Logistic planning and control of reworking perishable production defectives

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Abstract

We consider a production line that is dedicated to a single product. Produced lots may be non-defective, reworkable defective, or non-reworkable defective. The production line switches between production and rework. After producing a fixed number (N) of lots, all reworkable defective lots are reworked. Reworkable defectives are perishable, i.e., worsen while held in stock. We assume that the rework time and the rework cost increase linear with the time that a lot is held in stock. Therefore, N should not be too large. On the other hand, N should not be too small either, since there are set-up times and costs associated with switching between production and rework. For a given N, we derive an explicit expression for the average profit (sales revenue minus costs). Using that expression, the optimal value for N can be determined numerically.

Keywords: rework, perishability, batch production

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1 Introduction

In many production processes, defectives occur. Sometimes, part of these defectives can be reworked. Depending on the quality, reworked products can then be sold at either the same price as 'immediate non-defective products' or at a lower price. If input materials are expensive, or if rework times are much smaller than the initial production times, rework can be economically attractive. But there are other motives for rework, which have become increasingly important in recent years. New legislation and disposal bans force some producers to rework (part of) their defectives. Moreover, many companies want to have an environmentally friendly image, because the image of a company influences its sales.

There is quite a lot of literature on the logistic planning and control of production processes with rework. For a recent overview see Flapper and Jensen [5]. No attention has been paid, however, to situations where reworkable defective products are perishable.

We speak of perishability if products that are held in stock worsen or can be ruined or destroyed. Perishability is also known as deterioration. Excellent overviews of perishability are given in Nahmias [8] and Raafat [9]. More recent publications that also provide a short literature overview are Bose, Goswami, and Chaudhuri [2] and Chiu [3]. But none of the studies on perishability considers rework.

Industries where perishable reworkable defective products are quite common, include the food and the pharmaceutical industry. See e.g. Flapper et al. [4]. In this paper, we will study a specific situation with perishable reworkable defective products. We consider a production line that is dedicated to a single product. This product is produced in lots of a predetermined size, possibly one. Produced lots may be non-defective, reworkable defective, or non-reworkable defective. The production line switches between production and rework. After producing a fixed number (N) of lots, all reworkable defective lots are reworked.

A detailed description of this production line will be given in the next section. A key assumption is that the rework time and cost increase linear with the time that a lot is held in stock. So, reworkable lots gradually worsen while held in stock. The exact relation between the time that a reworkable defective lot awaits rework and the rework time/cost may differ from this in practice. But assuming linear relations might lead to reasonable approximations. Moreover, the explicit expression for the average profit, that we will derive, can be used to gain valuable insights. In cases where products worsen as a result of physical depletion, like the evaporation of volatile liquids such as gasoline and alcohol, linear relations might be almost exact.

2 Situation description

We consider a production line that is dedicated to a single product. Produced lots are non-defective (probability q_n), reworkable defective (probability q_r), or non-reworkable non-defective (probability $1 - q_n - q_r$). Rework is done on this same production line ('in-line rework').

The production time is T_P per lot. The switching time from production to rework is T_{PR}. The switching time from rework to production is T_{RP}. These are all fixed. The rework time for a lot increases linear with the length of the period during which it is held in stock and awaits rework. More exact, if that length is x, then the rework time is $T_0 + T_1 x$.

The objective is to minimize the average profit, i.e., the expected profit per time unit. There is a sales revenue p_n for each immediately non-defective lot, and a sales revenue p_r for each reworked lot. The cost for producing a lot, including the purchase cost for input materials, is c_p. The cost for reworking a lot is $c_0 + c_1 x$, where x is the length of the period during which the lot is held in stock. The cost for disposing of a non-reworkable defective lot is c_d. The cost rate for holding reworkable defective lots in stock is h per lot per time unit, which includes an out-of-pocket cost component and a capital/opportunity cost component. Finally, the summed costs for switching from production to rework and for switching back to production are c_s.

The overall optimal production/rework strategy is expected to be very complex. That strategy probably bases its switching (from production to rework) decisions on the exact times at which reworkable batches are produced. This optimal strategy will be difficult, if not impossible, to determine. Moreover, it is not very practical. We therefore restrict our attention to a more practical class of strategies. These switch to rework after producing a fixed number (N) of lots and then rework all reworkable defective lots. Of course, if none of the produced lots in a batch is reworkable defective, the next production batch starts immediately. This class of strategies has also been considered for non-deteriorating production defectives by e.g. Lee [6], Liu and Yang [7], Tay and Ballou [10], and Zargar [11].

The Last Come First Served (LCFS) rule is applied for determining the order in which reworkable defective lots are reworked. Using an adjacent pairwise interchange method, as described by Baker [1], it can easily be shown that the LCFS rule leads to the rework order that minimizes the total rework time and the total rework cost (if, as we assume, the rework time and cost increase linear with the time that a defective lot awaits rework). Hence, the LCFS rule maximizes the average profit.

The production line is illustrated in Figure 1 for $N = 5$, $T_P = 1$, $T_{PR} = 1$, $T_{RP} = 1$, $T_0 = 0.5$, and $T_1 = 0.1$.

Figure 1

For the first production batch in that figure, lots 5 and 1 turn out to be reworkable defective. Hence, the production line switches to rework. First lot 1 is reworked and then lot 5. When the rework on lot 5 is completed, the production line again switches, and the second production batch starts. Since none of the lots turn out to be reworkable defective, the production line then directly continuous with the third production batch. Out of that batch, lots 5, 3, and 2 turn out to be reworkable defective. Hence the production line switches again and reworks them in reverse order.

3 Calculation of the average profit

We start by stating and proving two useful lemmas. These focus on an arbitrary production cycle, which is defined at the period between the moments at which two successive production batches (of N lots) start. The production lots of that cycle are numbered backwards as $N, N-1, \ldots, 1$. By S_n is denoted the length of the period that begins when the production of lot n, $n = 1, 2, \ldots, N$, is started and ends when the work (production and possibly rework) on lots $1, 2, \ldots, n$ is completed. By H_n is denoted the length of the period during which lot n, $n = 1, 2, \ldots, N$, is held in the stock of reworkable products. See Figure 2.

Figure 2

Lemma 1 *For all $n = 1, 2, \ldots, N$ it holds that*

$$E[S_n] = \sum_{j=0}^{n-1} \left[\alpha^j \left(\beta + \gamma \delta^{n-1-j} \right) \right],$$

where $\alpha = 1 + q_r T_1$, $\beta = T_P + q_r T_0$, $\gamma = q_r T_{PR}(1 + T_1)$, and $\delta = 1 - q_r$.

Proof. We will prove this lemma by induction on n. For $n = 1$ we get

$$\begin{aligned} E[S_1] &= \alpha^0 \left(\beta + \gamma \delta^0 \right) \\ &= \beta + \gamma \\ &= T_P + q_r \left(T_0 + T_{PR}(1 + T_1) \right), \end{aligned}$$

which is clearly correct.

Now assume that the lemma holds for all $n = 1, 2, \ldots, l$ ($1 \leq l \leq N - 1$). We will complete the proof by showing that the lemma then holds for $n = l + 1$ also. It is easy to see that

$$S_{l+1} = \begin{cases} T_P + S_l & \text{if} \quad \text{lot } l+1 \text{ is not reworked} \\ T_P + S_l + T_0 + T_1 S_l & \text{if} \quad \text{lot } l+1 \text{ is reworked} \\ & \phantom{\text{if}} \quad \text{and at least one of the lots} \\ & \phantom{\text{if}} \quad 1, \ldots, l \text{ is reworked} \\ T_P + S_l + T_0 + T_1(S_l + T_{PR}) + T_{PR} & \text{if} \quad \text{lot } l+1 \text{ is reworked} \\ & \phantom{\text{if}} \quad \text{and none of the lots} \\ & \phantom{\text{if}} \quad 1, \ldots, l \text{ is reworked} \end{cases} . \tag{1}$$

Let $\mathcal{S}_l$ denote the set of all possible outcomes of S_l, and let $s_l^* \in \mathcal{S}_l$ be that outcome for which none of the lots $1, \ldots, l$ is reworked. Let R_{l+1} be a stochastic variable that is 1 if lot $l + 1$ is reworked and 0 otherwise. Let $\Pr(.)$ denote the probability that an event occurs. Using (1) we get

$$\begin{aligned} E\left[S_{l+1}\right] = & \sum_{s_l \in \mathcal{S}_l} \left[\Pr(R_{l+1} = 0, S_l = s_l)(T_P + s_l) \right] \\ & + \sum_{s_l \in \mathcal{S}_l \setminus s_l^*} \left[\Pr(R_{l+1} = 1, S_l = s_l)(T_P + s_l + T_0 + T_1 s_l) \right] \\ & + \sum_{s_l = s_l^*} \left[\Pr(R_{l+1} = 1, S_l = s_l)(T_P + s_l + T_0 + T_1(s_l + T_{PR}) + T_{PR}) \right]. \end{aligned}$$

Since R_{l+1} and S_l are independent, and since $\Pr(R_{l+1} = 0) = 1 - q_r$ and $\Pr(R_{l+1} = 1) = q_r$, this can be rewritten as

$$\begin{aligned}
E\left[S_{l+1}\right] &= \sum_{s_l \in \mathcal{S}_l} \left[(1 - q_r) \Pr(S_l = s_l)(T_P + s_l)\right] \\
&\quad + \sum_{s_l \in \mathcal{S}_l \setminus s_l^*} \left[q_r \Pr(S_l = s_l)(T_P + s_l + T_0 + T_1 s_l)\right] \\
&\quad + \sum_{s_l = s_l^*} \left[q_r \Pr(S_l = s_l)(T_P + s_l + T_0 + T_1(s_l + T_{PR}) + T_{PR})\right] \\
&= \sum_{s_l \in \mathcal{S}_l} \left[\Pr(S_l = s_l)((T_P + s_l) + q_r(T_0 + T_1 s_l))\right] \\
&\quad + \sum_{s_l = s_l^*} \left[\Pr(S_l = s_l) q_r (T_1 T_{PR} + T_{PR})\right] \\
&= T_P + q_r T_0 + (1 + q_r T_1) E[S_l] + q_r T_{PR}(1 + T_1)(1 - q_r)^l \\
&= \beta + \alpha E[s_l] + \gamma \delta^l.
\end{aligned}$$

Since we assumed that the lemma holds for $n = l$, we get

$$\begin{aligned}
E\left[S_{l+1}\right] &= \beta + \gamma\delta^l + \alpha \sum_{j=0}^{l-1} \left[\alpha^j \left(\beta + \gamma\delta^{l-1-j}\right)\right] \\
&= \beta + \gamma\delta^l + \sum_{j=1}^{l} \left[\alpha^j \left(\beta + \gamma\delta^{(l+1)-1-j}\right)\right] \\
&= \sum_{j=0}^{l} \left[\alpha^j \left(\beta + \gamma\delta^{(l+1)-1-j}\right)\right].
\end{aligned}$$

So the lemma holds for $n = l + 1$, which completes the proof. □

Lemma 2 *For all $n = 1, 2, \ldots, N$ it holds that*

$$E[H_n] = q_r \left(\sum_{j=0}^{n-2} \left[\alpha^j \left(\beta + \gamma\delta^{n-2-j}\right)\right] + T_{PR}\delta^{n-1} \right),$$

where $\alpha = 1 + q_r T_1$, $\beta = T_P + q_r T_0$, $\gamma = q_r T_{PR}(1 + T_1)$, and $\delta = 1 - q_r$. An empty summation (for $n = 1$) is set to zero.

Proof. Since $H_1 = 0$ if lot 1 is not reworked and $H_1 = T_{PR}$ otherwise, we have $E[H_1] = q_r T_{PR}$. So, the lemma is correct for $n = 1$. What remains is to prove that it is correct for $n = 2, 3, \ldots, N$ also. It is easy to see that for all those values of n, it holds that

$$H_n = \begin{cases} 0 & \text{if} \quad \text{lot } n \text{ is not reworked} \\ S_{n-1} & \text{if} \quad \text{lot } n \text{ is reworked} \\ & \quad\quad \text{and at least one of the lots} \\ & \quad\quad 1, \ldots, n-1 \text{ is reworked} \\ S_{n-1} + T_{PR} & \text{if} \quad \text{lot } n \text{ is reworked} \\ & \quad\quad \text{and none of the lots} \\ & \quad\quad 1, \ldots, n-1 \text{ is reworked} \end{cases} .$$

Using a similar independence argument as was used in the proof of Lemma 1, it then follows that

$$E[H_n] = q_r \left(E[S_{n-1}] + T_{PR}(1-q_r)^{n-1} \right).$$

Combining this with Lemma 1 completes the proof. □

Using Lemmas 1 and 2, it is easy to find an expression for the average profit. This expression is given in the theorem below.

Theorem 1 *The average profit, if production takes place in batches of N lots and if reworkable defective lots are reworked in a LCFS order, is*

$$\frac{\epsilon - c_s(1-\delta^N) - (h+c_1)\sum_{n=1}^{N} \left[q_r \left(\sum_{j=0}^{n-2} \left[\alpha^j \left(\beta + \gamma \delta^{n-2-j} \right) \right] + T_{PR}\delta^{n-1} \right) \right]}{\sum_{j=0}^{N-1} \left[\alpha^j \left(\beta + \gamma \delta^{N-1-j} \right) \right] + T_{RP}(1-\delta^N)},$$

where $\alpha = 1 + q_r T_1$, $\beta = T_P + q_r T_0$, $\gamma = q_r T_{PR}(1+T_1)$, $\delta = 1 - q_r$, and $\epsilon = q_n N p_n + q_r N p_r - N c_p - q_r N c_0 - (1 - q_n - q_r) N c_d$.
An empty summation (for $n = 1$) is set to zero.

Proof. The production/rework process is a renewal process, that is renewed at the beginning of each cycle. The average length of a cycle is $E[S_N] + T_{RP}(1 - (1-q_r)^N)$. The average profit in a cycle is $q_n N p_n + q_r N p_r - N c_p - q_r N c_0 - (1 - q_n - q_r) N c_d - c_s(1 - (1-q_r)^N) - (h + c_1)\sum_{n=1}^{N} E[H_n]$. Applying Lemma 1, Lemma 2, and the renewal-reward theorem completes the proof. □

Using the above theorem, the optimal value of N can be determined numerically. We end by illustrating this for the following example: $q_n = 0.7$, $q_r = 0.3$, $p_n = 1$, $p_r = 1$, $T_P = 1$, $T_{PR} = 10$, $T_{RP} = 10$, $T_0 = 0.2$, $T_1 = 0.02$, $c_p = 0.6$, $c_0 = 0.1$, $c_1 = 0.003$, $c_d = 0.1$, $c_s = 0.1$, and $h = 0.001$. Figure 3 shows the average profits for $N = 1, 2, \ldots, 130$. These have been calculated using 'Mathcad 7 Professional'. The average profit curve seems to be concave in N, as for all the examples that we considered. The optimal batch size is $N = 49$ lots, and the associated average profit is 0.188.

Figure 3

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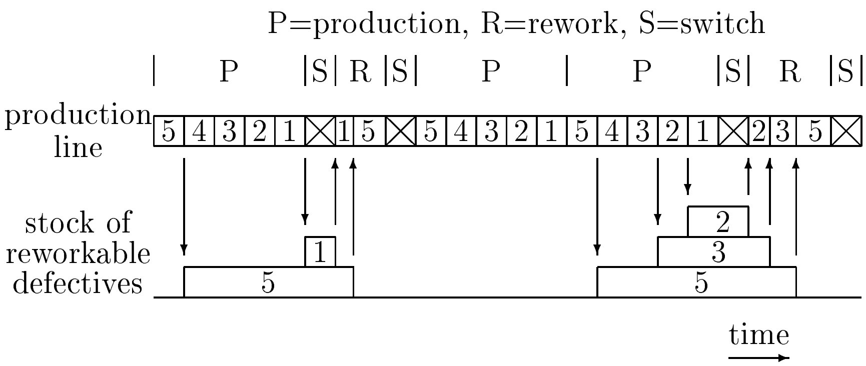

Figure 1: Graphical illustration of the production line for $N = 5$ (produced lots in a batch are numbered backwards from 5 to 1), $T_P = 1$, $T_{PR} = 1$, $T_{RP} = 1$, $T_0 = 0.5$, and $T_1 = 0.1$.

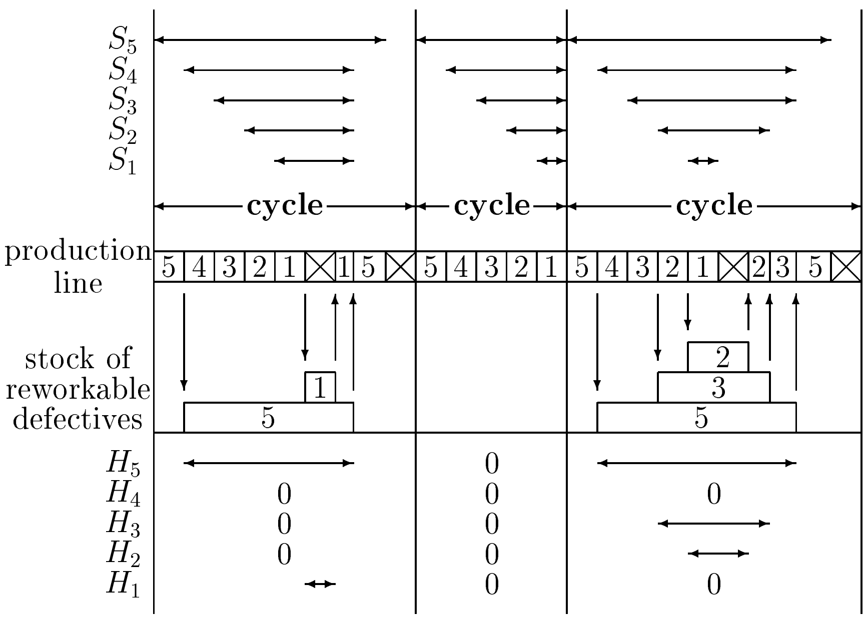

Figure 2: Graphical illustration of the definition of a cycle, and of the definitions of S_n and H_n in a cycle.

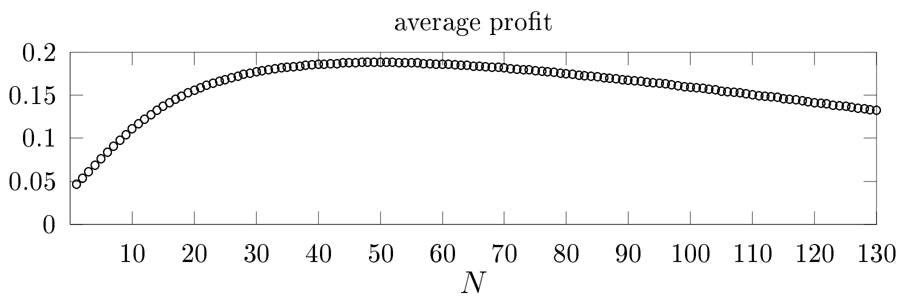

Figure 3: Average profit for $N = 1, 2, \ldots, 130$ if $q_n = 0.7$, $q_r = 0.3$, $p_n = 1$, $p_r = 1$, $T_P = 1$, $T_{PR} = 10$, $T_{RP} = 10$, $T_0 = 0.2$, $T_1 = 0.02$, $c_p = 0.6$, $c_0 = 0.1$, $c_1 = 0.003$, $c_d = 0.1$, $c_s = 0.1$, and $h = 0.001$.