Modelling product returns in inventory control - exploring the validity of general assumptions

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Abstract

The literature on stochastic models for inventory control with product returns commonly makes the following simplifying assumptions: demand and returns are each a homogeneous (compound) Poisson process, and the processes are independent of each other. In this paper we explore the validity of these assumptions by analysing real data on return flows. In addition, we discuss practical implications of our findings and we provide insights on information management for inventory systems with return flows.

Keywords

Inventory Control, Product Returns, Reverse Flows, Data Analysis

I. Introduction

Already for a long time, companies take back products, where products in good condition go back to inventory. Furthermore, environmental consciousness, legal and economic forces have brought more attention to systems with reverse flows, and to its control. Van der Laan et al. [34], Inderfurth & van der Laan [18], and Fleischmann et al. [11] are recent examples of scientific literature on inventory control in case of returns. From a modelling perspective, one of the consequences of reverse flows is the loss of monotonicity of inventory levels between replenishments of new products. That is, the inventory level does not only decrease because of demand but it may also increase in case of returns. Since this makes the analysis much more difficult than traditional inventory control, authors use simplifying assumptions regarding the return process. These assumptions typically are 1) the demand is a homogeneous Poisson process; 2) the return flow is similarly a homogeneous Poisson process, and 3) the return process is independent of the demand process (see [10] & [6]). However, there is nearly no (scientific) literature on empirical analysis of data with reverse flows. Thus, in this paper we explore the empirical validity of these common assumptions. First of all, we present a methodology to check empirically the assumptions. We describe actual practice in companies with respect to information storage on returns and inventory control. Moreover, we apply the methodology to real data and we discuss practical implications of our findings, for instance with respect to information management on inventory systems with returns. We employ data from three companies here referred to as CERN, MOC and RF. CERN, the database of the European Organization for Nuclear Research [8] and RF, a refinery, regard internal material returns. MOC, a mail-order company, handles customer product returns.

The remainder of the paper is organised as follows. The next section is dedicated to a survey of relevant issues concerning return handling in practice. Next, a review of the main assumptions in the literature when it comes to inventory models with return flows is presented. Then, in Section IV the methodology is put forward and in Section V the data is described. The analysis can be found in Section VI. The last section sums up the conclusions, practical implications and research needs.

II. Handling Returns in practice

Regarding product returns one may distinguish products which are in a good state and those which are not. The former can be put again in inventory within a short period of inspection, sorting, possible testing and repackaging. The latter ones usually go out of the company's system to be sold in another market, remanufactured, recycled or disposed. This paper studies product returns of the first group, e.g. products in good condition, whose handling is of relatively short duration and which are put again in the inventory of new goods (Figure 1).

One can distinguish several situations where this type of returns occurs:

- due to commercial agreements (supplier/retailer, retailer/customer);
- in-company warehouses (internal returns);
- in maintenance settings (e.g, spare-parts returns from maintenance engineers);
- lease of equipment;

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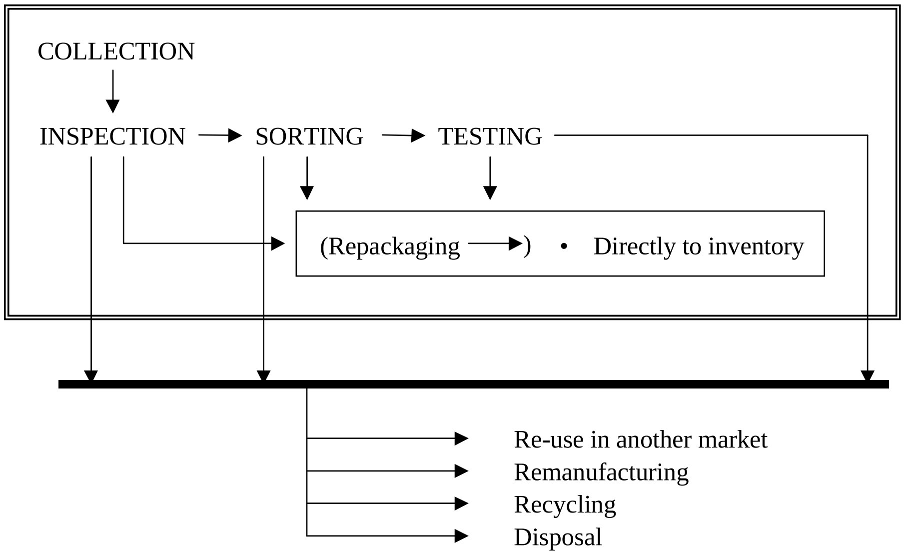

Fig. 1. Handling returns in practice.

Below we add more details to each of these situations contributing to illustrate the dimensions of product returns systems.

Take-back guarantees by suppliers to retailers are very common in case of products which become out-of-fashion before so long as newspapers or personal computers. Also, mail-order companies and electronic retailers (e-tails) allow their customers to return products within certain amount of days. For instance, *amazon.com* fully refunds products returned within 30 days after they have been received by the customer. In case of a self-owned warehouse, "customers" have more freedom to return products, as these products belong to the same organisation as the store. Another situation where returns in good condition occur refers to maintenance service organisations of geographically spread out equipment, e.g. photocopiers. In this business, it is common that spare-parts are delivered at night to the service engineers, according to their requisition. Very often, these engineers actually use fewer parts than requested and after some time they return the non-needed spare-parts. Yet, this may take some time, for instance until they are able to visit the base. In fact, in spite of the well organised supply of new parts, the handling of returns is not that well covered by maintenance organisations. Finally, returns in good condition, which can again be put on inventory, also happen in case of leasing equipment.

To handle return flows calls for a broad coordination of affairs, including the set-up of a logistics network, the use of information systems, the accounting of returns and the inventory control. This explains why many firms lack competence and lag behind in respect to management of product returns (see [23]). The Reverse Logistics Executive Council has reported that US firms have been loosing opportunities as well as billions of dollars per year due to not addressing properly their return flows [27]. There are fortunately successful cases to report as e.g. Estee Lauder. This firm was able to recover in the first year more than all the money invested in a software system which has focused on the managing of product returns (see [4]). Yet this success is an exception rather than the rule. Guide [14] concluded that most firms have been loosing opportunities because their information systems were not able to handle return flows well. This is specially true for e-tailers, where reverse logistics competence seems to be(come) a pre-requisite for survival (see [21] and [25]). The reason is that the size of return flows are estimated at 6% for general products and for mass products as high as 15%, while for catalogue and e-sells the numbers go up to 35% [13].

In conclusion, many companies have not yet realised that an efficient handling of returns can save them a lot of money. Therefore, they also have not invested in monitoring systems or data collection of returns. The scarcity of data and, in many cases, its confidentiality explain the lack of publications on empirical analysis of return flows. In this paper we were however able to use both data on demand and returns (internal or in-company returns) of the warehouse of CERN. In addition data from two other companies were used: a mail-order company and a refinery in The Netherlands.

III. Literature Review of inventory models with returns

Literature on inventory control models with return flows can be distinguished into two streams: 1) typical repair models; 2) other models with imperfect correlation between demand and returns. In the former models there is a perfect correlation between machine failure and the demand for a substitute, where the number of machines or parts in the system is constant. Our paper concentrates on product returns that fit in the second group. For recent surveys on repairable inventory theory we refer to Cho & Parlar [5] and to Guide & Srivastava [15]. The review follows in two sections: deterministic models and stochastic models.

A. Deterministic models

Schrady [29] considers a deterministic inventory model in which a certain percentage of sold products comes back, after a known period of time, to be repaired. Repaired items are put in inventory to be eventually re-used. Since the demand and the return processes are assumed to be continuous deterministic flows, the dependency relationship between the demand and return process is not explicitly modelled. Later, Richter [26] and Teunter [31] extended this model with the option of product disposal. As regards the demand and return processes, Schrady's assumptions remain the same in both extensions.

Also in a deterministic context, Minner and Kleber [22] discuss the optimal control for a production and remanufacturing system with continuous and non-stationary rates for both the demand and return processes.

B. Stochastic models

The stochastic models can be divided into two groups that typically have different assumptions: periodic review models and continuous review models.

B.1 Periodic review models

This category of models typically focuses on proving the structure of the optimal policy rather than finding optimal parameter values. Simpson [30] provides for instance the optimal policy structure for an inventory model with product returns in which product demands and returns can be stochastically dependent within the *same* period only. Demand and returns are known through a joint probability function, which can differ from period to period. Inderfurth [17] extends the previous model with non-zero (re)manufacturing and procurement lead times. All other assumptions equal the ones of Simpson. Inderfurth proves that there is a simple optimal control policy structure as long as the leadtime for manufacturing and remanufacturing differ at most one period.

Buchanan and Abad [3] consider a system with partial returns. Each period, a fixed fraction of products is lost while a stochastic fraction is returned. The authors establish an optimal policy for the case that the time until return is exponentially distributed.

Toktay et al. [33] study ordering policies for a business case of single-use Kodak's cameras. After using the camera, customers bring it to a photo laboratory where the film is developed. The laboratories next return the used cameras to Kodak (but sometimes they go to the so-called jobbers). Kodak dismantles the used cameras and reuses the flash circuit broad of every camera in the manufacturing of new ones. A closed queueing network model is applied to decide on periodic ordering decisions. Custom demand is treated as a stationary Poisson process from which a known percentage is returned. Time until return is modeled by an infinite Customer & Lab server with general processing time. Another important feature of this paper is the identification of the information's value according to different scenarios.

Kiesmuller and van der Laan [20] develop a periodic review inventory model where product returns depend on the demand process. Both the demand and the return streams follow a Poisson distribution. All returns depend on previous demands through a constant time until return, and two probabilities: the return probability (which underlying event is assumed to be known upon the demand), and the probability that a returned item is in a sufficiently good condition to be remanufactured. The authors compare this model with the situation of independent demands and returns. The outcome supports that it is worth to use information about the dependency structure between demands and returns.

B.2 Continuous review models

Heyman [16] analyses different disposal policies for a single-item inventory system with returns. He uses a model where demands and returns are independent compound renewal processes and all leadtimes are zero. An explicit expression for the optimal disposal policy is given when the processes are Poisson. Muckstadt and Isaac [24] investigate too the control of a single-item inventory system with independent demands and returns following a Poisson Process and derive some approximations.

Fleishmann et al. [11] derive an optimal policy and optimal control parameters for a basic inventory model with returns where demand and return are independent Poisson processes.

Yuan and Cheung [36] discuss an (s,S) inventory system with returns and analyse the impact of partial returns on rental systems. Demands are assumed to follow a homogeneous Poisson process and there are no lost customers. Customers keep the acquired item during an exponentially distributed time. When this period ends, the item is either returned or disposed, with a given probability which outcome is known upon demand. They also assume that the return period is already known at the time of demand.

Van der Laan et al.[34] deal with policies in the context of two inventory facilities, one of new products and the other of remanufactured items. The model considered is based on unit demand and unit returns with independent Poisson processes.

	Demand process	Return process	Processes' relations
		Deterministic models	
Schrady, '67	Continuous (constant) rate	Continuous (constant) rate	Not explicitly modeled
Richter, '94	idem	idem	idem
Teunter, '98	idem	idem	idem
Minner & Kleber, '01	Continuous (dynamic) rate	Continuous (dynamic) rate	idem
		Stochastic periodic review	
Simpson, '78	General non-stationary	General non-stationary	Dependency allowed in *same* period only
Inderfurth, '97	idem	idem	idem
Buchanan & Abad, '98	Random variable	A stochastic fraction of demand comes back after an exponentially distributed time.	
Toktay, '00	Homogeneous Poisson Process	Partial returns & general distribution	Dependency comprised through a queueing system
Kiesmuller & van der Laan '01	Homogeneous Poisson Process	Poisson distribution with rate depending on the prob. of having a remanufacturable return.	Returns are dependent of demand through a constant return probability and time until return.
		Stochastic continuous review	
Heyman, '77	Compound renewal process	Compound renewal process – explicit expressions for Poisson	(independence)
Muckstadt & Isaac, '81	Homogeneous Poisson Process	Homogeneous Poisson Process	idem
Fleishmann et al, '97	(Compound) Poisson Process	idem	idem
Yuan & Cheung, '98	Homogeneous Poisson process	It is known that a constant fraction of demand comes back after an exponentially distributed time.	
Van der Laan et al., '99	Homogeneous Poisson Process	Homogeneous Poisson Process	(independence)

Table 1.The assumptions of the literature of inventory models with returns.

Although the above does not constitute an exhaustive review of the literature on inventory models with returns, it serves to identify the common assumptions of these models. Table 1 resumes the main assumptions over the demand and return processes, as well as over the dependency structure of these two processes. Concluding, the bulk of the literature, especially when uncertainty is modelled, assumes the following: 1) the demand is a homogeneous (compound) Poisson process; 2) the return process is similarly a homogeneous (compound) Poisson process, and 3) the two processes are independent. These choices have to do with the tractability properties of the Poisson distribution. This is also the main reason behind the independence assumption. In fact, less restrictive assumptions complicate significantly the analysis. Besides, as the review of the literature shows, in order to pursue an exact analysis or to give explicit expressions for optimal policies one has to make these common assumptions. The independence assumption has also been motivated by the scarcity of individual data on product returns [11]. This motivates empirical analysis of real data.

IV. Methodology

The statistical analysis encompasses the following: the demand process, the return process, and the relation between these two processes.

In order to describe a testing framework for the demand and return processes, first the notation is introduced. Let $P(t)$ be a counting process that at time t ≥ 0 equals to:

$$P(t) = \sum_{k=1}^{N(t)} Q(k) \tag{1}$$

where N(t) $\in Z$ is the number of customer arrivals in the interval of time (0,t), and Q(k) is the amount of items ordered (or returned) by client k.

$P(t)$ is a Homogeneous Compound Poisson Process (HCPP) if [32]:

- N(t), t$\geq$ 0 is a Poisson Process;
- Q(k) is identically and independently distributed;
- N(t) and Q(k) are independent processes;

Let the subscript D refer to a demand process and the R to a return process and let L be the random variable indicating the time to return. It is clear that if the return probability $p = 1$ and L is negative exponentially distributed then $P_D(t)$ being a HCPP implies that $P_R(t)$ is a HCPP which is independent of $P_D(t)$. However, in case p is strictly smaller than 1, then $P_R(t)$ is not a homogeneous compound poisson process given $P_D(t)$. To see the latter consider a demand y time units ago. The probability that this demand will trigger a return is

$$\frac{p.e^{-\lambda.y}}{p.e^{-\lambda.y} + (1-p)} \tag{2}$$

where λ is the scale parameter of the exponential distribution of the lag L. Since it depends on y, the correspondent process is not a HCPP. One may still opt for approximating $P_R(t)$ with a HCPP if there are demands occuring regularly. To evaluate whether a process $P(t)$ behaves as a HCPP consists in checking the three sufficient and necessary characteristics mentioned before. In order to first check whether it is acceptable that N(t) is a Poisson Process, one can then check if 1) N(t) follows the Poisson distribution and 2) if N(t) has independent increments. The last implies that the occurrence of events after any time t is independent of the previous occurrences. For the first aspect, a chi-squared goodness-of-fit test can be used (see [1]). To employ this test the potential outcome space has to be split into intervals. For a good test performance, the number of intervals shall be as many as possible as long as the expected number of observations is at least about 5 per interval. The second investigation can be done through a Ljung-Box Q test which directly checks if the autocorrelations between increments are significant or not. This test can be found in statistical packages, which can be carried out without sophisticated knowledge of time series theory (see [2]). The investigation over Q(k) can be carried out by applying a Wald-Wolfowitz runs test for randomness (see [35]) and again a Ljung-Box Q test. For the former test one finds in the literature tables of critical values from sample size 6 onwards. Finally, the test of independence based on Pearson's coefficient can be used to trace if N(t) and Q(k) are independent [1]. This test is to be used when the sample is larger than 10 data observations. The appendix embraces more details on the aforementioned statistical tests. The choice of those tests was made targeting the following objectives: quality testing performance against small sample sizes, simplicity, utility and parsimony.

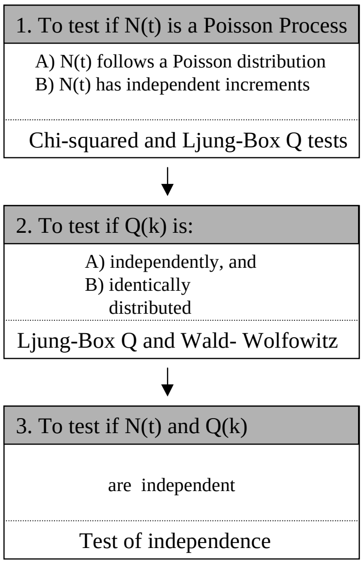

Fig. 2. Statistical analysis to test if a process $P(t) = \sum_{k=1}^{N(t)} Q(k)$ is a HCPP.

Besides the assumptions in the literature regarding demand and return being each one a HCPP, these two processes are commonly assumed independent. In the proposed methodology there is no direct investigation about whether the demand and return processes can be dealt with as independent processes, because the data to be analysed is likely to have only a small amount of returns. Instead, it is investigated whether it is reasonable to model the *time until return* with an exponential distribution. The advantage of this is that one may pool over multiple products as the return lag is likely to be the outcome of a logistical process rather than being a product specific aspect.

The exponential assumption also appears in the literature (see Section III). So, indirectly it is tested how dependent returns are of previous demand occurrences. This has important consequences for inventory management, which are to be discussed later in the paper. *Time until return* is the period from the moment a product is acquired, until it is returned. A goodness-of-fit statistical test, k^2, constructed by Gan & Koehler [12], is used for this investigation (See appendix). In accordance with the authors, this test fairly detects the non-exponentiality in the presence of data from a Weibull, a LogNormal, or Normal truncated, among about two dozens of other distributions.

V. Data

A. Case 1 - CERN

CERN is a very large research center with more than 7 000 scientists. These scientists have several warehouses at their disposal with more than 15 000 stock keeping units. Scientists have two ways of acquiring the products they need for their experiments: by going to the self-service stores or, by ordering via the web from the main warehouse. The first is meant for low-value regularly used items and the latter for all other items. In both situations, scientists provide the budget code of the project they are working on. This code is therefore registered and the project is charged for the delivery [7]. For already 10 years, CERN registers inventory transactions through the LIMS package which is used on top of Baan's TRITON system. The LIMS package was developed by Dr. B. Schorr and Dr. M. Krever. [9]. The database encompasses the following general information for each transaction occurrence:

- Product identification by means of a code and a name.
- Product quantity and its monetary value.
- Store identification (main vs. self-service store).
- Transaction type, e.g. replenishment from supplier, delivery to client, and returned by client.

In case of products issued to scientists, the date of order placement, date of planned and actual delivery, budget number and client code are in principle registered. In case of a return, the date of transaction, the budget number and client code can also be registered. Originally, an attribute was created in CERN's database to register the reason for returning the product, but in practice it is not used. The reason codes included wrong delivery regarding quantity or material; quality problem due to corrosion damages on transportation, or technical fault; delivered too late or even never ordered by the client. If the transaction is replenishment, the supplier code and name is recorded in the database.

CERN's data is not only extraordinary by its dimension and life-span, but also due to the individual registration of transaction occurrences, including the identification of returns, client codes and budget numbers. In most other commercial packages, transaction data are stored only for a short time and are aggregated to e.g. monthly or yearly demand figures. Accordingly, when a product is returned at the CERN, it is possible to trace back its date of purchase. The procedure we used in this respect is as follows. If the client number is registered, the return is associated with the most recent purchase by this client whose amount is larger than the quantity returned. If not, but the budget number is known, the inspection is now done through the issues purchased under this budget number. The rationale behind this is that sometimes the client who receives the product is not the one who returns it. Still, it is most likely that it is a client within the same project, i.e. the same budget number. In several cases of returns it is not possible to make a link with any previous delivery, because the client and budget identification are missing. In other cases, there is no registration of previous deliveries to these clients. Possible explanations are that the products are being returned under another project, or that they were purchased before the implementation of the database.

B. Case 2 - mail-order company MOC

The second set of data comes from a mail-order company active in the Dutch market, here referred to as MOC. The main product line is fashion, but household appliances and furniture are also found in MOC's catalogues. Customers can order by mail, phone, fax or internet. Merchandise can be returned at no cost up to a limited amount of time after delivery, yet the company sometimes also accepts later returns.

MOC sends out two catalogues per year: spring/summer and autumn/winter. Before the season starts, the catalogue is sent to a subset of recurrent customers, which can pose their orders even though the products are not delivered before more than one month has passed. These orders help the expert's committee to point out an estimation of how many of each single item will be sold over the all season. This estimated value is periodically updated together with the return percentage, as more information becomes available on both sold and returned items.

C. Case 3 - refinery RF

The third case concerns a refinery in the Netherlands. The warehouse keeps thousands of materials in stock. The majority of these ,materials are owned by the warehouse. A minority is owned by "customers". In this case, though materials are kept in stock in the warehouse of the refinery, there are no monetary exchanges directly associated with a particular demand or material return. Nevertheless, the transactions are registered in the database.

We consider the spare parts inventory, which mainly consist of slow movers with a consumption in a maximum of four months per year. Returns may be caused for instance by maintenance personnel finding out that they actually did not need the spares or by rotatables, which have been repaired to an as good as new condition. The company uses SAP R/3 material management module for their inventory control. In this system information on demands is stored on a monthly basis. Accordingly, the immediate returns are netted. Only returns that occur in future months are visible (unless they are netted with new demands). The return lag was determined by identifying for each return the lates month with a positive netted demand. Unlike the CERN case, we did not have client or budget codes for tracing . So, the tracing can not be done with the certainty of the CERN's case. The inventory control policy of SAP employs basically the net demand and ignores returns. The database consists of data from the last 4 years. We have analysed data from some 800 materials.

VI. Data analyses

A. *Case 1 - CERN data analysis*

Data from CERN is now applied to the tests discussed in Section IV. Other products than only the ones here discussed were investigated. However, it was not always possible to trace back return occurrences (see Section V.A), and sometimes there were no sufficient observations to pursue satisfactory testing. Nevertheless, the results put forward give room to discussion and the findings are of practical relevance. For each product, the results of the analysis mainly state whether it is reasonable to accept that:

- the demand process is a HCPP.
- the return process is a HCPP.
- the time elapsing between return and demand occurrences follows an exponential distribution.

The decision to reject, or not, in each of the tests is based on a error Type I (α)=0.05, i.e. the probability of rejecting the hypothesis given that it is valid, equals 0.05. The results of the analysis of the first two products can be found in Table 2. One of the products is a *stabiliser gas*, and the other is catalogued as a *large agenda*. Both of these products have return rates around 5% of the issued quantities and a large number of issues. One of the reasons these products were chosen for analysis is the possibility of tracking a sufficiently number of observations to pursue the required tests. In the case of unit purchases and by assuming no lost or backlogged demand, one may opt for carrying out the tests only on the demand and time until return. If both hypotheses are not rejected, one can consider the system with an M/M/∞ queueing structure. For such a system the output flow will eventually behave as a Poisson Process [28], dispensing in this way the need for testing it.

	Demand Process			Return Process			Time until return		
Product	Nº. obs.	Period	Decision	Nº. obs.	Period	Decision	Nº. obs.	Period	Decision
Stab. gas	283 36 26	88-98 96-98 97-98	Rejected Rejected Not Rej.	12	88-97	Not Rejected	12	88-97	Not Rejected
Large agenda	1043 1009 362 253	88-99 88-98 96-98 97-98	Rejected Rejected Rejected Rejected	14	89-98	Not Rejected	17	89-99	Not Rejected

Table 2. Results of the statistical analysis of two products of CERN's database.

In the case of the stabiliser, it is rejected that the demand process follows a homogeneous Poisson Process for the periods 88-98 and 96-98, but not when only data from 97 and 98 is taken into account. Thus, demand does not remain stable for a period of 10 or even 3 years, but the opposite is acceptable for a couple of years (see Figure 3). Regarding the return process, a HCPP is not rejected. It is also statistically not rejected that the time until return is exponentially distributed.

The product agenda is a seasonal product, as observed in Figure 4. Therefore, for all the periods considered, the assumption of a HCPP demand is rejected. Actually the demand occurrences do not even behave as a Poisson Process. However, it is accepted that the time in market is exponentially distributed and that the return process is a HCPP. One can explain this by the fact that there are always enough products in the market. Making an analogy with queueing systems this means that the server is always busy, so the output flow does not depend so much on the demand process, but on the service time.

Figure 5 is a dotplot of the ordered quantities of the product agenda. One can notice that there are on the one hand many orders of low quantities and on the other hand a few orders of large quantities. This puts forward a division in two groups, namely small and large orders. Other graphical representations, among which the histogram of Figure 6 helped us to make the division at the order quantity 20: large orders are the ones of quantities larger or equal than 20

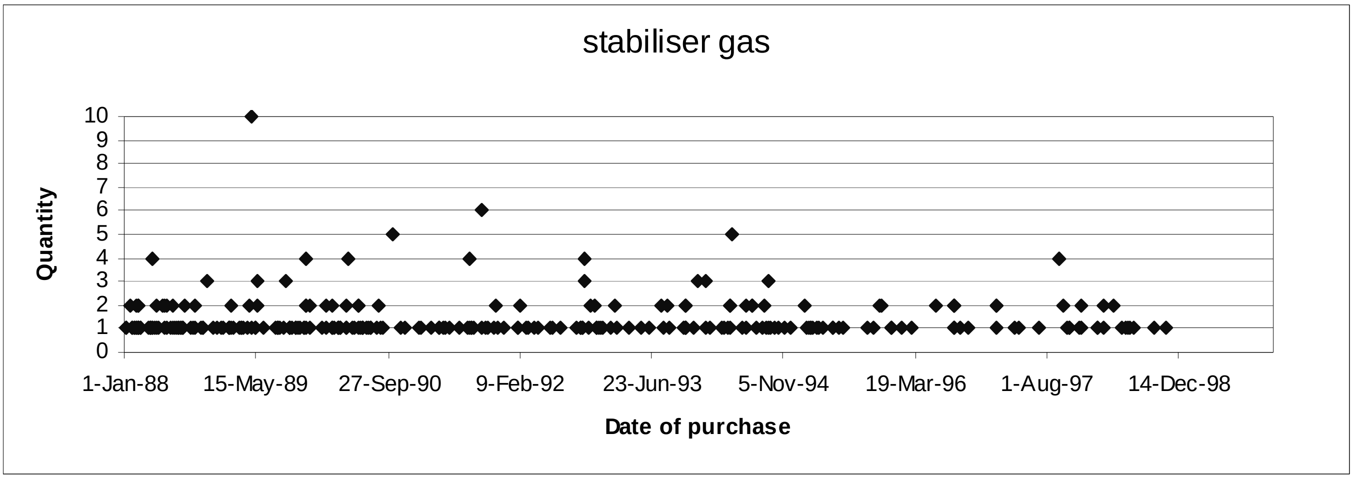

Fig. 3. Stabiliser gas: date of purchase vs. purchased quantity.

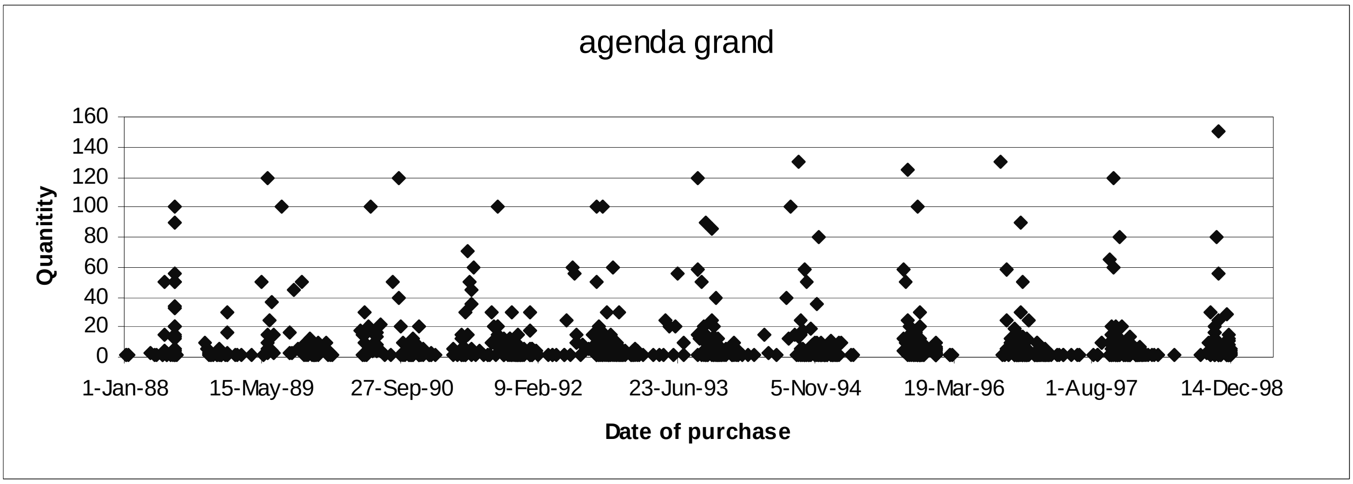

Fig. 4. Large agenda: date of purchase vs. purchased quantity.

(100 out of 1043). We proceed with testing whether it is acceptable a HCPP on the group of smaller order quantities. The hypothesis is rejected. The seasonality is likely to be the reason for this outcome. Regarding the large order, a tailor-made procedure to integrate these orders in a possible ordering policy is advisable.

Due to insufficient data of individual products, three groups of products (aggregated data) are investigated next. The existence of many, but infrequent individual orders, justifies that the demand follows a Poisson Process [32]. Therefore, the statistical analysis deals with the time until return alone. Group 1 is constituted of several gases, Group 2 by products with traceable returns and Group 3 includes the previous, as well as the returns of the product agenda grand (Table 3).

	Group 1	Group 2		Group 3	
		A	B	B	C
N^{o}.obs.	18	40	37	53	35
Decision	Not Rejected	Rejected	Not Rej.	Rejected	Not Rej.

Table 3. Outcome of the the investigation whether time until return of 3 groups of products is exponentially distributed. (A: total number of observations; B: A without a few very large observations; C: the products which

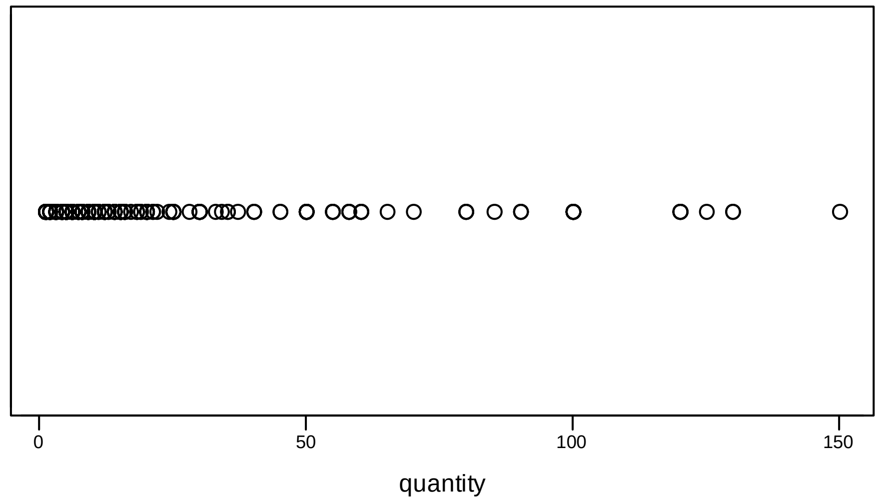

Fig. 5. Large agenda: dot-plot of the purchased quantity.

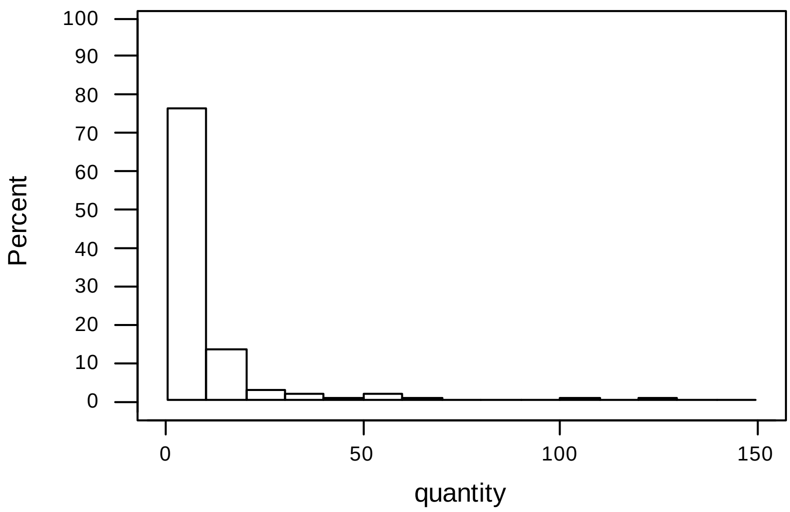

Fig. 6. Large agenda: histogram of the purchased quantity.

returned within 1 week were removed from sample B)

It is accepted that the sample of 18 return occurrences in the first group of products follows an exponential distribution. For the total of 40 return occurrences of products of the second group, the exponentiality is however rejected. To investigate whether the data could reasonably be from other distribution, four probability-plots of the exponential and other distributions were traced (Figure 7). The plot for the exponential points out that the three large observations are disturbing a good adjustment of this distribution. Indeed, it is not rejected that the data is exponentially distributed if the three large return times are removed. These return times fall between 6 months and one year after purchase, while all these other returns occur within less than 3 months after acquisition. These returns may be the result of termination of projects and a separated treatment is recommendable. Yet, to decide how to deal with these situations in practice, it would be helpful to really have registered the return reason, as it is conceived by CERN's database, but regrettably not used.

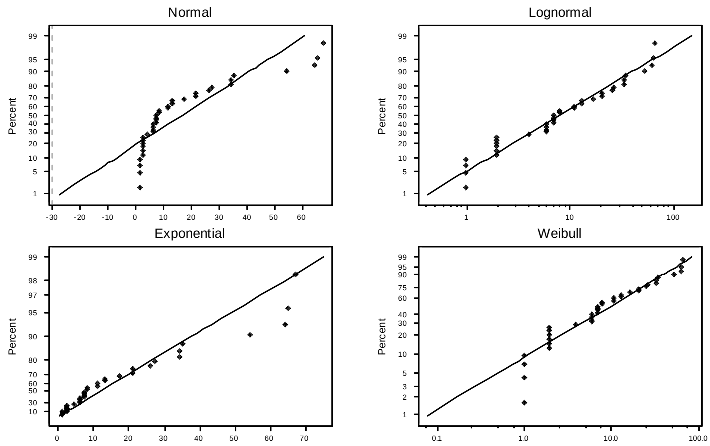

Fig. 7. Group 2 of products: four probability plots.

For Group 3 of products, the very large observations were put aside in analogy to the analysis of Group 2. Still, it is not statistically acceptable that these products come back after an exponentially distributed lag

There is one aspect that has not been considered until now. From an inventory perspective, when products come back almost immediately, it is like the demand did not occur in the first place. Therefore, one can distinguish between short-lag returns and the longer lag returns. The longer lag return are the ones likely to have more impact on inventory control, since they may give to think about annulment of order replenishments by the warehouse. The short-lag returns can be simply subtracted from the demand process. The issues discussed in this paragraph give emphasis to the importance of recording return reasons. Moreover, one would be then able to identify the returns which feed demand, and those which not (e.g. wrong delivery, no longer needed). This information could later be used to forecast returns and for the design of inventory policies.

Following the above arguments, it was investigated for the Group 3 of products whether longer lag returns go along with an exponential distribution. Short-lag returns were those of one week or less return time. The hypothesis was then not rejected.

B. Case 2 - MOC data analysis

The demand in distance-selling environments (and specially on fashion) is known to fluctuate considerably through time, dispensing the test to evaluate whether the demand behaves as an HCPP, or not.

MOC provided us with estimated values for the average return percentage per period since the demand both for fashion and hardware for the years 2000 and 2001. For example, the estimated return percentages for fashion products in the year 2000 are respectively 5%, 70%, 22%, 2% and 1% for the first five periods after sales. The unit of measure is confidential in this case. Because MOC protects detailed information on product demands and returns, we constructed a testing sample. The size of the sample is 100, which is not an unreasonable number of returns in this setting. We tested whether the time elapsing between return and demand occurrences follows an exponential distribution.

For all the four cases tested (fashion-2000; fashion-2001; hardware-2000; hardware-2001) the hypothesis has been rejected. Figure 8 is a histogram of the return lag for Fashion in the year 2000. A tentative explanation for non-exponentiality is the presence of the short-lags. However, when the long-lags are tested the hypothesis is still rejected. This is because the thick tail spread in time manifested in Figure 8 is still perceptible without the short-lags.

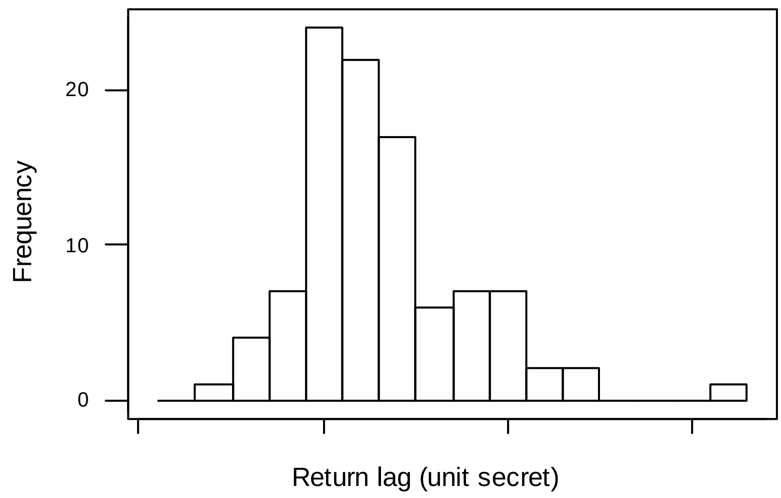

Fig. 8. MOC Fashion 2000: histogram of the return lag.

C. Case 3 - RF data analysis

In the case of the refinery, it was not tested if demand follows a HCPP because of the lack of demand occurrences per product and because the data is aggregated in monthly netted demand.

We scanned about 800 materials and found 137 return occurrences. From those, about 8% could not be traced back, which we therefore left out of consideration. Out of the traced returns, around 20% were materials owned by the customer.

We tested whether the time elapsing between return and demand occurrences follows an exponential distribution, when the lags are pooled over all the products. The assumption is rejected for $\alpha \geq 0.005$, so it is rejected for the usual values. We also have tested the hypothesis for non-owned products only. The rationale behind this test has to do with the non existence of monetary exchanges by occasion of returns of materials that are in any case owned by the customer. The customer is not going to be refunded when returns materials. Thus, there is no monetary incentive to return as soon as possible. Therefore, we suspected that this would lead to a spread of a thick tail, conflicting with the exponential. Figure 9 illustrates the existence tail. However, the tail effect remains even when only non-owned materials are considered.

From the data we noticed that for some products the stock level was above the desired maximum level (as specified by RF) for a substantial amount of time. The fact that returns are not taken into account in the inventory control is the cause of this overstock.

VII. Conclusions

A. Summary of findings

In this paper we explored the validity of the general assumptions in the literature on inventory modelling with product returns. We have set a framework for the statistical testing of these assumptions, i.e. whether

- the demand process is a Homogeneous Compound Poisson Process (HCPP).
- the return process is a Homogeneous Compound Poisson Process (HCPP).
- the time elapsing between return and demand occurrences follows a negative exponential distribution.

Afterwards, we used the framework with three sets of real data:

Case 1: data from an in-company warehouse with internal returns in Switzerland (CERN)
Case 2: data from a mail-order company with commercial returns in the Netherlands (referred to as MOC)
Case 3: data from an in-company warehouse in the Netherlands referred to as RF).

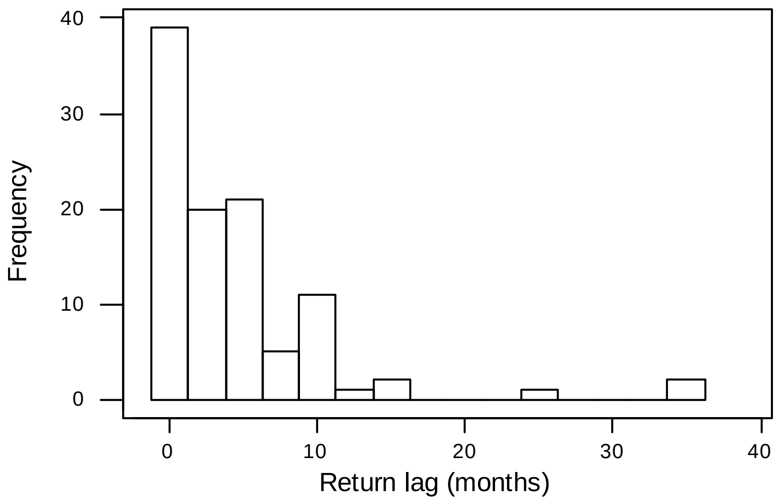

Fig. 9. RF: histogram of the return lag.

We found products for which the common assumptions on the theoretical models fit reasonably, as opposed to products for which they do not. Regarding the (non-)validity of the common assumptions, we empirically observed that :

A) Demand may not behave as a HCPP due to seasonality or over long periods of time (e.g. agenda and stabiliser gas- CERN).

B) In spite of a non-stationary environment, we can not reject that the return process behaves as an HCPP (e.g. large agenda- CERN).

C) In some cases we can not reject that the time to return behaves as negative exponentially distributed (e.g. stabiliser gas & Group 1- CERN).

D) Time to return does not seem to behave as an exponential distribution due to a long tail (e.g. Group 2- CERN).

E) Time to return does not seem to behave as an exponential distribution on account of the short-lag returns (e.g. Group 3- CERN)

With the developed methodology it was possible to carry out the desired testing including small (below 20) sample sizes. Only for extremely small sample sizes, the methodology does not work. It is unambiguous that in the last case, rigour can not be guaranteed by either the proposed or any another statistical framework.

In spite of the specific sets of data, the findings and its implications are transferable to a much larger group of demand-return processes. We employed data on products of an in-company warehouses (CERN and RF) with internal returns and a mail-order company with commercial returns. In case of equipment leasing, we conjecture that although the tail of the time to return may behave as an exponential, the same is unlikely regarding the short-lag returns. In maintenance settings, we believe that the validity of assumptions depends on whether the maintenance is corrective or preventive. In the first case this research findings are likely to apply.

B. Practical implications

Returns can be said to be a "necessary evil" as they complicate inventory control. Important is to know how much will come back and when. This knowledge can be used to avoid superfluous replenishment orders and outdating of products. Therefore it is important that companies monitor returns, investigate return reasons and get an idea of the return lag. Next, companies will have to look how the lag can be shortened, especially for the long lags. In the context of inventory control, we put forward the following suggestions:

- To separate short-lag returns from long-lag returns. In the presence of the former, proceed to netting ($D^{'}$= D-R), so it is as returns have never occurred. Especially for long-lag returns, monitor return lag distributions, return rates and

register return reasons. The last helps to find return patterns that may be useful to potential shortening of the lag and to inventory control (e.g. cancellation of replenishments when many products are expected to return).
• When investing on product return information (e.g. tracking and tracing) and its storage, the following questions should be taken into account:

1) Does the return lag behave as a negative exponential distribution?
2) Is the environment likely to be stationary?

An exponentially distributed time to return is in itself less demanding concerning information than other distributions. In a stationary environment, high investments on tracking and tracing may not pay off. On the other hand, we conjecture that the same happens in highly unstable environments. Thus, the most gains of information are likely to be in medium well-behaved environments.

C. Research needs

This study and the reflections pursued here allow us to identify some of the actual research needs, and to point out some directions. First of all, there is a need to break with the traditional assumptions in the literature on inventory modelling with returns. A necessary direction is to develop models suitable for the non-stationary (real) situations, as for instance seasonality of products. In this last case products are especially troublesome if they return at the end of the season. This should be considered in research on inventory modelling with product returns.

We have used several real examples to verify whether the time to return behaves as a negative exponential. For some instances the hypothesis was accepted, for others rejected. The impact on inventory control of mis-assuming an exponential lag remains a topic for further research.

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Appendix

I. Statistical tests

Here there are more details on the statistic tests discussed in Section IV which were used to carry out the statistical analysis in Section V.

Chi-squared goodness-of-fit test [1]

Test hypothesis: 1 a): N(t), t$\geq$ 0 follows a Poisson distribution;

The procedure is as follows. To divide the all potential outcome space into c intervals $A_1, A_2, ..., A_c$; Let o_j be the number of observations within interval j and p_j the probability of an observation falling in A_j and $ej = n * p_j$ the expected number in the jth interval with n being the sample size. For a error Type I equal to α, we reject the hypothesis if the value of test statistic is larger than the (1-α) quantile of a chi-squared distribution with c-2 degrees of freedom., i.e. if

$$\sum_{j=1}^{c} = \frac{(o_j - e_j)^2}{e_j} > \chi^2_{1-\alpha}(c-2) \tag{3}$$

To have an accurate test $e_j \geq 5$, or so, should be assured per cell. As many cells as possible is desired because it increases the degrees of freedom.

For the test on the demand process we observed the number of arrivals per month, and for the return process the number of returns per year. After, the chi-squared test was employed.

Ljung-Box Q-test [2]

Test hypotheses:

1 b): N(t) has independent increments;

2 a): Q(k) is independently distributed;

The test is based on a chi-squared test statistic, which is the approximate distribution of the autocorrelations.

The MINITAB statistical packaged was employed to carry on this test.

Wald-Wolfowitz test [35]

Test hypothesis: 2 b): Q(k) is identically distributed.

To apply this test, one has to divide the sample in two mutually exclusive categories, e.g. above and below the median. The test statistic is the number of runs, i.e. groups of one or more observations of the same category. The test detects deviance in randomness and it may suggest non randomness due to trends or periodicity. To test the hypothesis, one may rather divide the sample in two and then test if the distributions are identical. However, these tests are only suitable for samples large enough [19].

Test of independence [1]

Test hypotheses: 3): N(t) and Q(k) are independent.

The test is based on Pearson's correlation coefficient and the samples used were the inter-time between arrival of clients and the amount ordered, or returned. For very small sample sizes, less than 10 observations, a nonparametric rank test is preferable, e.g. the Spearman's rank correlation test.

k^2 goodness-of-fit test [12]

Test hypotheses: "Time until return" has an exponential distribution.

The test is based on the P-P probability plot. This plot is build drawing $Z_i = F(X_i/\widehat{\lambda})$ versus p_i, where

- F is the cumulative distribution function;
- X_i is the i^{th} lower ordered observation, i=1,...,n with n being the sample dimension;
- $\widehat{\lambda}$ is the estimated scale parameter;
- p_i is an appropriate plotting position;

The degree of fit is decided upon the linearity featured in the graph. The statistic proposed is based on measures of linearity:

$$k^2 = \frac{\left(\sum_{i=1}^{n}\left(Z_i - \bar{Z}\right)(p_i - \bar{p})\right)^2}{\sum_{i=1}^{n}\left(Z_i - \bar{Z}\right)^2 \sum_{i=1}^{n}(p_i - \bar{p})^2} \tag{4}$$

Approximations for the lower p^{th} percentile are obtained through the formula below:

$$1 - k_p^2 = \frac{1}{\alpha_p + n\beta_p} \tag{5}$$

A table with the coefficients α_p and β_p is supplied in [12].

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