The Effects of Institutional and Technological Change and Business Cycle Fluctuations on Seasonal Patterns in Quarterly Industrial Production Series*

Dick van Dijk†
Econometric Institute
Erasmus University Rotterdam

Birgit Strikholm‡
Department of Economic Statistics
Stockholm School of Economics

Timo Teräsvirta§
Department of Economic Statistics
Stockholm School of Economics

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Abstract

Changes in the seasonal patterns of macroeconomic time series may be due to the effects of business cycle fluctuations or to technological and institutional change or both. We examine the relative importance of these two sources of change in seasonality for industrial production series of the G7 countries. We find compelling evidence that the effects of gradual institutional and technological change are much more important than the effects attributable to the business cycle.

Key words: Nonlinear time series, seasonality, smooth transition autoregression, structural change, time-varying parameter.

JEL Classification Code: C22, C52, E32

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†Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, NL-3000 DR Rotterdam, The Netherlands, `email: djvandijk@few.eur.nl` (corresponding author)

‡Department of Economic Statistics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden, `email: Birgit.Strikholm@hhs.se`

§Department of Economic Statistics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden, `email: Timo.Terasvirta@hhs.se`

1 Introduction

Seasonal fluctuations are an important source of variation in many macroeconomic time series. When such monthly or quarterly series are modelled, it is often assumed that the seasonal pattern of the series is constant over time, in which case it may be characterized by seasonal dummy variables, see Miron (1996) and Miron and Beaulieu (1996), among others. On the other hand, it has been known for a long time that seasonality in a series may change. As Kuznets (1932) remarked:

> "For a number of years statisticians have been concerned with the problem of measuring changes in the seasonal behaviour of time series."

Possible causes for such changes have also been a longstanding object of interest. After examining a number of employment series from various countries and regions, Gjermoe (1931) wrote (in Norwegian):

> "The strength of seasonal fluctuations has to do with the *level* of business activity. A month in a year of low employment is more affected by seasonality than the same month in a year of high employment." (Author's italics)

The possibility that seasonality is affected by the business cycle has been reconsidered in the more recent literature. For example, Canova and Ghysels (1994) investigated it using quarterly US output. Their autoregressive model with seasonal dummies for the first difference of the logarithmic series contained a set of extra seasonal dummies that obtained nonzero values only if the NBER business cycle indicator defined the corresponding quarter to be a recession quarter. Whether the business cycle influences the seasonal cycle was examined by testing the null hypothesis that the coefficients of the extra dummies were zero. Franses (1996, pp. 86-87) later argued that a more appropriate null hypothesis to test was that these coefficients were equal but possibly non-zero. In that case, the autoregressive model contained a business cycle influenced intercept under the null hypothesis. The empirical results of these authors suggested that the seasonal pattern in the quarterly US output series is indeed affected by the business cycle. See Cecchetti, Kashyap and Wilcox (1997) and Krane and Wascher (1999) for other recent investigations of the effect of the business cycle on seasonal patterns in US production, inventories and employment.

Business cycle fluctuations are not the only possible reason for changes in the seasonal pattern of output or employment series. In particular, technological change and changes in institutions and habits may cause changes in seasonality as well. As an example of the former, in the construction industry it has become possible to keep a construction site going in the winter in countries where, for a few decades ago, work was interrupted for the winter months. As to the latter type of change, the increase in paid leasure over the last few decades has gradually changed people's vacation habits, at least in some Scandinavian countries. It has become customary to spend a week of the annual holiday in the winter, and this in turn has affected output and consumption in the first quarter of the year. Yet another example may be the increasingly efficient use of capital and just-in-time production techniques. Many factories in Europe no longer close down for the summer vacation but the production process is kept running without interruption. In all these examples, the result has been that the seasonal pattern of output series has changed over time.

Our aim is to compare the effects on seasonality of gradual institutional and technological change with the effects attributable to the business cycle. As for the former, there do not seem to exist reliable aggregate measures for these changes. We allow for the possibility that the aggregate change is steady and continuous and simply use time as a proxy variable for it. This means that we in fact contrast "Kuznets-type" unspecified change in seasonality with the "Gjermoe-type" caused by fluctuations in economic activity. The main question we ask is: which one of the two is more prominent in practice, if any? We shall investigate the problem using industrial production series of the world's leading market economies, the G7 countries.

The plan of the paper is as follows. In Section 2, we describe the output series for the G7 countries, focusing on the properties of their seasonal cycles. In Section 3, we present our statistical tool, the time-varying smooth transition autoregressive [TV-STAR] model. In Section 4, we use the TV-STAR framework for answering the question of whether the changes in the seasonal patterns in the output series are due to the effects of business cycle fluctuations or to technological and institutional change or both. For all 7 series, we find convincing evidence that "Kuznets-type" unspecified change is much more important than "Gjermoe-type" business cycle-induced change. In Section 5 we specify and estimate TV-STAR models for our series to gain further insight into when and how seasonality in the output series has changed. Section 6 contains final remarks.

2 Preliminaries

2.1 Data

Our data set consists of quarterly seasonally unadjusted industrial production volume indexes for the G7 countries, taken from the OECD *Main Economic Indicators*.

The sample period runs from 1960.1 until 1999.2, except for Canada for which the series is available only from 1961.1. Obvious outliers in 1963.1 and 1968.2 for France and in 1969.4 for Italy are replaced by the average of the index values in the same quarter of the previous and the following year.

The importance of seasonal variation in the industrial production series may be illustrated by regressing the first differences of the logarithmic series on a set of four seasonal dummy variables. The coefficients of determination from these regressions lie between 0.75 and 0.92 for the European series and Canada and are appreciably lower only for Japan (0.29) and the United States (0.06). Incidentally, the correlation between the seven standard deviations of the fitted values (the "seasonal cycle") and the corresponding residual standard deviations (the "business cycle" and noise) equals 0.81. This accords well with the finding of Beaulieu, MacKiemason and Miron (1992) that countries with large seasonal cycles tend to have large business cycles as well.

We may also inspect the seasonal patterns in the series visually. Figures 1-7 show graphs of the level, the first difference per quarter, and the seasonal difference of the log industrial production series, as well as the range, defined as the maximum intra-year variation in the first differences. The differenced and seasonally differenced series are multiplied by 100 to express the changes in percentage points.

- insert Figures 1-7 about here -

A common feature for Canada and three European countries, Germany, France and the UK, is that the seasonal variation in the industrial output series is dampened over time, see panels (b) and (c). In particular, the drop in output in the third quarter and the fourth-quarter peak have become less pronounced over time. This is not true, however, for the remaining European country, Italy, where rather the opposite occurs. This corresponds with results in Canova and Hansen (1995), who test for structural change in the seasonal patterns of the four European series (over the period 1960-1989) and find that rejections of the null hypothesis of constancy are concentrated in the third and fourth quarters. The Japanese and the US series do not show a third-quarter summer holiday decrease, and there is no visible tendency in the amplitude of seasonal fluctuations in these countries. In the US, the quarterly growth in the 1990s is actually highest in the third quarter and lowest in the fourth quarter.

2.2 Deterministic and Stochastic Seasonality

In the case of nonstationary time series, time-varying seasonal patterns may often be conveniently characterized by seasonal unit roots, see Hylleberg (1994). Autoregressive models of seasonally differenced data are capable of generating series in which the seasonal pattern evolves over time. For example, in realizations from such models "summer may become winter" or, in general, seasons may "change places." Hylleberg *et al.* (1990) developed statistical tests for testing the seasonal unit root hypothesis. When their HEGY test is applied to our time series, we can only reject the presence of some seasonal unit roots for Japan, the UK and the US, see the upper panel of Table 1. On the other hand, the picture is quite different if a single break is allowed for in the deterministic seasonal pattern under the null hypothesis. In that case, applying the test of Franses and Vogelsang (1998), the number of rejections increases as shown in the lower panel of Table 1.

- insert Table 1 about here -

Structural time series models offer another way of modelling stochastically time-varying seasonality; see Harvey (1989, Chapter 6). In this approach, the time series is divided into components, of which the seasonal one is represented by a linear combination of trigonometric functions with stochastic coefficients. If these coefficients have zero variance, seasonality is deterministic.

Neither one of these alternatives, seasonal differencing or decomposition of time series, is directly applicable to our situation. The reason is that we intend to consider two types of time-varying seasonality, variation due to technological and institutional change ("unspecified change") and variation induced by cyclical fluctuations in the economic activity, simultaneously. This requires a model within which we can distinguish these two types of changes from each other and thus compare the relative importance of these two different sources of variation on the seasonal pattern of our quarterly output series. Next we shall present such a model and discuss some of its properties.

3 The TV-STAR Model

The model that we use to investigate the source of changes in seasonality in the G7 output series is the TV-STAR model introduced by Lundbergh, Teräsvirta and van Dijk (2000). To suit our purposes, we augment the model by seasonal dummies, such that for our quarterly time series it has the following form:

$$\begin{aligned}\Delta y_t = {} & [(\boldsymbol{\phi}_1'\mathbf{x}_t + \boldsymbol{\delta}_1'\mathbf{D}_t)(1 - G_1(w_t)) + (\boldsymbol{\phi}_2'\mathbf{x}_t + \boldsymbol{\delta}_2'\mathbf{D}_t)G_1(w_t)][1 - G_2(t^*)] \\ & + [(\boldsymbol{\phi}_3'\mathbf{x}_t + \boldsymbol{\delta}_3'\mathbf{D}_t)(1 - G_1(w_t)) + (\boldsymbol{\phi}_4'\mathbf{x}_t + \boldsymbol{\delta}_4'\mathbf{D}_t)G_1(w_t)]G_2(t^*) + \varepsilon_t, \quad (1)\end{aligned}$$

where y_t is the log-level of the industrial production index, w_t a stochastic transition variable, Δ denotes the first differencing operator, defined by $\Delta_k y_t \equiv y_t - y_{t-k}$ for all

$k \neq 0$ and $\Delta \equiv \Delta_1$, $\mathbf{x}_t = (1, \widetilde{\mathbf{x}}_t')'$, $\widetilde{\mathbf{x}}_t = (\Delta y_{t-1}, \ldots, \Delta y_{t-p})'$, $\mathbf{D}_t = (D_{1,t} - D_{4,t}, D_{2,t} - D_{4,t}, D_{2,t} - D_{4,t})'$, $D_{s,t}$, $s = 1, \ldots, 4$ are seasonal dummy variables, with $D_{s,t} = 1$ when time t corresponds with season s and $D_{s,t} = 0$ otherwise, and $t^* \equiv t/T$ with T denoting the sample size. The transition functions $G_j(s_t) \equiv G_j(s_t; \gamma_j, c_j)$, $j = 1, 2$, are assumed to be given by the logistic function

$$G_j(s_t; \gamma_j, c_j) = \left(1 + \exp\left\{-\gamma_j(s_t - c_j)/\sigma_{s_t}\right\}\right)^{-1}, \qquad \gamma_j > 0, \tag{2}$$

where the transition variable $s_t = w_t$ ($j = 1$) or $s_t = t^*$ ($j = 2$), and $\sigma_{s_t} = [\text{var}(s_t)]^{1/2}$ makes γ_j scale-free. As s_t increases, the logistic function changes monotonically from 0 to 1, with the change being symmetric around the location parameter c_j, as $G_j(c_j - z; \gamma_j, c_j) = 1 - G_j(c_j + z; \gamma_j, c_j)$ for all z. The slope parameter γ_j determines the smoothness of the change in the value of the logistic function. As $\gamma_j \to \infty$, the logistic function $G_j(s_t; \gamma_j, c_j)$ approaches the indicator function $\mathtt{I}[s_t > c_j]$ and, consequently, the change of $G_j(s_t; \gamma_j, c_j)$ from 0 to 1 becomes instantaneous at $s_t = c_j$. When $\gamma_j \to 0$, $G_j(s_t; \gamma_j, c_j) \to 0.5$ for all values of s_t.

The TV-STAR model distinguishes four regimes corresponding with combinations of $G_1(w_t)$ and $G_2(t^*)$ being equal to 0 or 1. The transition variable w_t in (1) is assumed to be a lagged seasonal difference, $w_t = \Delta_4 y_{t-d}$, $d > 0$. As this variable tracks the business cycle quite closely for our quarterly industrial production series (see panels (d) of Figures 1-7), and because the logistic function $G_j(s_t)$ is a monotonic transformation of s_t, the regimes associated with $G_1(\Delta_4 y_{t-d}) = 0$ and 1 will roughly correspond with recessions and expansions, respectively. Thus, using $\Delta_4 y_{t-d}$ as transition variable ensures that the TV-STAR model allows for "Gjermoe-type" change in the seasonal pattern of y_t. On the other hand, the function $G_2(t^*)$ enables the model to describe "Kuznets-type" unspecified change as well. It may be argued, however, that GNP is a more representative and more commonly used indicator of the business cycle than the industrial production. In fact, we repeated our tests described in Section 4 using lagged seasonal differences of GNP instead of $\Delta_4 y_{t-d}$ as the transition variable. The results were very similar to the ones obtained by using $\Delta_4 y_{t-d}$ and will therefore be omitted.

The reason for defining the elements of $\mathbf{D}_t$ as $D_{s,t} - D_{4,t}$, $s = 1, 2, 3$, is that it effectively separates the deterministic seasonal fluctuations from the overall intercept. For example, the coefficients in $\boldsymbol{\delta}_1 = (\delta_{11}, \delta_{12}, \delta_{13})'$ measure the difference between the intercept in the first three quarters of the year and the overall intercept, given by the first element of $\boldsymbol{\phi}_1$, in the regime $G_1(\Delta_4 y_{t-d}) = 0$ and $G_2(t^*) = 0$. The difference for the fourth quarter is obtained as $-\sum_{s=1}^{3} \delta_{1s}$. This parameterization makes it easy, for example, to test constant seasonality while allowing for a business cycle influenced intercept under the null hypothesis, cf. Franses (1996, pp. 86-87).

The general TV-STAR model in (1) allows both the dynamics and the seasonal properties of the growth rate of industrial production to vary both over the business cycle and over time. By imposing appropriate restrictions on either the autoregressive parameters or the seasonal dummy parameters or both, more restrictive models can be obtained. Of particular interest here are models in which seasonality only varies either over time or over the business cycle. A model in which seasonality is constant over time is obtained if $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_3$ and $\boldsymbol{\delta}_2 = \boldsymbol{\delta}_4$ in (1). Similarly, a model in which seasonality is constant over the business cycle is obtained by setting $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_2$ and $\boldsymbol{\delta}_3 = \boldsymbol{\delta}_4$. When $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_2 = \boldsymbol{\delta}_3 = \boldsymbol{\delta}_4$, seasonality is linear and constant over time. Imposing analogous restrictions on $\boldsymbol{\phi}_i$, $i = 1, \ldots, 4$ results in models with constant but nonlinear, linear but time-varying, and linear and constant autoregressive dynamics. If both the seasonal patterns and the autoregressive dynamic structure are constant either over time or over the business cycle, the TV-STAR model reduces to a STAR or TV-AR model, respectively. All these restrictions are testable, and testing will be discussed in the next section.

Often a useful restricted TV-STAR model is an additive one, containing a nonlinear and a time-varying component. For example, a model in which the seasonal dummy coefficients vary over time and the autoregressive parameters enter nonlinearly can be written as

$$\Delta y_t = \boldsymbol{\phi}_1^{*\prime}\mathbf{x}_t + \boldsymbol{\delta}_1^{*\prime}\mathbf{D}_t + \boldsymbol{\phi}_2^{*\prime}\mathbf{x}_t G_1(w_t) + \boldsymbol{\delta}_2^{*\prime}\mathbf{D}_t G_2(t^*) + \varepsilon_t. \quad (3)$$

In Section 5 we will use this form for the models for the industrial production series.

Finally, the TV-STAR model (1) is restrictive in the sense that it requires any nonlinearity or structural change to be common across the autoregressive dynamics and seasonal dummies. For example, if the coefficients of both the lagged growth rates and the seasonal dummies are time-varying, the structural change is centred at the same point in time, $t^* = c_2$, and occurs at the same speed, as determined by γ_2. It is straightforward to extend or modify the model to allow for different timing and/or speed of the structural change of the two sets of coefficients. For example, models of type (3) in which G_1 and G_2 have the same transition variable are not excluded a priori.

4 Changes in the Seasonal Pattern and Their Causes

4.1 Testing Linearity and Parameter Constancy in the TV-STAR Framework

The question posed in the Introduction about the causes of fluctuations in the seasonal pattern is addressed in the framework of the TV-STAR model (1), in particular by testing hypotheses about the coefficients of the model. In the previous section, it was emphasized that linearity or parameter constancy in the TV-STAR model (1) may be achieved by imposing zero or equality restrictions on certain coefficient vectors $\boldsymbol{\delta}_i$ and/or $\boldsymbol{\phi}_i$. Note however that linearity or parameter constancy of both the seasonal pattern and the dynamic autoregressive structure also results if the smoothness parameter γ_j in the corresponding transition function G_j is set equal to zero. This is an indication of an identification problem present in the model: the TV-STAR model is only identified under the alternative, not under the null hypothesis. For a general discussion, see Hansen (1996). In this paper, we follow the approach of Lundbergh *et al.* (2000), see also Luukkonen, Saikkonen and Teräsvirta (1988), and circumvent the identification problem by approximating the transition functions by their first-order Taylor expansions.

Let the null hypothesis be $\mathsf{H}_0 : \gamma_1 = \gamma_2 = 0$, which is to be tested against the alternative hypothesis $\mathsf{H}_1 : \gamma_1 > 0$ and/or $\gamma_2 > 0$. Under H_0, model (1) reduces to a seasonality-augmented linear autoregressive model, which we assume to be stationary and ergodic. Furthermore, we assume that the moment condition $\mathsf{E}[(\Delta y_t)^2(\Delta_4 y_t)^2] < \infty$ is satisfied, which is necessary for the asymptotic inference. The first-order Taylor expansion of (1) around H_0 becomes

$$y_t = \boldsymbol{\phi}_1^{*\prime}\mathbf{x}_t + \boldsymbol{\delta}_1^{*\prime}\mathbf{D}_t + (\boldsymbol{\phi}_2^{*\prime}\mathbf{x}_t + \boldsymbol{\delta}_2^{*\prime}\mathbf{D}_t)t^* + \sum_{i=1}^{r}(\boldsymbol{\phi}_{3,i}^{*\prime}\mathbf{x}_t + \boldsymbol{\delta}_{3,i}^{*\prime}\mathbf{D}_t)\Delta_4 y_{t-i} + \sum_{i=1}^{r}(\boldsymbol{\phi}_{4,i}^{*\prime}\mathbf{x}_t + \boldsymbol{\delta}_{4,i}^{*\prime}\mathbf{D}_t)t^*\Delta_4 y_{t-i} + R(\gamma_1,\gamma_2) + \varepsilon_t, \quad (4)$$

where $R(\gamma_1, \gamma_2)$ is a remainder from the two Taylor expansions. Under the null hypothesis of linearity and parameter constancy, $R(\gamma_1, \gamma_2) \equiv 0$, such that this remainder does not affect the distribution theory.

Equation (4) is linear in parameters. Furthermore, and this is crucial, the parameter vectors $\boldsymbol{\phi}_2^* = \gamma_2\widetilde{\boldsymbol{\phi}}_2^*(\boldsymbol{\theta})$ and $\boldsymbol{\delta}_2^* = \gamma_2\widetilde{\boldsymbol{\delta}}_2^*(\boldsymbol{\theta})$, $\boldsymbol{\phi}_3^* = \gamma_1\widetilde{\boldsymbol{\phi}}_3^*(\boldsymbol{\theta})$ and $\boldsymbol{\delta}_3^* = \gamma_1\widetilde{\boldsymbol{\delta}}_3^*(\boldsymbol{\theta})$, and $\boldsymbol{\phi}_4^* = \gamma_1\gamma_2\widetilde{\boldsymbol{\phi}}_4^*(\boldsymbol{\theta})$ and $\boldsymbol{\delta}_4^* = \gamma_1\gamma_2\widetilde{\boldsymbol{\delta}}_4^*(\boldsymbol{\theta})$ where $\widetilde{\boldsymbol{\phi}}_j^*(\boldsymbol{\theta})$, $\widetilde{\boldsymbol{\delta}}_j^*(\boldsymbol{\theta})$, $j = 2, 3, 4$, are nonzero functions of the parameters $\boldsymbol{\theta} = (\boldsymbol{\phi}_1', \ldots, \boldsymbol{\phi}_4', \boldsymbol{\delta}_1', \ldots, \boldsymbol{\delta}_4')'$. In view of this, the original null hypothesis becomes

$$\mathsf{H}_0' : \boldsymbol{\phi}_2^* = \boldsymbol{\phi}_{3,i}^* = \boldsymbol{\phi}_{4,i}^* = \mathbf{0},\ \boldsymbol{\delta}_2^* = \boldsymbol{\delta}_{3,i}^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r$$

in the transformed equation (4). In testing model (1) we only assume $d \in \{1, \ldots, r\}$, that is, the true delay is unknown. This leads to the linear combination of lags of $\Delta_4 y_{t-d}$ in (4); see Luukkonen *et al.* (1988). The tests reported below all set $r = 4$. The standard χ^2 statistic for testing H_0' has an asymptotic χ^2 distribution with

$(p+4)(1+2r)$ degrees of freedom under the null hypothesis. In practice, an F-version of the test is recommended because its size properties in small and moderate samples are much better than those of the χ^2-based test statistic. It should be noted that, depending on the values of p and r, certain terms $\phi^*_{2,i,0}\Delta_4 y_{t-i}$ and $\phi^*_{2,i,j}\Delta y_{t-j}\Delta_4 y_{t-i}$ should be excluded from (4) to avoid perfect multicollinearity.

In order to keep the notation simple, we so far have discussed the case of the standard logistic function (2) being the transition function. It is useful to generalize this slightly. Let

$$G_j(s_t;\gamma_j,\mathbf{c}_j) = (1+\exp\{-\frac{\gamma_j}{\sigma^k_{s_t}}\prod_{i=1}^{k}(s_t-c_{ji})\})^{-1}, \qquad \gamma>0, c_{j1}\leq ... \leq c_{jk}. \tag{5}$$

This function allows more flexibility in the transition. When we test linearity against the TV-STAR model (1) with (5), a first-order Taylor expansion of (5) leads to terms with higher powers of $\Delta_4 y_{t-j}$ and t^* in equation (4); see, for example, Luukkonen *et al.* (1988), Granger and Teräsvirta (1993, Chapter 6) or Lundbergh *et al.* (2000). The dimension of the null hypothesis increases compared to the case $k=1$, and the moment condition required for the test becomes $\mathsf{E}[(\Delta y_t)^2(\Delta_4 y_t)^{2k}]<\infty$. Below we report results for $k=1$, 2 and 3, and denote the corresponding statistics as LM_k.

Finally, it should be pointed out that the lag length p in (1) is unknown. It is selected from the linear seasonality-augmented autoregressive model using BIC with the maximum order set equal to $p_{\max}=12$. As remaining residual autocorrelation may be mistaken for nonlinearity, we apply the Breusch-Godfrey LM test to examine joint significance of the first 12 residual autocorrelations in the model that is selected by the BIC. If necessary, the lag length p is increased until the null hypothesis of no error autocorrelation can no longer be rejected at the 5% significance level. Testing is carried out conditionally on the selected lag length $\widehat{p}$.

4.2 Testing Hypotheses of Interest

The test just described is a general linearity test within our maintained TV-STAR model (1). In this paper, however, the main interest lies in testing a set of sub-hypotheses that place restrictions on seasonal dummy variables. We may also set certain parameter vectors to zero (null vectors) *a priori*. This leads to a maintained model that is a submodel of (1). For example, we may test constant seasonality against the alternative that the seasonal pattern changes smoothly over time, conditional on the assumption that seasonality is not affected by the business cycle and that the autoregressive structure does not change. In terms of the parameters in

(4), the corresponding null hypothesis is

$$\mathsf{H}_0^{\text{TV-AR}}: \boldsymbol{\delta}_2^* = \mathbf{0} \mid \boldsymbol{\phi}_2^* = \boldsymbol{\phi}_{3,i}^* = \boldsymbol{\phi}_{4,i}^* = \mathbf{0},\ \boldsymbol{\delta}_{3,i}^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r.$$

This is one of our hypotheses of interest. Another interesting one is

$$\mathsf{H}_0^{\text{STAR}}: \boldsymbol{\delta}_{3,i}^* = \mathbf{0} \mid \boldsymbol{\phi}_2^* = \boldsymbol{\phi}_{3,i}^* = \boldsymbol{\phi}_{4,i}^* = \mathbf{0},\ \boldsymbol{\delta}_2^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r.$$

In this case, we test constant seasonality against the alternative that the seasonal pattern is affected by the business cycle only. A test against the joint alternative of smooth change and fluctuations ascribed to the business cycle may be formed accordingly. The corresponding null hypothesis is denoted as $\mathsf{H}_0^{\text{TV-STAR}}$.

These tests are based on the assumption of linearity. But then, the first difference of the volume of industrial production may be a nonlinear or time-varying process. One way of accounting for this possibility could be to relax the conditions in the above tests. This variant of $\mathsf{H}_0^{\text{TV-AR}}$ becomes

$$\mathsf{H}_0^{\text{TV-AR}^*}: \boldsymbol{\delta}_2^* = \mathbf{0} \mid \boldsymbol{\delta}_{3,i}^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r.$$

Similarly, we have

$$\mathsf{H}_0^{\text{STAR}^*}: \boldsymbol{\delta}_{3,i}^* = \mathbf{0} \mid \boldsymbol{\delta}_2^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r.$$

While testing these hypotheses is not difficult in practice, this may not be an optimal way to proceed. Instead it may be better to test our two competing hypotheses concerning seasonality within a model which allows the autoregressive structure to change, either as a function of time (TV-AR) or as a function of the business cycle (STAR). In that case, we may begin by testing linearity against STAR and TV-AR. The relevant null hypotheses (assuming constant seasonality and unknown delay d) are

$$\mathsf{H}_0^{\text{STAR-ns}}: \boldsymbol{\phi}_{3,i}^* = \mathbf{0} \mid \boldsymbol{\phi}_2^* = \boldsymbol{\phi}_{4,i}^* = \mathbf{0},\ \boldsymbol{\delta}_2^* = \boldsymbol{\delta}_{3,i}^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r,$$

and

$$\mathsf{H}_0^{\text{TV-AR-ns}}: \boldsymbol{\phi}_2^* = \mathbf{0} \mid \boldsymbol{\phi}_{3,i}^* = \boldsymbol{\phi}_{4,i}^* = \mathbf{0},\ \boldsymbol{\delta}_2^* = \boldsymbol{\delta}_{3,i}^* = \boldsymbol{\delta}_{4,i}^* = \mathbf{0},\ i = 1, \ldots, r,$$

respectively. Assume for a moment that $\mathsf{H}_0^{\text{STAR-ns}}$ is rejected and $\mathsf{H}_0^{\text{TV-AR-ns}}$ is not. This implies that the dynamic behaviour of the process, excluding seasonality, may be adequately characterized by a STAR model. We subsequently specify, estimate and evaluate a STAR model for Δy_t. The issue is now the constancy of the coefficients of the seasonal dummy variables in the STAR model. The maintained model may be written as follows:

$$\Delta y_t = \boldsymbol{\phi}_1' \mathbf{x}_t + \boldsymbol{\phi}_2' \mathbf{x}_t G(s_t) + \{\boldsymbol{\delta}_1 + \boldsymbol{\delta}_2 H_1(\Delta_4 y_{t-e}) + \boldsymbol{\delta}_3 H_2(t^*) + \boldsymbol{\delta}_4 H_1(\Delta_4 y_{t-e}) H_2(t^*)\}' \mathbf{D}_t + \varepsilon_t, \quad (6)$$

where the transition functions $H_1(\Delta_4 y_{t-e})$, $e > 0$, and $H_2(t^*)$ are logistic functions as in (5). Note that we can choose either $s_t = \Delta_4 y_{t-d}$ or $s_t = t^*$ in (6). The relevant parameter constancy hypotheses can now be formulated within equation (6) in terms of the slope parameters in the transition functions $H_1(\Delta_4 y_{t-e})$, $e > 0$, and $H_2(t^*)$ or in terms of the coefficient vectors $\boldsymbol{\delta}_2$, $\boldsymbol{\delta}_3$ and $\boldsymbol{\delta}_4$. Asymptotic theory for inference requires the assumption that the null model, (6) with $\boldsymbol{\delta}_2 = \boldsymbol{\delta}_3 = \boldsymbol{\delta}_4 = \mathbf{0}$, is stationary and ergodic. Testing is based on the Taylor approximation of $H_1(\Delta_4 y_{t-e})$ and $H_2(t^*)$ as described in Section 4.1; for a general account of STAR model misspecification tests, see, for example, Teräsvirta (1998).

4.3 Results

Table 2 reports p-values of the F-statistics for testing $\mathsf{H}_0^{\text{TV-AR}}$, $\mathsf{H}_0^{\text{STAR}}$ and $\mathsf{H}_0^{\text{TV-STAR}}$ based on a linear null model. The column headings LM_1, LM_2 and LM_3 correspond to the tests based on the first-order Taylor expansion of the transition function (5) with $k = 1$, 2 and 3, respectively. The rows correspond to tests involving both the seasonal pattern and autoregressive structure ($D_{s,t}$, Δy_{t-j}), the seasonal pattern only ($D_{s,t}$) and the autoregressive coefficients only (Δy_{t-j}). All tests are computed with the maximum value of the unknown delay r set equal to 4. A broad classification of the results by the magnitude of the p-values appears in the upper panel of Table 3.

- insert Tables 2 and 3 about here -

This panel contains plenty of evidence to support the argument that seasonality is changing for unspecified reasons, including institutional and technological change, proxied by the time variable. The results for the LM_1 statistic are mixed, but LM_2 rejects the null hypothesis $\mathsf{H}_0^{\text{TV-AR}}$ at the 0.01 level in all seven cases, and LM_3 in six cases out of seven. On the other hand, there is much less evidence to support the notion that seasonality varies with the business cycle. The p-values for the tests corresponding to $\mathsf{H}_0^{\text{STAR}}$ are considerably larger. The only occasions in which a p-value lies below 0.01 are LM_2 for France and LM_3 for Japan. For Japan, there is in fact substantial evidence of both nonlinearity and parameter nonconstancy in the series. For the other five countries, it seems that business cycle fluctuations are not a major cause for changes in the seasonal pattern.

Finally, another fact obvious from Table 2 and the upper panel of Table 3 worth mentioning is that testing against both types of changes in seasonality jointly has an adverse effect on power. More information is gained by looking at the two alternatives separately. Similarly, testing linearity of the seasonal pattern and the autoregressive structure simultaneously can weaken the power and, furthermore, complicate the interpretation of results.

Two objections may be made at this point. First, seasonality may not be fully explained by the seasonal dummy variables. A part of the seasonal variation may be absorbed in (explained through) the autoregressive dynamic structure of model (1). Pierce (1978) discussed this possibility in connection with seasonal adjustment of economic time series. This variation may be related to the business cycle. Second, results on testing linearity against STAR in Table 2 (cells (Δy_{t-j}, STAR)), suggest that the dynamic behaviour of some of the industrial production series may be nonlinear. For some other series a case can be made for a TV-AR process, that is, the dynamic behaviour may be time-varying because of phenomena proxied by time. It may therefore be argued that the results just presented are affected by misspecification of the basic model and that in order to avoid this, the null model should already accommodate non-seasonal nonlinearity.[1]

In order to consider this possibility we proceed as follows. Consider the row of linearity tests (Δy_{t-j}) in Table 2. We choose the type of transition variable, either a lag of $\Delta_4 y_t$ or t^*, for each series by comparing the p-values of the tests against STAR and against TV-AR. If the p-value of the test against STAR is smaller than the one for TV-AR, we choose a STAR model, otherwise we proceed with a TV-AR model. In this case, all three LM_k statistics yield the same result for France, Japan, the UK (TV-AR), Germany and the US (STAR). For Canada and Italy, we obtain conflicting results but, considering the three statistics jointly, it appears that the evidence for TV-AR is stronger than the evidence for STAR for both countries. Next, we specify and estimate a STAR/TV-AR model for Δy_t with the seasonal dummies only entering linearly, following the modelling strategy described, for example, in Teräsvirta (1998). We then test the constancy of the coefficients of the seasonal dummy variables within this model. The p-values of the appropriate test statistics can be found in Table 4, where the first column of the table gives the transition variable of the model.

[1] An obvious modification of the tests presented in Table 2 would be to allow the overall intercept to be affected by the business cycle when testing for "Gjermoe"-type changes in seasonality, and to allow for the intercept to be time-varying when testing for "Kuznets"-type changes in seasonality, cf. Franses (1996, pp. 86-87). Results from these tests are very similar to the ones shown in Table 2 and are therefore omitted.

- insert Table 4 about here -

It is seen that the basic message is still the same, although the p-values are somewhat higher than before. Obviously, some of the seasonal variation has been absorbed by the re-specified dynamic structure of the model. Nevertheless it seems that the seasonal parameters are still changing over time for unspecified reasons rather than as a function of the cyclical fluctuations in the economy. The lower panel of Table 3 gives a summary of the results. At the 5% level, $\mathsf{H}_0^{\text{STAR}}$ can only be rejected for some tests for Canada, France, Japan and the UK. Rejections against TV-AR are still the rule, in particular when the test is LM_3.

Canada is an interesting case. It is seen that LM_1 now rejects constancy quite strongly against "Gjermoe-type" change. It appears possible that both institutional and technological change on the one hand and business cycle fluctuations on the other may have affected the seasonal pattern of the Canadian output series. The null model thus influences our view of the situation. Attention may also be drawn to the US. The STAR model obviously characterizes most of the systematic variation in the series. There is some evidence left suggesting that the seasonal pattern varies over time due to unspecified reasons, but it is not very strong: the p-values of the LM_k tests equal 0.11, 0.081, and 0.11, for $k = 1$, 2 and 3, respectively.

Putting all this together, our general conclusion is that the institutional, technological and other changes proxied by time are the main cause for changes in seasonal pattern in the output series of G7 countries. The results of this section also illustrate the fact that our conclusions to some extent depend on the model used for carrying out the relevant tests. This may not be surprising, and mentioning this fact may even sound trivial. But then, we may also argue that our general conclusion seems remarkably robust to the choice of the null model.

5 Modelling Changing Seasonal Patterns by TV-STAR Models

Our test results in the previous section clearly show that seasonal patterns in the G7 ouput series are not constant over time. In this section, our aim is to characterize this change with a parametric model, instead of just demonstrating its existence through a number of hypothesis tests. We will attempt to build an adequate TV-STAR model for each of the series and focus on the components related to seasonal variation.

As the TV-STAR model is a rather flexible nonlinear model, we need a coherent modelling strategy or modelling cycle in order to arrive at an acceptable parameterization. We choose the "specific-to-general" strategy of Lundbergh *et al.* (2000). The main features of this modelling cycle are the following. First, starting with a seasonality-augmented linear autoregressive model, test linearity against STAR ($\Delta_4 y_{t-d}$ being the transition variable) and TV-AR (t^* being the transition variable). Choose the submodel against which the rejection is strongest (if it is strong enough, otherwise accept the linear model). Estimate the chosen model; this involves repeated estimation while reducing the size of the model through exclusion and equality restrictions on parameters. Evaluate the estimated model by subjecting it to a number of misspecification tests. The results may either indicate that the estimated model is adequate or they may point at the necessity of extending the model further, for example towards a full TV-STAR model. A detailed account of the modelling strategy can be found in Lundbergh *et al.* (2000). The misspecification test are described, for example, in Teräsvirta (1998).

The estimated models are given in full detail in Appendix A together with a brief account of the most important modelling events or decisions during the modelling cycle. The appendix also contains a table with the results of the misspecification tests of the final models. Here we concentrate on the deterministic seasonal components of the models, as shown in panel (b) of Figures 8-14. The figures also contain the deterministic components of the seasonality-augmented linear AR models. A striking feature apparent in all figures is that the latter have a much smaller amplitude than the corresponding components in the nonlinear models. Obviously, in a linear model, the parameters of the dynamic structure have to explain a greater part of the seasonality as the deterministic structure is assumed constant over time. Also, in linear models a part of seasonal variation may remain unexplained. In this respect, we note that the residual standard deviation of the nonlinear model is in all cases about 70-80% of that of the corresponding linear AR model.

Of the individual graphs, those for France and the UK show a slowly changing seasonal pattern with decreasing amplitude. The amplitude also ultimately decreases for Germany. The start of the decrease in 1978 is rather abrupt and follows a slow increase during the first part of the sample period. On the other hand, the German unification has hardly affected the seasonal pattern of the industrial output of the country. A similar abrupt shift occurs in the model for the US, but we have to keep in mind that seasonal variation in the US industrial output is small compared to the European G7 countries or Canada. The deterministic component obtained from the Canadian output model accords with the visual information in Figure 1. The amplitude of the seasonal pattern first slowly increases until a reasonably rapid decrease takes place in the late 1980s.

As mentioned above, seasonality in a series need not be fully deterministic. It may even be argued that, within a STAR model, seasonality may sometimes change from principally stochastic to mainly deterministic and vice versa. Consider the UK TV-STAR model. The stochastic component of model (A.18) disappears almost completely over time. Would it contain seasonality? One way of finding out is to consider the roots of the lag polynomial

$$z^9+(0.40z^5-0.19z^3+0.19z^2+0.44)(1-G_1(t^*,54.9,0.36))+0.10z^4G_1(t^*,54.9,0.36)=0$$

at various values of the transition function $G_1(t^*, 54.9, 0.36)$. For $G_1 = 0$ (the beginning of the period) the dominating root is a real root 0.975, but there exists a complex pair of roots $0.08 \pm 0.95i$ with modulus 0.95 and period 4.22 quarters. It may be interpreted as representing a stochastic seasonal component that gradually becomes less persistent and finally disappears. For example, for $G_1 = 0.5$ the modulus of the corresponding pair of roots only equals 0.87 whereas the period length equals 4.32 quarters. This is the only case among our models in which stochastic seasonality is distinct in the sense that the estimated period length lies reasonably close to four quarters while the component is at the same time persistent (the modulus at least at some point of time exceeding 0.9).

Finally, the deterministic component in the industrial output of Japan shows a very rich structure of change. Obviously, the seasonal pattern is affected by a number of factors pulling in different directions at different times.

6 Final Remarks

The results of this paper suggest that seasonal patterns in quarterly industrial production series for the G7 countries change over time. On the other hand, business cycle fluctuations do not seem to be the main cause for this change. Our findings are in contrast with Canova and Ghysels (1994) and Franses (1996), who considered US output and concluded that the business cycle influences the seasonal cycle. Similarly, Cecchetti *et al.* (1997) found that in the US seasonal fluctuations in production and inventories vary with the state of the business cycle. There are at least two reasons for differences between our results and the ones of the above authors. First, they only considered US series and include the GNP and inventories. The second, and perhaps the more important, reason is that those authors did not consider other causes than business cycle fluctuations. Less restrictive considerations appear to lead to rather different conclusions.

As the "Kuznets-type" unspecified change in seasonal patterns is in our work proxied by time, we cannot give a definite answer to the question of what kind of change, technological, institutional, or "other", has been important in the industrial output series we have investigated. Nevertheless, some speculation may be allowed. There is evidence for changes in inventory management affecting the seasonal pattern of industrial output. Carpenter and Levy (1998) showed that inventory investment and output are highly correlated not only at business cycle frequencies but also at seasonal frequencies. Given the importance of inventories for (changes in) fluctuations in output (see Sichel, 1994, and McConnell and Perez Quiros, 2000, among others), it may well be that changes in inventory management such as the use of "just-in-time" techniques have affected the seasonal cycle in inventory investment and thereby changed the seasonal cycle in production. This would seem a plausible explanation or a part of it in cases where the amplitude of the seasonal pattern has decreased over time. On the other hand, very abrupt changes, such as the one in the German industrial output series in 1978, may perhaps best be ascribed to the agency producing the data, unless other information about the nature of the change is available. In general, it may sometimes be relatively easy to suggest individual causes for shifts in the seasonal pattern at the industry level. Because of aggregation this becomes more difficult when the volume of the total industrial output is concerned.

As a whole, our research shows that seasonal patterns in output series tend to change over time. This fact may also raise questions about the consequences of using seasonally adjusted series in macroeconomic modelling. Investigating them in the present context, however, has to be left for future work.

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Appendix A Estimated TV-STAR Models

All models reported below are estimated over the sample period 1964:2-1999:2 (141 observations). Misspecification tests are given in Table 5. Figures 8-14 depict the value of the deterministic seasonal components and the residuals from the TV-STAR model and the best fitting subset AR model, and the transition functions.

Canada

In a seasonality-augmented linear AR model for Canada, linearity of the seasonal dummy coefficients is rejected most strongly against "Kuznets"-type unspecified change. A TV-AR model with a standard logistic transition function (2) does not satisfactorily capture the variation in the seasonal pattern. This is not surprising given the way the seasonal pattern evolves, as shown in panel (b) of Figure 1. A TV-AR model with a generalized logistic function (5) with $k = 2$ also is inadequate, because the decline in the amplitude of seasonal fluctuations after 1978 is different (both in terms of magnitude and speed) from the increase during the first part of the sample, as shown in panel (c) of Figure 1. We therefore use a TV-AR model with two standard logistic functions. After sequentially deleting lagged first differences with insignificant coefficients and increasing the maximum lag order to 11 to capture remaining autocorrelation in the residuals, we obtain the model

$$\begin{aligned}\Delta y_t = {} & \underset{(0.080)}{0.36}\ \Delta y_{t-1} + \underset{(0.074)}{0.33}\ \Delta y_{t-2} - \underset{(0.087)}{0.14}\ \Delta y_{t-3} - \underset{(0.077)}{0.21}\ \Delta y_{t-5} + \underset{(0.071)}{0.16}\ \Delta y_{t-10} \\ & - \underset{(0.069)}{0.24}\ \Delta y_{t-11} - \underset{(1.06)}{0.37}\ D_{1,t} + \underset{(1.44)}{1.39}\ D_{2,t} + \underset{(0.94)}{0.045} D_{3,t} + \underset{(1.07)}{3.90}\ D_{4,t} \\ & + (\underset{(1.51)}{4.78}\ D_{1,t} - \underset{(1.96)}{9.70}\ D_{2,t} - \underset{(1.45)}{4.44}\ D_{3,t} + \underset{(1.62)}{5.77}\ D_{4,t}) \times G_1(t^*; \gamma_1, c_1) \\ & + (-\underset{(0.78)}{1.33}\ D_{1,t} + \underset{(0.83)}{4.05}\ D_{2,t} + \underset{(0.79)}{1.32}\ D_{3,t} - \underset{(0.73)}{3.66}\ D_{4,t}) \times G_2(t^*; \gamma_2, c_2) + \widehat{\varepsilon}_t, \end{aligned} \qquad \text{(A.1)}$$

$$G_1(t^*; \gamma_1, c_1) = (1 + \exp\{-\underset{(1.00)}{3.15}\ (t^* - \underset{(0.033)}{0.25})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.2)}$$

$$G_2(t^*; \gamma_2, c_2) = (1 + \exp\{-\underset{(33.1)}{34.3}\ (t^* - \underset{(9.8\text{E}-3)}{0.73})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.3)}$$

$\widehat{\sigma}_\varepsilon = 1.34$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.82$, SK $= -0.13$, EK $= 0.42$, JB $= 1.40(0.50)$, $\text{LM}_{\text{SC}}(1) = 0.059(0.81)$, $\text{LM}_{\text{SC}}(4) = 1.10(0.36)$, $\text{LM}_{\text{SC}}(12) = 1.05(0.41)$, ARCH$(1) = 0.88(0.35)$, ARCH$(4) = 4.24(0.37)$, AIC $= -0.24$, BIC $= -0.011$,

where OLS standard errors are given in parentheses below the parameter estimates, $\widehat{\varepsilon}_t$ denotes the regression residual at time t, $\widehat{\sigma}_\varepsilon$ is the residual standard deviation, $\widehat{\sigma}$(TV-STAR/AR)is the ratio of the residual standard deviations in the estimated TV-STAR and AR models, SK is skewness, EK excess kurtosis, JB the Jarque-Bera test of normality of the residuals, $\text{LM}_{\text{SC}}(j)$ is the LM test for no residual autocorrelation up to and including lag j, ARCH(q) is the LM test of no ARCH effects up to order q, and AIC and BIC are differences between the Akaike and Schwarz Information Criteria, respectively, of the estimated TV-STAR and the AR models. The numbers in parentheses following the test statistics are p-values.

France

In the linear model for France, parameter constancy is rejected for both the seasonal dummies and the lagged first differences (although rejection is stronger for the deterministic terms). It turns out that the timings of these changes are different and hence we include two logistic transition functions with time as transition variable. One of these is related to the seasonal dummy parameters and the other to the autoregressive parameters. The diagnostic tests for this model reject the hypothesis of no remaining nonlinearity for the lagged autoregressive parameters, with $\Delta_4 y_{t-1}$ indicated as the appropriate transition variable. Adding such a nonlinear component, recursively deleting insignificant coefficients and imposing $\pm$ restrictions on the parameters of Δy_{t-2}, Δy_{t-7}, $D_{1,t}$ and $D_{2,t}$, we arrive at the following specification:

$$\begin{aligned}
\Delta y_t = &- \underset{(0.070)}{0.31}\ \Delta y_{t-5} + \underset{(0.11)}{0.14}\ \Delta y_{t-6} + \underset{(1.49)}{3.43}\ D_{1,t} - \underset{(1.91)}{0.58}\ D_{2,t} - \underset{(0.79)}{20.0}\ D_{3,t} + \underset{(0.94)}{19.9}\ D_{4,t} \\
&+ (- \underset{(0.069)}{0.31}\ \Delta y_{t-2} - \underset{(0.042)}{0.080}\ \Delta y_{t-7}) \times G_1(t^*;\gamma_1,c_1) \\
&+ (- \underset{(1.49)}{3.43}\ D_{1,t} - \underset{(1.54)}{0.061}\ D_{2,t} + \underset{(1.41)}{10.1}\ D_{3,t} - \underset{(1.00)}{8.25}\ D_{4,t}) \times G_2(t^*;\gamma_2,c_2) \\
&+ (\underset{(0.069)}{0.31}\ \Delta y_{t-2} + \underset{(0.049)}{0.21}\ \Delta y_{t-5} - \underset{(0.074)}{0.22}\ \Delta y_{t-6} + \underset{(0.042)}{0.080}\ \Delta y_{t-7}) \times G_3(\Delta_4 y_{t-1};\gamma_3,c_3) + \widehat{\varepsilon}_t,
\end{aligned} \quad \text{(A.4)}$$

$$G_1(t^*;\gamma_1,c_1) = (1+\exp\{- \underset{(2.35)}{4.55}\ (t^* - \underset{(0.036)}{0.36}\ )/\sigma_{t^*}\})^{-1}, \quad \text{(A.5)}$$

$$G_2(t^*;\gamma_2,c_2) = (1+\exp\{- \underset{(0.89)}{3.00}\ (t^* - \underset{(0.030)}{0.64}\ )/\sigma_{t^*}\})^{-1}, \quad \text{(A.6)}$$

$$G_3(\Delta_4 y_{t-1};\gamma_3,c_3) = (1+\exp\{- \underset{(-)}{500}\ (\Delta_4 y_{t-1} + \underset{(-)}{2.57}\ )/\sigma_{\Delta_4 y_{t-1}}\})^{-1}, \quad \text{(A.7)}$$

$\widehat{\sigma}_\varepsilon = 1.50$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.77$, SK $= -0.32$, EK $= 1.16$, JB $= 10.6(5.0\text{E}-3)$, $\text{LM}_{\text{SC}}(1) = 0.056(0.81)$, $\text{LM}_{\text{SC}}(4) = 0.47(0.76)$, $\text{LM}_{\text{SC}}(12) = 0.80(0.65)$, ARCH(1) $=$ $0.11(0.74)$, ARCH(4) $= 1.52(0.82)$, AIC $= -0.42$, BIC $= -0.25$.

Germany

For Germany, the results from the LM-type misspecification tests in the linear model indicate that the seasonal dummy coefficients may be varying for unspecified reasons and the autoregressive dynamics may be varying with the business cycle. The evidence for the latter disappears, however, once we allow the seasonal dummies to vary over time. To capture the variation in the seasonal pattern, we find that three TV components with standard logistic functions are required. The final specification is

$$\begin{aligned}
\Delta y_t = \; & \underset{(0.080)}{0.26}\ \Delta y_{t-1} + \underset{(0.074)}{0.074}\ \Delta y_{t-2} + \underset{(0.061)}{0.068}\ \Delta y_{t-3} - \underset{(0.070)}{0.090}\ \Delta y_{t-4} - \underset{(0.059)}{0.17}\ \Delta y_{t-7} \\
& - \underset{(1.06)}{7.58}\ D_{1,t} + \underset{(1.67)}{10.0}\ D_{2,t} - \underset{(1.45)}{6.64}\ D_{3,t} + \underset{(1.02)}{11.3}\ D_{4,t} \\
& + (-\underset{(1.11)}{0.97}\ D_{1,t} - \underset{(0.67)}{5.42}\ D_{2,t} - \underset{(2.27)}{8.80}\ D_{3,t} + \underset{(1.37)}{7.38}\ D_{4,t}) \times G_1(t^*;\gamma_1,c_1) \\
& + (\underset{(1.11)}{0.97}\ D_{1,t} - \underset{(0.67)}{5.42}\ D_{2,t} + \underset{(1.61)}{13.3}\ D_{3,t} - \underset{(1.37)}{7.38}\ D_{4,t}) \times G_2(t^*;\gamma_2,c_2) \\
& + (\underset{(1.11)}{0.97}\ D_{1,t} + \underset{(0.67)}{5.42}\ D_{2,t} + \underset{(1.75)}{1.88}\ D_{3,t} - \underset{(1.37)}{7.38}\ D_{4,t}) \times G_3(t^*;\gamma_3,c_3) + \widehat{\varepsilon}_t, \qquad (A.8)
\end{aligned}$$

$$G_1(t^*;\gamma_1,c_1) = (1+\exp\{-\underset{(0.96)}{3.36}\ (t^* - \underset{(0.042)}{0.22})/\sigma_{t^*}\})^{-1}, \qquad (A.9)$$

$$G_2(t^*;\gamma_2,c_2) = (1+\exp\{-\underset{(-)}{500}\ (t^* - \underset{(-)}{0.42})/\sigma_{t^*}\})^{-1}, \qquad (A.10)$$

$$G_3(t^*;\gamma_3,c_3) = (1+\exp\{-\underset{(-)}{3.58}\ (t^* - \underset{(0.039)}{0.89})/\sigma_{t^*}\})^{-1}, \qquad (A.11)$$

$\widehat{\sigma}_\varepsilon = 1.61$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.70$, SK $= -0.24$, EK $= 0.80$, JB $= 5.26(0.072)$, $\text{LM}_{\text{SC}}(1) = 4.12(0.045)$, $\text{LM}_{\text{SC}}(4) = 1.12(0.35)$, $\text{LM}_{\text{SC}}(12) = 1.66(0.086)$, ARCH$(1) = 14.5(1.4\text{E}-4)$, ARCH$(4) = 16.5(2.4\text{E}-3)$, AIC $= -0.53$, BIC $= -0.24$.

Even though the model contains three smooth transition components, the number of parameters is not excessively large as the parameters for the first, second and fourth quarter in these components are restricted by equality and ± restrictions. Note that not all autoregressive parameters in (A.8) are significant - these are retained to keep the residual autocorrelations small.

Italy

For Italy, constancy of the seasonal dummy parameters is rejected most strongly against unspecified change. Misspecification tests of a TV-AR model in which only the seasonal dummy coefficients are time-varying reject constancy of the coefficients of the lagged first differences. Allowing these to be time-varying as well we find that the exclusion restrictions (the combined parameter equals zero when $G_1 = 1$) on the autoregressive parameters cannot be rejected. This means that the industrial production growth rate is a white noise series with seasonal means after the smooth transition has been completed. The estimated model has the form

$$\begin{aligned}\Delta y_t = & \underset{(0.10)}{0.24}\,\Delta y_{t-2} - \underset{(0.11)}{0.53}\,\Delta y_{t-5} - \underset{(0.12)}{0.31}\,\Delta y_{t-6} + \underset{(2.35)}{6.07}\,D_{1,t} + \underset{(2.15)}{5.32}\,D_{2,t}\\ & - \underset{(0.82)}{11.0}\,D_{3,t} + \underset{(1.61)}{7.36}\,D_{4,t} + (-\underset{(0.10)}{0.24}\,\Delta y_{t-2} + \underset{(0.11)}{0.53}\,\Delta y_{t-5} + \underset{(0.12)}{0.31}\,\Delta y_{t-6}\\ & - \underset{(2.39)}{4.16}\,D_{1,t} - \underset{(2.20)}{3.65}\,D_{2,t} - \underset{(0.98)}{9.65}\,D_{3,t} + \underset{(1.67)}{11.2}\,D_{4,t}) \times G_1(t^*;\gamma_1,c_1) + \widehat{\varepsilon}_t, \end{aligned} \qquad \text{(A.12)}$$

$$G_1(t^*;\gamma_1,c_1) = (1+\exp\{-\underset{(4.76)}{10.2}\,(t^* - \underset{(0.016)}{0.40})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.13)}$$

$\widehat{\sigma}_\varepsilon = 2.16$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.85$, SK $= 0.052$, EK $= 0.022$, JB $= 0.069(0.97)$, $\text{LM}_{\text{SC}}(1) = 0.20(0.66)$, $\text{LM}_{\text{SC}}(4) = 0.38(0.82)$, $\text{LM}_{\text{SC}}(12) = 0.68(0.77)$, ARCH$(1) = 10.4(1.2\text{E}-3)$, ARCH$(4) = 12.5(0.014)$, AIC $= -0.27$, BIC $= -0.18$.

Japan

For Japan, both linearity and parameter constancy are forcefully rejected for both the lagged autoregressive parameters and the seasonal dummy coefficients. Starting with a linear model with time-varying seasonal dummy coefficients, it appears necessary to have two structural change components. In the resulting TV-AR model, linearity of the lagged autoregressive parameters is rejected by the diagnostic tests. Accounting for this by including a nonlinear component with $\Delta_4 y_{t-1}$ as transition variable, we finally obtain the specification

$$\begin{aligned}\Delta y_t = & \underset{(0.077)}{0.072}\,\Delta y_{t-2} + \underset{(0.38)}{0.47}\,\Delta y_{t-3} - \underset{(0.64)}{1.70}\,\Delta y_{t-4} - \underset{(0.59)}{2.30}\,\Delta y_{t-5} + \underset{(0.070)}{0.032}\,\Delta y_{t-7} - \underset{(0.56)}{2.20}\,\Delta y_{t-10}\\ & - \underset{(1.71)}{0.80}\,D_{1,t} - \underset{(1.97)}{0.60}\,D_{2,t} + \underset{(1.27)}{1.18}\,D_{3,t} + \underset{(0.88)}{3.78}\,D_{4,t} + (-\underset{(3.13)}{6.02}\,D_{1,t} + \underset{(4.15)}{7.15}\,D_{2,t}\\ & - \underset{(2.42)}{3.57}\,D_{3,t} + \underset{(1.58)}{0.061}\,D_{4,t}) \times G_1(t^*;\gamma_1,c_1)\\ & + (\underset{(13.9)}{13.4}\,D_{1,t} - \underset{(22.3)}{22.4}\,D_{2,t} + \underset{(12.3)}{12.0}\,D_{3,t} - \underset{(7.54)}{7.55}\,D_{4,t}) \times G_2(t^*;\gamma_2,c_2)\\ & + (\underset{(0.080)}{0.50}\,\Delta y_{t-1} - \underset{(0.38)}{0.47}\,\Delta y_{t-3} + \underset{(0.64)}{1.70}\,\Delta y_{t-4} + \underset{(0.59)}{2.30}\,\Delta y_{t-5} - \underset{(0.074)}{0.089}\,\Delta y_{t-8}\\ & + \underset{(0.070)}{0.037}\,\Delta y_{t-9} + \underset{(0.56)}{2.21}\,\Delta y_{t-10} + \underset{(0.061)}{0.19}\,\Delta y_{t-11}) \times G_3(\Delta_4 y_{t-1};\gamma_3,c_3) + \widehat{\varepsilon}_t, \end{aligned} \qquad \text{(A.14)}$$

$$G_1(t^*;\gamma_1,c_1) = (1+\exp\{-\underset{(1.25)}{3.24}\,(t^* - \underset{(0.052)}{0.21})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.15)}$$

$$G_2(t^*;\gamma_2,c_2) = (1+\exp\{-\underset{(1.36)}{1.86}\,(t^* - \underset{(0.25)}{0.96})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.16)}$$

$$G_3(\Delta_4 y_{t-1};\gamma_3,c_3) = (1+\exp\{-\underset{(1.03)}{3.08}\,(\Delta_4 y_{t-1} + \underset{(1.77)}{8.57})/\sigma_{\Delta_4 y_{t-1}}\})^{-1}, \qquad \text{(A.17)}$$

$\widehat{\sigma}_\varepsilon = 1.30$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.72$, SK $= -0.55$, EK $= 0.59$, JB $= 9.29(9.6\text{E}-3)$, $\text{LM}_{\text{SC}}(1) = 2.21(0.14)$, $\text{LM}_{\text{SC}}(4) = 0.86(0.49)$, $\text{LM}_{\text{SC}}(12) = 1.35(0.21)$, ARCH$(1) = 1.01(0.31)$, ARCH$(4) = 3.80(0.43)$, AIC $= -0.41$, BIC $= -0.060$.

Note that the model contains some insignificant lagged first differences. These are retained because removing them results in significant autocorrelation in the residuals.

United Kingdom

The test results for the UK are similar to the ones for France: constancy is rejected for both the seasonal dummy parameters and the lagged autoregressive terms (although rejection is stronger for the deterministic terms). Again it turns out that the timing of these changes is significantly different and, hence, we include two logistic transition functions with time as transition variable, one operating on the seasonal dummy coefficients and the other on the coefficients of the lagged first differences. After deleting insignificant lagged first differences and imposing ± restrictions on the autoregressive parameters and the coefficients for $D_{1,t}$, the final specification is

$$\begin{aligned}\Delta y_t = & -\underset{(0.091)}{0.40}\,\Delta y_{t-4} + \underset{(0.088)}{0.19}\,\Delta y_{t-6} - \underset{(0.088)}{0.19}\,\Delta y_{t-7} - \underset{(0.098)}{0.44}\,\Delta y_{t-9} + \underset{(1.84)}{7.07}\,D_{1,t} \\ & - \underset{(1.15)}{4.98}\,D_{2,t} - \underset{(1.85)}{11.0}\,D_{3,t} + \underset{(1.61)}{13.5}\,D_{4,t} + (\underset{(0.091)}{0.40}\,\Delta y_{t-4} - \underset{(0.068)}{0.10}\,\Delta y_{t-5} \\ & - \underset{(0.088)}{0.19}\,\Delta y_{t-6} + \underset{(0.088)}{0.19}\,\Delta y_{t-7} + \underset{(0.098)}{0.44}\,\Delta y_{t-9}) \times G_1(t^*;\gamma_1,c_1) \\ & + (-\underset{(1.84)}{7.07}\,D_{1,t} + \underset{(1.52)}{0.22}\,D_{2,t} + \underset{(2.24)}{8.47}\,D_{3,t} - \underset{(1.94)}{5.52}\,D_{4,t}) \times G_2(t^*;\gamma_2,c_2) + \widehat{\varepsilon}_t, \end{aligned} \tag{A.18}$$

$$G_1(t^*;\gamma_1,c_1) = (1+\exp\{-\underset{(82.0)}{54.9}\,(t^* - \underset{(0.010)}{0.36})/\sigma_{t^*}\})^{-1}, \tag{A.19}$$

$$G_2(t^*;\gamma_2,c_2) = (1+\exp\{-\underset{(0.67)}{1.78}\,(t^* - \underset{(0.072)}{0.46})/\sigma_{t^*}\})^{-1}, \tag{A.20}$$

$\widehat{\sigma}_\varepsilon = 1.81$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.83$, SK $= 8.7\text{E}-3$, EK $= 1.19$, JB $= 8.58(0.014)$, $\text{LM}_{\text{SC}}(1) = 0.090(0.76)$, $\text{LM}_{\text{SC}}(4) = 0.40(0.80)$, $\text{LM}_{\text{SC}}(12) = 1.00(0.45)$, ARCH$(1) = 0.044(0.83)$, ARCH$(4) = 13.6(8.8\text{E}-3)$, AIC $= -0.29$, BIC $= -0.15$.

United States

For the US, constancy of the seasonal dummy coefficients and linearity of the autoregressive parameters are rejected. Starting with a TV-AR model allowing for changing seasonal parameters only, the hypothesis of linearity of the coefficients of the lagged first differences is still rejected at conventional significance levels, most strongly if $\Delta_4 y_{t-3}$ is used as transition variable. Adding such a nonlinear component and deleting insignificant variables, we arrive at the TV-STAR model

$$
\begin{aligned}
\Delta y_t = &\underset{(0.068)}{0.38}\;\Delta y_{t-1} - \underset{(0.099)}{0.66}\;\Delta y_{t-2} + \underset{(0.065)}{0.19}\;\Delta y_{t-3} - \underset{(0.097)}{0.26}\;\Delta y_{t-4} - \underset{(0.065)}{0.28}\;\Delta y_{t-5} \\
&- \underset{(0.14)}{0.68}\;\Delta y_{t-7} - \underset{(0.14)}{0.44}\;\Delta y_{t-8} - \underset{(0.39)}{0.69}\;D_{1,t} + \underset{(0.42)}{2.51}\;D_{2,t} - \underset{(0.43)}{1.26}\;D_{3,t} + \underset{(0.38)}{1.23}\;D_{4,t} \\
&+ (\underset{(0.099)}{0.66}\;\Delta y_{t-2} + \underset{(0.14)}{0.49}\;\Delta y_{t-4} + \underset{(0.14)}{0.68}\;\Delta y_{t-7} + \underset{(0.14)}{0.44}\;\Delta y_{t-8}) \times G_1(\Delta_4 y_{t-3}; \gamma_1, c_1) \\
&+ (\underset{(0.49)}{1.23}\;D_{1,t} - \underset{(0.42)}{2.51}\;D_{2,t} + \underset{(0.56)}{2.71}\;D_{3,t} - \underset{(0.53)}{2.14}\;D_{4,t}) \times G_2(t^*; \gamma_2, c_2) + \widehat{\varepsilon}_t, \qquad \text{(A.21)}
\end{aligned}
$$

$$
G_1(\Delta_4 y_{t-3}; \gamma_1, c_1) = (1 + \exp\{-\underset{(-)}{500}\;(\Delta_4 y_{t-3} - \underset{(-)}{0.61})/\sigma_{\Delta_4 y_{t-3}}\})^{-1}, \qquad \text{(A.22)}
$$

$$
G_2(t^*; \gamma_2, c_2) = (1 + \exp\{-\underset{(-)}{500}\;(t^* - \underset{(0.16)}{0.44})/\sigma_{t^*}\})^{-1}, \qquad \text{(A.23)}
$$

$\widehat{\sigma}_\varepsilon = 1.18$, $\widehat{\sigma}$(TV-STAR/AR)$= 0.77$, SK $= -0.88$, EK $= 2.87$, JB $= 68.4(1.4\text{E} - 15)$, $\text{LM}_{\text{SC}}(1) = 0.72(0.40)$, $\text{LM}_{\text{SC}}(4) = 1.56(0.19)$, $\text{LM}_{\text{SC}}(12) = 1.03(0.43)$, ARCH$(1) = 0.18(0.68)$, ARCH$(4) = 1.81(0.77)$, AIC $= -0.37$, BIC $= -0.17$.

The autoregressive order is increased to 8 in order to avoid rejections of the diagnostic tests of no remaining nonlinearity in the autoregressive coefficients. The model is made more parsimonious by imposing $\pm$ restrictions on the coefficients of Δy_{t-2}, Δy_{t-7}, Δy_{t-8}, and $D_{2,t}$, which are supported by the data. The skewness and excess kurtosis of the residuals are caused entirely by the observations for 1975.1 and 1980.2, where large negative residuals occur. The diagnostics tests reported in Table 5 show that parameter constancy is rejected for the seasonal dummy coefficients. A model with a second time-varying component did not render plausible estimates, however.

Table 1: Seasonal unit root tests

Country	k	t_1	t_2	F_{34}	Roots	Break date
			HEGY tests			
Canada	8	-2.791	-1.571	6.091^{*}	± 1, $\pm i$	
France	5	-1.899	-0.828	2.051	± 1, $\pm i$	
Germany	7	-2.036	-1.037	4.902	± 1, $\pm i$	
Italy	8	-2.495	-2.588^{*}	3.983	± 1, $\pm i$	
Japan	2	-2.390	-0.806	18.554^{***}	± 1	
United Kingdom	5	-4.283^{***}	-1.486	6.993^{**}	-1	
United States	4	-3.586^{**}	-1.661	18.296^{***}	-1	
			Franses-Vogelsang tests			
Canada	8	-2.626	-2.886	7.762	± 1, $\pm i$	1976.4
France	3	-1.490	-3.000	13.677^{**}	± 1	1984.4
Germany	1	-2.430	-7.354^{***}	18.767^{***}	1	1978.2
Italy	2	-2.719	-3.692^{*}	27.713^{***}	± 1	1976.4
Japan	8	-2.833	-3.370	9.659	± 1, $\pm i$	1992.2
United Kingdom	0	-3.609	-7.079^{***}	64.820^{***}	1	1983.2
United States	2	-3.981^{**}	-5.699^{***}	18.121^{***}	–	1980.1

The upper panel contains results from the seasonal unit root tests of Hylleberg *et al.* (1990) [HEGY], which are based upon the regression

$$\Delta_4 y_t = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{j=1}^{k} \phi_j \Delta_4 y_{t-j} + \varepsilon_t, \tag{24}$$

where

$$\begin{aligned} \mu_t &= \mu_1 D_{1,t} + \mu_2 D_{2,t} + \mu_3 D_{3,t} + \mu_4 + \mu_5 t, \\ y_{1,t} &= (1 + B + B^2 + B^3) y_t, \\ y_{2,t} &= -(1 - B + B^2 - B^3) y_t, \\ y_{3,t} &= -(1 - B^2) y_t, \end{aligned}$$

with B denoting the backshift operator. If $\pi_1 = 0$, a non-seasonal unit root is present in y_t; $\pi_2 = 0$ corresponds with a seasonal unit root -1 at the bi-annual frequency; and $\pi_3 = \pi_4 = 0$ corresponds with a pair of seasonal unit roots $\pm i$ at the annual frequency. One-sided t-tests are used to determine the significance of the π_1 and π_2 coefficients, denoted as t_i, $i = 1, 2$, and an F-test is used for the joint significance of π_3 and π_4, denoted F_{34}. The number of lagged seasonal differences k is determined using the procedure of Ng and Perron (1995): starting with $k_{\max} = 8$ lagged seasonal differences, this number is reduced until the last lag included is significant at the 10% significance level (using asymptotic standard normal critical values).
The lower panel contains results from the seasonal unit root tests of Franses and Vogelsang (1998), which modify the HEGY-tests to allow for a one-time change in the deterministic terms by augmenting the regression (24) with regressors $D_{s,t}\mathtt{I}[t > c]$ and $\Delta_4 D_{s,t}\mathtt{I}[t > c]$, $s = 1, \ldots, 4$, where $\mathtt{I}[A]$ is the indicator function for the event A. The time of the break is chosen to maximize the F-test for the significance of the coefficients of $D_{s,t}\mathtt{I}[t > c]$. The estimated break date is shown in the right-most column. The same data-dependent method for choosing k is used.
Entries marked with *, ** and *** are significant at the 10, 5 and 1% level, respectively.

Table 2: Testing linearity and parameter constancy

Parameters tested	STAR LM$_1$	STAR LM$_2$	STAR LM$_3$	TV-AR LM$_1$	TV-AR LM$_2$	TV-AR LM$_3$	TV-STAR LM$_1$	TV-STAR LM$_2$	TV-STAR LM$_3$
Canada ($\widehat{p}=9$)									
$D_{s,t}, \Delta y_{t-j}$	0.32	0.12	0.15	0.57	0.028	0.018	0.52	–	–
$D_{s,t}$	0.14	0.27	0.71	0.25	1.3E−4	9.4E−5	0.24	4.7E−3	0.13
Δy_{t-j}	0.15	0.43	0.56	0.55	0.18	0.062	0.48	–	–
France ($\widehat{p}=8$)									
$D_{s,t}, \Delta y_{t-j}$	5.0E−3	0.18	0.58	2.5E−3	7.4E−4	2.7E−4	0.11	–	–
$D_{s,t}$	0.019	7.1E−3	0.046	3.4E−4	1.3E−5	1.8E−6	5.2E−4	3.6E−3	6.6E−4
Δy_{t-j}	0.062	0.13	0.42	2.9E−3	1.3E−4	4.0E−4	0.012	0.10	–
Germany ($\widehat{p}=7$)									
$D_{s,t}, \Delta y_{t-j}$	0.011	0.030	0.068	0.021	4.5E−3	6.5E−3	0.030	–	–
$D_{s,t}$	0.074	0.12	0.50	0.018	1.7E−3	4.9E−4	0.11	0.094	0.14
Δy_{t-j}	8.3E−4	0.027	0.013	0.015	0.050	0.15	0.028	0.12	–
Italy ($\widehat{p}=11$)									
$D_{s,t}, \Delta y_{t-j}$	0.20	0.20	–	0.030	7.5E−3	0.019	0.43	–	–
$D_{s,t}$	0.19	0.25	0.28	0.068	1.2E−4	1.8E−4	0.43	0.11	0.43
Δy_{t-j}	0.057	0.32	0.82	0.19	8.7E−3	0.053	0.14	–	–
Japan ($\widehat{p}=5$)									
$D_{s,t}, \Delta y_{t-j}$	0.010	7.9E−3	4.2E−3	4.4E−3	1.1E−5	1.1E−4	1.1E−3	0.57	–
$D_{s,t}$	0.029	0.010	3.1E−4	0.053	5.8E−6	2.8E−5	7.7E−4	2.9E−5	4.3E−3
Δy_{t-j}	0.025	6.7E−3	0.016	0..015	1.5E−3	0.011	1.5E−3	2.2E−3	0.053
United Kingdom ($\widehat{p}=9$)									
$D_{s,t}, \Delta y_{t-j}$	0.073	0.095	0.40	0.015	0.070	0.028	0.33	–	–
$D_{s,t}$	0.040	0.11	0.078	5.1E−3	5.9E−4	1.9E−3	0.029	1.2E−3	1.7E−3
Δy_{t-j}	0.070	0.15	0.13	0.012	0.038	0.020	0.11	–	–
United States ($\widehat{p}=7$)									
$D_{s,t}, \Delta y_{t-j}$	0.014	0.010	2.4E−3	0.13	0.17	0.56	0.11	–	–
$D_{s,t}$	0.028	0.16	0.17	3.4E−3	5.0E−3	0.010	0.044	0.19	0.26
Δy_{t-j}	4.1E−3	0.012	0.017	0.87	0.40	0.80	0.36	0.56	–

The table contains p-values of F-variants of the LM$_k$, $k = 1, 2, 3$, tests of linearity and parameter constancy within the TV-STAR model (1) with $w_t = \sum_{=1}^{r} \alpha_i \Delta_4 y_{t-i}$. The null hypotheses of the different tests are linearity conditional on parameter constancy [STAR], constancy conditional on linearity [TV], and linearity and constancy [TV-STAR]. The tests involving either the seasonal dummies ($D_{s,t}$) or the lagged growth rates (Δy_{t-j}) only are performed conditional on assuming that the remaining parameters enter linearly and with constant parameters. A dash indicates that the test could not be computed due to a shortage in degrees of freedom.

Table 3: Summary of test results

Parameters tested	p-value	STAR LM_1	STAR LM_2	STAR LM_3	TV-AR LM_1	TV-AR LM_2	TV-AR LM_3	TV-STAR LM_1	TV-STAR LM_2	TV-STAR LM_3
					Linear null model					
	> 0.05	3	3	4	2	3	1	5	1	0
$D_{s,t}$, Δy_{t-j}	$0.01-0.05$	3	2	0	3	0	3	1	0	0
	< 0.01	1	1	2	2	4	3	1	0	0
	> 0.05	3	5	5	3	0	0	3	3	4
$D_{s,t}$	$0.01-0.05$	4	1	1	1	0	1	2	0	0
	< 0.01	0	1	1	3	7	6	2	4	3
	> 0.05	4	4	4	3	2	4	4	3	1
Δy_{t-j}	$0.01-0.05$	1	2	3	3	2	2	2	0	0
	< 0.01	2	1	0	1	3	1	1	1	0
					Nonlinear null model					
	> 0.05	4	4	6	5	2	2	6	5	5
$D_{s,t}$	$0.01-0.05$	3	3	0	1	2	1	0	0	2
	< 0.01	0	0	1	1	3	4	1	2	0

The Table reports the number of series for which the p-values of the tests of linearity and parameter constancy, shown in Tables 2 and 4, fall within the indicated category. The null hypotheses of the different tests are linearity conditional on parameter constancy [STAR], constancy conditional on linearity [TV], and linearity and constancy [TV-STAR].

Table 4: Testing linearity and parameter constancy in STAR models

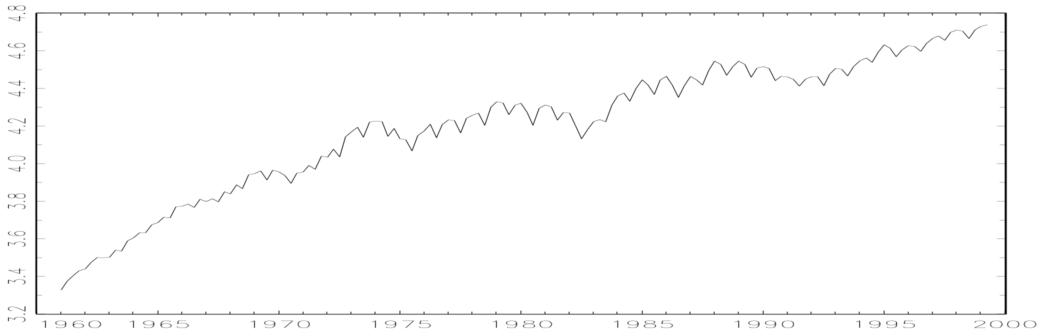
Transition variable	STAR			TV-AR			TV-STAR		
	LM_1	LM_2	LM_3	LM_1	LM_2	LM_3	LM_1	LM_2	LM_3
Canada									
$s_t = t^*$	0.020	0.15	0.27	0.35	0.022	0.072	0.19	0.47	0.21
France									
$s_t = t^*$	0.033	0.043	0.24	0.29	0.052	0.039	0.061	0.11	0.067
Germany									
$s_t = \Delta_4 y_{t-1}$	0.35	0.059	0.19	0.043	1.7E−3	3.5E−4	0.21	0.061	0.35
Italy									
$s_t = t^*$	0.63	0.57	0.51	0.90	3.9E−3	9.7E−3	0.87	0.10	0.32
Japan									
$s_t = t^*$	0.083	0.021	5.3E−3	5.9E−3	1.1E−6	1.3E−6	5.1E−3	1.8E−4	0.012
United Kingdom									
$s_t = t^*$	0.040	0.030	0.11	0.15	0.023	3.2E−3	0.16	8.2E−3	0.029
United States									
$s_t = \Delta_4 y_{t-3}$	0.16	0.36	0.58	0.11	0.081	0.11	0.12	0.30	0.51

The table contains p-values of F-variants of the LM_k, $k = 1, 2, 3$, tests of linearity of the seasonal pattern within the STAR model (6) with $w_t = \sum_{=1}^{r} \alpha_i \Delta_4 y_{t-i}$. The null hypotheses of the different tests are linearity conditional on parameter constancy [STAR], constancy conditional on linearity [TV], and linearity and constancy [TV-STAR].

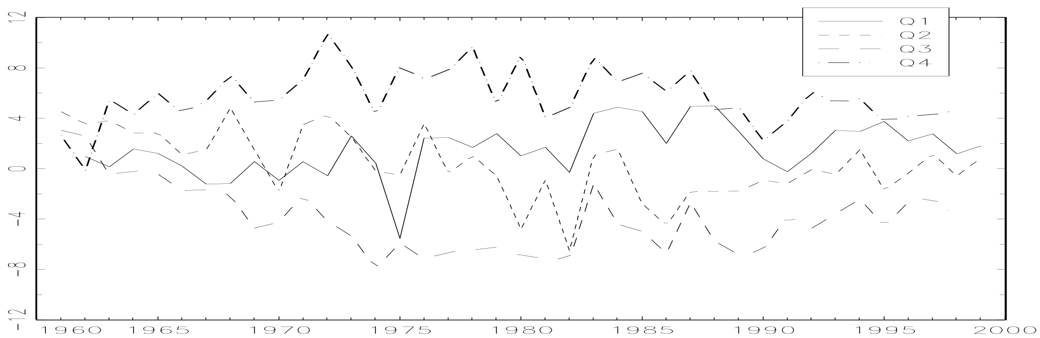
Table 5: Diagnostic tests of parameter constancy and no remaining nonlinearity in TV-STAR models

Transition	$D_{s,t}$			Δy_{t-j}			σ^2_ε		
variable	LM_1	LM_2	LM_3	LM_1	LM_2	LM_3	LM_1	LM_2	LM_3
Canada									
t	0.73	0.63	0.93	0.81	0.85	0.91	0.50	0.039	0.084
$\Delta_4 y_{t-1}$	0.54	0.57	0.54	0.40	0.30	0.44	0.49	0.69	0.59
$\Delta_4 y_{t-2}$	0.56	0.26	0.15	0.21	0.40	0.48	0.65	0.90	0.74
$\Delta_4 y_{t-3}$	0.84	0.76	0.84	0.35	0.58	0.34	0.72	0.76	0.72
$\Delta_4 y_{t-4}$	0.51	0.70	0.78	0.41	0.58	0.63	0.73	0.89	0.38
France									
t	0.83	0.31	0.55	0.74	0.43	0.71	0.87	0.46	0.64
$\Delta_4 y_{t-1}$	0.13	0.36	0.65	0.35	0.42	0.27	0.82	0.53	0.70
$\Delta_4 y_{t-2}$	0.35	0.69	0.76	0.23	0.70	0.62	0.61	0.52	0.42
$\Delta_4 y_{t-3}$	0.87	0.72	0.60	0.95	0.94	0.80	0.73	0.83	0.77
$\Delta_4 y_{t-4}$	0.96	0.96	0.92	0.99	0.79	0.73	0.72	0.86	0.92
Germany									
t	0.79	0.59	0.27	0.72	0.97	0.97	0.45	0.19	0.35
$\Delta_4 y_{t-1}$	0.73	0.36	0.41	0.20	0.46	0.71	0.076	0.20	0.25
$\Delta_4 y_{t-2}$	0.34	0.17	0.19	0.41	0.33	0.59	0.26	0.53	0.74
$\Delta_4 y_{t-3}$	0.64	0.15	0.47	0.67	0.71	0.84	0.37	0.59	0.74
$\Delta_4 y_{t-4}$	0.79	0.58	0.83	0.53	0.58	0.87	0.28	0.47	0.54
Italy									
t	0.31	0.30	0.45	0.40	0.81	0.87	0.40	0.22	0.16
$\Delta_4 y_{t-1}$	0.62	0.74	0.46	0.14	0.29	0.55	0.37	0.26	0.32
$\Delta_4 y_{t-2}$	0.83	0.90	0.89	0.21	0.46	0.48	0.17	0.26	0.28
$\Delta_4 y_{t-3}$	0.58	0.64	0.60	0.24	0.58	0.81	0.12	0.28	0.39
$\Delta_4 y_{t-4}$	0.75	0.89	0.83	0.64	0.86	0.63	0.36	0.63	0.82
Japan									
t	0.84	0.76	0.28	0.95	0.99	0.99	0.91	0.98	0.99
$\Delta_4 y_{t-1}$	0.93	0.80	0.37	0.95	0.81	0.82	0.76	0.28	0.37
$\Delta_4 y_{t-2}$	0.39	0.38	0.14	0.80	0.78	0.82	0.26	0.39	0.52
$\Delta_4 y_{t-3}$	0.44	0.18	0.50	0.78	0.83	0.69	0.16	0.36	0.56
$\Delta_4 y_{t-4}$	0.64	0.81	0.84	0.95	0.56	0.66	0.12	0.30	0.44
United Kingdom									
t	0.34	0.22	0.10	0.29	0.24	0.16	0.32	0.041	0.094
$\Delta_4 y_{t-1}$	0.25	0.13	0.22	0.26	0.55	0.41	0.49	0.76	0.89
$\Delta_4 y_{t-2}$	0.99	0.88	0.78	0.94	0.88	0.72	0.91	0.73	0.88
$\Delta_4 y_{t-3}$	0.96	0.90	0.95	0.80	0.63	0.54	0.78	0.43	0.62
$\Delta_4 y_{t-4}$	0.72	0.85	0.87	0.72	0.39	0.65	0.26	0.46	0.57
United States									
t	4.8E-4	2.2E-3	5.7E-3	0.86	0.42	0.28	0.33	0.067	0.095
$\Delta_4 y_{t-1}$	0.66	0.88	0.97	0.81	0.90	0.63	0.052	0.090	0.13
$\Delta_4 y_{t-2}$	0.48	0.75	0.93	0.74	0.91	0.64	0.20	0.41	0.47
$\Delta_4 y_{t-3}$	0.16	0.49	0.59	0.31	0.46	0.26	0.46	0.76	0.87
$\Delta_4 y_{t-4}$	0.23	0.61	0.71	0.62	0.63	0.80	0.63	0.88	0.90

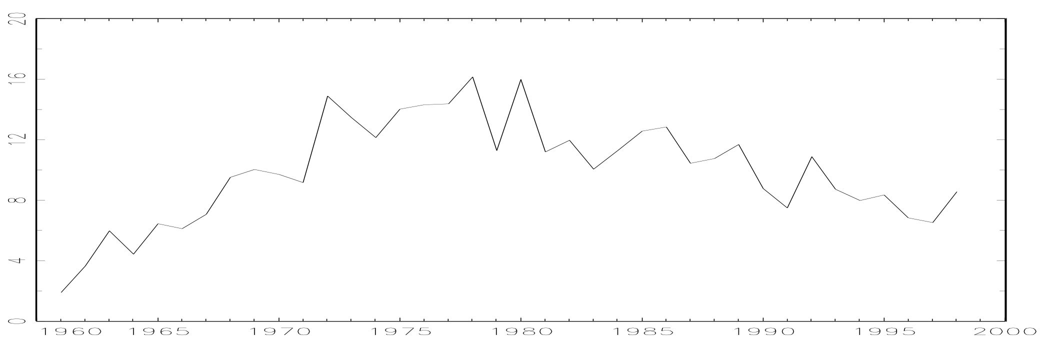
Diagnostic tests of parameter constancy and no remaining nonlinearity of seasonal dummy coefficients (columns headed $D_{s,t}$), autoregressive parameters (columns headed Δy_{t-j}), and residual variance (columns headed σ^2_ε) in estimated TV-STAR models.



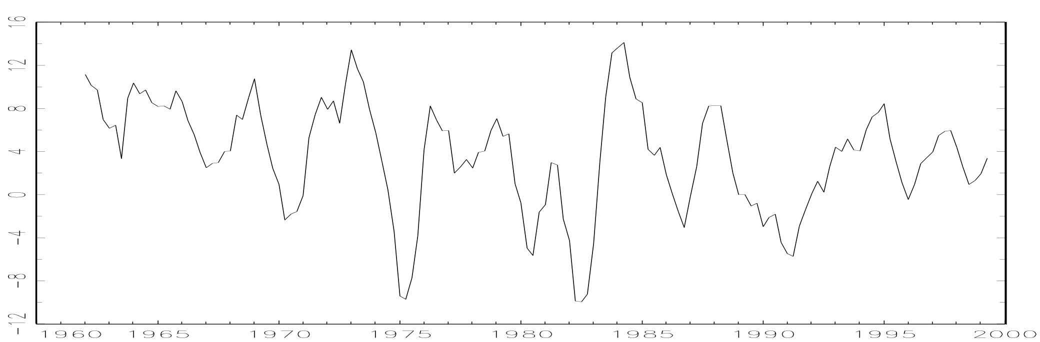
(a) Level



(b) First difference per quarter

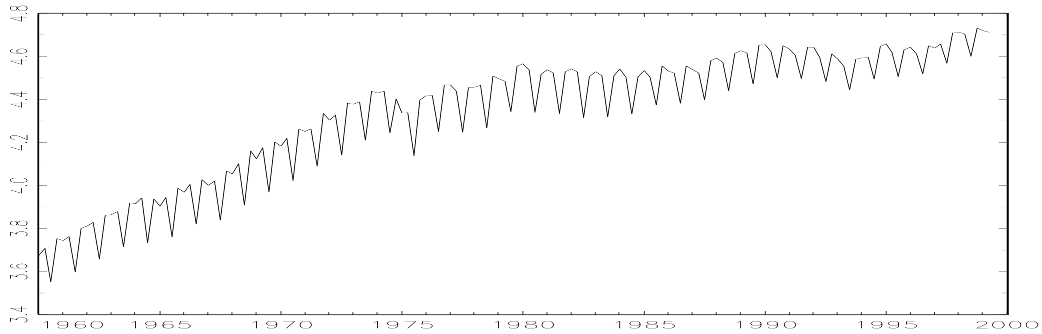


(c) Range

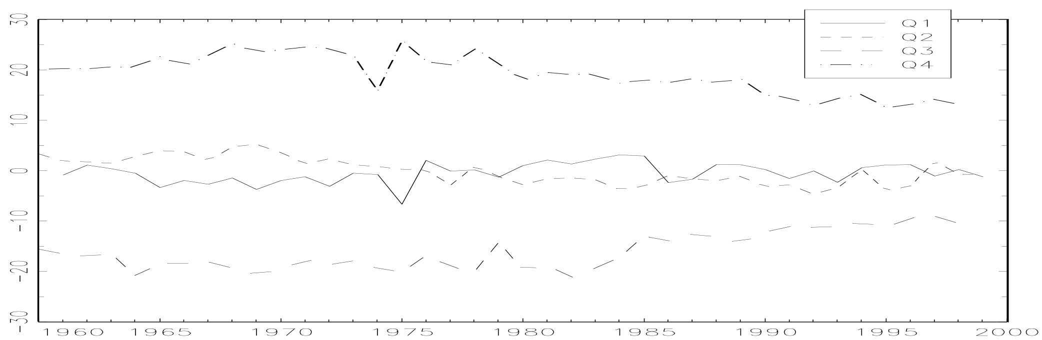


(d) Seasonal difference

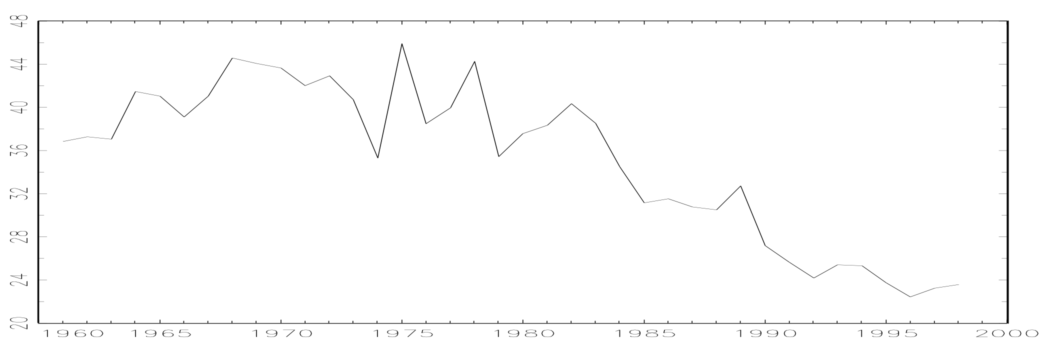
Figure 1: Industrial production Canada



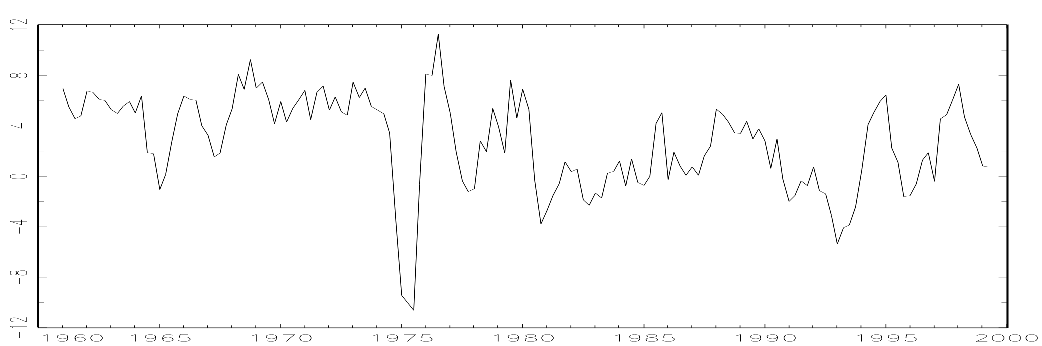
(a) Level



(b) First difference per quarter

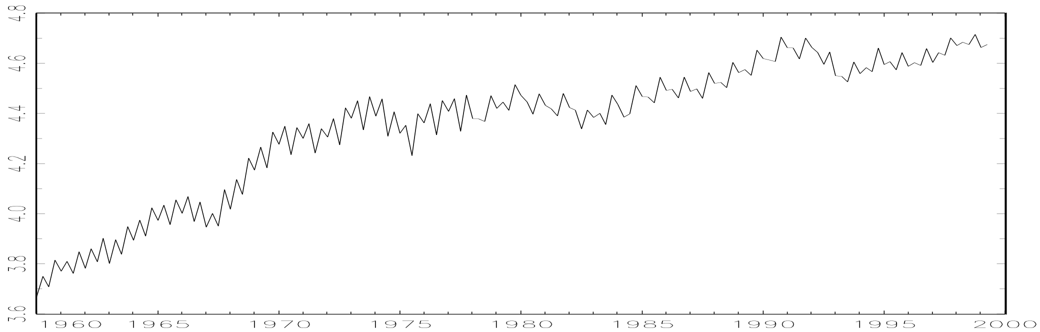


(c) Range

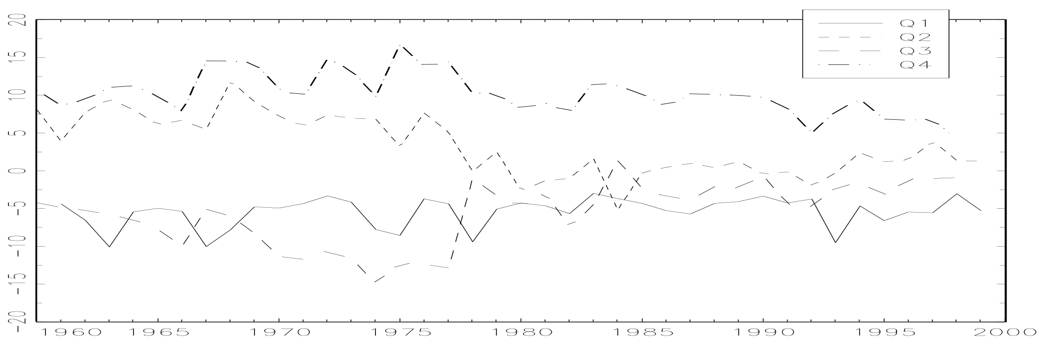


(d) Seasonal difference

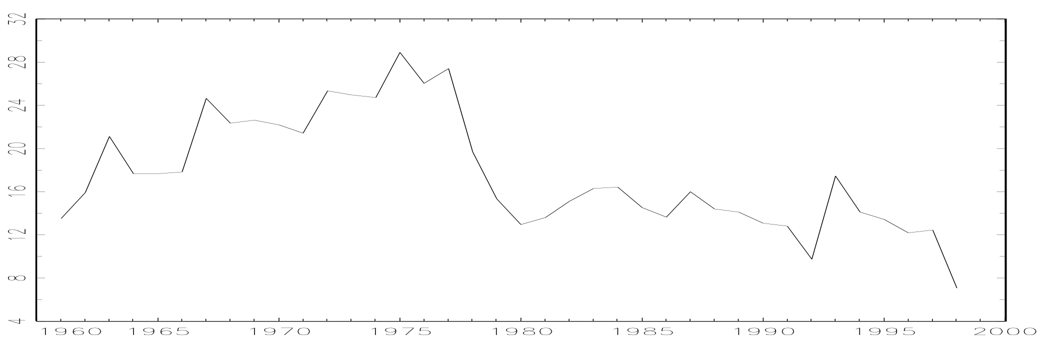
Figure 2: Industrial production France



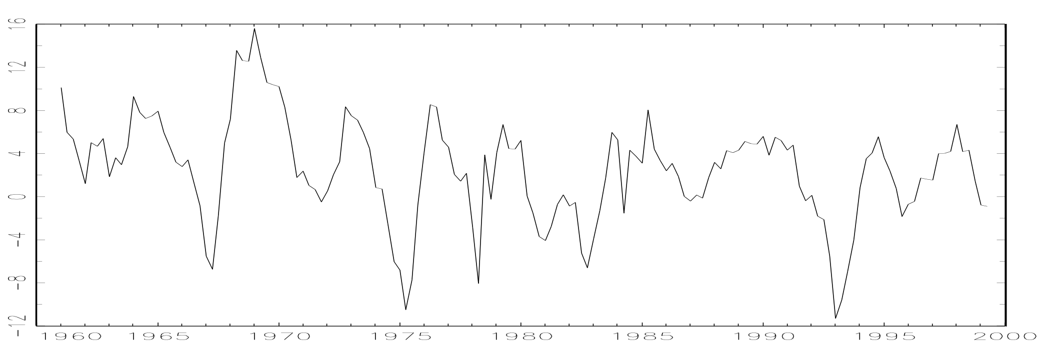
(a) Level



(b) First difference per quarter

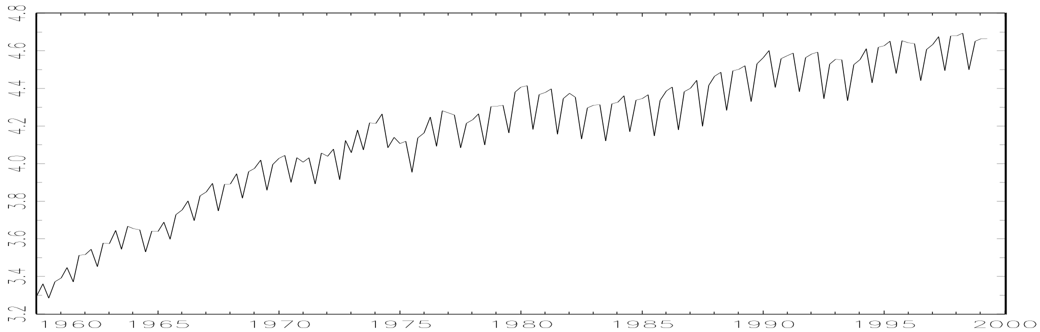


(c) Range

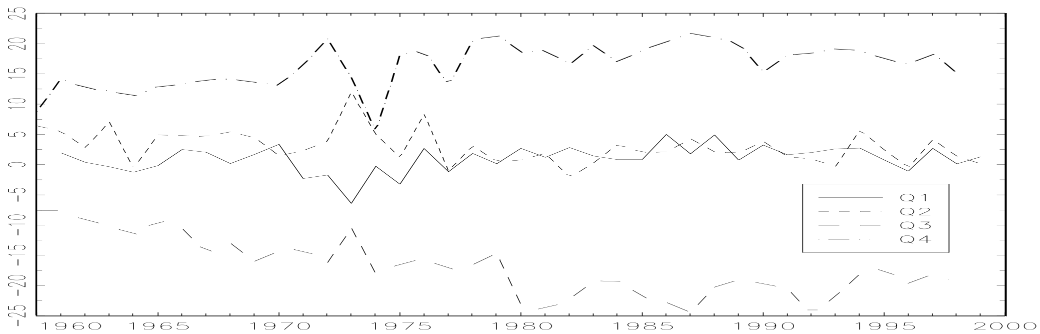


(d) Seasonal difference

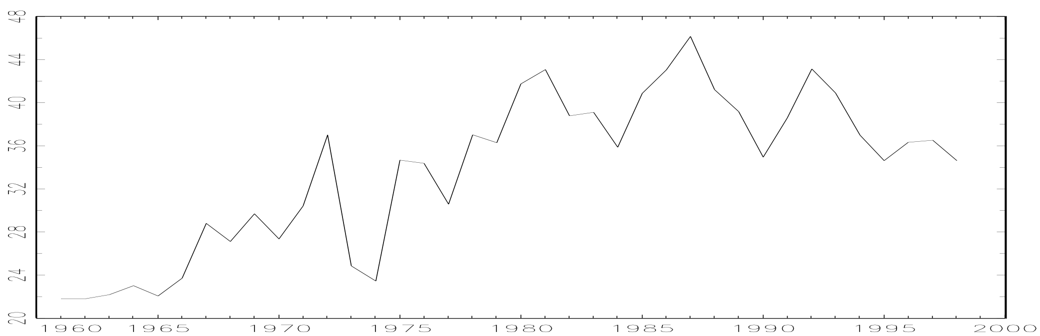
Figure 3: Industrial production Germany



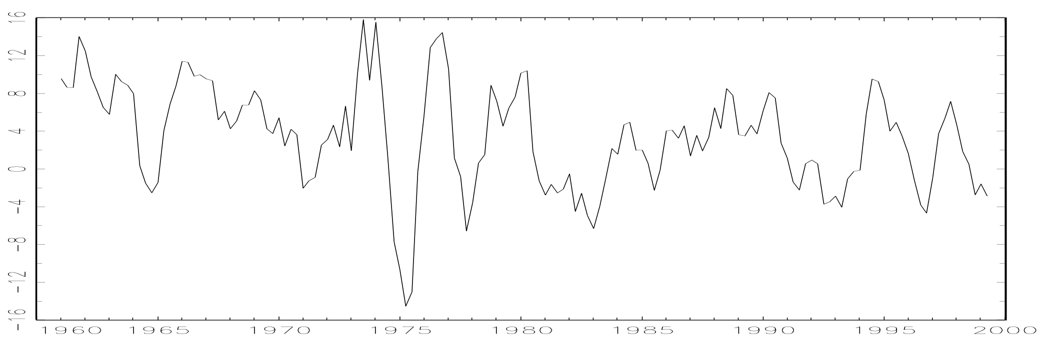
(a) Level



(b) First difference per quarter

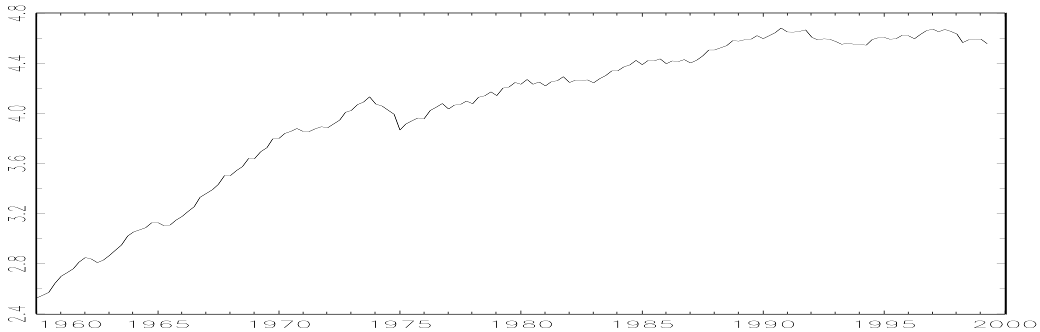


(c) Range

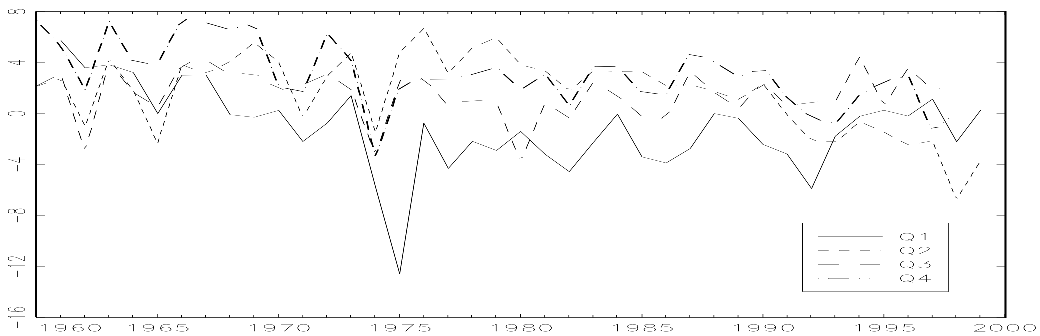


(d) Seasonal difference

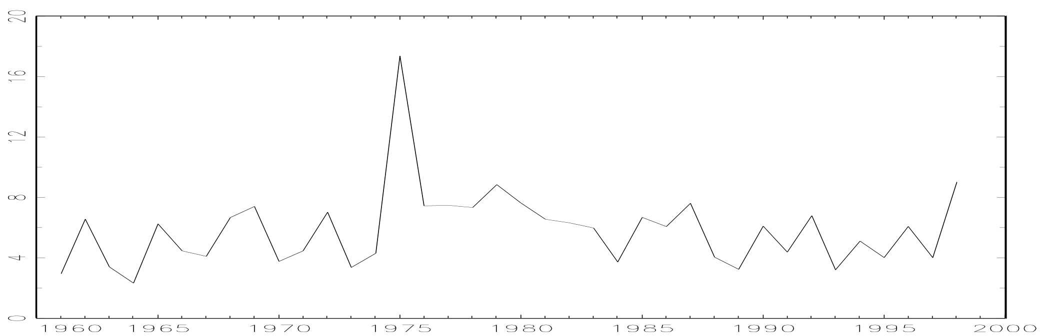
Figure 4: Industrial production Italy



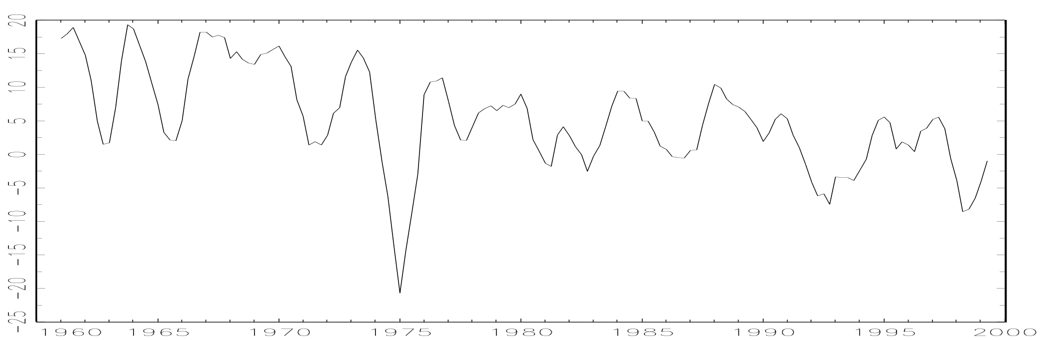
(a) Level



(b) First difference per quarter

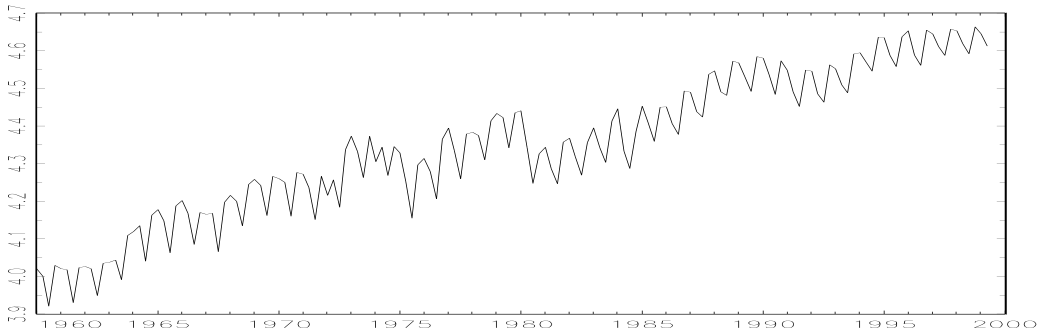


(c) Range

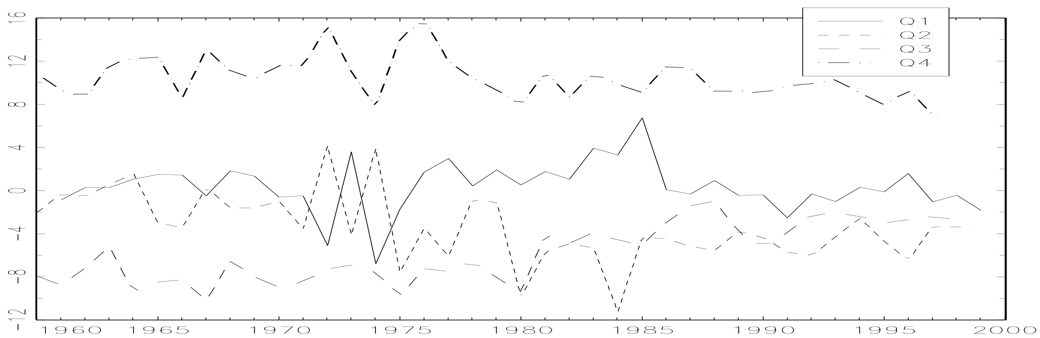


(d) Seasonal difference

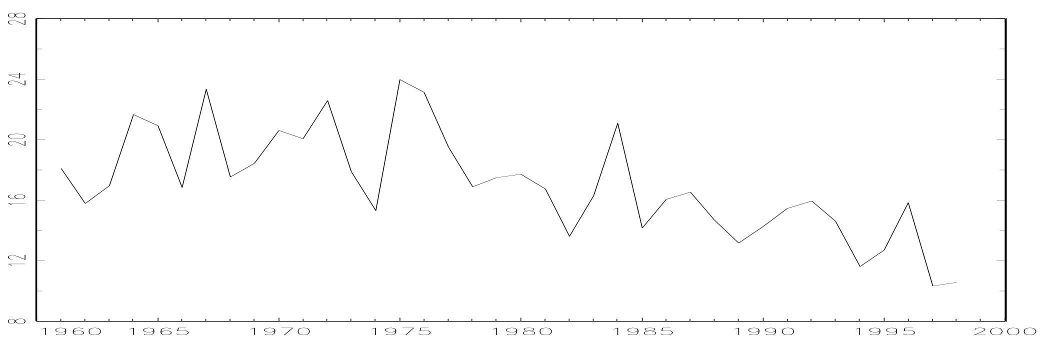
Figure 5: Industrial production Japan



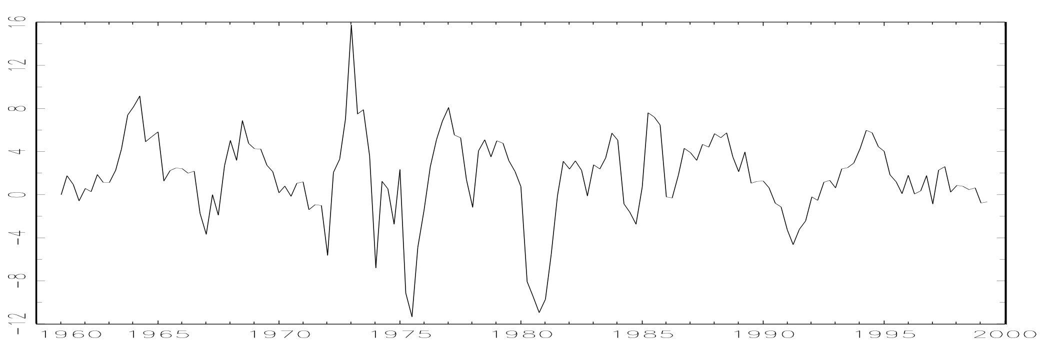
(a) Level



(b) First difference per quarter

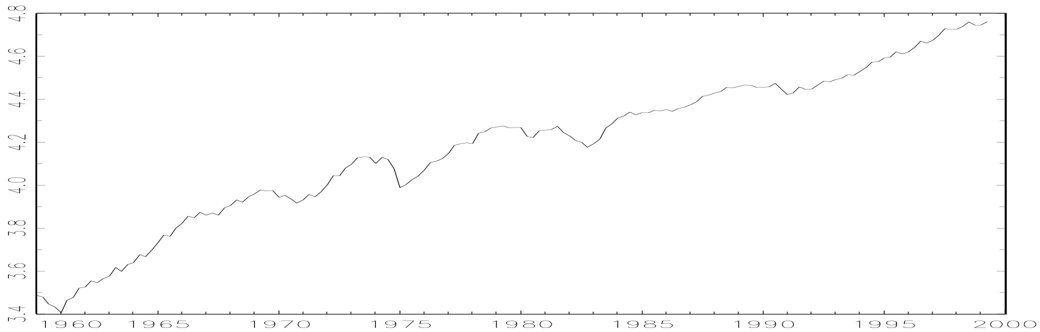


(c) Range

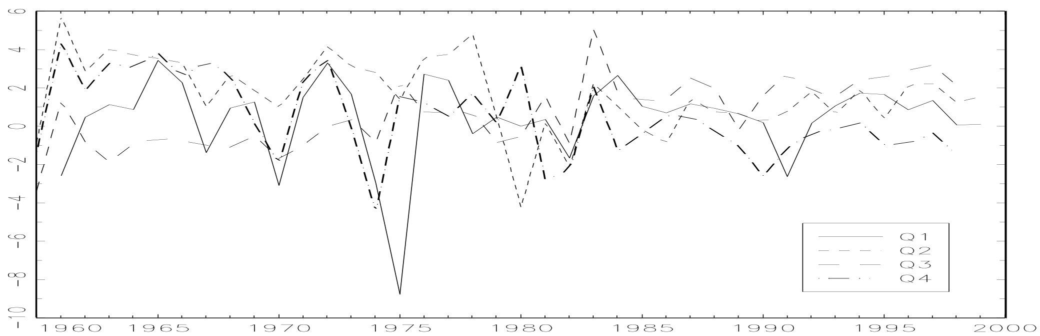


(d) Seasonal difference

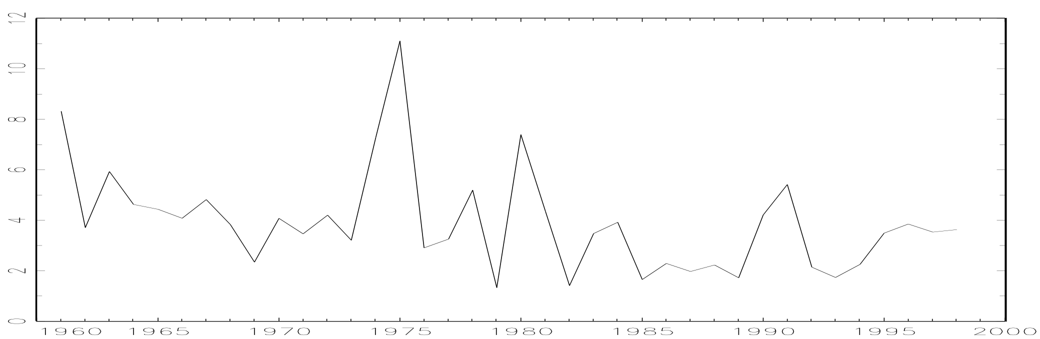
Figure 6: Industrial production United Kingdom



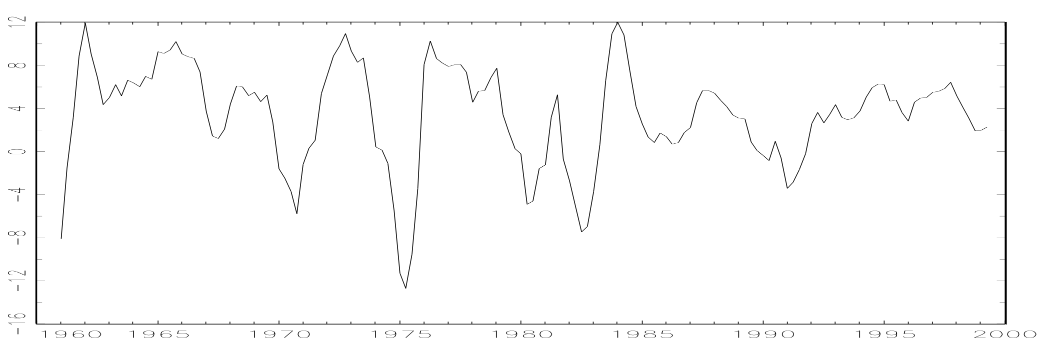
(a) Level



(b) First difference per quarter

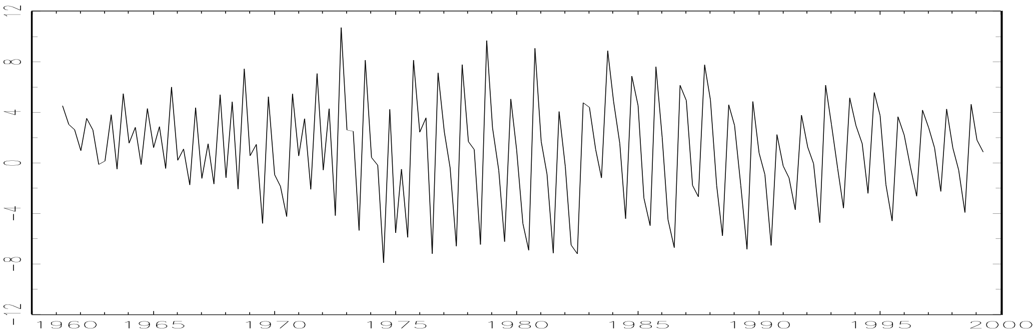


(c) Range

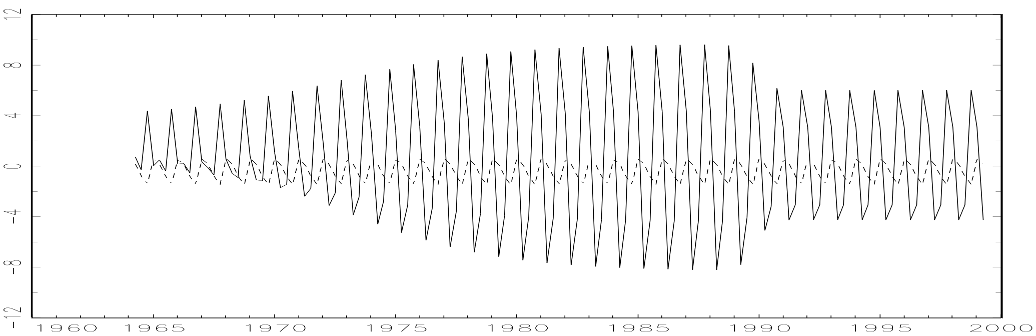


(d) Seasonal difference

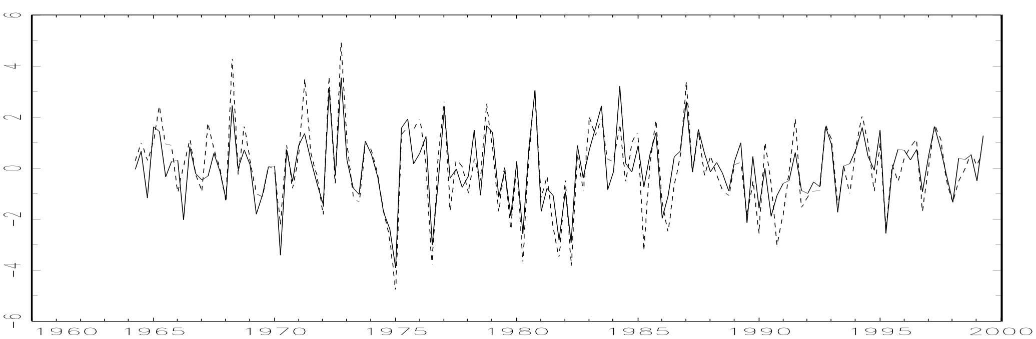
Figure 7: Industrial production United States



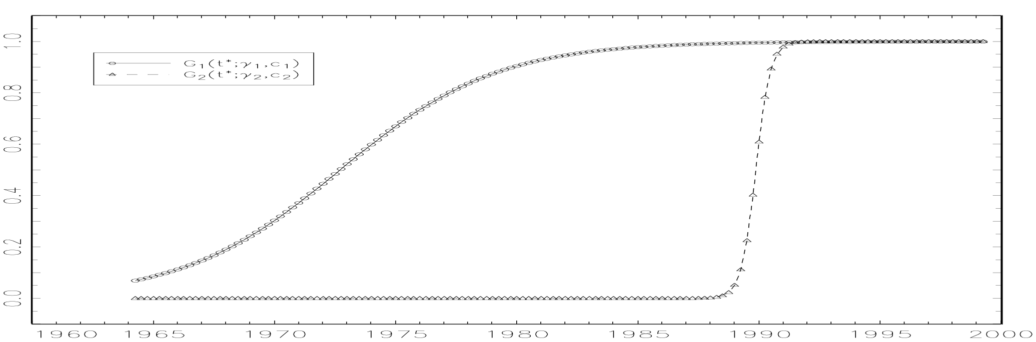
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

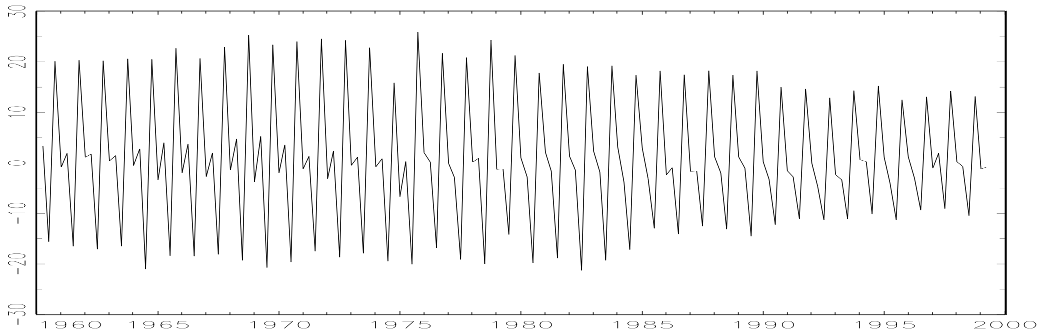


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

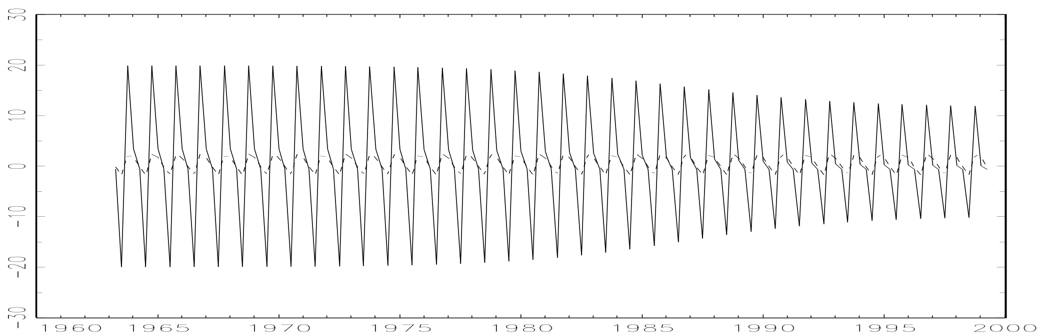


(d) Transition functions in TV-STAR model

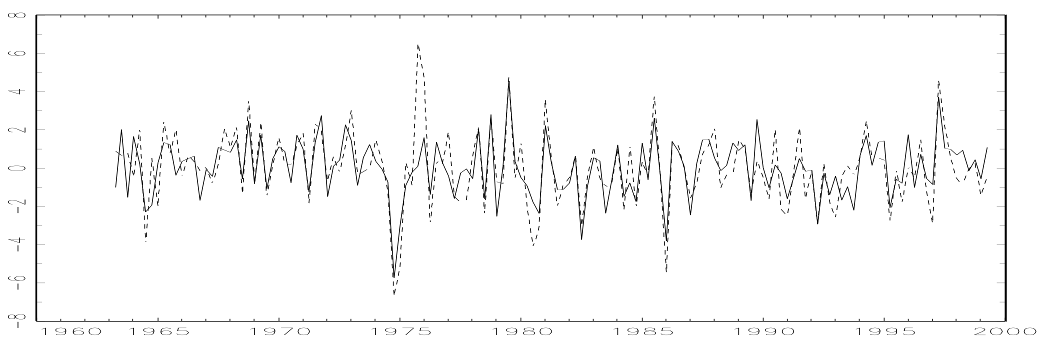
Figure 8: Characteristics of TV-STAR model for Canada



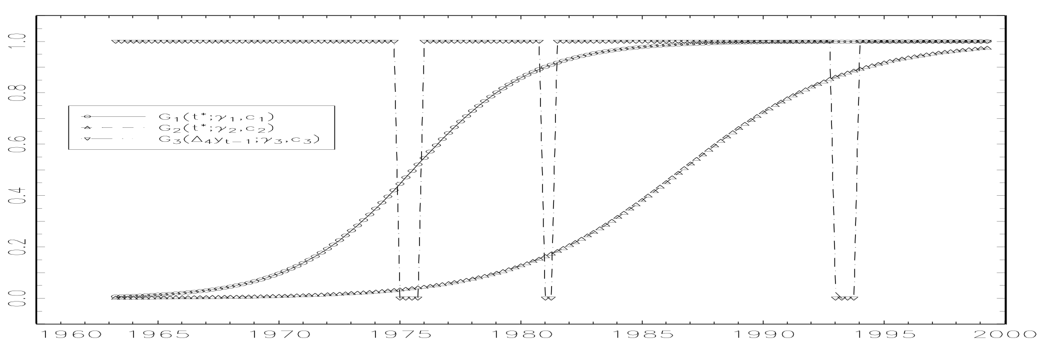
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

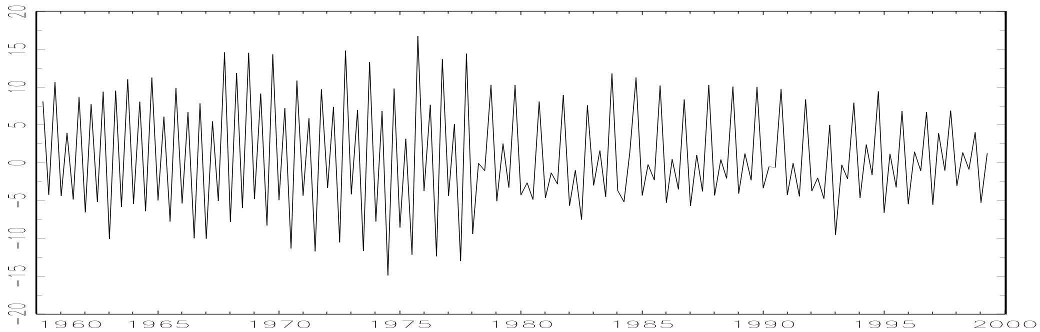


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

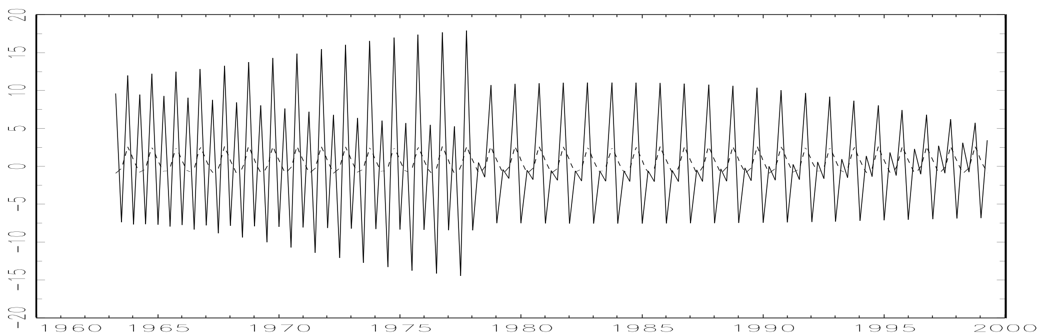


(d) Transition functions in TV-STAR model

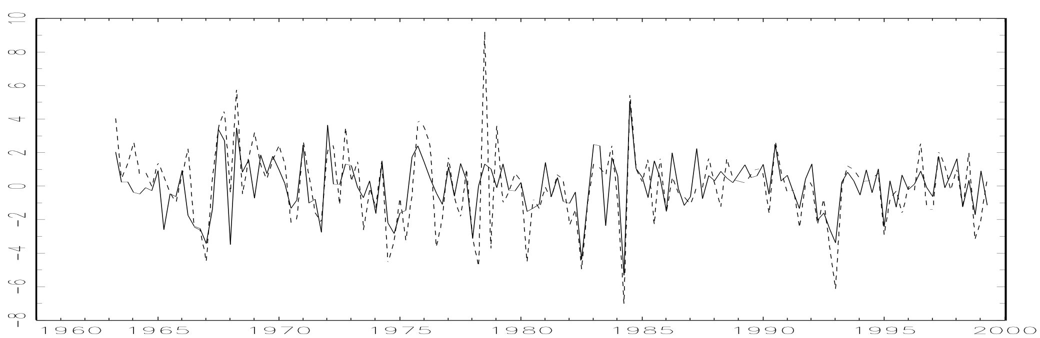
Figure 9: Characteristics of TV-STAR model for France



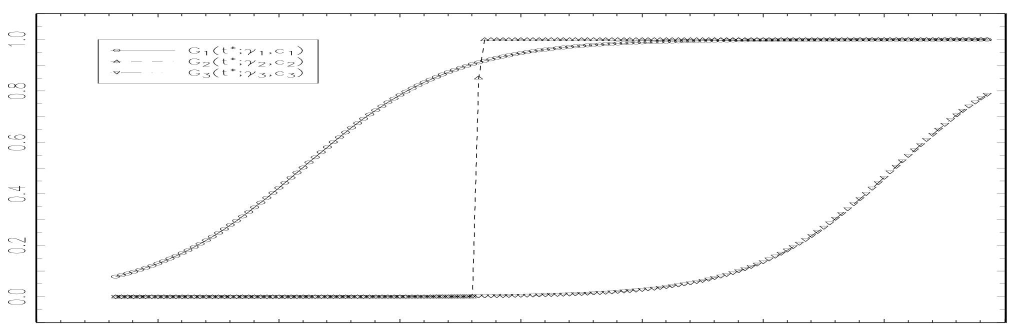
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

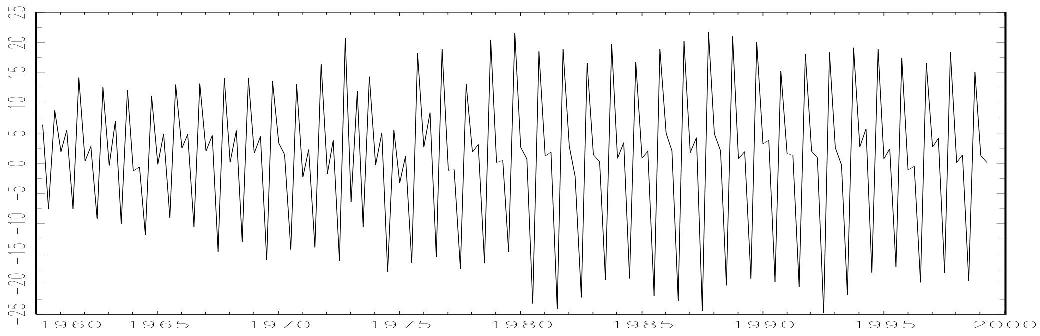


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

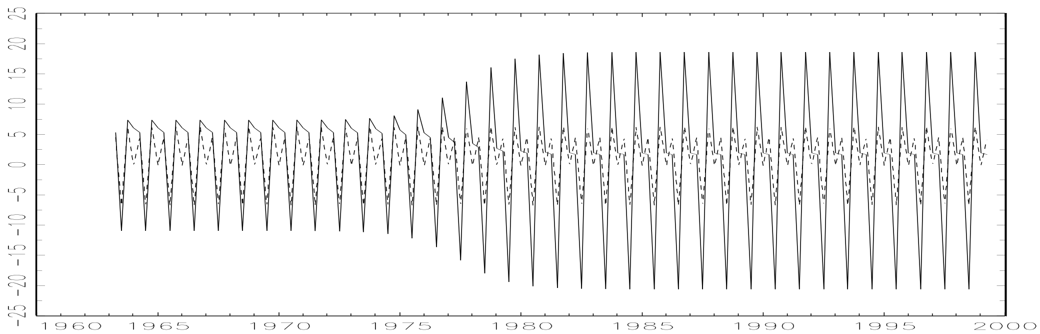


(d) Transition functions in TV-STAR model

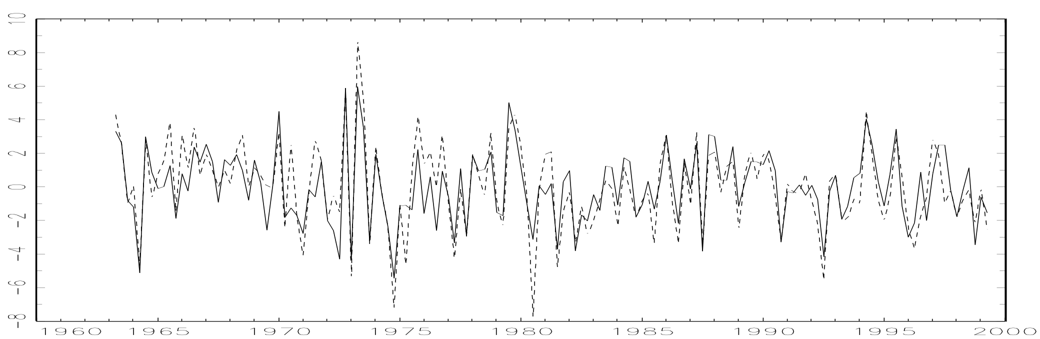
Figure 10: Characteristics of TV-STAR model for Germany



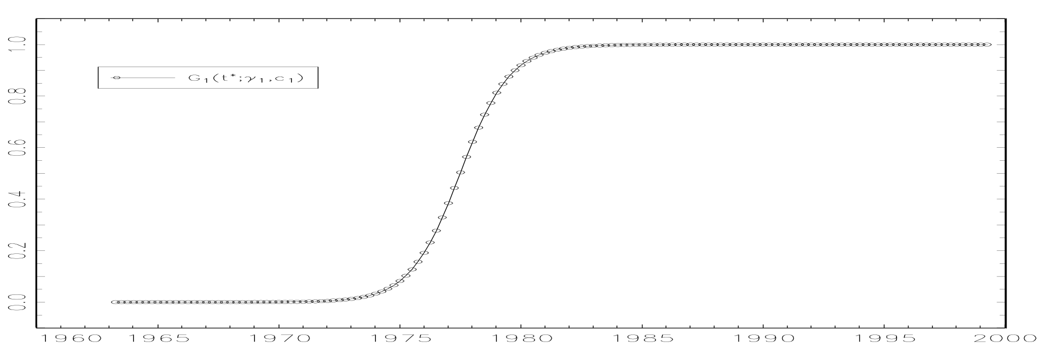
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

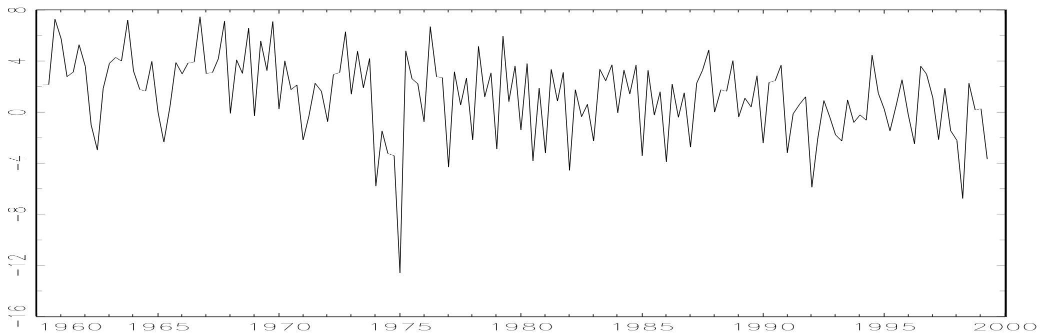


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

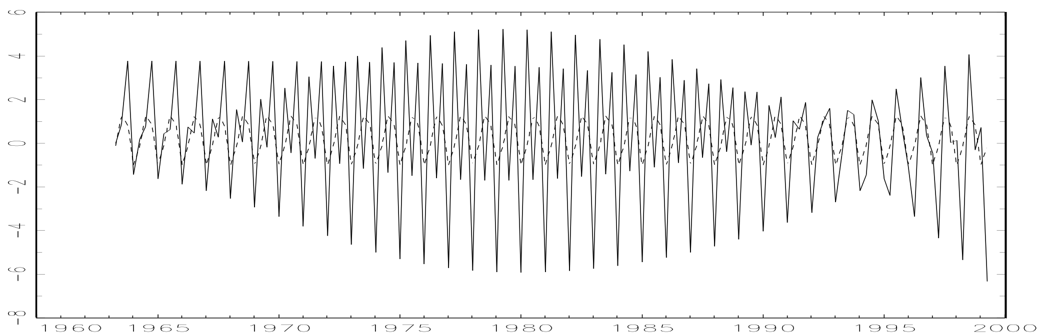


(d) Transition functions in TV-STAR model

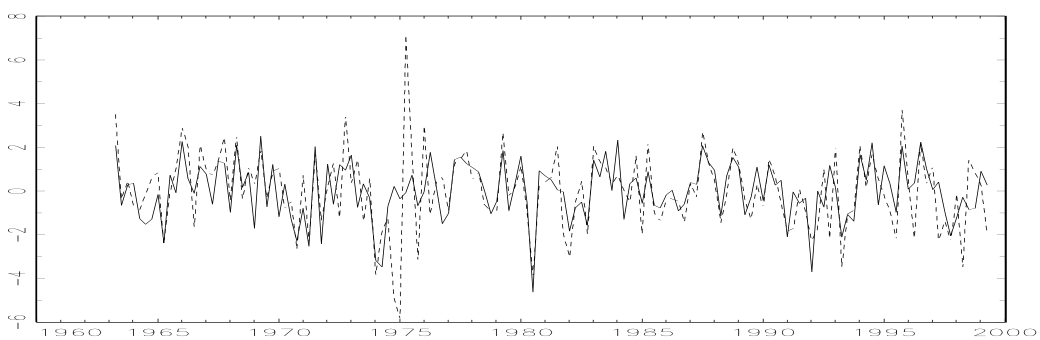
Figure 11: Characteristics of TV-STAR model for Italy



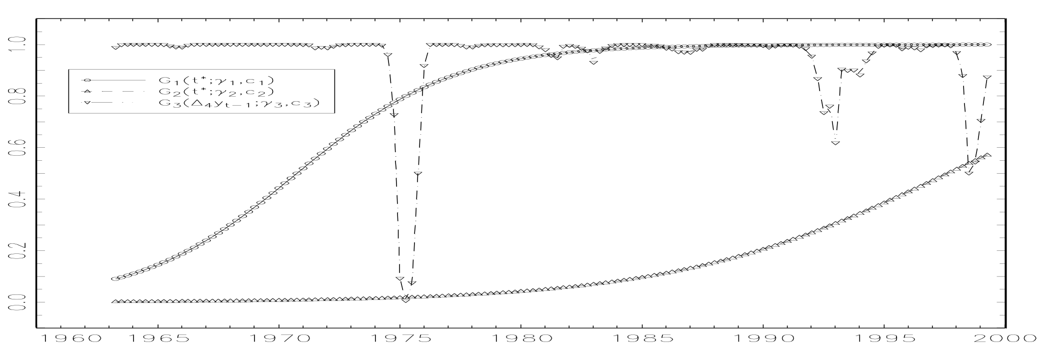
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

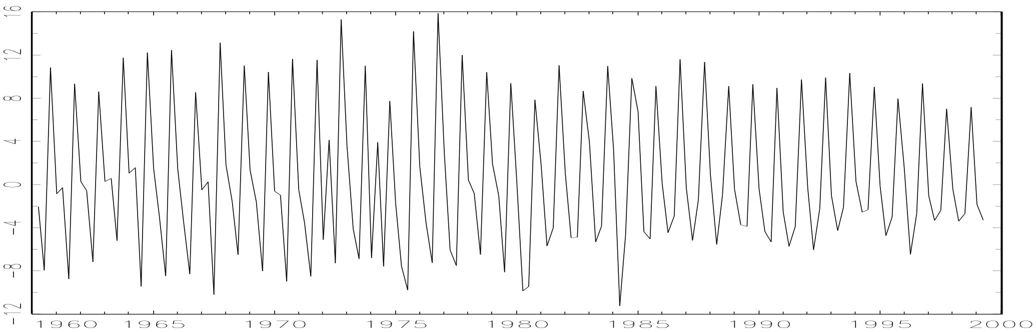


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

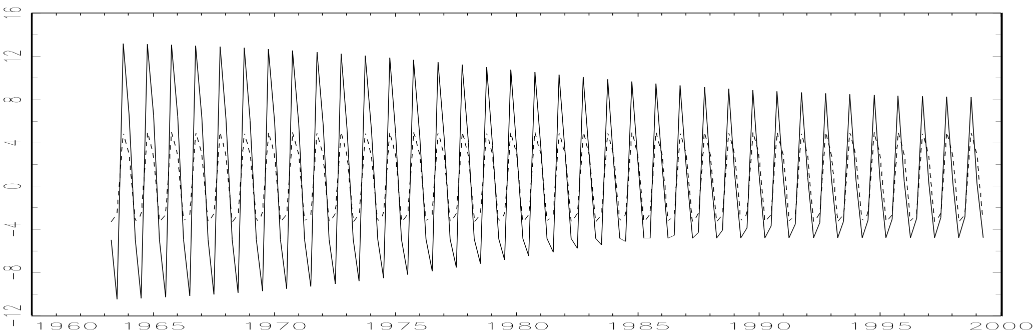


(d) Transition functions in TV-STAR model

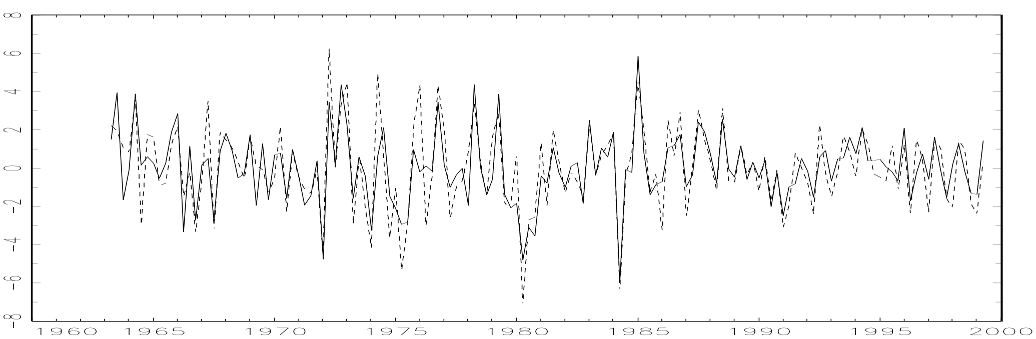
Figure 12: Characteristics of TV-STAR model for Japan



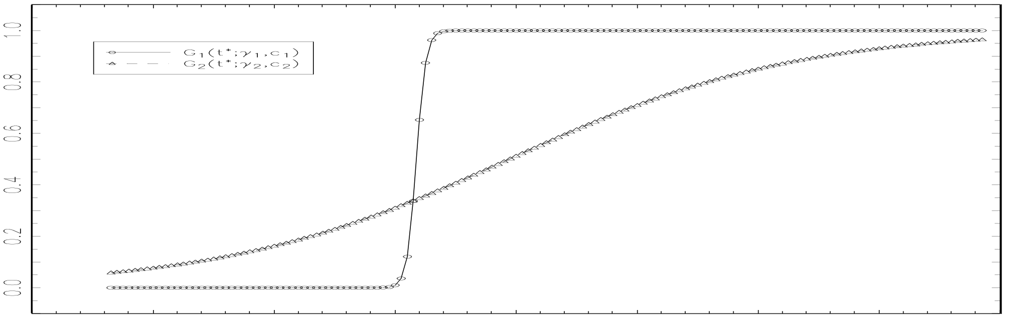
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)

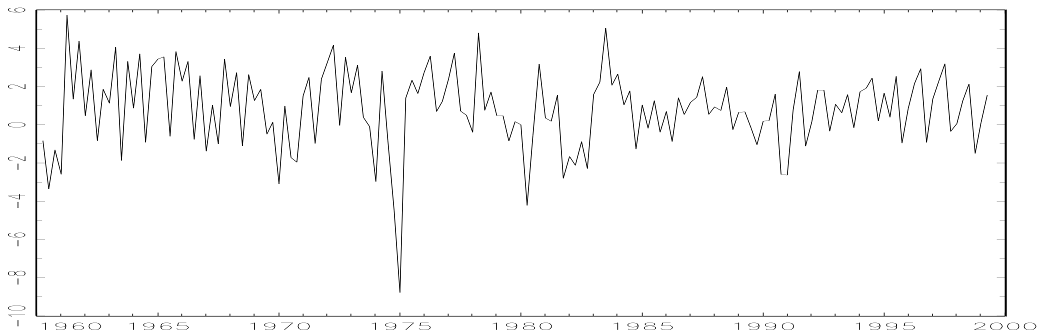


(c) Residuals from AR (dashed line) and TV-STAR model (solid line)

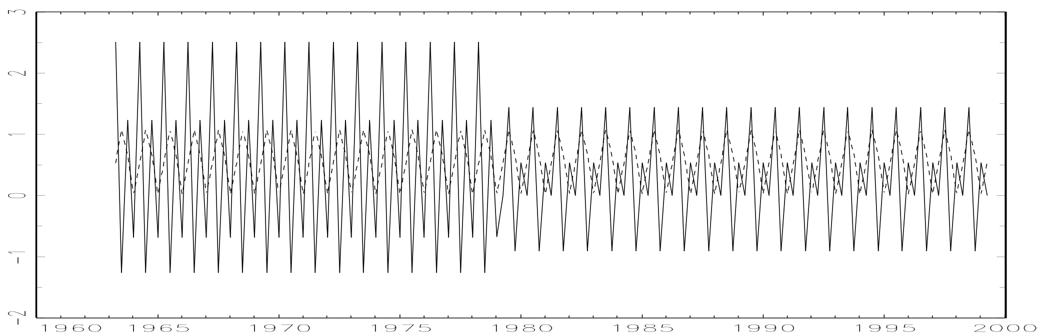


(d) Transition functions in TV-STAR model

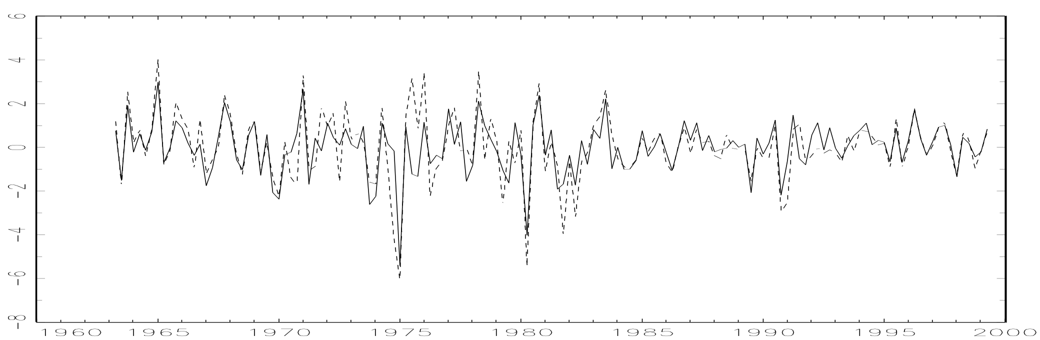
Figure 13: Characteristics of TV-STAR model for United Kingdom



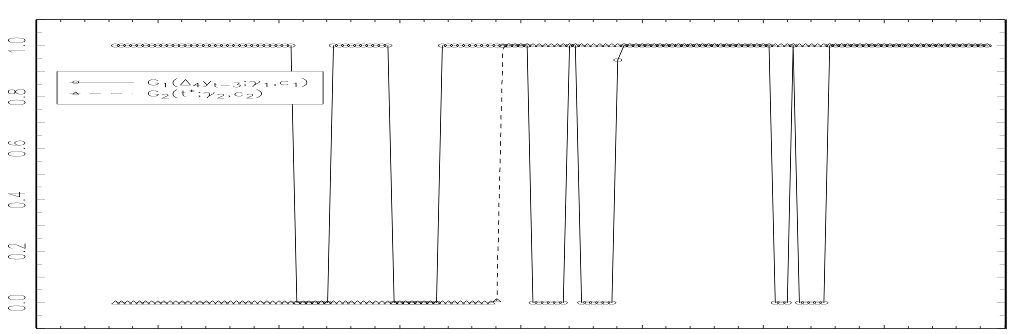
(a) First difference



(b) Seasonal intercepts AR (dashed line) and TV-STAR model (solid line)



(c) Residuals from AR (dashed line) and TV-STAR model (solid line)



(d) Transition functions in TV-STAR model

Figure 14: Characteristics of TV-STAR model for United States