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Technical Report

The Logic of Logical Relativism

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Abstract We explicate the thesis of logical relativism (people of different cultures may have different logics) in logical terms. Our illustrations come from the field of paraconsistent logic.

1 Introduction

Logical relativism is the claim that

People of different cultures may have specifically different logics (for example, [there may be] a peculiarly Chinese logic distinct from Western logics). [15]

Logical relativism was rather popular in anthropological circles before the Second World War [10, 13, 14]. It then went out of fashion [7]. The thesis is currently undergoing a revival in the so-called Strong Programme in the sociology of knowledge [2, 12].

There is no evidence that there actually exist cultures which adhere to different logics than we do [23, 24]. The thesis of logical relativism is nevertheless interesting in itself. What does it mean to say that somebody follows some particular logic? What does it mean to say that people follow different logics? How should such claims be understood? These are the issues which we shall address. Our enterprise is uncommon in that we shall try to clarify these issues in *logical* terms. To the best of our knowledge, this is something which anthropologists, logicians and sociologists of knowledge have never tried to do.

2 Standard Doxastic Logic

When talking about the logical properties of other peoples' opinions, it is natural to turn one's attention to doxastic logic, the logic of belief [11, 18].

It is, however, impossible to make sense of the relativists' claim in terms of standard doxastic logic.

Standard doxastic logic is based on modal system \mathbf{K} . This system has, amongst others, the following rules. \mathbf{C} stands for classical propositional logic.

$$\text{R1 } \vdash_{\mathbf{C}} \phi \implies \vdash_{\mathbf{K}} \Box\phi;$$

$$\text{R2 } \vdash_{\mathbf{C}} \phi \rightarrow \psi \implies \vdash_{\mathbf{K}} \Box\phi \rightarrow \Box\psi;$$

$$\text{R3 } \vdash_{\mathbf{C}} \phi \equiv \psi \implies \vdash_{\mathbf{K}} \Box\phi \equiv \Box\psi.$$

Reading $\Box\phi$ as “the agent believes that ϕ ,” these rules imply (1) that all agents believe all classical tautologies, and (2) that agents' beliefs are closed under \mathbf{C} -provable consequence, and (3) that if an agent believes that ϕ , he also believes all propositions which are \mathbf{C} -provably equivalent with ϕ . Thus standard doxastic logic portrays all doxastic agents as followers of classical logic. When one wants to deal with logical relativism, one has to adopt a less parochial outlook.

Romane Clark has written that belief-ascription is

mainly a matter of keeping the references and concepts of those of us who are scribes, recording the occurrences of psychological happenings, distinct from those of the agents to whom we ascribe mental events. [6]

The logical relativist seems committed to the claim that belief-ascription involves keeping the agents' and our own *logics* distinct as well. Just as $\forall x\forall y(x = y \rightarrow (\Box Fx \equiv \Box Fy))$ is not an acceptable principle of doxastic logic, so $\vdash_{\mathbf{C}} \phi \equiv \psi \implies \vdash \Box\phi \equiv \Box\psi$ is, from the relativist's perspective, not acceptable either.

3 Adhering to a Particular Logical System

What does it mean to say that somebody adheres to a particular logical system? In the literature about logical relativism, this expression is used in at least two quite different senses. First, it is sometimes said that people may adhere to different logics in the sense of embracing different sets of logical truths (see, e.g., [24]). Second, it is sometimes said that people may adhere to different logics in the sense of following different logical rules (see, e.g., [12]). The first conception of adherence to a logic seems related to R1 above, whereas the second seems closer to R2 and R3. We accordingly propose the following three definitions:

Def. 1 The agent adheres₁ to logical system X according to logical system Y iff $\vdash_{\mathsf{X}} \phi \implies \vdash_{\mathsf{Y}} \Box \phi$.

Def. 2 The agent adheres₂ to logical system X according to logical system Y iff $\vdash_{\mathsf{X}} \phi \rightarrow \psi \implies \vdash_{\mathsf{Y}} \Box \phi \rightarrow \Box \psi$.

Def. 3 The agent adheres₃ to logical system X according to logical system Y iff $\vdash_{\mathsf{X}} \phi \equiv \psi \implies \vdash_{\mathsf{Y}} \Box \phi \equiv \Box \psi$.

These senses of “adherence to a logic” should be carefully distinguished from each other, as the following example makes vivid.

Let $\overline{\mathsf{C}}$ be the complement of the propositional calculus, i.e., the set of all non-theorems of C . This system has been axiomatized as follows ([4]; see also [3, 25, 26]).

A1 $p \rightarrow \neg p$ (p atomic)

A2 $\neg p \rightarrow p$ (p atomic)

R1a $\phi/p \rightarrow \phi$ (p atomic, p does not occur in ϕ)

R1b $\phi/\neg p \rightarrow \phi$ (p atomic, p does not occur in ϕ)

R2 $\phi \rightarrow \psi/\phi \rightarrow (\phi \rightarrow \psi)$

R3 $\phi \rightarrow \psi/(\chi \rightarrow \phi) \rightarrow \psi$

R4 $\neg \phi \rightarrow \psi/(\phi \rightarrow \chi) \rightarrow \psi$

R5 $\neg \phi \rightarrow \psi/\phi$

R6 $\phi \rightarrow \psi/\neg \neg \phi \rightarrow \psi$

R7 $\phi \rightarrow (\psi \rightarrow \chi)/\psi \rightarrow (\phi \rightarrow \chi)$

R8 $\phi \rightarrow S, \neg \psi \rightarrow S/\neg(\phi \rightarrow \psi) \rightarrow S$, where S is of the form $S = S_i$ or $S = S_1 \rightarrow (S_2 \rightarrow \dots (S_{n-1} \rightarrow S_n) \dots)$, with $S_i = p_i$ or $S_i = \neg p_i$, $p_i \neq p_k$ for $i \neq k$, and $p \in \{p_1, p_2, \dots, p_n\}$ for all p which occur in $\phi \rightarrow \psi$.

$\overline{\mathsf{C}}$ is *perfectly unsound* and *completely antitautological* in the sense that $\vdash_{\overline{\mathsf{C}}} \phi$ iff $\not\vdash_{\mathsf{C}} \phi$ [4]. It will be clear that $\overline{\mathsf{C}}$ is paraconsistent (it contains all logical falsehoods but no logical truths) and non-monotonic.

Now define $\mathsf{C} + \Box \overline{\mathsf{C}}$ as follows.

- Axioms: All classical tautologies.

- Rules:
 1. $\vdash_{\bar{C}} \phi \implies \vdash_{C+\square\bar{C}} \square\phi$;
 2. $\not\vdash_{\bar{C}} \phi \implies \vdash_{C+\square\bar{C}} \neg\square\phi$;
 3. Modus Ponens.

It will be clear that an agent whose beliefs are described by this system does not adhere to classical logic in the sense of Def. 1. Nor does he adhere to classical logic in the sense of Def. 2, for we have $\vdash_C \perp \rightarrow \top$ but not $\vdash_{C+\square\bar{C}} \square\perp \rightarrow \square\top$. The agent does however adhere to classical logic in the sense of Def. 3. For suppose that $\vdash_C \phi \equiv \psi$. It follows that $\vdash_C \phi$ iff $\vdash_C \psi$, whence $\vdash_{\bar{C}} \phi$ iff $\vdash_{\bar{C}} \psi$. So either (1) $\vdash_{\bar{C}} \phi$ and $\vdash_{\bar{C}} \psi$ or (2) $\not\vdash_{\bar{C}} \phi$ and $\not\vdash_{\bar{C}} \psi$. In the first case, we have $\vdash_{C+\square\bar{C}} \square\phi$ and $\vdash_{C+\square\bar{C}} \square\psi$, whence $\vdash_{C+\square\bar{C}} \square\phi \wedge \square\psi$ and hence $\vdash_{C+\square\bar{C}} \square\phi \equiv \square\psi$; in the second case, we have $\vdash_{C+\square\bar{C}} \neg\square\phi$ and $\vdash_{C+\square\bar{C}} \neg\square\psi$, whence $\vdash_{C+\square\bar{C}} \neg\square\phi \wedge \neg\square\psi$ and hence $\vdash_{C+\square\bar{C}} \square\phi \equiv \square\psi$. So we have $\vdash_{C+\square\bar{C}} \square\phi \equiv \square\psi$ in any case, QED.

It is not difficult to contrive systems in which an agent adheres to a particular logic in the sense of one of the other definitions. One may for example use the logic of awareness discussed in [11], §9.5.

4 Adherence to Different Logical Systems

What does it mean to say that two agents adhere to different logics? We propose the following three definitions, corresponding to Defs. 1–3 above. $\square_i\phi$ means that agent i believes that ϕ .

Def. 4 Agents i and k adhere₁ to different logics according to logical system X iff it is not the case that $\vdash_X \square_i\phi \iff \vdash_X \square_k\phi$.

Def. 5 Agents i and k adhere₂ to different logics according to logical system X iff it is not the case that $\vdash_X \square_i\phi \rightarrow \square_i\psi \iff \vdash_X \square_k\phi \rightarrow \square_k\psi$.

Def. 6 Agents i and k adhere₃ to different logics according to logical system X iff it is not the case that $\vdash_X \square_i\phi \equiv \square_i\psi \iff \vdash_X \square_k\phi \equiv \square_k\psi$.

The following systems illustrate the notion of several agents each following his own logic in the senses of all three definitions. They are simplified versions of the systems discussed in [16].

Each system $C_i+(\square C_k)_{0 \leq k < \omega}$, $0 \leq i < \omega$, is defined as follows.

- Axioms: All axioms of C_i , i.e., the i -th system in Da Costa's well-known paraconsistent hierarchy [8], plus $\square_k(\phi \rightarrow \psi) \rightarrow (\square_k\phi \rightarrow \square_k\psi)$, for all k , $0 \leq k < \omega$.

- Rules: Modus Ponens and $\vdash_{C_k} \phi \implies \vdash \Box_k \phi$, for all k , $0 \leq k < \omega$.

It is obvious that in each system $C_i + (\Box C_k)_{0 \leq k < \omega}$, all agents k , m , $k \neq m$, adhere to different logics in the senses of Defs. 4–6.

Actually, the agents adhere to different logics in an even stronger sense of the word. Following [21], one may call Γ a theory of X iff Γ is closed under conjunction and Modus Ponens. One may then observe that each agent k adheres to C_k in the sense that for all theories Γ of $C_i + (\Box C_k)_{0 \leq k < \omega}$ and all sequences Σ of doxastic operators, $\{\phi: \Sigma \Box_k \phi \in \Gamma\}$ is a theory of C_k . Agents adhere to classical logic in the same strong sense in standard doxastic logic.

We think that the just-described systems capture the thesis of logical relativism in a particularly clear way. Borrowing a term from [21], we may say that the operators \Box_k have a “guarding” function in these systems: they serve to isolate the agents’ logics from each other. The systems may of course be extended to deal with adherents of Łukasiewicz’s multi-valued calculi, intuitionists, and so on and so forth.

We want to emphasize that it is no accident that we have twice chosen to illustrate logical relativism by means of paraconsistent systems. The literature about logical relativism is usually concerned with the acceptance or rejection of contradictions. As Lévy-Bruhl wrote, “the primitive mind is not constrained above all else, as ours is, to avoid contradictions. What to our eyes is impossible or absurd, it sometimes will admit without seeing any difficulty” [14]. “It does not bind itself down, as our thought does, to avoiding contradiction” [13]. Lévy-Bruhl would apparently have regarded paraconsistent logic and dialetheism as manifestations of primitive mentality!

5 Imaginary Worlds

In an “ontological” formulation, logical relativism is the claim that

people of other cultures live in other worlds, so that what is rational in their world may well appear irrational in ours... The relativist slogan, that people of different cultures live in different worlds, would be nonsense if understood as literally referring to physical worlds. If understood as referring to cognized worlds, it would overstate a very trivial point... If, however, the worlds referred to are *cognizable worlds*, then the claim need be neither empty nor absurd. [22]

We may explicate this claim in terms of the semantics of the systems we have described.

We do not have to refer to non-classical worlds when giving a semantical account of $C+\Box\bar{C}$. A variation on the usual neighborhood semantics of classical modal systems (see, e.g., [5]) suffices. For let us define a $C+\Box\bar{C}$ -model as a structure $\mathfrak{M} = (W, C, N, V)$, where W is a set, $C \subseteq W$, $N: W \mapsto \mathcal{P}\mathcal{P}(W)$, and $V: \text{WFF} \times W \mapsto \{0, 1\}$, which satisfies the following two conditions (NB: $|\phi| \stackrel{\text{df}}{=} \{w \in W: V(\phi, w) = 1\}$): (1) $N(w) = \{|\phi|: \exists \mathfrak{M}. |\phi|_{\mathfrak{M}} \neq W_{\mathfrak{M}}\}$; (2) if ϕ is *not* of the form $\Box\psi$, then $V(\phi, w)$ behaves like the valuation function of classical logic; on the other hand, if $\phi = \Box\psi$ and $w \in C$, then $V(\phi, w) = 1$ iff $|\psi| \in N(w)$. Definition: $\models_{\mathfrak{M}} \phi \stackrel{\text{df}}{=} C \subseteq |\phi|$. It will be clear that $\vdash_{C+\Box\bar{C}} \phi$ iff $\models_{\mathfrak{M}} \phi$ for all $C+\Box\bar{C}$ -models \mathfrak{M} .

In contrast to the semantics of $C+\Box\bar{C}$, those of $C_i+(\Box C_k)_{0 \leq k < \omega}$, $0 \leq i < \omega$, are most easily specified in terms of different classes of worlds, many of them non-classical. A $C_i+(\Box C_k)_{0 \leq k < \omega}$ -model is a structure $\mathfrak{M} = ((W_k)_{0 \leq k < \omega}, W_i, R, V)$, where each W_k is a set, $R \subseteq W \times W$, where $W = \bigcup_{0 \leq k < \omega} W_k$, and $V: \text{WFF} \times W \mapsto \{0, 1\}$ is such that if $w \in W_k$, then if ϕ is *not* of the form $\Box_i\psi$, then $V(\phi, w)$ is like the valuation function of Da Costa's system C_k (see [9]); on the other hand, if $\phi = \Box_i\psi$, then $V(\phi, w) = \min\{V(\psi, w'): w' \in W_i \text{ and } wRw'\}$. $\models_{\mathfrak{M}} \phi$ means that $V(\phi, w) = 1$ for all $w \in W_i$. It can be proven that $\vdash_{C_i+(\Box C_k)_{0 \leq k < \omega}} \phi$ iff $\models_{\mathfrak{M}} \phi$ for all $C_i+(\Box C_k)_{0 \leq k < \omega}$ -models \mathfrak{M} [16].

Thus, there are several types of worlds in the semantics of $C_i+(\Box C_k)_{0 \leq k < \omega}$. Only the worlds which are of the agent's own type matter as to what he believes. The other worlds are beyond his logical horizon. This seems a nice explication of both the relativists' claims about different "cognizable" worlds and Vasil'ev's speculations about "imaginary worlds" where our logic does not hold (see [1, 20, 27, 28]). The "ontological" relativist claim seems to come down to the assertion that other agents' beliefs may well have to be modelled by worlds which we deem impossible.

As Montaigne put it in a different context:

Or, s'il y a plusieurs mondes, comme Epicurus et presque toute la philosophie a pensé, que sçavons nous si les principes et les règles de cettuy touchent pareillement les autres? Ils ont à l'aventure autre visage et autre police. [19]

6 Adjacent Territory

Without going so far as to claim that everything is relative, we want to point out that the ideas we have presented are not only relevant in connection with doxastic logic. They may also be applied to, for instance, deontic logic

(different cultures may not only have different norms, e.g., in the way outlined in [17]: they may also have different ways of judging adherence to these norms), truth in fiction (one should not judge the works of, for example, intuitionists by *our*—classical or, as the case may be, paraconsistent—standards) and alethic modal logic (the concepts of necessity and possibility are logic-relative). Some of the details may be found in [16]; we leave the rest to the hopefully imaginative reader.

References

- [1] A. I. ARRUDA. On the imaginary logic of N. A. Vasil'ev. In A. I. ARRUDA, N. C. A. DA COSTA, & R. B. CHUAQUI, eds., *Non-Classical Logics, Model Theory, and Computability*, pp. 3–24. Amsterdam, 1977.
- [2] D. BLOOR. *Knowledge and Social Imagery*. London, 1976.
- [3] P. BONATTI & A. C. VARZI. On the meaning of complementary systems. In *Tenth International Congress of Logic, Methodology and Philosophy of Science: Volume of Abstracts*, p. 122. Florence, 1995.
- [4] X. CAICEDO. A formal system for the non-theorems of the propositional calculus. *Notre Dame Journal of Formal Logic*, 19:147–151, 1978.
- [5] B. F. CHELLAS. *Modal Logic: An Introduction*. Cambridge, 1980.
- [6] R. CLARK. Old foundations for a logic of perception. *Synthese*, 33:75–99, 1976.
- [7] M. COLE & S. SCRIBNER. *Culture and Thought: A Psychological Introduction*. New York, 1974.
- [8] N. C. A. DA COSTA. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, 15:497–510, 1974.
- [9] N. C. A. DA COSTA & E. H. ALVES. A semantical analysis of the calculi C_n . *Notre Dame Journal of Formal Logic*, 18:621–530, 1977.
- [10] E. E. EVANS-PRITCHARD. *Witchcraft, Oracles and Magic Among the Azande*. Oxford, 1937.
- [11] R. FAGIN, J. Y. HALPERN, Y. MOSES, & M. Y. VARDI. *Reasoning about Knowledge*. Cambridge, Mass., 1995.
- [12] R. C. JENNINGS. Zande logic and Western logic. *British Journal for the Philosophy of Science*, 40:275–285, 1989.

- [13] L. LÉVY-BRUHL. *Les fonctions mentales dans les sociétés inférieures*. Paris, 1910.
- [14] L. LÉVY-BRUHL. *La mentalité primitive*. Paris, 1922.
- [15] L. LÉVY-BRUHL. *Les carnets*. Paris, 1949.
- [16] G. J. C. LOKHORST. Multiply modal extensions of Da Costa's C_n , $0 \leq n \leq \omega$, logical relativism, and the imaginary. *The Journal of Non-Classical Logic*, 5 (2):7–22, 1989.
- [17] K. MENGER. *Moral, Wille und Weltgestaltung: Grundlegung zur Logik der Sitten*. Vienna, 1934.
- [18] J.-J. C. MEYER & W. VAN DER HOEK. *Epistemic Logic for AI and Computer Science*. Cambridge, 1995.
- [19] M. E. DE MONTAIGNE. *Apologie de Raimond Sebond*. Paris, 1580.
- [20] L. Z. PUGA & N. C. A. DA COSTA. On the imaginary logic of N. A. Vasil'ev. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 34:205–211, 1988.
- [21] P. K. SCHOTCH & R. E. JENNINGS. Inference and necessity. *Journal of Philosophical Logic*, 9:327–340, 1980.
- [22] D. SPERBER. Apparently irrational beliefs. In M. HOLLIS & S. LUKES, eds., *Rationality and Relativism*. Oxford, 1982.
- [23] T. TRIPLETT. Azande logic versus Western logic? *British Journal for the Philosophy of Science*, 39:361–366, 1988.
- [24] T. TRIPLETT. Is there anthropological evidence that logic is culturally relative? Remarks on Bloor, Jennings, and Evans-Pritchard. *British Journal for the Philosophy of Science*, 45:749–760, 1994.
- [25] A. C. VARZI. Complementary sentential logics. *Bulletin of the Section of Logic*, 19:112–116, 1990.
- [26] A. C. VARZI. Complementary logics for classical propositional languages. *Kriterion*, 4:20–24, 1992.
- [27] N. A. VASIL'EV. *Voobrazhaemaya logika. Izbrannye trudy*. Moscow, 1989.
- [28] E. ŻARNECKA-BIAŁY. Truth and logic in Vasil'ev's imaginary worlds. In *Typy logicheskikh sistem i problema istiny*, pp. 11–19. Sofia, 1985.