Maximizing remanufacturing profit using product acquisition management

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Abstract

The profitability of remanufacturing depends on the quantity and quality of product returns and on the demand for remanufactured products. The quantity and quality of product returns can be influenced by varying quality dependent acquisition prices, i.e., by using product acquisition management. Demand can be influenced by varying the selling price. We develop a framework for determining the optimal prices and the corresponding profitability.

1 Introduction

In recoverable product environments, products are reused rather than being discarded. Product recovery options include value-added recovery (remanufacturing), material recovery (recycling), and energy recovery (incineration). Product recovery reduces the requirements for virgin materials, energy consumption, and landfill space. Perhaps most importantly, from a business perspective, these systems can significantly contribute to the overall profitability of the firm. Lund (1998) reports that there are over 70,000 remanufacturing firms in the US with total sales of $53 billion (USD). These firms directly employ 350,000 workers and average profit margins exceed 20% (Nasr et al. 1998).

No matter what type of product recovery option is practiced, the firm must obtain used products to serve as inputs to the recovery system. Guide and Van Wassenhove (2000) discuss the implications of product acquisition management systems and describe two alternatives for product recovery. We will focus on the market-driven recovery system and develop an economic analysis for calculating the optimal (profit maximizing) price incentives for product returns and the optimal selling price for remanufactured products.

In the sections that follow, we discuss the economics of product recovery and present a case documenting the product recovery problem at a firm remanufacturing consumer electronics goods. We then present the economic model and a practical strategy for solving the problem.

2 The economics of product recovery

Guide and Van Wassenhove (2000) remark that, surprisingly, there is no literature on the economic analysis of the potential profitability of product recovery (see Guide 2000 and Fleischmann 2000 for comprehensive literature reviews). They develop a framework for analyzing the profitability of product remanufacturing. An important aspect of their framework is the ability of a remanufacturing firm to influence quality, quantity, and timing of product returns by offering users a quality dependent price incentive for turning in products. Empirical evidence shows that a number of remanufacturing firms in the US. have adopted such a market-driven product acquisition management approach (Guide 2000).

European firms, on the other hand, seem to rely on the waste stream for acquiring recoverable products. Firms using this approach passively accept all product returns. They are not involved in product recovery for economic reasons, but because of environmental legislation. Many of the firms operating under a waste stream approach consider their product recovery system to be a cost center rather than a profit center. Their objective is to minimize the costs associated with a product recovery system, rather than maximize the profit. Returned products in the waste-stream tend to be old and have a poor quality, and as a consequence, the recovery options for these products are often limited. Offering price incentives might be more profitable for a firm, if it leads to more high quality, low age returns.

The framework provided by Guide and Van Wassenhove (2000) is very general and provides a number of insights. Many different aspects of a product recovery system are affected by choices in product acquisition management. Some of the aspects considered are: system characteristics (machine utilization rates, process lead times, work in process), revenues and costs (material, labor, acquisition price, disposal), and assets (inventory, machines, buildings) and liabilities (trade payables, accrued expenses). The discussion is in general terms, and not expressed in functional relationships. Therefore, their framework cannot be used directly for calculating optimal price incentives. The framework is a motivation for the analysis developed here. In the next section we present a case study of product acquisition management.

3 Product acquisition management

We present the specifics of product acquisition management at a firm that recovers mobile cellular telephone handsets and accessories. ReCellular, Inc., was founded in 1991 in Ann Arbor, MI by Charles Newman to trade new, used, and remanufactured cellular handsets. The company offers remanufactured and graded as-is products as a high quality, cost effective alternative to new cellular handsets. Customer services include: grading and sorting, remanufacturing, logistics, trading, and product sourcing (all services are specific to cellular handsets and accessories). Grading the handsets is based on functional and cosmetic criteria. Handsets may be slated for remanufacturing, where they are restored to like new standards with respect to quality and cosmetic appearance. Remanufacturing is mainly limited to replacement of parts that have been damaged (e.g., scratched faceplates) or broken (e.g., antena). Handsets sold as-is are guaranteed to meet predetermined nominal quality standards ReCellular operates globally, buying and selling in markets around the world.

The cellular communications industry is a highly dynamic market where the demand for telephones changes daily. Demand may be influenced by the introduction of new technology (e.g., digital and analog), price changes in cellular airtime, promotional campaigns, the opening of new markets, churn (customers leaving present airtime providers), and the number of new cellular telephones manufactured. Additionally, there is no worldwide standard technology (e.g., Europe and the United States both use GSM, but at different bandwidths) and this necessitates dealing in a number of often disparate technologies and standards. These global differences make regional activities difficult since there may be no local market for certain types/models of phones, requiring a firm to manage global sales and procurement. Additionally, cellular airtime providers may limit the number of telephones supported by their system and the dropping of a phone model by a major carrier can greatly affect a local market.

A company offering used or remanufactured equipment faces numerous factors affecting the supply of used cellular phones. The same factors that complicate demand affect the availability of used handsets. The supply of used handsets is a volatile market, with volumes and prices in a constant state of flux.

The acquisition of used telephones is central to the success of a remanufacturing firm. The nature of product acquisitions is driven by what future demands (unknown) will be for phones. The lead times for delivery after used phones have been purchased are often lengthy and may be subject to large amount of variability. This has caused remanufacturers to have stocks of used phones on-hand to compete for sales.

ReCellular obtains used phones in bulk from a variety of sources, including cellular airtime providers and third-party collectors. Third-party collectors are often charitable foundations that act as consolidators by collecting used handsets and accessories from individuals. Cellular airtime providers also act as consolidators by collecting used phones from customers who have returned the phones at the end of service agreements, or customers upgrading to newer technology (cellular airtime providers are often buyers of the remanufactured products). Both these and other sources worldwide may offer a variety of handsets and accessories in varying condition for a wide range of prices and quantities. Due to the low cost (approximately $0.50 per phone using air transport) of bulk transportation of phones, using a worldwide network of suppliers of used phones is practical and cost-efficient. No individual returns are accepted since the channels required for direct returns from the consumer have too high a cost to be effective at this time.

Obtaining the best grade of used products for the best price is one of the key tasks necessary for the success of ReCellular. Deciding on a fair price to offer for the used phones is a difficult and complex task. Present state-of the-art is based primarily on expert judgement, which is acquired by trial and error.

The value of a used handset is highly dependant on future market demand for that particular model either in remanufactured or as-is form. The present demand for a graded as-is used cellular phone or a remanufactured phone is known for that instant in time, but due to the highly dynamic nature of the industry, these prices are not stable. The market forces discussed earlier may cause the value of a particular model of phone to drop or rise with little warning. An additional factor is that the selling price for remanufactured phones tends to drop over time, making the used phones a perishable product.

In the following section we describe a simple model to aid a decision maker in deciding how many used products to acquire and what to pay for them.

4 The economic model

In the sections that follow, we develop a simple economic mode, based on operations at ReCellular. However, we believe that the operations at ReCellular are representative of problems faced by remanufacturing firms in general. For the sake of simplicity and clarity we list the assumptions, in Table 1 below, required by our base model. We will justify these assumptions in the following sections.

A1	Perfect testing
A2	There exist a few, mutually exclusive, quality classes
A3	No capacity constraints
A4	No fixed costs
A5	Return rates are independent of sales rates
A6	The system is operating in steady state
A7	The model is for a single period only and not dependent on the product life cycle
A8	There are no supply or demand constraints

Table 1: Assumptions

There are N quality classes, numbered $1, 2, ..., N$, for returned products. These classes differ, for example, in preliminary testing results, physical damage and appearance. As a consequence, the classes have different associated expected remanufacturing costs. We assume that within a certain quality class, all returned products have the same associated expected remanufacturing cost. Note that this does not imply that testing is perfect or that the classes are non-overlapping in quality or in remanufacturing cost. We only assume that the expected remanufacturing cost for a returned product that falls into quality class i is known, and denote it by $c_i, i = 1, 2, ..., N$. For ease of notation, we will refer to a product return that falls into quality class i as a return of type i in the remainder of this paper.

To stimulate returns, price incentives are offered. The acquisition price for a return of type $i, i = 1, 2, ..., N$, is denoted by a_i, and $r_i(a_i)$ denotes the corresponding return rate (function). We assume that $r_i(a_i)$ is continuous, increasing, and twice differentiable. It is defined on $[\alpha_i, \infty)$ and starts at zero, that is, $r_i(\alpha_i) = 0$. We remark that α_i can be both negative and positive. High quality returns can only be expected if a positive acquisition price is offered ($\alpha_i > 0$). But for low quality product returns, especially products that contain toxic materials, users might be willing to pay a disposal fee ($\alpha_i < 0$). We order the classes in such a way (not necessarily unique) that $\alpha_1 + c_1 \leq \alpha_2 + c_2 \leq ... \leq \alpha_N + c_N$. This will turn out to be convenient in the analysis that follows. The first and second derivative of $r_i(a_i)$ are denoted by $r_i'(a_i)$ and $r_i''(a_i)$ respectively.

Our modelling of the returns implies independence of the return rates. That is, the acquisition price in one class does not influence the return rates in other classes. In cases like that of ReCellular, where used products are obtained in bulk from a number of collecting sources, this assumption is justified. Those sources will sell their on-hand stock of used products of a certain quality to the highest bidder, i.e., to the bidder that offers the highest price for products of that quality. The assumption might not be justified in cases where most used products are obtained from the users themselves, and where many of those users compare acquisition prices before deciding at which age (and corresponding quality class) to turn in their product.

Remanufactured products are sold at price p, and $d(p)$ denotes the corresponding demand rate (function). We assume that $d(p)$ is continuous, decreasing, and twice differentiable. It is defined on $[\alpha_1 + c_1, \beta]$ and ends at zero, that is, $d(\beta) = 0$. So β can be interpreted as the maximal price that customers are willing to pay for a remanufactured product. We assume that $\beta > \alpha_N + c_N$, because otherwise one or more types of returns could never be sold at a profitable price. The first and second derivative of $d(p)$ are denoted by $d'(p)$ and $d''(p)$ respectively. We remark that the shape of the demand curve will most of all depend on the type of market for selling remanufactured products, especially on the number of competing firms.

The goal is to determine the combination of a selling price p and acquisition prices $a_i, i = 1, 2, ..., N$, that maximizes the profit rate. Since the return rates are increasing in the acquisition price and the demand rate is decreasing in the selling price, we can restrict our attention to pricing strategies for which the demand rate is equal to the total return rate, i.e., $d(p) = \sum_{i=1}^{N} r_i(a_i)$. Therefore, we characterize a pricing strategy by the set of acquisition prices $\overline{a} = \{a_1, ..., a_N\}$ only and denote it by $\pi(\overline{a}) = \pi(a_1, ..., a_N)$. The selling price associated with such a strategy is denoted by $p(\overline{a}) = p(a_1, ..., a_N)$. The optimal values for $a_1, ..., a_N$ are denoted by $a_1^*, ..., a_N^*$.

Our economic model focuses on a specific point in time, i.e., we assume that the demand and return rate functions are known. Given those functions, and the remanufacturing costs, our objective is to determine the selling price and acquisition prices that maximize the profit rate. Of course, the model can be modified at any time, re-estimating demand and return rate functions and recalculating the optimal strategy. In fact, the model could be used to decide when the remanufacturing of a certain product should be initiated and terminated.

We finally remark that this continuous time model can easily be modified to a single period discrete time model. The functions $d(p)$ and $r_i(a_i), i = 1, 2, ..., N$, then denote the total demand and return in that period, and the objective is to determine the selling price and acquisition prices that maximize the total profit.

5 The optimal strategy

Let $P(\overline{a})$ denote the profit rate of pricing strategy $\pi(\overline{a})$, which can be expressed as

$$P(\overline{a}) = p(\overline{a}) \sum_{i=1}^{N} r_i(a_i) - \sum_{i=1}^{N} \left(r_i(a_i) \left(a_i + c_i\right)\right),$$

and let $\frac{\partial}{\partial a_j} P(\overline{a})$ and $\frac{\partial^2}{\partial a_j^2} P(\overline{a})$ denote the first and second order partial derivatives of $P(\overline{a})$ with respect to $a_j, j = 1, 2, ..., N$. The first order optimality conditions for a pricing strategy $\pi(\overline{a})$ are

$$\frac{\partial}{\partial a_j} P(\overline{a}) = 0 \text{ for all } j \in \{1, 2, ..., N\} \text{ such that } a_j > \alpha_j \text{ and} \tag{1}$$

$$\frac{\partial}{\partial a_j} P(\overline{a}) \leq 0 \text{ for all } j \in \{1, 2, ..., N\} \text{ such that } a_j = \alpha_j. \tag{2}$$

The second order optimality conditions are

$$\frac{\partial^2}{\partial a_j^2} P(\overline{a}) < 0 \text{ for all } j \in \{1, 2, ..., N\} \text{ such that } a_j > \alpha_j. \tag{3}$$

Expressions for the first order and the second order partial derivatives are derived in Appendix A. Using those expressions, and recalling that $r(\alpha_j) = 0$ and that $\sum_{i=1}^{N} r_i(a_i) = d(p(\overline{a}))$, we can rewrite the first order optimality conditions (1) and (2) as

$$\frac{r_j(a_j)}{r_j'(a_j)} + (a_j + c_j) = \frac{d(p(\overline{a}))}{d'(p(\overline{a}))} + p(\overline{a}) \text{ if } a_j > \alpha_j \text{ and} \tag{4}$$

$$(a_j + c_j) \geq \frac{d(p(\overline{a}))}{d'(p(\overline{a}))} + p(\overline{a}) \text{ if } a_j = \alpha_j, \tag{5}$$

and the second order optimality conditions (3) (assuming that (4) holds) as

$$\frac{2r_j'(a_j)}{d'(p(\overline{a}))} - \frac{d''(p(\overline{a}))r_j'(a_j)d(p(\overline{a}))}{(d'(p(\overline{a})))^3} + \frac{r_j(a_j)r_j''(a_j)}{\left(r_j'(a_j)\right)^2} < 2 \text{ if } a_j > \alpha_j. \tag{6}$$

For notational ease we define

$$g(p) := \frac{d(p)}{d'(p)} + p \tag{7}$$

and for all $j = 1, ..., N$,

$$f_j(a_j) := \frac{r_j(a_j)}{r_j'(a_j)} + (a_j + c_j). \tag{8}$$

These can be interpreted as the marginal revenue of selling one extra remanufactured product and the marginal cost of buying one extra used product of type $j, j = 1, ..., N$. If one extra product is sold, then the revenue associated with that product is $p(\overline{a})$, but the other $d(p(\overline{a}))$ products are sold at a $-1/d'(p(\overline{a}))$ lower price, which gives a total marginal revenue of $g(p(\overline{a}))$. If one extra used product of type j is bought, then the cost associated with that product is $(a_j + c_j)$, and the other $r_j(a_j)$ products of type j are bought at a $1/r_j'(a_j)$ higher price, which gives a total marginal cost of $f_j(a_j)$.

Using (7) and (8), the first order optimality conditions (4) and (5) simplify to

$$f_j(a_j) = g(p(\overline{a})) \text{ if } a_j > \alpha_j \text{ and} \tag{9}$$

$$f_j(a_j) \geq g(p(\overline{a})) \text{ if } a_j = \alpha_j. \tag{10}$$

Using the above results, the optimal strategy can be determined as follows. First, determine all strategies that satisfy (9) and (10). Then, check which of those strategies also satisfy (6) and are hence locally optimal. Obviously, at least one locally optimal strategy has to exist. Finally, compare the profit rates of the locally optimal strategies.

Based on a number of examples, we know that there can be several strategies that satisfy (9), (10) and (6) if the demand and return rate functions have complex shapes. Furthermore, those strategies can be difficult to determine. But if $f_j(a_j), j = 1, ..., N$, and $g(p)$ are all increasing, then there is only one strategy that satisfies (9), (10) and (6). In fact, there is only one strategy that satisfies (9) and (10), as is stated in Lemma 1. Moreover, that strategy can easily be determined using Theorem 1. The proofs of Lemma 1 and Theorem 1 are respectively given in Appendices B and C. Note that $f_j(a_j), j = 1, ..., N$, and $g(p)$ are all increasing if (but not only if) $r_j''(a_j) \leq 0, j = 1, ..., N$, and $d''(p) \leq 0$, that is, if $r_j(a_j), j = 1, ..., N$, and $d(p)$ are all concave.

Lemma 1 *If $f_j(a_j), j = 1, ..., N$, and $g(p)$ are all increasing, then there is only one strategy that satisfies (9) and (10). This strategy also satisfies (6), and is of the type $\pi(a_1, ..., a_M, \alpha_{M+1}, ..., \alpha_N)$ with $a_j > \alpha_j$ for $j = 1, ..., M$, where $M \in \{1, ..., N\}$.*

Theorem 1 *If $f_j(a_j), j = 1, ..., N$, and $g(p)$ are all increasing, then the optimal strategy is determined by the following procedure.*

Step 1: *$M := 1$.*

Step 2: *$h_M^M(a_M) := a_M$. If $M \geq 2$, then express $a_j, j = 1, 2, ..., M-1$, as a function $h_j^M(a_M)$ using $f_j(a_j) = f_M(a_M)$.*

Step 3: *Using $\sum_{i=1}^M r_i(h_i^M(a_M)) = d\left(p\left(h_1^M(a_M), ..., h_M^M(a_M), \alpha_{M+1}, ..., \alpha_N\right)\right)$, find the value a_M^* for a_M for which $f_M(a_M) = g\left(h_1^M(a_M), ..., h_M^M(a_M), \alpha_{M+1}, ..., \alpha_N\right)$.*

Step 4: *If either $M = N$ or $f_M(a_M^*) \leq f_{M+1}(\alpha_{M+1})$ then the optimal strategy is $\pi(h_1^M(a_M^*), ..., h_M^M(a_M^*), \alpha_{M+1}, ..., \alpha_N)$. Otherwise, $M := M + 1$ and repeat steps 2, 3, and 4.*

Remark 1 *If $r_j(a_j), j = 1, ..., M$, are 'simple' functions, for instance first or second degree polynomials, then closed-form expressions for $h_j^M(a_M), j = 1, ..., M-1$, are obtained in Step 2. In such a case, Steps 3 and 4 can be done analytically. If it is not possible to obtain closed-form expressions in Step 2, then Steps 3 and 4 have to be done numerically.*

In Example 1, we apply Theorem 1 to determine the optimal strategy for a simple case with linear (and hence concave) demand and return rate functions.

Example 1 *Let $N = 3$, $c_1 = 1$, $c_2 = 3$, $c_3 = 6$, $\alpha_1 = 4$, $\alpha_2 = 3$, $\alpha_3 = 2$, $r_1(a_1) = 60(a_1 - 4)$, $r_2(a_2) = 40(a_2 - 3)$, $r_3(a_3) = 100(a_3 - 2)$, $\beta = 10$, and $d(p) = 1000 - 100p$.*

It easily follows that $f_1(a_1) = 2a_1 - 3$, $f_2(a_2) = 2a_2$, and $f_3(a_3) = 2a_3 + 4$. Applying the procedure in Theorem 1 gives:

Step 1: $M := 1$.

Step 2: $h_1^1(a_1) := a_1$.

Step 3: Using $60(a_1 - 4) = 1000 - 100p(h_1^1(a_1), \alpha_2, \alpha_3)$ *we get* $g(h_1^1(a_1), \alpha_2, \alpha_3) = 10 - 0.02\,(60(a_1 - 4))$. *So* $f_1(a_1) = g(h_1^1(a_1), \alpha_2, \alpha_3)$ *becomes* $2a_1 - 3 = 10 - 1.2(a_1 - 4)$, *which is solved by* $a_1^* = 5.5625$.

Step 4: $f_1(a_1^*) = 8.125 > 6 = f_2(\alpha_2)$. *So* $M := M + 1 = 2$.

Step 2: $h_2^2(a_2) := a_2$. *Using* $f_1(a_1) = f_2(a_2)$ *we get* $a_1 = h_1^2(a_2) := a_2 + 1.5$.

Step 3: Using $60(a_2 - 2.5) + 40(a_2 - 3) = 1000 - 100p(h_1^2(a_2), h_2^2(a_2), \alpha_3)$ *we get* $g(h_1^2(a_2), h_2^2(a_2), \alpha_3) = 10 - 0.02\,(60(a_2 - 2.5) + 40(a_2 - 3))$. *So* $f_2(a_2) = g(h_1^2(a_2), h_2^2(a_2), \alpha_3)$ *becomes* $2a_2 = 10 - 2a_2 + 5.4$, *which is solved by* $a_2^* = 3.85$.

Step 4: $f_2(a_2^*) = 7.7 < 8 = f_3(\alpha_3)$. *So the optimal strategy is* $\pi(h_1^2(a_2^*), h_2^2(a_2^*), \alpha_3) = \pi(5.35, 3.85, 2)$. *The associated selling price is* 8.85 *and the profit rate is* $8.85\,(81 + 34) - 81\,(5.35 + 1) - 34\,(3.85 + 3) = 270.5$.

As mentioned before, it is complicated to determine the optimal strategy if the demand and return rate functions have complex shapes. In the next section, we therefore restrict our attention to a special class of pricing strategies. Within that intuitively appealing class, the optimal strategy can be determined more easily. Indeed, as we will show, that optimal strategy can be determined graphically, thereby avoiding the need to express the demand and return rate functions analytically.

6 A practical strategy

We restrict our attention to strategies $\pi(\overline{a})$ with a fixed per product profit rate. That is, we set

$$a_j = \max\left(\alpha_j, a_{\text{new}} - c_j\right), j = 1, 2, ..., N, \tag{11}$$

where a_{new} denotes the (theoretical) acquisition price that we are willing to pay for an as-good-as-new returned item that does not require manufacturing. We will denote such a strategy by $\pi^0(a_{\text{new}})$, the associated total return rate by $R^0(a_{\text{new}})$, and the associated selling price (for which the demand rate is equal to the total return rate) by $p^0(a_{\text{new}})$. Moreover, we will denote the optimal value for a_{new} by a_{new}^*. Rewriting (11) as

$$a_j + c_j = a_{\text{new}} \text{ if } a_j > \alpha_j \text{ and} \tag{12}$$

$$a_j + c_j \geq a_{\text{new}} \text{ if } a_j = \alpha_j, \tag{13}$$

we observe that a strategy of type $\pi^0(a_{\text{new}})$ shares some similarities with a strategy that satisfies the first order optimality conditions (4) and (5), but $\pi^0(a_{\text{new}})$ ignores the shapes of the demand and return rate functions.

Since strategies in the restricted class are characterized by a single strategy parameter a_{new}, the optimal strategy in that class is much easier to determine than the overall optimal strategy. In fact, as we will show in the remainder of this section, the optimal strategy of type $\pi^0(a_{\text{new}})$ can be determined graphically.

The total return rate

$$R^0(a_{\text{new}}) = \sum_{i;\alpha_i + c_i < a_{\text{new}}} r_i\left(a_{\text{new}} - c_i\right)$$

can graphically be determined by shifting each return curve $r_i(a_i)$ for type i returns c_i units to the right, and adding the shifted curves. Combining the so obtained total return rate curve with the demand rate curve, the profit rate

$$\left(p^0(a_{\text{new}}) - a_{\text{new}}\right) R^0(a_{\text{new}})$$

of strategy $\pi^0(a_{\text{new}})$ can be determined graphically. This is illustrated in Figure 1 for Example 1 of the previous section. Hence, this avoids the need for expressing the demand and return rate functions analytically, which is an important practical advantage. For Example 1, we did use the analytical expressions to determine the exact optimal strategy. This turned out to be $\pi^0(a^*_{\text{new}} = 6.55) = \pi(5.55, 3.55, 2)$. The associated selling price is 8.85 and the profit rate is $8.85\,(93 + 22) - 93\,(5.55 + 1) - 22\,(3.55 + 3) = 264.5$. Recall that the overall optimal strategy is $\pi(5.35, 3.85, 2)$ with profit rate 270.5. The acquisition prices and the corresponding return rates for both strategies are represented graphically in Figure 2.

So for Example 1, strategy $\pi^0(a^*_{\text{new}})$ performs quite well. Also in most other cases that we considered, the profit rate of strategy $\pi^0(a^*_{\text{new}})$ did not differ more than a few per cent from that of the overall optimal strategy $\pi(a^*_1, ..., a^*_N)$. Exceptions are formed by cases where the return rate functions are very different in shape, that is, if some of those functions are very steep whereas others are rather flat. Since a strategy with a fixed per product profit rate ignores such differences, this can result in a poor performance of strategy $\pi^0(a^*_{\text{new}})$. This is illustrated in Example 2.

Example 2 *Let* $N = 2$, $c_1 = 1$, $c_2 = 0.5$, $\alpha_1 = 0$, $\alpha_2 = 1.55$,

$$r_1(a_1) = \begin{cases} \frac{1}{2-a_1} - 500(0.1 - a_1)^3 & \textit{if} \quad a_1 \leq 0.1 \\ \frac{1}{2-a_1} & \textit{if} \quad a_1 > 0.1 \end{cases},$$

$r_2(a_2) = 40(a_2 - 1.55)$, $\beta = 3$, *and*

$$d(p) = \begin{cases} 1.15 - 0.05p & \textit{if} \quad p \leq 2.9 \\ 1.15 - 0.05p - 1000(p - 2.9)^3 & \textit{if} \quad p > 2.9 \end{cases}.$$

The demand and return rate functions are represented graphically in Figure 3. That figure also shows the acquisition prices, the selling price, and the corresponding return and demand rates for the optimal strategies of type π^0 *and* π. *These optimal strategies are* $\pi^0(a^*_{new} = 1.10)$ *and* $\pi(a^*_1 = 0.09, a^*_2 = 1.562)$, *with profit rates of respectively* 0.99 *and* 1.36. *So strategy* $\pi^0(a^*_{new})$ *is* 27 *per cent less profitable than the overall optimal strategy. The poor performance of strategy* $\pi^0(a^*_{new})$ *is due to its restricted focus on strategies with a fixed per product profit rate. For this example, due to the steepness of* $r_2(a_2)$, *it is much better to use a lower per product profit rate for class* 2 *returns.*

7 Conclusions and Future Research

We have provided a first attempt at viewing remanufacturing as a profitable business venture by positively influencing the incoming quality of used products. By controlling the quality of the acquired products that serve as the input to the remanufacturing process, a decision maker can determine the overall profitability of a remanufacturing system. The model we have developed is a simple treatment restricted to one category of sales. Obviously future models should examine multiple categories and examine non-linear relationships.

A Partial derivatives of the profit rate $P(\overline{a})$

Recall that the profit rate $P(\overline{a})$ of strategy $\pi(\overline{a})$ is

$$P(\overline{a}) = p(\overline{a}) \sum_{i=1}^{N} r_i(a_i) - \sum_{i=1}^{N} \left(r_i(a_i) \left(a_i + c_i \right) \right).$$

Differentiating with respect to a_j gives

$$\frac{\partial}{\partial a_j} P(\overline{a}) = \frac{\partial}{\partial a_j} p(\overline{a}) \sum_{i=1}^{N} \left(r_i(a_i) \right) + p(\overline{a}) r_j'(a_j) - r_j(a_j) - r_j'(a_j)(a_j + c_j).$$

It is easy to see that

$$\frac{\partial}{\partial a_j} p(\overline{a}) = \frac{r_j'(a_j)}{d'(p(\overline{a}))} \tag{14}$$

and hence we get

$$\begin{aligned}
\frac{\partial}{\partial a_j} P(\overline{a}) &= \frac{r_j'(a_j) \sum_{i=1}^{N} r_i(a_i)}{d'(p(\overline{a}))} + p(\overline{a}) r_j'(a_j) - r_j(a_j) - r_j'(a_j)(a_j + c_j) \\
&= r_j'(a_j) \left(\frac{\sum_{i=1}^{N} r_i(a_i)}{d'(p(\overline{a}))} + p(\overline{a}) - \frac{r_j(a_j)}{r_j'(a_j)} - (a_j + c_j) \right).
\end{aligned}$$

Differentiating again with respect to a_j gives

$$\begin{aligned}
&\frac{\partial^2}{\partial a_j^2} P(\overline{a}) \\
= \; & r_j''(a_j) \left(\frac{\sum_{i=1}^{N} r_i(a_i)}{d'(p(\overline{a}))} + p(\overline{a}) - \frac{r_j(a_j)}{r_j'(a_j)} - (a_j + c_j) \right) \\
& + r_j'(a_j) \left(\frac{r_j'(a_j) d'(p(\overline{a})) - d''(p(\overline{a})) \frac{\partial}{\partial a_j} p(\overline{a}) \sum_{i=1}^{N} r_i(a_i)}{\left(d'(p(\overline{a})) \right)^2} \right) \\
& + r_j'(a_j) \left(\frac{r_j'(a_j)}{d'(p(\overline{a}))} - \frac{\left(r_j'(a_j) \right)^2 - r_j(a_j) r_j''(a_j)}{\left(r_j'(a_j) \right)^2} - 1 \right).
\end{aligned}$$

Using (14) this can be rewritten as

$$\frac{\partial^2}{\partial a_j^2} P(\overline{a})$$

$$= r''(a_j)\left(\frac{\sum_{i=1}^{N} r_i(a_i)}{d'(p(\overline{a}))} + p(\overline{a}) - \frac{r_j(a_j)}{r_j'(a_j)} - (a_j + c_j)\right)$$
$$+r_j'(a_j)\left(\frac{2r_j'(a_j)}{d'(p(\overline{a}))} - \frac{d''(p(\overline{a}))r_j'(a_j)\sum_{i=1}^{N} r_i(a_i)}{(d'(p(\overline{a})))^3} + \frac{r_j(a_j)r_j''(a_j)}{\left(r_j'(a_j)\right)^2} - 2\right).$$

B Proof of Lemma 1

We first proof that there is only one strategy that satisfies (9) and (10). This is done by assuming that there are at least two such strategies and then deriving a contradiction. Let us denote these two strategies by $\pi(\overline{b}) = \pi(b_1, ..., b_N)$ and $\pi(\overline{c}) = \pi(c_1, ..., c_N)$. Without loss of generality, assume that $p(\overline{b}) \leq p(\overline{c})$. We will consider the cases $p(\overline{b}) = p(\overline{c})$ and $p(\overline{b}) < p(\overline{c})$ separately.

$p(\overline{b}) = p(\overline{c})$. Clearly, $g(p(\overline{b})) = g(p(\overline{c}))$. Because $f_j(a_j), j = 1, ..., N$, is increasing, (9) and (10) then imply that $b_j = c_j$ for all $j = 1, ..., N$. But this contradicts the assumption that $\pi(\overline{b})$ and $\pi(\overline{c})$ are different strategies.

$p(\overline{b}) < p(\overline{c})$. Because $g(p)$ is increasing, we have $g(p(\overline{b})) < g(p(\overline{c}))$. Using again that $f_j(a_j), j = 1, ..., N$, is increasing, (9) and (10) then imply that $b_j \leq c_j$ for all $j = 1, ..., N$. Hence $d(p(\overline{b})) = \sum_{i=1}^{N} r_i(b_i) \leq \sum_{i=1}^{N} r_i(c_i) = d(p(\overline{c}))$, since $r_j(a_j), j = 1, ..., N$, is increasing. But since $d(p)$ is decreasing, this contradicts the assumption that $p(\overline{b}) < p(\overline{c})$.

We next show that the strategy $\pi(\overline{a})$ that satisfies (9) and (10) always satisfies (6) also. Differentiating $g(p(\overline{a}))$ and $f_j(a_j)$ with respect to a_j gives (see (7) and (8))

$$\frac{\partial}{\partial a_j} g(p(\overline{a})) = 2\frac{\partial}{\partial a_j} p(\overline{a}) - \frac{d''(p(\overline{a}))\frac{\partial}{\partial a_j} p(\overline{a}) d(p(\overline{a}))}{(d'(p(\overline{a})))^2} \tag{15}$$

and

$$f_j'(a_j) = \frac{\left(r_j'(a_j)\right)^2 - r_j(a_j)r_j''(a_j)}{\left(r_j'(a_j)\right)^2} + 1 = -\frac{r_j(a_j)r_j''(a_j)}{\left(r_j'(a_j)\right)^2} + 2 \tag{16}$$

Using (14) we can rewrite (15) as

$$\frac{\partial}{\partial a_j} g(p(\overline{a})) = 2\frac{r_j'(a_j)}{d'(p(\overline{a}))} - \frac{d''(p(\overline{a}))r_j'(a_j)d(p(\overline{a}))}{(d'(p(\overline{a})))^3} \tag{17}$$

and we also get

$$\frac{\partial}{\partial a_j} g(p(\overline{a})) = g'(p(\overline{a}))\frac{\partial}{\partial a_j} p(\overline{a}) = g'(p(\overline{a}))\frac{r_j'(a_j)}{d'(p(\overline{a}))} \tag{18}$$

Combining (16), (17) and (18), we can rewrite (6) as

$$g'(p(\overline{a}))\frac{r_j'(a_j)}{d'(p(\overline{a}))} - f_j'(a_j) < 0 \text{ if } a_j > \alpha_j.$$

Clearly, this condition is satisfied if $f_j(a_j), j = 1, ..., N$, and $g(p)$ are all increasing (recall that $r_j(a_j), j = 1, ..., N$, are increasing and that $d(p)$ is decreasing).

What remains is to show that only strategies of type $\pi(a_1, ..., a_M, \alpha_{M+1}, ..., \alpha_N)$ with $a_j > \alpha_j$ for $j = 1, ..., M$, where $M \in \{1, ..., N\}$, can satisfy (9) and (10). This is equivalent to showing that a strategy $\pi(\overline{a})$ can never satisfy (9) and (10) if there are $j, k \in \{1, ..., N\}$ so that $j < k$, $a_j = \alpha_j$, and $a_k > \alpha_k$. Indeed, (9) and (10) would then imply that $f_j(a_j) \geq f_k(a_k)$. But $f_k(a_k)$ is increasing, and hence this would give $\alpha_j + c_j = f_j(a_j) \geq f_k(a_k) > \alpha_k + c_k$, which contradicts the assumption (see section 4) that $\alpha_1 + c_1 \leq \alpha_2 + c_2 \leq ... \leq \alpha_N + c_N$.

C Proof of Theorem 1

Applying Lemma 1, it follows that the optimal strategy is the unique strategy that satisfies the first order optimality conditions (9) and (10). Moreover, that strategy has to be of the type $\pi(a_1, ..., a_M, \alpha_{M+1}, ..., \alpha_N)$ with $a_j > \alpha_j$ for $j = 1, ..., M$, where $M \in \{1, ..., N\}$. Note that these are exactly the strategies that the procedure considers, starting with $M = 1$ in Step 1.

In Steps 2 and 3, for some fixed value of $M \in \{1, ..., N\}$, the strategy that satisfies (9) is determined. Since $f_j(a_j), j = 1, 2, ..., M-1$, is increasing, $f_j(\alpha_j) = \alpha_j + c_j \leq \alpha_M + c_M = f_M(a_M)$ and $\lim_{a_j \to \infty} f_j(a_j) = \infty$, it follows that for each value of $a_M \geq \alpha_M$ there is a unique value $a_j \in [\alpha_j, \infty)$ such that $f_j(a_j) = f_M(a_M)$. Hence, $a_j, j = 1, 2, ..., M-1$, can be expressed as a function $h_j^M(a_M)$, as is done in Step 2. Furthermore, that function $h_j^M(a_M)$ is increasing since both $f_j(a_j)$ and $f_M(a_M)$ are increasing.

So $f_M(a_M)$ and $h_j^M(a_M), j = 1, 2, ..., M$, are all increasing. Since we also have that $g(p)$ is increasing and that $p(\overline{a})$ is decreasing in all $a_j, j = 1, 2, ..., M$, (see (14)), it follows that there is only a single value a_M^* for a_M for which $f_M(a_M) = g\left(p\left(h_1^M(a_M), ..., h_M^M(a_M), \alpha_{M+1}, ..., \alpha_N\right)\right)$. In Step 4 it is checked whether strategy $\pi(h_1^M(a_M^*), ..., h_M^M(a_M^*), \alpha_{M+1}, ..., \alpha_N)$, that satisfies (9), satisfies (10) also. If so, then it is clear from the first part of this proof that it is the optimal strategy. If not, then the procedure tries the next value for M.

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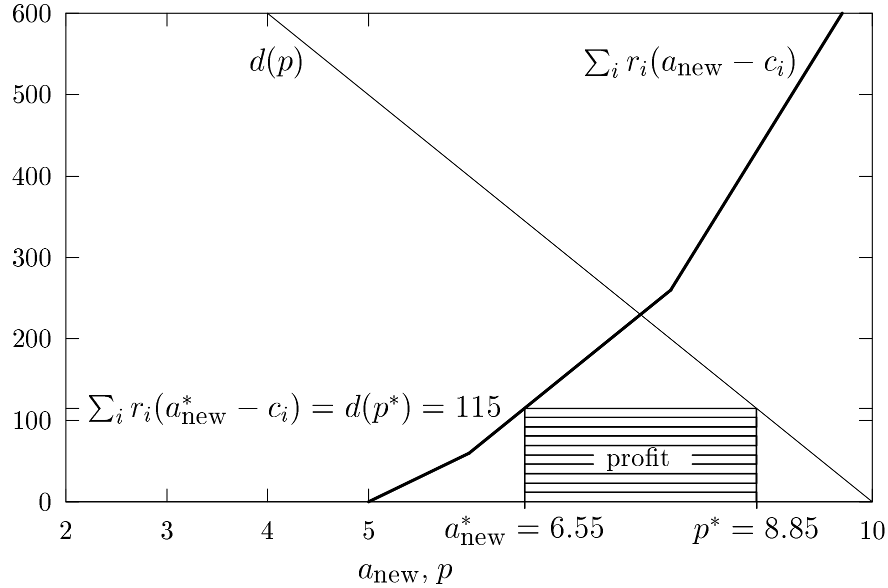

Figure 1: Graphical determination of the profit rate associated with strategy $\pi^0(a^*_{\rm new} = 6.55)$ for Example 1.

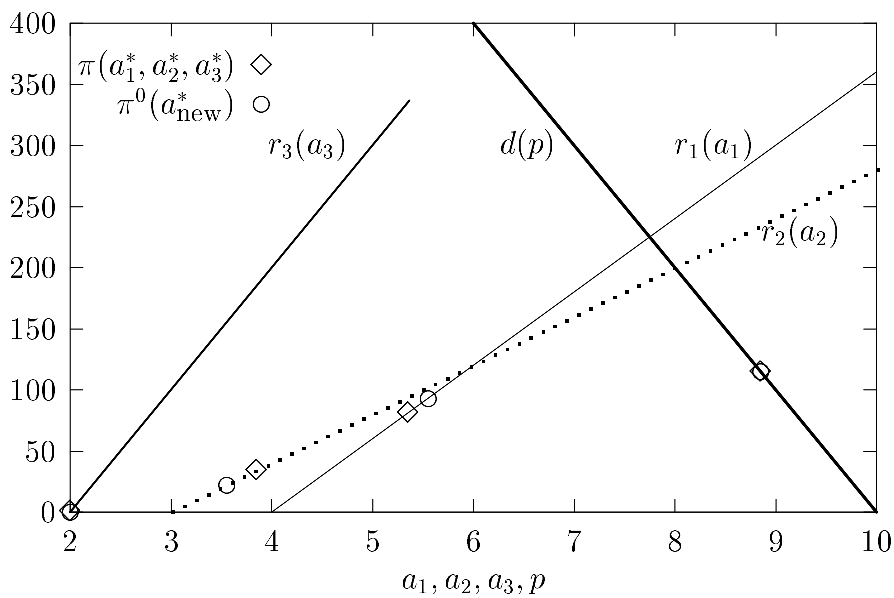

Figure 2: Comparison of the acquisition prices, the selling price, and the corresponding return and demand rates associated with strategies $\pi^0(a^*_{\rm new} = 6.55) = \pi(a_1 = 5.55, a_2 = 3.55, a_3 = 2)$ and $\pi(a_1^* = 5.35, a_2^* = 3.85, a_3^* = 2)$ for Example 1. The associated profit rates are 264.5 and 270.5, respectively.

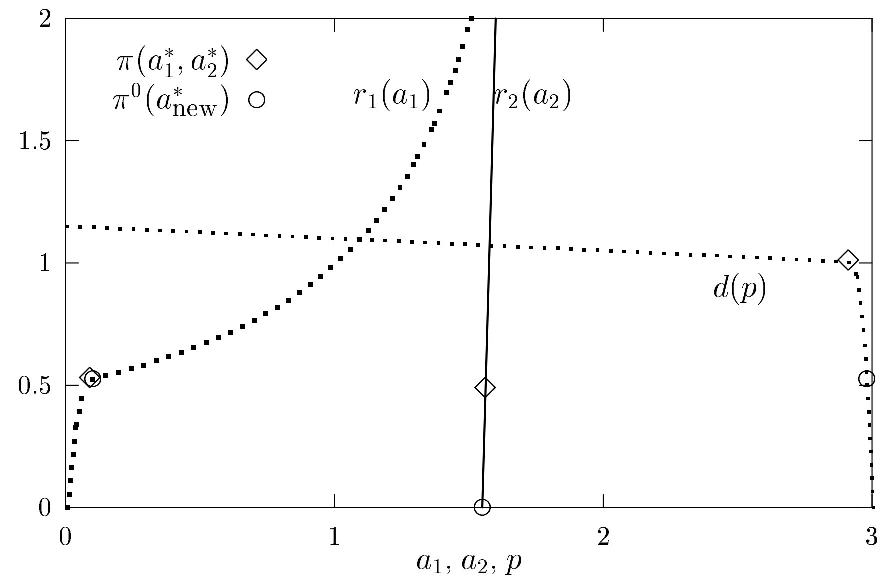

Figure 3: Comparison of the acquisition prices, the selling price, and the corresponding return and demand rates associated with strategies $\pi^0(a_{\text{new}}^* = 1.1) = \pi(a_1 = 0.1, a_2 = 1.55)$ and $\pi(a_1^* = 0.09, a_2^* = 1.562)$ for Example 2. The associated profit rates are 0.99 and 1.36, respectively.