04	2.0	0.5	0.8	0.1

In the problems 131, 141, 142 and 143, using models 13 and 14 respectively (cf. appendix II), a distinction between short-term and long-term reactions has been made. Short-term reactions may be said to be reactions during the same year, long-term reactions are those occurring after a few years.

The distinction applies first of all to the coefficient ε_1, related to the elasticity of demand for export products. For short-run calculations we will, in agreement with Dutch experiences, take it equal to 1, corresponding with an elasticity of 2; for long-term calculations a value of 2 will be assumed. Similarly we assume that the long-term value of π_2^x and π_2^e, the reaction of a unit change in wage rates on prices, will be higher than the short-term value: the reason being that in the long run independents tend to increase their income along with the general increase in income. The maximum value for π_2^x will be 0.67, being the total national income component in the price $\bar{p}^x$ (= 1). The situation as to the price formation of export products is more complicated. Short-run movements of export prices appear to react to changes in costs only half as intensively as do home price movements; in addition to this reaction there appears to be a reaction to competing prices, i.e. export prices of other, competing, countries. Accordingly the coefficients have the following short-term values:

$\pi_1{}^e = 0.05$ $\pi_2{}^e = 0.17$ $\pi_3{}^e = 0.17$ (being half the corresponding values for home market prices) and $\pi_4{}^e = 0.5$.

In the longer run it would seem probable that there be a more pronounced influence of costs, since prices cannot diverge too much from costs without causing either losses or excessive profits leading to adaptations of various types. At the same time it may, however, be maintained, that there will be a complete adaptation to competing world-market prices; this seeming contradiction being solved by a change in composition of exports tending to make both tendencies possible at the same time: those products will be preferred for which costs do not exceed world market prices and products for which costs are lowered in comparison to world market prices will be produced in increased quantities, so that costs rise again to that level. Since the essential point in our problems is to know how intensively the economy reacts upon her own instruments, we will choose the long-term values of the above coefficients equal to:

$$\pi_1{}^e = 0.1,\ \pi_2{}^e = 0.67,\ \pi_3{}^e = 0.33 \text{ and } \pi_4{}^e = 0.$$

The following table summarizes the values of the coefficients used in models 13 and 14:

Table 2 (App.) Coefficients used in models 13 and 14
(Central case C, with normal cyclical position)

Case	ξ_1	ξ_2	ε_1	$\pi_1{}^x$	$\pi_2{}^x$	$\pi_3{}^x$	$\pi_1{}^e$	$\pi_2{}^e$	$\pi_3{}^e$	$\pi_4{}^e$	ι
Csn	0.8	0.1	1	0.1	0.33	0.33	0.05	0.17	0.17	0.50	0.33
Cln	0.9	0.1	2	0.1	0.67	0.33	0.1	0.67	0.33	0	0.33

Variations in *cyclical positions* are assumed mostly to affect the value of coefficients $\pi_1{}^x$ and $\pi_1{}^e$; in boom conditions these will become high, expressing the tendency to steep price rises if full employment of capacity is approached. They have been assumed equal to five times their value under normal cyclical conditions, i.e. $\pi_1{}^x = 0.5$ and $\pi_1{}^e = 0.25$. Accordingly the following set of coefficients has been used in examples referring to short-term boom conditions (sb):

Table 3 (App.) Coefficients used in models 12, 13 and 14 in short-term boom conditions:

Case	ξ_1	ξ_2	ε_1	$\pi_1{}^x$	$\pi_2{}^x$	$\pi_3{}^x$	$\pi_1{}^e$	$\pi_2{}^e$	$\pi_3{}^e$	$\pi_4{}^e$	ι
Csb	0.8	0.1	1	0.5	0.33	0.33	0.25	0.17	0.17	0.50	0.33

The *incidental variations* in ξ_1 and ξ_2 used in model 12 are found in the table below:

Table 4 (App.) Coefficients used in 2 incidental variations on model 12 (central case C, normal cyclical position, short-term reactions)

Case	ξ_1	ξ_2	ε_1	$\pi_1{}^x$	$\pi_3{}^x$	$\pi_1{}^e$	$\pi_3{}^e$	ι
C'sn	0.9	0.05	1	0.10	0.33	0.05	0.17	0.33
C"sn	0.7	0.15	1	0.10	0.33	0.05	0.17	0.33

It might have been assumed that the high value of ξ_1 in C' would also occur in boom conditions, and the lower value in C" in a period of depression. It may be left to the reader to calculate examples with these values.

MODEL 15. OPEN, STATIC, MACRO, MONEY-FLOW AND ASSET MODEL

ACTORS *Number:* 3, national economy excl. banks (or: "public"), banks and "rest of the world".

Income: *public*, net national product; supply of products inelastic; *banks*, no income; *rest of the world*, not explicitly considered; inelastic supply of imports.

Spending habits: *public*, as in previous models, but limited by liquid assets available; *banks*, no current expenditure; *rest of the world*, demand for exports considered given.

Nature and origin of wealth: for al three actors consisting of bonds and liquid assets. Increase in wealth results from income surplus.

Investment habits: investment into bonds and liquid assets depends on national income and interest rates.

MARKETS *Number:* 5, one for national product (flow), and four markets for assets (stock character), namely bonds and liquidities in the national economy and the "rest of the world".

EQUATIONS

Income formation: $Y = X + E - I$ (1)

Income spending: $X = X_0 + \xi_1 Y - \xi_2 M^1 - \xi_3 m^1{}_M - \xi_4 m^1{}_B$ (2)

Destination of income surplus (or financing of income deficit):

$$\textit{national economy}\quad Y - X = \Delta B^{31} + \Delta B^{21} + \Delta M^{31} + \Delta M^{21} \quad (3)$$

$$\textit{rest of the world}\quad I - E = -\Delta B^{31} - \Delta B^{32} - \Delta M^{31} - \Delta M^{32} \quad (4)$$

$$\textit{Banks balance sheet variation:}\quad 0 = -\Delta B^{21} + \Delta B^{32} - \Delta M^{21} + \Delta M^{32} \quad (4')$$

Since equation (4′) is a consequence of (3) and (4) it is no new equation.
Equations (3)—(4)′ may be presented in the form of a "monetary survey".

Items		Sectors 1. Public	2. Banks	3. Rest of the world
Current receipts		Y	—	I
Current expenditure		X	—	E
Income surplus		$Y - X$	—	$I - E$
Increase in bond holding				
from sector	1	—	$-\Delta B^{21}$	$-\Delta B^{31}$
from sector	2	ΔB^{21}	—	$-\Delta B^{32}$
from sector	3	ΔB^{31}	ΔB^{32}	—
Increase in liquidity holding				
from sector	1	—	$-\Delta M^{21}$	$-\Delta M^{31}$
from sector	2	ΔM^{21}	—	$-\Delta M^{32}$
from sector	3	ΔM^{31}	ΔM^{32}	—

Equations (3), (4) and (4′) express the equality of income surplus and increase in holdings of bonds and liquid assets for each sector respectively. The sectors of origin and destination are indicated by the upper indices with the ΔB and ΔM figures. A minus sign indicates that the direction of movement is opposite to the order of the indices. Thus, instead of ΔB^{13}, the amount of bonds transferred from sector 1 to sector 3, the symbol $-\Delta B^{31}$ has been chosen in order to remember the reader of the necessary equality: $\Delta B^{13} = -\Delta B^{31}$. The sector of origin need not coincide with the sector which originally issued the bond or liquidity. Sectors of issue are not specified in this model at all. Bonds of all types are lumped together. The figure ΔB^{ik} may be positive as well as negative; a positive figure may represent an increase in bond holdings as well as a decrease in long-term debt.

$$\textit{Import demand:}\quad I = \iota Y \quad (5)$$

$$\textit{Investment decision:}\quad B^1 = f^1(M^1, Y, m^1{}_B, m^1{}_M) \quad (6)$$

$$B^2 = f^2(M^2, m^1{}_B, m^1{}_M) \quad (7)$$

$$B^3 = f^3(M^3, m^3{}_B, m^3{}_M) \quad (8)$$

$$\textit{Supply of bonds:}\quad B^1 + B^2 + B^3 = g^1\,(m^1{}_B,\, m^3{}_B) \tag{9}$$
$$\textit{Definition:}\quad D = I - E + \Delta B^{31} + \Delta B^{32} \tag{10}$$
$$\Delta B^1 = \Delta B^{21} + \Delta B^{31} \tag{11}$$
$$\Delta B^2 = -\Delta B^{21} + \Delta B^{32} \tag{12}$$
$$\Delta B^3 = -\Delta B^{31} - \Delta B^{32} \tag{13}$$
$$\Delta M^1 = \Delta M^{21} + \Delta M^{31} \tag{14}$$
$$\Delta M^2 = -\Delta M^{21} + \Delta M^{32} \tag{15}$$
$$\Delta M^3 = -\Delta M^{31} - \Delta M^{32} \tag{16}$$

Equations (11)—(16) define the total increase in holdings of bonds and liquid assets in each sector; together with the holdings already in the hands of each sector they form the end-of-period holdings B^1, B^2 etc. These holdings will, according to equations (6) to (8) inclusive show a certain optimum composition, depending on the relevant interest rates. It has been assumed that in the home economy the interest rates are different from those in the "rest of the world". Holdings of bonds are also dependent on the level of national income, which for the home economy is a variable; for the "rest of the world" it has not been explicitly introduced. Equation (9) expresses a supply relation, for all issuing agencies of bonds taken together of bonds in the national economy and in the "rest of the world".

Equation (10) gives one possible definition of the concept of balance of payments deficit; it rests on the assumption that the items ΔM^{31} and ΔM^{32} are balancing items. Other definitions might have been preferred; the precise choice does not influence the logic of our model.

MODEL 16. OPEN, STATIC, MICRO, MONEY, PRODUCT AND FACTOR FLOW AND PUBLIC FINANCE MODEL

ACTORS

Number: 4, wage earners, independents, government and "rest of the world".

Income: *wage earners:* total wages; supply of labour inelastic;
independents: net national income minus wages; supply of products not inelastic;
government: direct and indirect tax receipts minus expenditure;
rest of the world: not specified.

Spending habits: *wage earners* spend all income on consumer goods; *independents* spend all income after direct tax, in a fixed proportion between consumer and investment goods;
government spending is independent of tax

receipts (but if exact balance of payments equilibrium is a target of policy it follows that there will be equality between income and expenditure);

rest of the world: demand for export products not inelastic.

MARKETS *Number:* 5, consumer goods and investment goods sold at home; export goods sold abroad, import commodity and labour.

TECHNICAL RELATIONS Imports and labour assumed to be linearly dependent on volume of production.

EQUATIONS All variables are deviations from initial situation.

Income formation: *total income:*

$$Y = C^F + J + E - I \qquad (1)$$

Because of the distinction made between consumer and investment goods the term X in previous models has now been split into $C^F + J$. Because of the existence of indirect taxes (only levied from home-sold consumer goods) a distinction between the value of home-sold consumer goods at market prices C and at factor cost C^F has to be made. The relation between these two variables is given by eq. (10) and (11).

wages: $L = \bar{L}\,(a + l)$ (2)

The form of this equation differs from equations like (8)–(10) in model 12 (where they first appeared), because a is not measured in the same way as e.g. x in equation (8) in model 12; $\bar{\bar{a}}$ is an index number of employment with its base value $\bar{a} = 1$.

independents' income: $Z = Y - L$ (3)

Demand; consumer goods at home market:

$$C = C_0 + \gamma\,(1 - \bar{\theta})\,Z - \gamma\bar{Z}\theta + L \qquad (4)$$

In this equation the first term represents government expenditure and the last term consumer expenditure by workers. The two remaining terms derive from the expression for consumption expenditure by independents C_z of which the absolute amount is given by

$$\bar{\bar{C}}_z = \gamma\,(1 - \bar{\bar{\theta}})\,\bar{\bar{Z}}$$

i.e. the proportion γ of income after direct tax. The assumptions involved are relatively simple, e.g. a proportionate tax on $\bar{\bar{Z}}$ and a fixed average propensity to consume. It would not be difficult to generalize these assumptions; this may be left to the reader.

investment goods at home market:

$$J = (1 - \gamma)(1 - \bar{\theta}) Z - (1 - \gamma) \bar{Z}\theta \qquad (5)$$

This equation expresses that the remainder of income after tax $\bar{\bar{Z}}(1 - \bar{\bar{\theta}})$, after deducting consumption from income, is invested.

export goods: $e = -\varepsilon_1 p$ (6)

Here the autonomous component in export demand e_0 has been assumed to be unchanged ($e_0 = 0$); further it is assumed that prices of all products (consumer goods, investment goods and export goods) are the same ("national price level" p) and that no indirect taxes are paid on export goods.

Supply; price fixation equation:

$$p = \pi_1 v + \pi_2 l \qquad (7)$$

Since foreign prices and the exchange rate are assumed not to vary, this equation takes its simplest form. It would not be difficult to introduce equations with different coefficients for consumer goods, investment goods and export goods.

Technical: $i = \iota v$ (8)

$$a = \alpha v \qquad (9)$$

These equations express proportionality between changes in production volume on the one hand, and imports as well as volume of labour employed on the other hand.

Definition:

$$C^F = \bar{c}p + c \qquad (10)$$

$$C = \bar{c}(p + \tau) + (1 + \bar{\tau})c \qquad (11)$$

$$J = \bar{j}p + j \qquad (12)$$

$$I = i \qquad (13)$$

$$E = \bar{e}p + e \qquad (14)$$

$$D = I - E \qquad (15)$$

$$v = c + j + e \qquad (16)$$

Social equilibrium coefficient:

$$L = \lambda Z \tag{17}$$

This equation expresses the fact that a certain preconceived ratio between increases in labour income and "non-labour" income may be aimed at.

Alternatively this equation will be replaced by another defining the home price level for consumer goods:

$$p' = p + \tau \tag{17'}$$

The numerical values of the coefficients and constants will be chosen in a way similar to the one followed with model 14, but slightly different only where the simpler structure of 16 makes this necessary. This refers primarily to the neglect of the difference between home prices and export prices, as to their response to competing prices abroad. Apart from this feature, short-term reaction coefficients have been chosen for a country with imports normally equal to one-half of net national income. Model 16 is more complicated than model 14 as regards the fiscal structure; accordingly some new assumptions as to this sector have been necessary. All price indices are equal to 1 in the base period.

We have chosen $\bar{L} = \bar{Z} = \bar{e} = 0.5$, meaning that in the base period net national income at factor cost $\bar{Y} = 1$ and is distributed in equal parts between workers and independents; in addition, exports (and imports $\bar{i}$) $= 0.5$. The factor value of consumption $\bar{C}^F = \bar{c} = 0.9$, that of investment $\bar{j} = 0.1$. The tax rates are $\bar{\tau} = 0.1$, meaning an indirect tax of some 11 Pct on home consumption and $\bar{\theta} = 0.3$, meaning a tax of 30 Pct on non-labour income. Taxes on labour income are taken as zero. The marginal propensity to consume γ for independents, calculated on the basis of income *after* tax, $= 0.7$, and the marginal propensity to invest $= 0.3$, almost equal to the average propensity in the base period

$$\frac{0.1}{0.7 \times 0.5}$$

The elasticity of demand for export products has been again taken $= 2$, making $\varepsilon_1 = 1$; the flexibility of the home price level is taken as 0.15, equivalent with $\pi_1 = 0.1$; the wage coefficient in prices $\pi_2 = 0.33$ exactly corresponds to the average labour quota in the base period

$$\frac{\bar{L}}{\bar{v}} = \frac{0.5}{1.5}$$

The marginal import quota ι has also been taken equal to the average quota

$$0.33\left(=\frac{0.5}{1.5}\right)$$

Finally the marginal labour quota in equation (9) has been taken less than the average labour quota, actually about 0.55 times that quota, in accordance with numerous investigations [1]; $\alpha = 0.40$ seems to be a fair estimate. That the wage coefficient in prices is nevertheless equal to the average labour quota probably has to be explained by the custom of calculating some of the entrepreneurial income on the basis of the prevailing wage level.

MODEL 17. OPEN, STATIC, MICRO, MONEY FLOW AND BANKING MODEL

ACTORS *Number:* 4, central bank, private banks, rest of the economy and "rest of the world".

Income: for banks not considered explicitly; for rest of the economy net national product; for rest of the world not considered explicitly.

Spending habits: for rest of the economy as in previous models (cf. model 11).

Wealth, categories considered:

Central Bank, assets: gold (Au), rediscounts (B^R);
Central Bank, liabilities: money in circulation (M), Bankers' "reserves" (R).
Private Banks, assets: bills and advances (B^B), reserves with Central Bank (R);
Private Banks, liabilities: deposits (M'), rediscounts (B^R).

Wealth, origin: gold stock increases or decreases as a consequence of balance of payments position. Rediscounts: decision of private banks (eq. (3)). Money in circulation and deposits: decision of rest of the economy, (eqs. (6) and (7)). Bankers' reserves: balancing item with Central Bank. Bills and advances: balancing item with private banks.

MARKETS *Number:* 3, central bank credit, private credit and national product.

[1] P. J. Verdoorn, Praeadvies 1952 voor de Vereniging voor de Staathuishoudkunde, where a slightly lower figure has been chosen. Our figure has to be considered as a round figure.

TECHNICAL RELATIONS reserve requirements, taking the form of boundary conditions:
for Central Bank: $Au \geq aM$
for private banks: $R \geq \varrho M'$

EQUATIONS

Balance sheet,

Central Bank: $M + R = Au + B^R$ (1)

private banks: $M' + B^R = B^B + R$ (2)

These balance sheets are of course simplified. They might easily be made more complicated; an attempt is here being made to present some essential features in the simplest form conceivable.

Rediscounting decision of private banks:

$B^R = \beta_1 (m' - m) + \beta_0$ (3)

Here β indicates the intensity with which the private banks react on an interest difference between the private discount rate m' they earn on B^B, and the official discount rate m they have to pay if they, by rediscounting with the Central Bank, are able to maintain a certain level of lending. Negative values of B^R may be interpreted as additional reserves.

Supply of credit, given the form of price fixing equation:

Central Bank: $m = \mu\ (aM - Au) + \mu_0$ (4)

private banks: $m' = \mu'\ (\varrho M' - R) + \mu'_0$ (5)

According to these equations the official discount rate is raised or lowered in parallel with the difference between required gold stock and actual gold stock. There may be an autonomous element in the rate represented by μ_0. Similarly the private discount rate, taken here to represent the whole structure of private rates, moves parallel with the difference between required reserves and actual reserves and shows an autonomous element μ'_0.

Demand for (Central-Bank) money:

$M = \mu_1 Y - \mu_2 m + \mu_3$ (6)

Demand for deposits: $M' = \mu'_1 Y - \mu'_2 m' + \mu'_3$ (7)

Both types of money are assumed to be demanded in quantities which depend on national income as well as on the corresponding rates of interest.

National income formation: $Y = \eta_1 X_0 - \eta_2 m' + \eta_3$ (8)

Here X_0 represents autonomous national expenditure, as far as considered a datum, and η_1 the Keynesian multiplier; it is, moreover, assumed that autonomous expenditure is negatively influenced by the private discount rate.

Gold accumulation: $Au = Au_{-1} + E - \iota Y + \alpha' (m - m_{-1})$ (9)

The equation expresses that the increase of gold stock over its previous-year value Au_{-1} equals the balance of exports E over imports, assumed to be proportional to national income, plus an influx of foreign short-term capital, assumed to be proportional to the rise in discount rate since the previous time unit. This assumption is equivalent to supposing that holdings of national debt titles by the citizens and institutions of the "rest of the world" are dependent on the rate of interest they earn on such holdings.

About the numerical values of the coefficients much less can be said than in the preceding models. The relations in the financial sphere have not been so well investigated as those in the general models; partly because some attempts seem to have shown that their influence on the changes in the main variables is only limited. [1]

MODEL 18. OPEN, STATIC, MICRO, HORIZONTAL, MONEY, PRODUCT AND FACTOR FLOW MODEL

ACTORS *Number:* 4, two industries, all households of the economy, "rest of the world".

Income: *industries*, no income; supply of products not inelastic;

households, net national product; supply of services elastic;

rest of the world, not explicitly considered;

[1] Cf. J. Tinbergen, Business Cycles in the U.S. 1919–1932, League of Nations, Geneva 1939, and Business Cycles in the United Kingdom 1870–1914, Royal Netherlands Academy of Sciences, Amsterdam 1951.

Spending habits: households, total amount spent on each product dependent only on income (i.e. price elasticities $= 1$);
rest of the world, demand for export products depends on price of product.

Wealth: not explicitly considered.

MARKETS *Number:* 2, each product being sold at home and abroad at the same price.

Character: flows.

TECHNICAL RELATIONS It is assumed that the quantities of imports and of labour needed per unit of product in each industry have been given, and that a given change in these constants occurs.

EQUATIONS Symbols indicate deviations from initial situation

Income formation:	$Y = X^1 + X^2 + E^1 + E^2 - I$	(1)
Demand:	$X^1 = \xi^1 Y$	(2)
	$X^2 = \xi^2 Y$	(3)
	$e^1 = -\varepsilon^1 p^1$	(4)
	$e^2 = -\varepsilon^2 p^2$	(5)
Supply:	$p^1 = \alpha^1$	(6)
	$p^2 = \alpha^2$	(7)
Technical:	$i^1 = \iota^1 v^1$	(8)
	$i^2 = \iota^2 v^2$	(9)
	$a^1 = \bar{\alpha}^1 v^1 + \bar{v}^1 \alpha^1$	(10)
	$a^2 = \bar{\alpha}^2 v^2 + \bar{v}^2 \alpha^2$	(11)
Definition:	$x^1 = X^1 - \bar{x}^1 p^1$	(12)
	$x^2 = X^2 - \bar{x}^2 p^2$	(13)
	$E^1 = e^1 + \bar{e}^1 p^1$	(14)
	$E^2 = e^2 + \bar{e}^2 p^2$	(15)
	$v^1 = x^1 + e^1$	(16)
	$v^2 = x^2 + e^2$	(17)
	$a = a^1 + a^2$	(18)
	$D = -E^1 - E^2 + I$	(19)
	$I = i^1 + i^2$	(20)

Since in this model the coefficients α^1 and α^2 are assumed to change, we have applied the same notation as with variables, namely, $\bar{\alpha}^1$ and $\bar{\alpha}^2$ to indicate initial values and α^1 and α^2 to indicate changes.

MODEL 19. OPEN, STATIC, MICRO, HORIZONTAL, MONEY AND PRODUCT FLOW MODEL

ACTORS

Number: $H + 3$, namely H industries, all households of the economy, the government and the "rest of the world".

Income: *industries:* sales of their products the supply of which is assumed to be inelastic;
government: not considered explicitly;
households: derives from total expenditure; the supply of services is supposed to be inelastic;
rest of the world: not considered explicitly.

Spending habits: *industries:* not considered explicitly;
households: demand for the various products depends on income and on all prices;
government: expenditure autonomous;
rest of the world: demand for exports depends on price level of product.

Wealth: not considered explicitly.

MARKETS

Number: $2H + 1$, for each of the products at home and abroad; and labour.

Character: flows.

TECHNICAL RELATIONS Imports and labour needed per unit of product constants for each product.

EQUATIONS Symbols indicate deviations from initial situation

Income formation:	$Y = \Sigma_k (X^{Fk} + E^k) - I + X_0$		(1)
Income spending:	$X^h = \xi^h Y + \Sigma_k \xi^{hk} p'^k$	$h = 1 \ldots H$	(2)
Demand for exports:	$e^h = -\varepsilon^h p^h$	$h = 1 \ldots H$	(3)
Supply:	$p'^h = \pi^h l + \tau^h$	$h = 1 \ldots H$	(4)
	$p^h = \pi^h l$	$h = 1 \ldots H$	(5)
Technical:	$i^h = \iota^h v^h$	$h = 1 \ldots H$	(6)
	$a^h = \alpha^h v^h$	$h = 1 \ldots H$	(7)
Definition:	$x^h = X^{Fh} - \bar{x}^h p^h$	$h = 1 \ldots H$	(8)
	$E^h = e^h + \bar{e}^h p^h$	$h = 1 \ldots H$	(9)
	$I = \Sigma i^h$		(10)
	$D = I - \Sigma_k E^k$		(11)
	$v^h = x^h + e^h$	$h = 1 \ldots H$	(12)
	$X^h = X^{Fh} (1 + \bar{\tau}^h) + \bar{X}^{Fh} \tau^h$	$h = 1 \ldots H$	(13)

MODEL 20. OPEN, DYNAMIC, MACRO, INVESTMENT PROJECT APPRAISAL MODEL

ACTORS *Number:* 3, program sector (consisting of a "program" of projects to be selected), rest of the economy and rest of the world.

Income: for private and public sector: net product of those sectors; for rest of the world not explicitly considered; supply of products inelastic; for private sector determined by capital available; for public sector dependent on choice of projects to be selected.

Spending habits: a fraction ξ of all income, plus capital import spent; foreign demand for national product dependent on price level.

Wealth: does not influence demand for products.

MARKETS *Number:* 3, national product and the scarce factors, capital and foreign exchange.

Character: flow markets.

TECHNICAL RELATIONS For private sector, factor demand assumed to be proportional to volume of production; for public sector assumed given and different for each project.

EQUATIONS All variables in equation (1) to (15) should carry a lower index t and are supposed to apply to each time unit; the first period considered is $t = 1$.

Income formation:

$$Y^o = p\, v^o - p^i\, i^o - \delta\, p\, b^o \tag{1}$$

$$Y^h = p\, v^h - p^i\, i^h - \delta\, p\, b^h - K^{ih}\, m^i \tag{2}$$

The projects of which the program sector is composed are indicated by the index h; a certain number of h's has to be selected by the policy-maker. The rest of the economy is indicated by the index 0. Equation (1) is constructed according to the usual definition of a sector's contribution to national income (where no inter-sector supplies have been assumed). The last term on the right-hand side represents depreciation allowances. Equation (2) in addition contains a deduction of interest on foreign debts K^{ih} incurred for the execution of project h.

Income distribution: $Z^o = \xi^o Y^o$ (3)

$Z^h = \xi^h Y^h$ (4)

The proportion of income paid out to "independents", chosen here to represent those who save part of their income, is assumed to vary from project to project.

Demand:

$$x = \xi \frac{Y^o + \Sigma Y^h + \Sigma \dot{K}^{ih}}{p} \qquad (5)$$

This demand covers demand for all types of goods. It is assumed that a fraction ξ of income plus capital imports is spent.

$e = e^* - \varepsilon p$ (6)

Here e^* represents the autonomous component in export demand.

$S = \sigma^1 (Z^o + \Sigma Z^h - T^d) + \sigma^2 T$ (7)

Private savings are derived from private independents' income after direct taxes with a marginal propensity to save σ^1 and public saving from total taxes T with a propensity σ^2.

$i = \iota^o v^o$ (8)

Supply: $\beta v^o = b^o$ (9)

This is an approximation to the production function where β is the "capital coefficient" for gross product.

$$\dot{K}^{ih} = p^i i^h - \frac{K^{ih}}{\tau} \qquad (10)$$

Here $\dot{K}^{ih}$ represents net capital imports for each project, it being assumed that gross foreign credits supplied in each year are equal to the imports needed for the project.

Balance: $v^o + \Sigma v^h = e + x + \delta p (b^o + \Sigma b^h)$ (11)

$p (\dot{b}^o + \Sigma \dot{b}^h) = S + \Sigma \dot{K}^{ih}$ (12)

This equation represents the balance equation for capital; the left-hand side representing net demand for and the right-hand side net supply of savings.

Definition: $\tilde{Y}^0 = p\, v^0 - \tilde{k}^{-1}\, p^i\, i^0 - \delta\, p b^0$ (13)

$\tilde{Y}^h = p\, v^h - \tilde{k}^{-1}\, (p^i\, i^h + K^{ih}\, m^i) - \delta p\, b^h$ (14)

These expressions represent the contributions to national income of the rest of the economy and each of the projects, respectively, calculated at "accounting prices" $\tilde{k}^{-1} = \frac{1}{\tilde{k}}$ for foreign currency, which may differ from the official price (taken equal to 1). The accounting price has to be chosen in such a way as to equilibrate the balance of payments; since that price influences the choice of the projects and hence the imports i^h it can indeed satisfy this condition, at least within certain limits.

$$E = p\, e \qquad (15)$$

$$\tilde{Y}_0 = \Sigma^h\, \tilde{Y}_0{}^h + \tilde{Y}_0{}^0 \qquad (16)$$

$$\tilde{Y}_0{}^h = \sum_1^{\infty}{}^t \frac{\tilde{Y}_t{}^h}{\prod\limits_{t'=2}^{t} (1 + n_{t'})} \qquad (17)$$

$\tilde{Y}_0{}^h$ represents the discounted value, at middle of time period 1, of all future contributions of project h to national income, at accounting prices; $\tilde{Y}_0$ represents the total for all projects chosen, plus the private sector. It is this expression which should be made by a proper choice of projects, a maximum.

MODEL 21. GROUP OF ECONOMIES, STATIC, MONEY AND PRODUCT FLOW MODEL

ACTORS *Number:* $2H$, namely H national economies and H governments.

Income: *national economies:* approximated by volume of production. Supply of national products inelastic; supply price considered an instrument of economic policy;
governments: not explicitly considered.

Spending habits: *national economies,* depending on real income; *governments:* expenditure autonomous and an instrument of economic policy.

Wealth: not explicitly considered.

MARKETS *Number:* H, the national products, sold at one price each in all countries.

TECHNICAL RELATIONS Each country imports the national products of all other countries and uses them as raw material to its own national product; the import quota of each country with respect to each other country, in the absence of price differences, is equal to μ; it reacts on price differences between the importing country and the country of origin with an elasticity equal for all with respect to all.

EQUATIONS The symbol Σ' indicates summation from 1 to H inclusive, but exclusive of h.

Demand: home demand: $x^{hh} = \xi y^h + x_0^h \qquad h = 1 \ldots H \qquad (1)$

Here ξ stands for the propensity to spend and x_0^h is the autonomous component of national expenditure

imports: $x^{hh'} = \mu y^{h'} + \varepsilon (p^h - p^{h'}) \qquad h, h' = 1 \ldots H, \; h \neq h' \qquad (2)$

In these equations μ indicates the import quota with respect to any one of the other countries and ε is related to the price elasticity of foreign demand.

Supply: $p^h = \mu \sum'_{h'} p^{h'} + p_0^h \qquad h = 1 \ldots H \qquad (3)$

Here p_0^h indicates the national component in product prices, depending, among other things, of the level of efficiency incomes.

Definition: $y^h = \sum_{h'} x^{hh'} \qquad h = 1 \ldots H \qquad (4)$

$$D^h = \sum'_{h'} x^{h'h} p^{h'} - p^h \sum'_{h'} x^{hh'} \qquad h = 1 \ldots H \qquad (5)$$

In the examples to be treated with the aid of this model we choose $H = 10$, $\mu = 0.02$, $\xi = 0.7$ and $\varepsilon = -0.27$; the meaning of these figures being that there are ten countries, with total import quota of $9\mu = 0.18$, a marginal propensity to spend of 0.7 and a price elasticity of foreign demand of —2. [1]

[1] Particulars will be found in: J. Tinbergen, Centralization and Decentralization in Economic Policy, Amsterdam 1954, pp. 28–34.

APPENDIX 4

EXPLANATION OF SYMBOLS USED

Below are listed the symbols used to indicate economic variables and instrument variables. The main list does not apply to model 08, for which a separate list is added. It does not contain the Greek symbols used to indicate coefficients; these are explained in the model where they first appear. They correspond to the variable which occurs on the left-hand side of the equation in which they appear. The list below does not mention separately either the additive constants contained in a number of equations and indicated as a rule by a lower index 0 added to the symbol used for the variable on the left-hand side of the equation.

A higher index h indicates sector (industry or country) h.

Barred symbols: single bar, initial value; double bar: absolute value in problem.

In models 10, 12, 13, 14, 16, 18 and 19 symbols without bar indicate deviations from initial value; e.g. $a = \bar{\bar{a}} - \bar{a}$.

Details about the units used are to be found in the description of the model (cf. especially model 14).

Lower index added to variable (except index 0) indicates time.

Stock variables are indicated by *.

The sign $\sim$ indicates values at "accounting prices" (model 20).

LIST OF VARIABLES AND INSTRUMENTS FOR ALL MODELS EXCEPT 08

(Alphabetical order)

*: stock variables.

a volume of employment
* Au gold stock
* b quantity of other factors of production, e.g. capital
* B quantity of bonds in circulation
* B^B bills and advances at private banks balance sheet
* B^R rediscounts at Central Bank balance sheet

c volume of consumption
C value of consumption at market prices
C^F value of consumption at factor cost
C^z value of consumption by independents (i.e. persons other than employees)
d depreciation allowances
D deficit on balance of payments
e volume of exports
E value of exports
h, h' number of sector (industry or country)
h'' as an index indicates average value of some variable for all values of h' except h
i volume of imports
I value of imports
j volume of gross investment
J value of gross investment
k exchange rate (price of national currency)
* K^i foreign debt

l wage rate
L total wages
m interest rate (model 17: Central Bank rate)
m' private discount rate
* M money in circulation (model 17: bank notes only)
* M' bank deposits
* M^i money held by sector i

ΔM^{ik} money flow from sector i to sector k
p price level of national product (excl. indirect taxes)

p'	home price level of national product (incl. indirect taxes)
p^x	price level of home sales
p^e	export price level
p^i	import price level
p^w	world market price level of export goods
q	price of "other" products
r	volume of replacement
* R	reserves of private banks with central bank
s	real savings
S	value of savings
t	time
T	tax receipts (model 09: life time of equipment)
v	volume of gross product
* w	stock of equipment
x	real national expenditure (or home sales)
X	national expenditure (at market prices)
X^G	public expenditure
X^P	private expenditure
y	real national income
Y	national income at factor cost
Z	income of "independents" (before direct tax)

APPENDIX 4

SYMBOLS USED IN MODEL 08

(i) *Latin symbols*

i number of "ability" or "property" (0: family size; 1: speed or intensity of work; I: total number)
l income per occupied person
L total income paid out by production "organizers"
L_i coefficients occurring in income scale l
m frequency distribution of "required properties"
M total number of jobs ($= N$)
n frequency distribution of properties available
N total number of occupied persons ($= M$)
p product of one person
P total product of all occupied
s_i intensity or degree of property i required for job
$\bar{s}_i$ average of all s_i
t_i intensity or degree of property i available in population
$\bar{t}_i$ average of all t_i

(ii) *Greek symbols*

γ tax rate
λ_i coefficients appearing in utility function
π_i coefficients appearing in production function
σ_i standard deviation of s_i
τ_i standard deviation of t_i
φ disutility function of speed or intensity of work
ψ coefficient appearing in income scale
ω utility function

APPENDIX 5

READING SUGGESTIONS

This list does not claim to be complete, but rather indicates some of the outstanding books and reports that deal with the subject matter of economic policy and of econometric models. Further literature will be found in several of these books and reports.

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